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An Analysis of the Effects of Lifestyle Changes by Using a Fractional-Order Population Model of the Overweight/Obese Diabetic Population

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Abstract: Unbalanced lifestyles and other underlying medical conditions are responsible for the worrying pace at which diabetes mellitus is becoming a global health crisis. Recent studies suggest that placing a diabetic patient into remission through a rigorous lifestyle change program can normalize blood glucose levels. This research focuses on fractional order derivative-based mathematical modeling and analysis of the diabetes mellitus model with remission parameters. Firstly, the existence and uniqueness of the solution of the diabetes mellitus model are discussed. Non-negativity and boundedness are also examined. Afterward, the concept of the Jacobian matrix is used to investigate the stability of the model's equilibrium points. The Daftardar-Gejji and Jafari Method has finally been applied to approximate the solutions. The conclusions drawn from numerical simulations of the diabetic model with fractional-order derivatives show a clear dependence on the remission parameters and fractional-order derivative.

Keywords: diabetes mellitus; fractional modeling; fractional stability; numerical study; simulation

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1. Introduction

Diabetes mellitus is a chronic disease that develops when the pancreas either fails to create enough insulin, the body cannot utilize insulin effectively, or the pancreas fails to make insulin entirely. The blood sugar regulation hormone, insulin, is essential for preserving the appropriate blood sugar level. One of the most common complications of untreated diabetes is elevated blood glucose levels, which occur when there is a problem with insulin hormone release. Diabetes is becoming a serious public health issue that poses a significant financial and social burden on every nation. Generally, diabetes falls into two types.

- When $\beta-$cells that synthesize and release insulin and amylin in pancreatic islets become extinct, it results in type 1 diabetes, which is also known as insulin-dependent diabetes (IDD), which is characterized by insufficient insulin production and hyperglycemia.
- Type 2 diabetes, also known as non-insulin-dependent diabetes (NIDD), has a variety of causes, with heredity and lifestyle being two of the most crucial causes.

The World Health Organization (WHO) and the International Diabetes Federation (IDF) are institutions that gather data and develop diabetes policies on a global scale. The most recent reports from the WHO [1] and the IDF [2,3] raise concerns. Age-specific diabetes mortality rates increased by 3% between 2000 and 2019. In 2019, diabetes and kidney disease associated with diabetes were estimated to cause 2 million deaths. The direct cause of 1.5 million deaths in 2019 was diabetes, and 48% of these deaths occurred in people under the age of 70 years. Elevated blood glucose is responsible for 20% of

cardiovascular mortality, and diabetes contributes to a further 460,000 deaths from kidney disease [4]. The IDF recently released data for 2021, indicating that 6.7 million deaths, or one death every five seconds, were attributable to diabetes [2,3]. Additionally, by 2045, this figure is projected to reach 784 million [2].

More than 80% of all type 2 diabetes cases are correlated with lifestyles that lead to overweight and obesity [5]. With so many children and teenagers currently overweight or obese, an increase in diabetes among younger individuals is predicted. Dietary factors causing an excess of calories to be deposited as body fat are the primary cause of obesity. Sedentary behavior and lack of physical activity result in fewer calories being burned off, leading to an increase in body weight. One way healthcare providers and legislators can collaborate to improve diabetes prevention efforts is by focusing on the diabetic population. For years, researchers have been attracted to studying population-based diabetic models. Boutayeb et al. [6] developed the first diabetic population framework using ordinary differential equations as follows:

$$\begin{cases} \dfrac{dD(t)}{dt} = I - (\alpha + \omega)D(t) + \varsigma C(t) \\ \dfrac{dC(t)}{dt} = \alpha D(t) - (\varsigma + \omega + \vartheta + \theta)C(t), \end{cases} \tag{1}$$

where D represents the number of individuals with diabetes with no complications, C represents the number of diabetic individuals with complications, I stands for the incidence of diabetics with the assumption that there are no complications at this stage, α represents the rate at which complications develop, ς represents the complication recovery rate, ϑ represents the rate at which complications caused by diabetes lead to fatalities, and θ represents the rate at which C experiences permanent disability. The (1) structure has since been formulated into multiple research models that consider other aspects, including prediabetic and healthy people, awareness campaigns, genetics, and lifestyle and social influences.

In recent research, type II diabetes has been taken into account. Sweatman and Hassell [7] provided a framework for obtaining time-based health and sickness courses and examined the impact of changing the CHO and fat content of the diet on these time courses. A mathematical framework describing the endocrine glucose–insulin metabolic regulation feedback loop was put forth by Al-Hussein et al. [8], and its numerical investigation was provided. Kouidere et al. have developed a mathematical approach for the optimal regulation of diabetes mellitus[9]. They implemented four controllers in the model system, including treatment, psychological support, and awareness campaigns through media and education. By considering the impact of general public knowledge about diabetes mellitus in deterministic and stochastic situations, Mollah et al. created mathematical models [10]. Their findings demonstrated that a public awareness campaign could reduce the prevalence of diabetes mellitus. Additionally, Mollah et al. created a model centered on interactions between the density of diabetes awareness programs and the number of diabetic patients [11].

Several dynamical structures are modeled in fractional order instead of integer order because of the effect of memory or a non-local aspect. Additionally, fractional-order differential equations (FDEs) are thought to be incredibly reliable and precise. With the additional degree of freedom that fractional derivative offers, the real system or phenomena occurring in daily life (individuals with diabetes and their recovery) can be accurately approximated while employing fewer modeling parameters [12]. This is clear from [13], where the authors replaced several parameters with fractional derivatives and obtained findings that were reasonably accurate to the data. The fractional-order products are also utilized to broaden the system's stability zone. By using two immune effectors, Ahmed et al. established a fractional-order cancer model [14]. They noted that the order of fractional derivatives controlled the system's behavior. The same model with an RL derivative was also studied by Srivastava et al., who solved it using the homotopy decomposition

method [15]. The population model of diabetes of healthy individuals, overweight/obese people, and diabetics with and without complications in Morocco was modeled in 2022 by Mohamed Lamlili E.N. et al. [16].

$$\begin{cases} \frac{dP(t)}{dt} = \eta - (\varepsilon + \delta_1 + \varpi_1 + \theta)P + \varpi_2 W \\ \frac{dW(t)}{dt} = \varpi_1 P - (\varpi_2 + \varpi_3 + \varpi_4 + \theta)W \\ \frac{dD(t)}{dt} = \delta_1 P + \varpi_3 W - (\delta_2 + \theta)D \\ \frac{dC(t)}{dt} = \varepsilon P + \varpi_4 W + \delta_2 D - (\theta + \nu)C, \end{cases} \tag{2}$$

where $P(t)$—Population of adults without diabetes and overweight;
$W(t)$—Population of overweight/obese adults;
$D(t)$—Adult population of diabetics without complications;
$C(t)$—Adult population of diabetics with complications;
η—The prevalence of the population of adults;
ε—The frequency of healthy people suffering from difficulties;
δ_1—The rate at which healthy people develop diabetes;
δ_2—The frequency of complications emerging in persons with diabetes;
ϖ_1—The ratio of healthy people who become overweight or obese;
ϖ_2—The ratio of overweight or obese people who become healthy;
ϖ_3—The ratio of overweight or obese people who become diabetic;
ϖ_4—The ratio of overweight or obese people who develop complications;
θ—The rate of natural mortality;
ν—The mortality rate due to complications.

They found that educating policymakers about a healthy lifestyle, which includes consuming nutritious food, engaging in physical activity, and maintaining a suitable weight, is the most effective technique for reducing their risk of becoming overweight or obese.

Since fractional order derivatives can provide greater accuracy with actual world occurrences compared to integral order derivatives and have the property of historical memory, this study expands the concept and utilizes fractional order derivatives in the diabetes population model [16]. Moreover, we incorporated remission parameters into the population model to eliminate individuals with diabetes who consistently lead healthy lives. The diabetic mellitus population model with overweight/obesity has not been analyzed using fractional order derivatives so far. The presented study emphasizes the effect on the diabetic mellitus population model when a fractional order derivative is used. We incorporate two additional characteristics into the framework of the diabetic population model. We aim to illustrate the impact of adopting a healthy lifestyle (a style of life that incorporates nutritious food, regular exercise, meditation, and other healthy practices into everyday routines to reduce the chance of becoming seriously ill or dying prematurely) on remission. We then examine the impact of the fractional order σ derivative on the model of the population with diabetes regardless of a remission parameter.
Therefore, the proposed model is defined as

$$\begin{aligned} {}^{C}D_0^{\sigma} A(t) &= \eta - (\alpha_1 + \beta_1 + r_1 + \mu)A(t) + \alpha_3 I(t), \\ {}^{C}D_0^{\sigma} I(t) &= \alpha_1 A(t) - (\alpha_2 + \alpha_3 + \alpha_4 + \mu)I(t), \\ {}^{C}D_0^{\sigma} D(t) &= \beta_1 A(t) + \alpha_2 I(t) - (\beta_2 + r_2 + \mu)D(t), \\ {}^{C}D_0^{\sigma} C(t) &= \alpha_4 I(t) + \beta_2 D(t) - (\delta + \mu)C(t), \end{aligned} \tag{3}$$

where $A(t)$—Number of people following a healthy lifestyle (not overweight/obese) on a regular basis;
$I(t)$—Number of people not following a healthy lifestyle (overweight/obese) on a regular

basis;
$D(t)$—Number of people with diabetes without any complication;
$C(t)$—Number of people with diabetes with complications;
$N(t) = A(t) + I(t) + D(t) + C(t)$
η—The prevalence of the population;
α_1—The rate at which individuals following a healthy lifestyle are unable to maintain it regularly and become obese;
α_2—The ratio of obese people who do not follow a healthy lifestyle regularly and become diabetic;
α_3—The rate at which individuals following a healthy lifestyle are able to control obesity;
α_4—The ratio of obese/overweight people who do not follow a healthy lifestyle and develop diabetes with complications;
β_1— The rate at which people following a healthy lifestyle on a daily basis develop diabetes without complications;
β_2— The frequency of complications emerging in persons with diabetes; μ—The rate of natural mortality;
δ—The rate at which diabetics with complications die or are removed due to permanent disability;
r_1—The probability rate at which people following a healthy lifestyle go into the remission category can be seen as the rate at which pre-diabetic individuals do not become diabetic by adopting a healthy lifestyle;
r_2—The probability rate at which diabetic patients without complications fall into the remission category by regularly applying a healthy lifestyle.

Sections of this article are divided as follows: In Section 2, preliminaries and notations that are used throughout this work are provided. Existence and uniqueness, non-negativity and boundedness, and the identification and stability analysis of equilibrium points for the provided model are discussed in Section 3. Section 4 presents the approximate solution and graphical interpretation. Finally, the conclusion of this work is provided in Section 5.

2. Preliminaries and Notations

Definition 1. *Let $\xi(z) \in C[a,b], a < z < b$ be a real-valued function, then for any $\sigma \in R^+$, the Riemann–Liouville fractional integral of order σ for the function $\xi(z)$, stated as*

$$\mathrm{I}_a^{\sigma}\,\xi(z) = \frac{1}{\Gamma(\sigma)}\int_a^z (z-\mu)^{\sigma}\xi(\mu)d\mu\ ,\ z > a.$$

- *For $\sigma = 0$, the integral is called the identity operator as $\mathrm{I}_a^0\,\xi(z) = \xi(z)$.*
- *Unless specified, we will use $a = 0$.*
- *The integral for the power function is defined as*

$$\mathrm{I}_a^{\sigma}\,(z-\alpha)^{\omega} = \frac{\Gamma(\omega+1)}{\Gamma(\omega+\sigma+1)}(z-\alpha)^{\omega+\sigma},\ \omega > -1.$$

Definition 2. *Let $\xi(z) \in C[a,b], a < z < b$ be a real-valued function. Then, for any $\sigma \in R^+$, the Riemann–Liouville fractional derivative of order σ for the function $\xi(z)$ is defined as*

$$\begin{aligned} {}^{RL}D_a^{\sigma}\,\xi(z) &= \tfrac{d^k}{dz^k}\left(\mathrm{I}_a^{k-\sigma}\,\xi(z)\right)\ ,\ k-1 < \sigma \leq\ k. \\ &= \left\{ \begin{array}{ll} \frac{1}{\Gamma(k-\sigma)}\frac{d^k}{dz^k}\left(\int_a^z (z-\mu)^{k-\sigma-1}\xi(\mu)d\mu\right) & ,\ k-1 < \sigma < k \\ \frac{d^k}{dz^k}\xi(z) & ,\ k = \sigma \end{array} \right\}, \end{aligned}$$

- *The Riemann–Liouville fractional derivative of the constant is*

$$^{RL}D_a^{\sigma}\, Const = \frac{z^{\sigma}}{\Gamma(1-\sigma)} Const \neq 0.$$

- *The Riemann–Liouville σ order fractional derivative for the power function is defined as*

$$^{RL}D_a^{\sigma}\,(z-\alpha)^{\omega} = \frac{\Gamma(\omega+1)}{\Gamma(\omega-\sigma+1)}(z-\alpha)^{\omega-\sigma},\ \omega > -1.$$

Definition 3. *Let $\xi(z) \in C^k[a,b], a < z < b$ be a real-valued function. Then, for any $\sigma \in R^+$, the Caputo fractional σ order derivative for function $\xi(z)$ is stated as*

$$^{C}D_a^{\sigma}\,\xi(z) = I_a^{k-\sigma}\left(\frac{d^k}{dz^k}\xi(z)\right),\ k-1 < \sigma \leq k.$$
$$= \left\{ \begin{array}{ll} \frac{1}{\Gamma(k-\sigma)}\left(\int_a^z (z-\mu)^{k-\sigma-1}\left(\frac{d^k}{dz^k}\xi(\mu)\right)d\mu\right) & ,\ k-1 < \sigma < k \\ \frac{d^k}{dz^k}\xi(z) & ,\ k = \sigma \end{array} \right\}.$$

Remark 1. *The Caputo fractional derivative for the power function is defined as*

$$^{C}D_a^{\sigma}\,(z-\alpha)^{\omega} = \frac{\Gamma(\omega+1)}{\Gamma(\omega-\sigma+1)}(z-\alpha)^{\omega-\sigma},\ \omega > k-1,\ k-1 < \sigma \leq k. \tag{4}$$

Definition 4. *Let $f(t,X) : [t_0,\infty) \times \Omega \to R^n,\ \Omega \subseteq R^n$ be a real-valued continuous function. Then, $f(t,X)$ is said to satisfy the Lipschitz condition, with respect to X, if there exists a positive real number L, such that*

$$|f(t,X_1) - f(t,X_2)| \leq L|X_1 - X_2|,$$

for $X_1, X_2 \in \Omega \subseteq R^n$

Lemma 1. *By using [17], for the fractional differential system*

$$^{C}D_0^{\sigma}\,X(t) = f(t,X),\ t > t_0,\ X(t_0) > 0,\ \sigma \in (0,1], \tag{5}$$

where $f(t,X) : [t_0,\infty) \times \Omega \to R^n,\ \Omega \subseteq R^n$. If $f(t,X)$ satisfies the Lipschitz condition over $[t_0,\infty) \times \Omega$ w.r.t. X, then the existence and uniqueness of (5) are guaranteed.

Lemma 2. *By [18], for $0 < \sigma \leq 1$, if $X(t)$ satisfies $X(t) \in C[a,b]$ and $^{C}D_0^{\sigma}\,X(t) \in C(a,b]$, then we have*

$$X(t) = X(a) + \frac{(t-a)^{\sigma}}{\Gamma(\sigma)}\,^{C}D_0^{\sigma}\,X(\tau),\ \forall\, t \in (a,b] \text{ and } a \leq \tau \leq b.$$

Corollary 1. *Let $\kappa(t) \in C[a,b]$ and $D^{\sigma}\kappa(t) \in C(a,b]$, where $0 < \sigma \leq 1$. If $D^{\sigma}\kappa(t) \geq 0$ for all $t \in (a,b)$, then $\kappa(t)$ will be non-decreasing for all t. Also, if $D^{\sigma}\kappa(t) \leq 0$ for all $t \in (a,b)$, then $\kappa(t)$ will be non-increasing for all t.*

Lemma 3. *By [19], let $f(t)$ be a continuous function on $[t_0,\infty)$ and satisfy*

$$\left\{ \begin{array}{c} {}^{C}D_0^{\sigma}\,f(t) \leq -\lambda f(t) + \mu\,, \\ f(t_0) = f_0\,, \end{array} \right.$$

where $\lambda, \mu \in R, \lambda \neq 0, t_0 \geq 0$ and $0 < \sigma < 1$. Then

$$f(t) \leq \left(f_0 - \frac{\mu}{\lambda}\right)E_{\sigma}\left[-\lambda(t-t_0)^{\sigma}\right] + \frac{\mu}{\lambda},$$

for E_σ to be the Mittag-Leffler function.

3. Model Validation and Stability Analysis

3.1. Existence and Uniqueness

Consider system (3) of fractional differential equations:

$$\begin{aligned}
{}^{C}D_0^{\sigma} A(t) &= \eta - (\alpha_1 + \beta_1 + r_1 + \mu)A(t) + \alpha_3 I(t),\\
{}^{C}D_0^{\sigma} I(t) &= \alpha_1 A(t) - (\alpha_2 + \alpha_3 + \alpha_4 + \mu)I(t),\\
{}^{C}D_0^{\sigma} D(t) &= \beta_1 A(t) + \alpha_2 I(t) - (\beta_2 + r_2 + \mu)D(t),\\
{}^{C}D_0^{\sigma} C(t) &= \alpha_4 I(t) + \beta_2 D(t) - (\delta + \mu)C(t).
\end{aligned}$$

We will discuss the existence, uniqueness, non-negativity, and boundedness of the model (3) in the $\Omega \times [0, T]$ region, where

$$\Omega = \Big\{(A, I, D, C) \in R^4 : \max(|A|, |I|, |D|, |C|) < \tau\Big\},$$

for $T < \infty$, and τ is sufficiently large.

Theorem 1. *For system (3) of fractional order differential equations, there exists a unique solution, say $X(t) = (A(t), I(t), D(t), C(t)) \in \Omega$ for all $t \geq 0$, satisfying the initial conditions $X_0 = (A_0, I_0, D_0, C_0)$.*

Proof. For $X = (A, I, D, C)$ and $\overline{X} = (\overline{A}, \overline{I}, \overline{D}, \overline{C})$, we define the map $f : \Omega \to R^4$ as

$$f(X) = (f_1(X), f_2(X), f_3(X), f_4(X)),$$

where

$$\begin{aligned}
f_1(X) &= \eta - (\alpha_1 + \beta_1 + r_1 + \mu)A(t) + \alpha_3 I(t),\\
f_2(X) &= \alpha_1 A(t) - (\alpha_2 + \alpha_3 + \alpha_4 + \mu)I(t),\\
f_3(X) &= \beta_1 A(t) + \alpha_2 I(t) - (\beta_2 + r_2 + \mu)D(t),\\
f_4(X) &= \alpha_4 I(t) + \beta_2 D(t) - (\delta + \mu)C(t).
\end{aligned}$$

Now, for any arbitrary $X, \overline{X} \in \Omega$, consider

$$\begin{aligned}
\big\|f(X) - f(\overline{X})\big\| &= \big|f_1(X) - f_1(\overline{X})\big| + \big|f_2(X) - f_2(\overline{X})\big|\\
&+ \big|f_3(X) - f_3(\overline{X})\big| + \big|f_4(X) - f_4(\overline{X})\big|\\
&= \big|-(\alpha_1 + \beta_1 + r_1 + \mu)A + \alpha_3 I + (\alpha_1 + \beta_1 + r_1 + \mu)\overline{A} - \alpha_3 \overline{I}\big|\\
&+ \big|\alpha_1 A - (\alpha_2 + \alpha_3 + \alpha_4 + \mu)I - \alpha_1 \overline{A} + (\alpha_2 + \alpha_3 + \alpha_4 + \mu)\overline{I}\big|\\
&+ \big|\beta_1 A + \alpha_2 I - (\beta_2 + r_2 + \mu)D - \beta_1 \overline{A} - \alpha_2 \overline{I} + (\beta_2 + r_2 + \mu)\overline{D}\big|\\
&+ |\alpha_4 I + \beta_2 D - (\delta + \mu)C - \alpha_4 I - \beta_2 D + (\delta + \mu)C|\\
&= (2\alpha_1 + 2\beta_1 + r_1 + \mu)\big|A - \overline{A}\big| + (2\alpha_2 + 2\alpha_3 + 2\alpha_4 + \mu)\big|I - \overline{I}\big|\\
&+ (2\beta_2 + r_2 + \mu)\big|D - \overline{D}\big| + (\delta + \mu)\big|C - \overline{C}\big|.
\end{aligned}$$

Therefore, we obtain

$$\big\|f(X) - f(\overline{X})\big\| \leq M\big\|X - \overline{X}\big\|,$$

where

$$M = \max\{2\alpha_1 + 2\beta_1 + r_1 + \mu,\ 2\alpha_2 + 2\alpha_3 + 2\alpha_4 + \mu,\ 2\beta_2 + r_2 + \mu,\ \delta + \mu\}.$$

Hence, f satisfies Lipschitz conditions. Therefore, using Lemma 1, system (3) has a unique solution. □

3.2. Non-Negativity and Boundedness

Theorem 2. *All the solutions for system (3) with the initial conditions being* $X_0 = (A_0, I_0, D_0, C_0)$ *are uniformly bounded and non-negative for each* $t \geq 0$.

Proof. By employing the contradiction, we initially demonstrate that $A(t) \geq 0$ for all $t \geq 0$. Let it be possible that $A(t) \geq 0$ for all $t \geq 0$ does not hold. Then we must have at least one $P^* > 0$ satisfying

$$\left\{ \begin{array}{l} A(t) > 0, \ 0 \leq t < P^* \\ A(t) = 0, \ t = P^* \\ A(t) < 0, \ t > P^* \end{array} \right\}.$$

Also, from the first equation of system (3), we have

$${}^{C}D_0^{\sigma} A(t)\Big|_{t=P^*} = 0.$$

By using Lemma 2 and the corresponding Corollary 1, we have $A(t) = 0$ for all $t > P^*$, a contradiction to $A(t) > 0$ for all $t > P^*$. Therefore, it follows that $A(t) \geq 0$ for all $t \geq 0$. In a similar way, $I(t), D(t), C(t) \geq$ *are* 0 for all $t \geq 0$. Now, to prove that solutions for system (3) are bounded, we define the map

$$T(t) = A(t) + I(t) + D(t) + C(t),$$

then

$$\begin{aligned} {}^{C}D_0^{\sigma} T(t) &= {}^{C}D_0^{\sigma} A(t) + {}^{C}D_0^{\sigma} I(t) + {}^{C}D_0^{\sigma} D(t) + {}^{C}D_0^{\sigma} C(t) \\ &= \eta - (r_1 + \mu)A(t) - \mu I(t) - (r_2 + \mu)D(t) - (\delta + \mu)C(t) \\ &= \eta - r_1 A(t) - r_2 I(t) - \delta C(t) - \mu T(t) \\ &\leq \eta - (\kappa + \mu)T(t) \end{aligned},$$

where $\kappa = \min\{r_1, r_2, \delta\}$. Also, it can be written as

$${}^{C}D_0^{\sigma} T(t) \leq \eta - \nu T(t),$$

for $\nu = \mu + \kappa$. By applying Lemma 3, it follows that

$$T(t) \leq T(0)E_{\sigma}(-\nu t^{\sigma}) + \frac{\eta}{\nu}(1 - E_{\sigma}(-\nu t^{\sigma})).$$

According to the Mittag-Leffler function properties, $0 < E_{\sigma}(-\nu t^{\sigma}) \leq 1$ implies that $0 < 1 - E_{\sigma}(-\nu t^{\sigma}) \leq 1$. Therefore,

$$T(t) \leq T(0) + \frac{\eta}{\nu}.$$

Hence, all the solutions for system (3) satisfy non-negativity and uniformly boundedness for every $t \geq 0$. □

3.3. Equilibrium Points and Stability Analysis

Equilibrium points were identified by comparing each equation of system (3) with zero. Also, the stability at each equilibrium point was checked by using the Jacobian matrix method [20]. From (3), an equilibrium point exists when

$$\begin{aligned} {}^{C}D_0^{\sigma} A(t) &= 0, \\ {}^{C}D_0^{\sigma} I(t) &= 0, \\ {}^{C}D_0^{\sigma} D(t) &= 0, \\ {}^{C}D_0^{\sigma} C(t) &= 0, \end{aligned}$$

implies that

$$\begin{aligned} \eta - (\alpha_1 + \beta_1 + r_1 + \mu)A(t) + \alpha_3 I(t) &= 0, \\ \alpha_1 A(t) - (\alpha_2 + \alpha_3 + \alpha_4 + \mu)I(t) &= 0, \\ \beta_1 A(t) + \alpha_2 I(t) - (\beta_2 + r_2 + \mu)D(t) &= 0, \\ \alpha_4 I(t) + \beta_2 D(t) - (\delta + \mu)C(t) &= 0, \end{aligned} \tag{6}$$

on solving the second equation of system (6), we obtain

$$I(t) = \frac{\alpha_1}{(\alpha_2 + \alpha_3 + \alpha_4 + \mu)} A(t),$$

upon substituting this value in the first equation of system (6), we find that

$$\eta - (\alpha_1 + \beta_1 + r_1 + \mu)A(t) + \frac{\alpha_1 \alpha_3}{(\alpha_2 + \alpha_3 + \alpha_4 + \mu)} A(t) = 0,$$

$$\frac{(\alpha_1 + \beta_1 + r_1 + \mu)(\alpha_2 + \alpha_3 + \alpha_4 + \mu) - \alpha_1 \alpha_3}{(\alpha_2 + \alpha_3 + \alpha_4 + \mu)} A(t) = \eta,$$

indicates that

$$A^* = \frac{\eta(\alpha_2 + \alpha_3 + \alpha_4 + \mu)}{(\alpha_1 + \beta_1 + r_1 + \mu)(\alpha_2 + \alpha_3 + \alpha_4 + \mu) - \alpha_1 \alpha_3}, \tag{7}$$

and using this,

$$I^* = \frac{\eta \alpha_1}{(\alpha_1 + \beta_1 + r_1 + \mu)(\alpha_2 + \alpha_3 + \alpha_4 + \mu) - \alpha_1 \alpha_3}, \tag{8}$$

again, by using (7) and (8) in the third equation of system (6), we have

$$D^* = \frac{\beta_1 A^* + \alpha_2 I^*}{(\beta_2 + r_2 + \mu)},$$

hence,

$$D^* = \frac{\eta \beta_1 (\alpha_2 + \alpha_3 + \alpha_4 + \mu) + \eta \alpha_1 \alpha_2}{((\alpha_1 + \beta_1 + r_1 + \mu)(\alpha_2 + \alpha_3 + \alpha_4 + \mu) - \alpha_1 \alpha_3)(\beta_2 + r_2 + \mu)}, \tag{9}$$

finally, by using (7)–(9) in the fourth equation of system (6), we obtain

$$C^* = \frac{\alpha_4 I^* + \beta_2 D^*}{(\delta + \mu)},$$

therefore,

$$C^* = \frac{\eta \alpha_1 \alpha_4 (\beta_2 + r_2 + \mu) + \eta \beta_1 \beta_2 (\alpha_2 + \alpha_3 + \alpha_4 + \mu) + \eta \alpha_1 \alpha_2}{((\alpha_1 + \beta_1 + r_1 + \mu)(\alpha_2 + \alpha_3 + \alpha_4 + \mu) - \alpha_1 \alpha_3)(\beta_2 + r_2 + \mu)(\delta + \mu)}. \tag{10}$$

Therefore, the equilibrium point for system (3) is $E^* = (A^*, I^*, D^*, C^*)$, where A^*, B^*, D^* and C^* are defined by Equations (5)–(8). Now, we will use the Jacobian matrix for stability, considering system (3) as

$$\begin{aligned} f_1 &= \eta - (\alpha_1 + \beta_1 + r_1 + \mu)A(t) + \alpha_3 I(t), \\ f_2 &= \alpha_1 A(t) - (\alpha_2 + \alpha_3 + \alpha_4 + \mu)I(t), \\ f_3 &= \beta_1 A(t) + \alpha_2 I(t) - (\beta_2 + r_2 + \mu)D(t), \\ f_4 &= \alpha_4 I(t) + \beta_2 D(t) - (\delta + \mu)C(t). \end{aligned}$$

Then, the Jacobian matrix is defined by

$$J = \begin{pmatrix} \frac{\partial f_1}{\partial A} & \frac{\partial f_1}{\partial I} & \frac{\partial f_1}{\partial D} & \frac{\partial f_1}{\partial C} \\ \frac{\partial f_2}{\partial A} & \frac{\partial f_2}{\partial I} & \frac{\partial f_2}{\partial D} & \frac{\partial f_2}{\partial C} \\ \frac{\partial f_3}{\partial A} & \frac{\partial f_3}{\partial I} & \frac{\partial f_3}{\partial D} & \frac{\partial f_3}{\partial C} \\ \frac{\partial f_4}{\partial A} & \frac{\partial f_4}{\partial I} & \frac{\partial f_4}{\partial D} & \frac{\partial f_4}{\partial C} \end{pmatrix},$$

implies that

$$J = \begin{pmatrix} -(\alpha_1 + \beta_1 + r_1 + \mu) & \alpha_3 & 0 & 0 \\ \alpha_1 & -(\alpha_2 + \alpha_3 + \alpha_4 + \mu) & 0 & 0 \\ \beta_1 & \alpha_2 & -(\beta_2 + r_2 + \mu) & 0 \\ 0 & \alpha_4 & \beta_2 & -(\delta + \mu) \end{pmatrix}.$$

The eigenvalues of this matrix at equilibrium point $E^* = (A^*, I^*, D^*, C^*)$ are

$$\left\{ -(\beta_2 + r_2 + \mu),\ -(\delta + \mu),\ -\frac{1}{2}\left(C_1 \pm \sqrt{(C_1)^2 - 4(C_2 + C_3)} \right) \right\}, \tag{11}$$

where

$$\begin{aligned} C_1 &= \alpha_1 + \beta_1 + r_1 + 2\mu + \alpha_2 + \alpha_3 + \alpha_4 > 0, \\ C_2 &= (\alpha_2 + \alpha_3 + \alpha_4 + \mu)(\beta_2 + \mu) > 0, \\ C_3 &= (r_1 + \alpha_1)(\alpha_2 + \mu) + r_1\alpha_3 + \alpha_1\alpha_4 > 0. \end{aligned}$$

It can be seen from Equation (11) that the first two eigenvalues are always negative (independent of parameters), and for all parametric values, the last two eigenvalues will either be negative real numbers or complex conjugates with negative real parts. Hence, by [21], system (3) is asymptotically stable.

4. Solution and Simulation

4.1. Approximate Solution

System (3) has a singular solution, as mentioned in Theorem 1. Using the new algorithm [22] toward the Daftardar-Gejji and Jafari method [23], we will approximate the solution. By [22], for the general functional equation

$$W(x_1, x_2, ..., x_n) = f + N(W(x_1, x_2, ..., x_n)), \tag{12}$$

where W is a function of n-variables, with $n \in N$, f is a known function, and N denotes a nonlinear operator. Then the approximate solution for system (12) by using the new iterative scheme of Daftardar-Gejji and Manoj Kumar is defined as

$$\left\{ \begin{array}{l} \Psi_0(x_1, x_2, ..., x_n) = f \\ \Psi_1(x_1, x_2, ..., x_n) = \Psi_0 + N(\Psi_0(x_1, x_2, ..., x_n)) \\ \Psi_2(x_1, x_2, ..., x_n) = \Psi_0 + N(\Psi_1(x_1, x_2, ..., x_n)) \\ \Psi_{k+1}(x_1, x_2, ..., x_n) = \Psi_0 + N(\Psi_k(x_1, x_2, ..., x_n)),\ k \geq 2, k \in N \end{array} \right\}, \tag{13}$$

and satisfies the convergence

$$\lim_{k \to \infty} \Psi_k(x_1, x_2, ..., x_n) = W(x_1, x_2, ..., x_n).$$

Now, consider system (3)

$$\begin{aligned} {}^{C}D_0^{\sigma} A(t) &= \eta - B_1 A(t) + \alpha_3 I(t), \\ {}^{C}D_0^{\sigma} I(t) &= \alpha_1 A(t) - B_2 I(t), \\ {}^{C}D_0^{\sigma} D(t) &= \beta_1 A(t) + \alpha_2 I(t) - B_3 D(t), \\ {}^{C}D_0^{\sigma} C(t) &= \alpha_4 I(t) + \beta_2 D(t) - B_4 C(t), \end{aligned}$$

with $B_1 = \alpha_1 + \beta_1 + r_1 + \mu$, $B_2 = \alpha_2 + \alpha_3 + \alpha_4 + \mu$, $B_3 = \beta_2 + r_2 + \mu$ and $B_4 = \delta + \mu$. By [24], the corresponding system of the Volterra integral equations is

$$\begin{aligned}
A(t) &= A(0) + \eta I_0^\sigma(1) - B_1 I_0^\sigma(A(t)) + \alpha_3 I_0^\sigma(I(t)),\\
I(t) &= I(0) + \alpha_1 I_0^\sigma(A(t)) - B_2 I_0^\sigma(I(t)),\\
D(t) &= D(0) + \beta_1 I_0^\sigma(A(t)) + \alpha_2 I_0^\sigma(I(t)) - B_3 I_0^\sigma(D(t)),\\
C(t) &= C(0) + \alpha_4 I_0^\sigma(I(t)) + \beta_2 I_0^\sigma(D(t)) - B_4 I_0^\sigma(C(t)),
\end{aligned}$$

upon comparing these equations with (12), we obtain

$$\begin{aligned}
A_0^* &= A(0) + \frac{\eta t^\sigma}{\Gamma(\sigma+1)},\ A_0 = A(0),\\
I_0 &= I(0),\ D_0 = D(0),\ C_0 = C(0),\\
N_A &= -B_1 I_0^\sigma(A(t)) + \alpha_3 I_0^\sigma(I(t)),\\
N_I &= \alpha_1 I_0^\sigma(A(t)) - B_2 I_0^\sigma(I(t)),\\
N_D &= \beta_1 I_0^\sigma(A(t)) + \alpha_2 I_0^\sigma(I(t)) - B_3 I_0^\sigma(D(t)),\\
N_C &= \alpha_4 I_0^\sigma(I(t)) + \beta_2 I_0^\sigma(D(t)) - B_4 I_0^\sigma(C(t)),
\end{aligned}$$

and by applying the iterative scheme defined in (13)

$$\begin{aligned}
A_{k+1} &= A_0^* + N_A(A_k, I_k, D_k, C_k),\ k = 0, 1, 2, \cdots\\
I_{k+1} &= I_0 + N_I(A_k, I_k, D_k, C_k),\ k = 0, 1, 2, \cdots\\
D_{k+1} &= D_0 + N_D(A_k, I_k, D_k, C_k),\ k = 0, 1, 2, \cdots\\
C_{k+1} &= C_0 + N_C(A_k, I_k, D_k, C_k),\ k = 0, 1, 2, \cdots.
\end{aligned} \tag{14}$$

For the first iteration of the approximate solution, putting $k = 0$ in (14) implies that

$$\begin{aligned}
A_1 &= A_0^* + N_A(A_0^*, I_0, D_0, C_0)\\
&= A_0 + \frac{\eta t^\sigma}{\Gamma(\sigma+1)} - B_1 I_0^\sigma\left(A_0 + \frac{\eta t^\sigma}{\Gamma(\sigma+1)}\right) + \alpha_3 I_0^\sigma(I_0)\\
&= A_0 + \frac{\eta t^\sigma}{\Gamma(\sigma+1)} - B_1\left(\frac{A_0}{\Gamma(\sigma+1)} t^\sigma + \frac{\Gamma(\sigma+1)}{\Gamma(\sigma+1)\Gamma(2\sigma+1)} t^{2\sigma}\right) + \alpha_3 \frac{I_0}{\Gamma(\sigma+1)} t^\sigma\\
&= A_0 + \frac{(\eta - B_1 A_0 + \alpha_3 I_0)}{\Gamma(\sigma+1)} t^\sigma - \frac{B_1 \eta}{\Gamma(2\sigma+1)} t^{2\sigma},
\end{aligned}$$

similarly

$$\begin{aligned}
I_1 &= I_0 + \frac{(\alpha_1 A_0 - B_2 I_0)}{\Gamma(\sigma+1)} t^\sigma + \frac{\alpha_1 \eta}{\Gamma(2\sigma+1)} t^{2\sigma},\\
D_1 &= D_0 + \frac{(\beta_1 A_0 + \alpha_2 I_0 - B_3 D_0)}{\Gamma(\sigma+1)} t^\sigma + \frac{\beta_1 \eta}{\Gamma(2\sigma+1)} t^{2\sigma},\\
C_1 &= C_0 + \frac{(\alpha_4 I_0 + \beta_2 D_0 - B_4 C_0)}{\Gamma(\sigma+1)} t^\sigma.
\end{aligned}$$

For the second iteration of the approximate solution, put $k = 1$ in (14); we have

$$\begin{aligned}
A_2 &= A_0 + \frac{(\eta - B_1A_0 + \alpha_3I_0)}{\Gamma(\sigma+1)}t^{\sigma} + \frac{(-B_1(\eta - B_1A_0 + \alpha_3I_0) + \alpha_3(\alpha_1A_0 - B_2I_0))}{\Gamma(2\sigma+1)}t^{2\sigma} \\
&+ \frac{\left((B_1)^2\eta + \alpha_1\alpha_3\eta\right)}{\Gamma(3\sigma+1)}t^{3\sigma}, \\
I_2 &= I_0 + \frac{(\alpha_1A_0 - B_2I_0)}{\Gamma(\sigma+1)}t^{\sigma} + \frac{(\alpha_1(\eta - B_1A_0 + \alpha_3I_0) - B_2(\alpha_1A_0 - B_2I_0))}{\Gamma(2\sigma+1)}t^{2\sigma} \\
&+ \frac{(\alpha_1B_1\eta + \alpha_1B_2\eta)}{\Gamma(3\sigma+1)}t^{3\sigma}, \\
D_2 &= D_0 + \frac{(\beta_1A_0 + \alpha_2I_0 - B_3D_0)}{\Gamma(\sigma+1)}t^{\sigma} \\
&+ \frac{(\beta_1(\eta - B_1A_0 + \alpha_3I_0) + \alpha_2(\alpha_1A_0 - B_2I_0) - B_3(\beta_1A_0 + \alpha_2I_0 - B_3D_0))}{\Gamma(2\sigma+1)}t^{2\sigma} \\
&+ \frac{(\eta(\alpha_1\alpha_2 - \beta_1B_1 - \beta_1B_3))}{\Gamma(3\sigma+1)}t^{3\sigma}, \\
C_2 &= C_0 + \frac{(\alpha_4I_0 + \beta_2D_0 - B_4C_0)}{\Gamma(\sigma+1)}t^{\sigma} \\
&+ \frac{(\alpha_4(\alpha_1A_0 - B_2I_0) + \beta_2(\beta_1A_0 + \alpha_2I_0 - B_3D_0) - B_4(\alpha_4I_0 + \beta_2D_0 - B_4C_0))}{\Gamma(2\sigma+1)}t^{2\sigma} \\
&+ \frac{(\beta_1\beta_2\eta + \alpha_1\alpha_4\eta)}{\Gamma(3\sigma+1)}t^{3\sigma}.
\end{aligned}$$

Finally, for the third iteration of the approximate solution, we put $k = 2$ in (14) and simplify by using the Riemann–Liouville integral properties defined in Definition 1; we obtain

$$\begin{aligned}
A_3 &= A_0 + (\eta - B_1A_0 + \alpha_3I_0)\frac{t^{\sigma}}{\Gamma(\sigma+1)} \\
&+ [-B_1(\eta - B_1A_0 + \alpha_3I_0) + \alpha_3(\alpha_1A_0 - B_2I_0)]\frac{t^{2\sigma}}{\Gamma(2\sigma+1)} \\
&+ [\alpha_3\{\alpha_1(\eta - B_1A_0 + \alpha_3I_0) - B_2(\alpha_1A_0 - B_2I_0)\} \\
&- B_1\{-B_1(\eta - B_1A_0 + \alpha_3I_0) + \alpha_3(\alpha_1A_0 - B_2I_0)\}]\frac{t^{3\sigma}}{\Gamma(3\sigma+1)} \\
&+ \left[\alpha_3\{\alpha_1\eta(B_1 + B_2)\} - B_1\left\{(B_1)^2\eta + \alpha_1\alpha_3\eta\right\}\right]\frac{t^{4\sigma}}{\Gamma(4\sigma+1)}, \\
I_3 &= I_0 + (\alpha_1A_0 - B_2I_0)\frac{t^{\sigma}}{\Gamma(\sigma+1)} + [\alpha_1(\eta - B_1A_0 + \alpha_3I_0) - B_2(\alpha_1A_0 - B_2I_0)]\frac{t^{2\sigma}}{\Gamma(2\sigma+1)} \\
&+ [\alpha_1\{-B_1(\eta - B_1A_0 + \alpha_3I_0) + \alpha_3(\alpha_1A_0 - B_2I_0)\} \\
&- B_2\{\alpha_1(\eta - B_1A_0 + \alpha_3I_0) - B_2(\alpha_1A_0 - B_2I_0)\}]\frac{t^{3\sigma}}{\Gamma(3\sigma+1)} \\
&+ \left[\alpha_1\left\{(B_1)^2\eta + \alpha_1\alpha_3\eta\right\} - B_2\{\alpha_1\eta(B_1 + B_2)\}\right]\frac{t^{4\sigma}}{\Gamma(4\sigma+1)},
\end{aligned}$$

$$
\begin{aligned}
D_3 = {} & D_0 + (\beta_1 A_0 + \alpha_2 I_0 - B_3 D_0)\frac{t^{\sigma}}{\Gamma(\sigma+1)} \\
& + [\beta_1(\eta - B_1 A_0 + \alpha_3 I_0) + \alpha_2(\alpha_1 A_0 - B_2 I_0) - B_3(\beta_1 A_0 + \alpha_2 I_0 - B_3 D_0)]\frac{t^{2\sigma}}{\Gamma(2\sigma+1)} \\
& + [\beta_1\{-B_1(\eta - B_1 A_0 + \alpha_3 I_0) + \alpha_3(\alpha_1 A_0 - B_2 I_0)\} \\
& + \alpha_2\{\alpha_1(\eta - B_1 A_0 + \alpha_3 I_0) - B_2(\alpha_1 A_0 - B_2 I_0)\} \\
& - B_3\{\beta_1(\eta - B_1 A_0 + \alpha_3 I_0) + \alpha_2(\alpha_1 A_0 - B_2 I_0) - B_3(\beta_1 A_0 + \alpha_2 I_0 - B_3 D_0)\}]\frac{t^{3\sigma}}{\Gamma(3\sigma+1)} \\
& + \left[\beta_1\left\{(B_1)^2\eta + \alpha_1\alpha_3\eta\right\} - \alpha_2\{\alpha_1\eta(B_1 + B_2)\} - B_3\{\eta(\alpha_1\alpha_2 - \beta_1 B_1 - \beta_1 B_3)\}\right]\frac{t^{4\sigma}}{\Gamma(4\sigma+1)}, \\
C_3 = {} & C_0 + (\alpha_4 I_0 + \beta_2 D_0 - B_4 C_0)\frac{t^{\sigma}}{\Gamma(\sigma+1)} \\
& + [\alpha_4(\alpha_1 A_0 - B_2 I_0) + \beta_2(\beta_1 A_0 + \alpha_2 I_0 - B_3 D_0) - B_4(\alpha_4 I_0 + \beta_2 D_0 - B_4 C_0)]\frac{t^{2\sigma}}{\Gamma(2\sigma+1)} \\
& + [\alpha_4\{\alpha_1(\eta - B_1 A_0 + \alpha_3 I_0) - B_2(\alpha_1 A_0 - B_2 I_0)\} \\
& + \beta_2\{\beta_1(\eta - B_1 A_0 + \alpha_3 I_0) + \alpha_2(\alpha_1 A_0 - B_2 I_0) - B_3(\beta_1 A_0 + \alpha_2 I_0 - B_3 D_0)\} \\
& - B_4\{\alpha_4(\alpha_1 A_0 - B_2 I_0) + \beta_2(\beta_1 A_0 + \alpha_2 I_0 - B_3 D_0) - B_4(\alpha_4 I_0 + \beta_2 D_0 - B_4 C_0)\}]\frac{t^{3\sigma}}{\Gamma(3\sigma+1)} \\
& + [\alpha_4\{-\alpha_1\eta(B_1 + B_2)\} + \beta_2\{\eta(\alpha_1\alpha_2 - \beta_1 B_1 - \beta_1 B_3)\} - B_4\{\eta(\beta_1\beta_2 + \alpha_1\alpha_4)\}]\frac{t^{4\sigma}}{\Gamma(4\sigma+1)}.
\end{aligned}
$$

4.2. Results and Discussion

This paper's main goal is to investigate the mathematical behavior of the fractional order dynamic model of diabetics by varying the order of the derivative and the impact of parameters r_1 and r_2 in the diabetes population model. We considered our parameter values from [16] as our model is an extension of the integer order model proposed by Mohamed Lamlili E.N. et al., and compared our results to the classical case, i.e., for $\sigma = 1$. The following are the parametric values used for the simulation:

$$
\begin{aligned}
\eta = 53000,\ \alpha_1 = 0.06,\ \alpha_2 = 0.007,\ \alpha_3 = 0.001,\ \alpha_4 = 0.005,\ \beta_1 = 0.005, \\
\beta_2 = 0.01,\ \mu = 0.012,\ \delta = 0.012,\ N = 24 \times 10^6,\ I(0) = 0.5 \times N, \\
D(0) = 0.11 \times N,\ C(0) = 0.5 \times D(0),\ A(0) = N - D(0) - C(0) - I(0).
\end{aligned}
$$

We consider $r_1 = r_2 = r$ for the concise description of graphs. According to different values of r, Figures 1–4 depict the behaviors of the integer-order population model from people following healthy lifestyles to people having diabetes with complications. According to Figures 1–4, the diabetic patient's number decreases as r rises; the same phenomenon is supported by comparable findings described in [25]. The authors concluded that diabetes patients might be sent into the remission category for longer using healthy lifestyle parameters, such as diet and exercise, leading to a significant decrease in the diabetic population.

Figures 5–8 show the impact of the fractional order population model without the remission characteristics. It is clear from the graphs that when the order of the derivatives decreases in the fraction portion, the population to drops, and A grows. Additionally, it is evident from the graphs that the effects of fractional order derivatives begin to be felt between t = 1 and t = 2, which is relatively consistent with real-world events.

This study concludes that using fractional order derivatives reduces the population of individuals living unhealthy lives, increases the population of people living good lifestyles, and lowers the population of people with complications from diabetes. Similar types of results are found in [16].

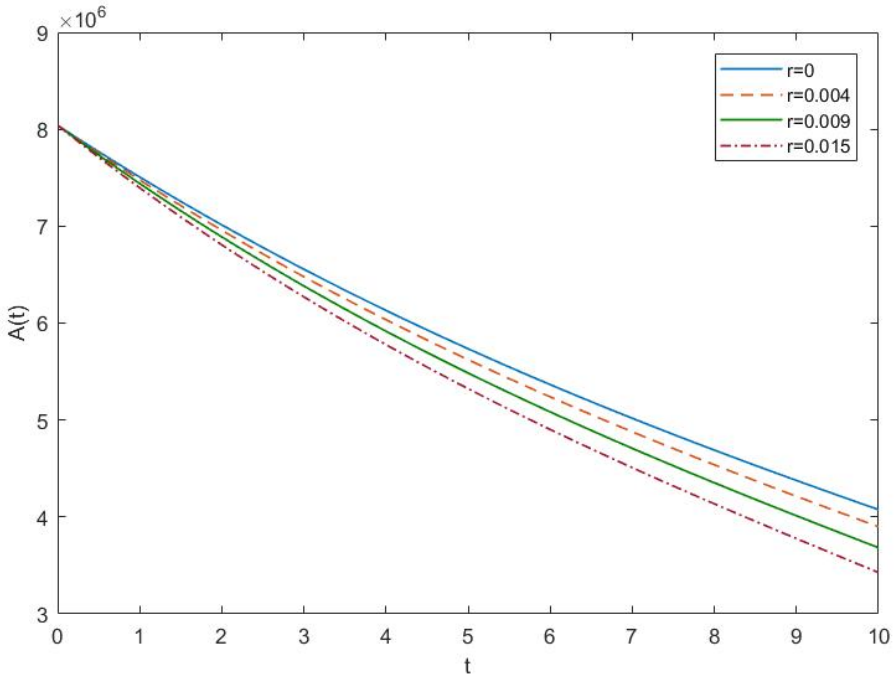


Figure 1. Impact of the remission parameter on the population following healthy lifestyles.

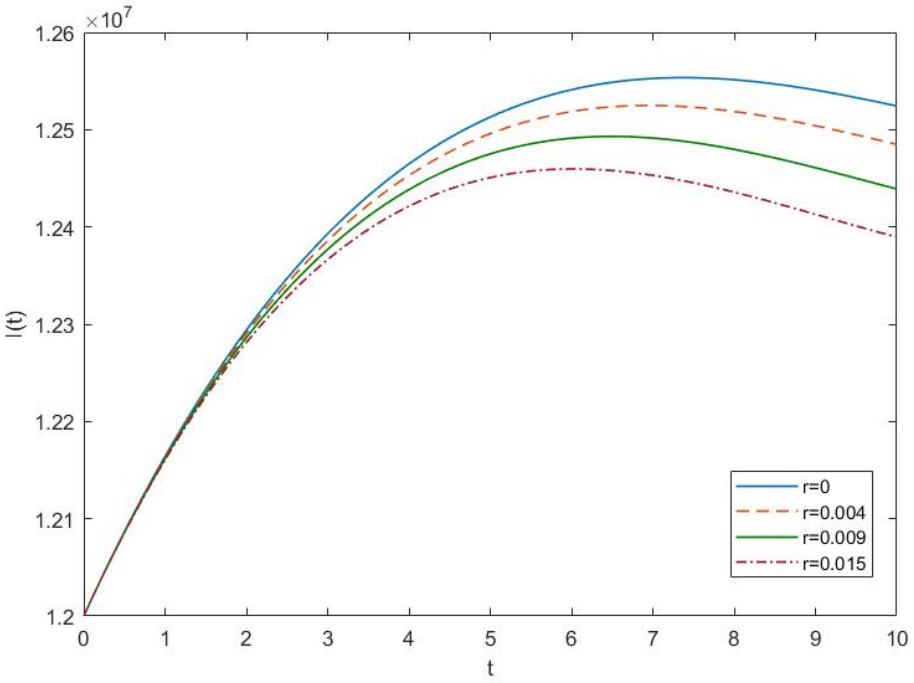


Figure 2. Impact of the remission parameter on the population following unhealthy lifestyles.

Mohamed Lamlili E.N. et al. concluded that their classical model of the diabetic population, i.e., for $\sigma = 1$, predicts a reduction in the population of individuals living unhealthy lives and an increase in the population of people living good lifestyles. Our findings for fractional order agree with those of the study by Mohamed Lamlili E.N. et al. for $\sigma = 1$. Also, similar types of results can be seen in [15].

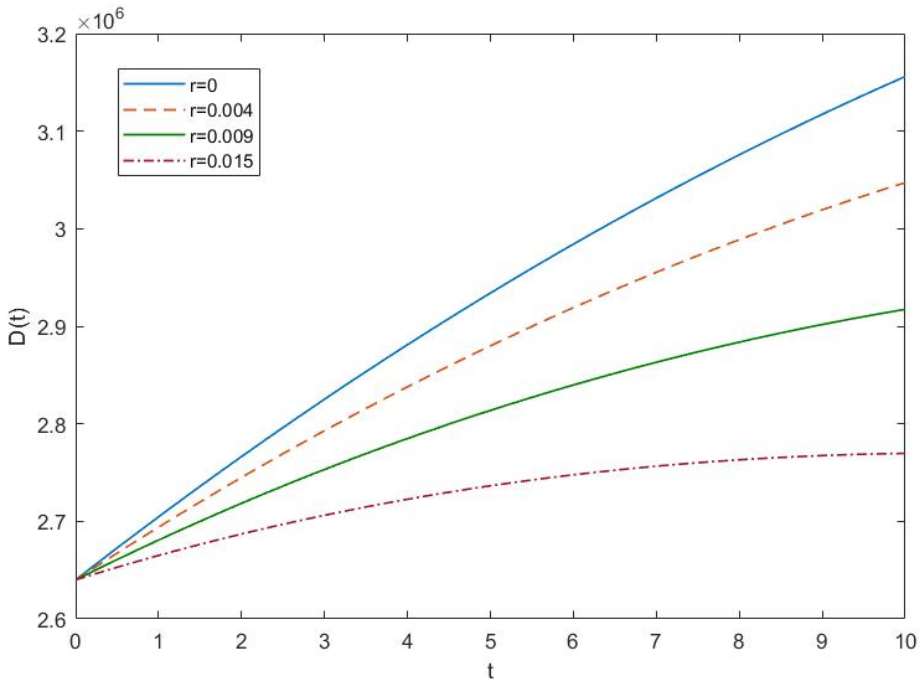


Figure 3. Effects of the remission parameter on diabetics without complications.

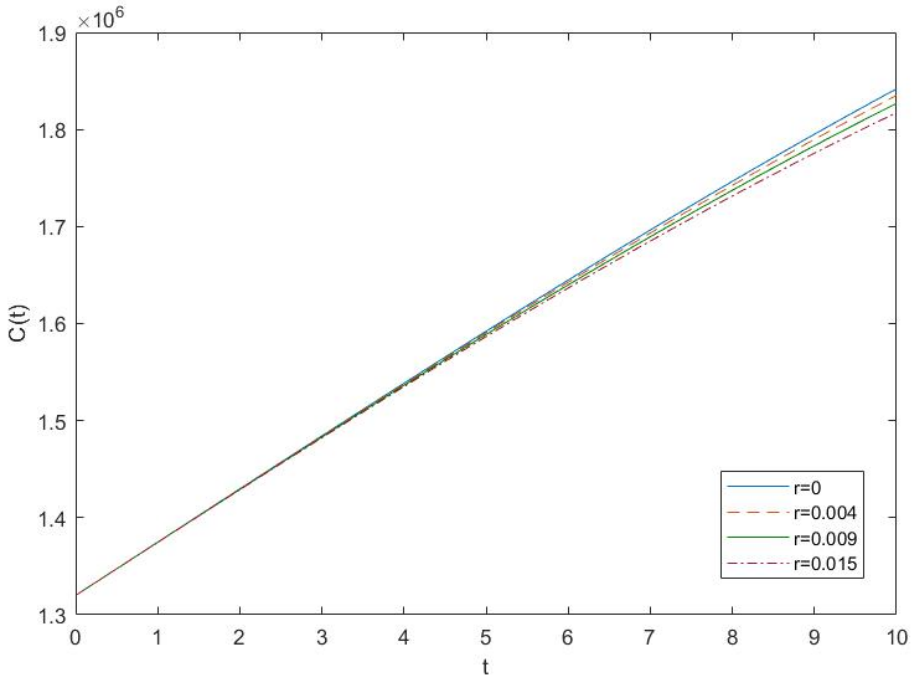


Figure 4. Effects of the remission parameter on diabetics without complications.

Figures 9–12 show the impact of the fractional order population model without the remission characteristics. It is clear from the graphs that when the order of the derivatives decreases in the fraction portion, the populations corresponding to D, I, and C drop, and A grows. Additionally, it is evident from the graphs that the effects of the fractional order derivatives begin to be felt between t = 1 and t = 2, which is relatively consistent with real-world events.

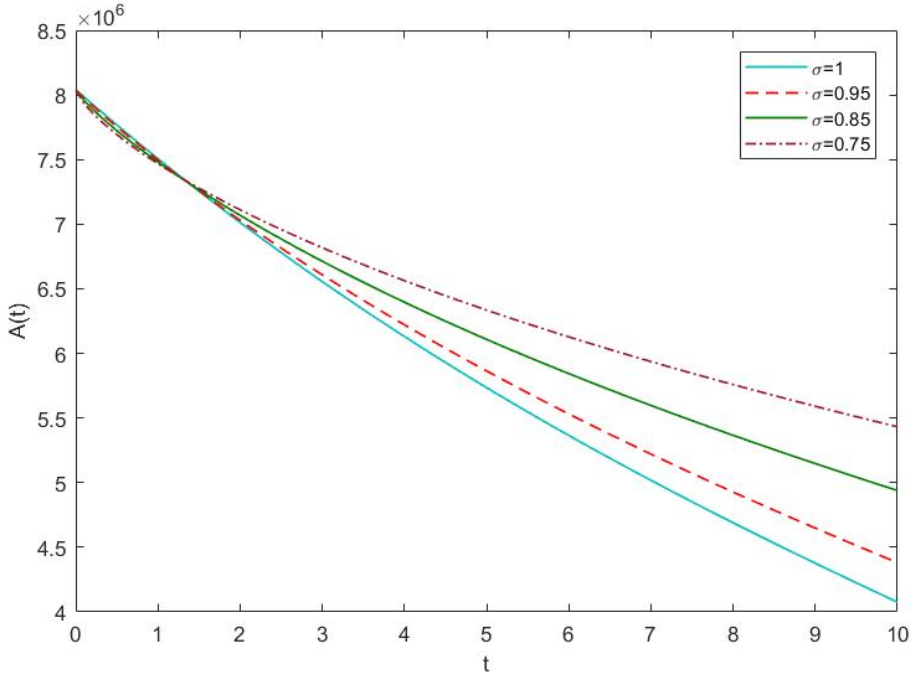


Figure 5. Impact of the fractional order on the population following healthy lifestyles, with $r = 0$.

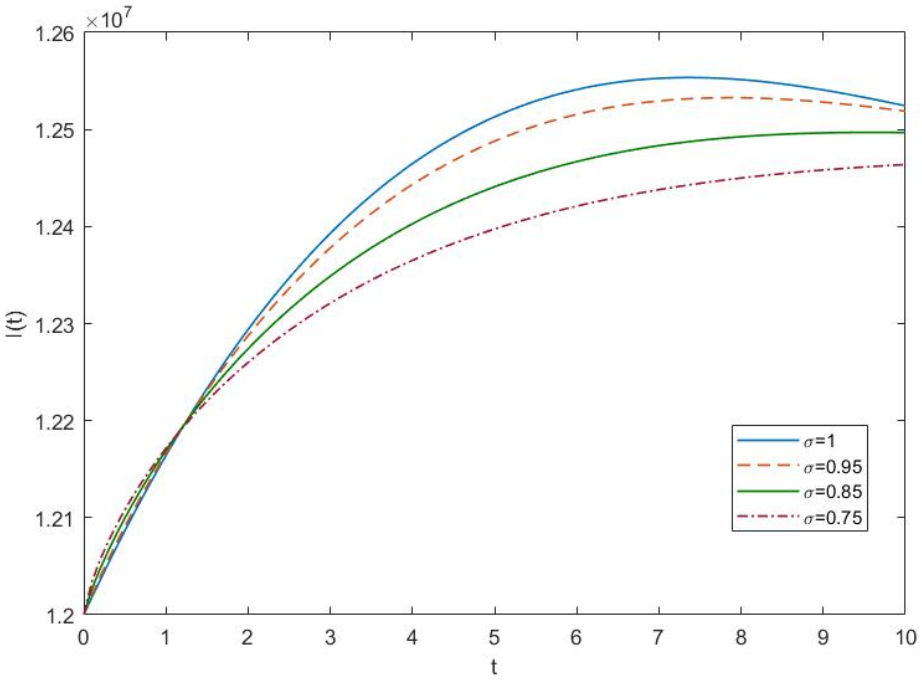


Figure 6. Impact of the fractional order on the population following unhealthy lifestyles, with $r = 0$.

This study concludes that using fractional order derivatives reduces the population of individuals living unhealthy lives, increases the population of people living good lifestyles, and lowers the population of people with complications from diabetes. Similar types of results were found in [16].

Mohamed Lamlili E.N. et al. concluded that their classical model of the diabetic population, i.e., for $\sigma = 1$, indicates a reduction in the population of individuals living unhealthy lives and an increase in the population of people adopting healthier lifestyles.

Our findings for the fractional order are in agreement with those of the Mohamed Lamlili E.N. et al. study for $\sigma = 1$. Also, similar types of results can be seen in [15].

Figures 9–12 show the same outcomes as Figures 5–8, but the population of persons leading unhealthy lives decreases more quickly, and diabetic complications are quickly cured as a result of the addition of the remission parameter.

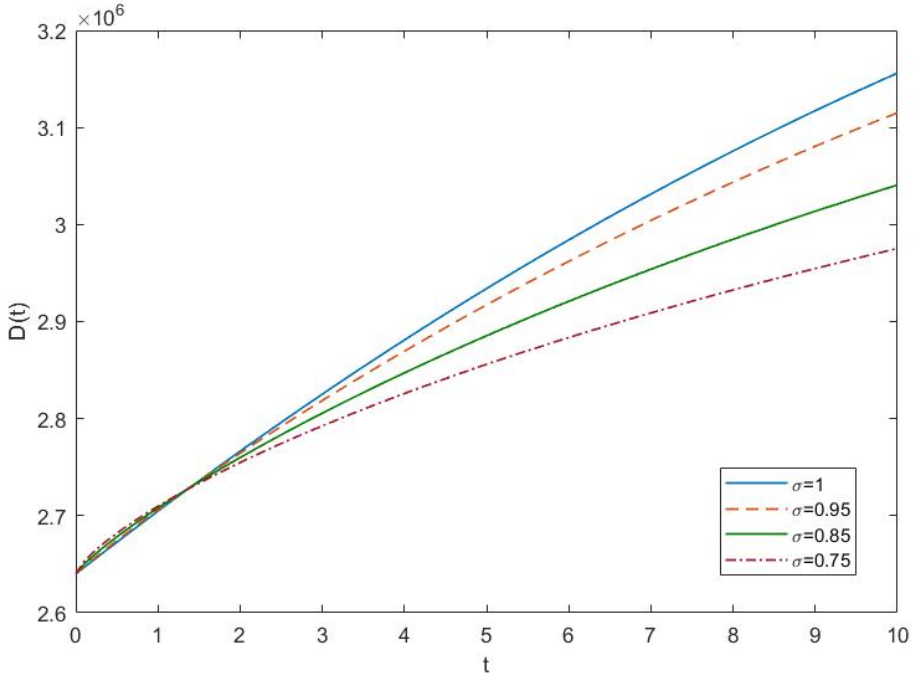


Figure 7. Effects of the fractional order on diabetics without complications with $r = 0$.

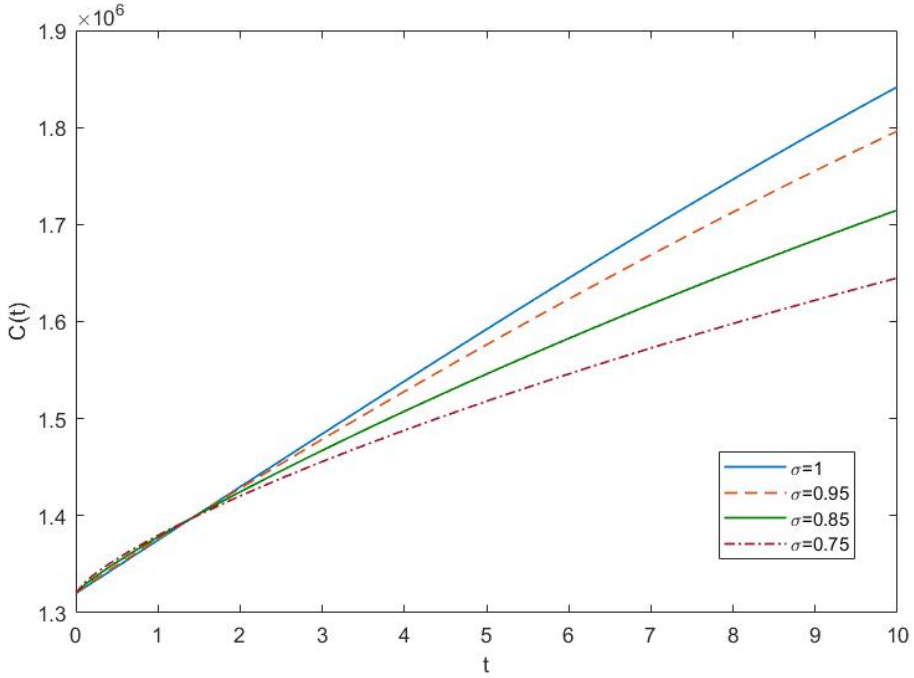


Figure 8. Effects of the fractional order on diabetics without complications with $r = 0$.

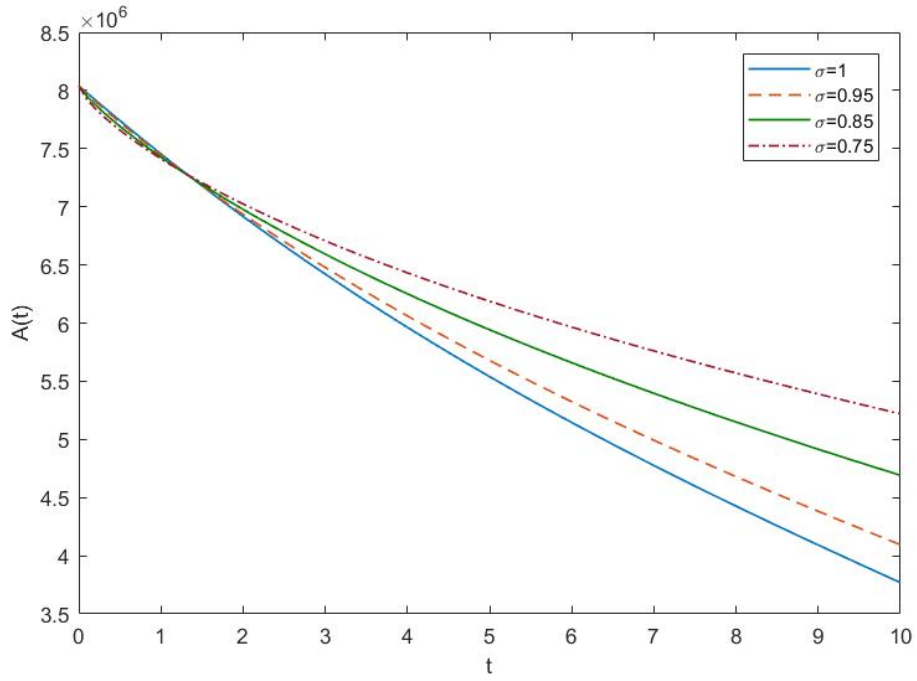


Figure 9. Impact of the fractional order and remission parameter on the population following healthy lifestyles, with $r = 0.007$.

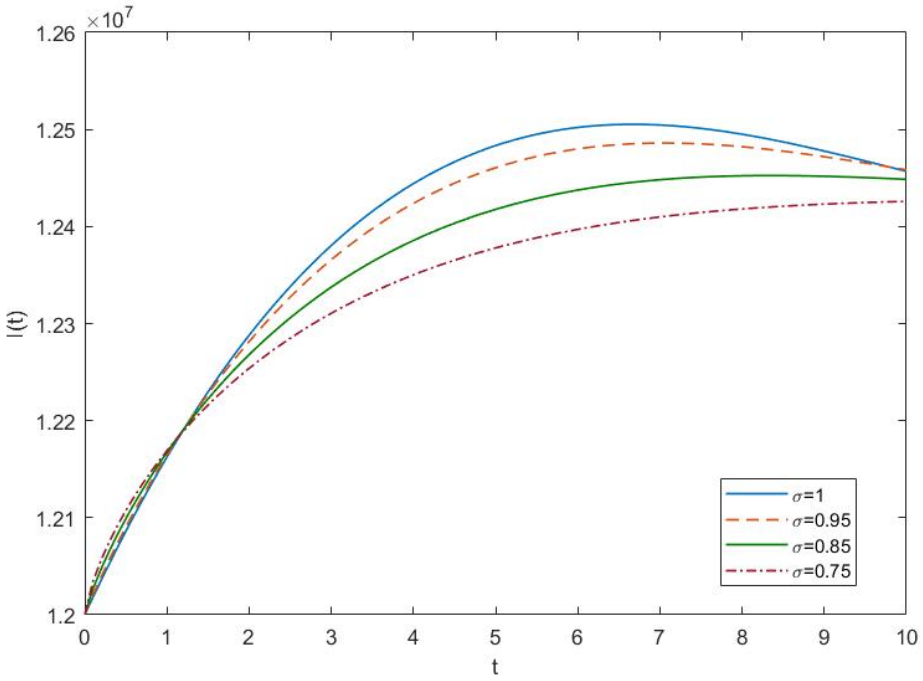


Figure 10. Impact of the fractional order and remission parameter on the population following unhealthy lifestyles, with $r = 0.007$.

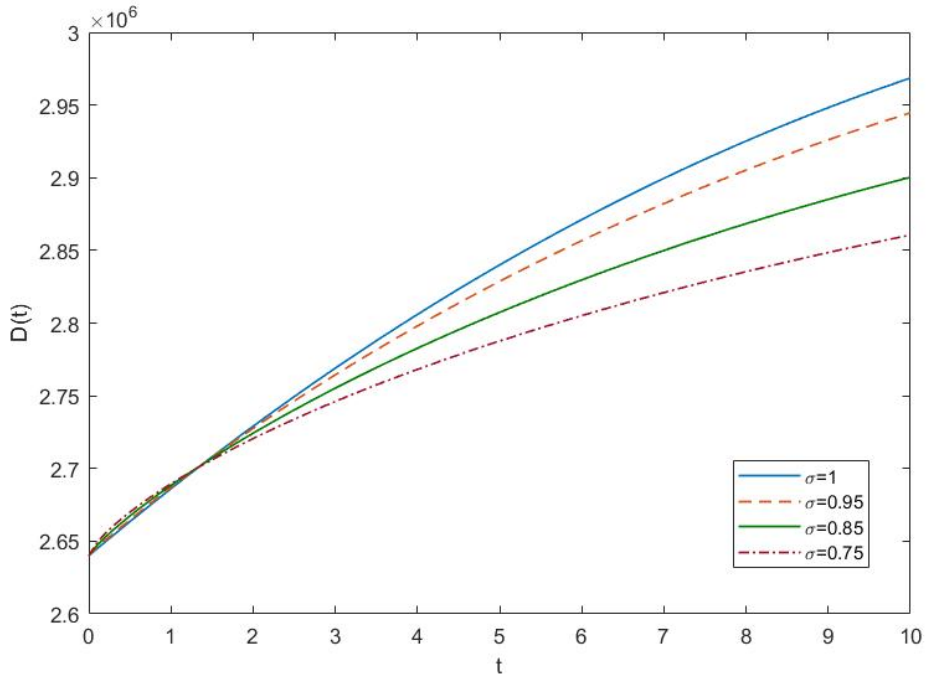


Figure 11. Effects of the remission parameter and fractional order on diabetics without complications, with $r = 0.007$.

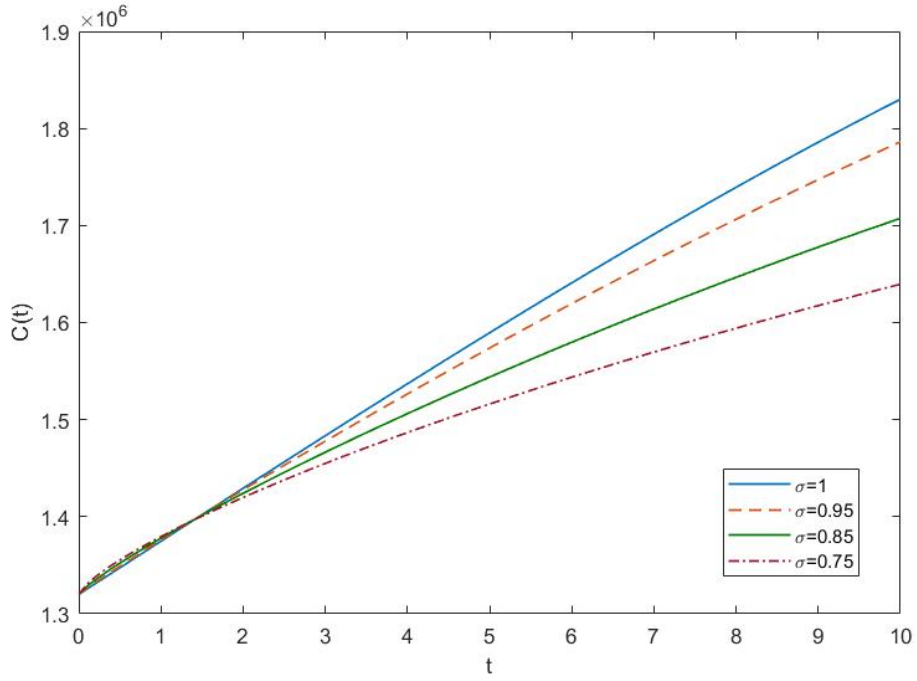


Figure 12. Effects of the remission parameter and fractional order on diabetics without complications, with $r = 0.007$.

5. Conclusions

This study presented a diabetic mellitus population model with fractional order derivatives. The analysis of the proposed fractional order model with overweight/obesity was conducted. We attempted to demonstrate the influence of a healthy lifestyle on remission by incorporating two additional parameters into the framework of the diabetic population. Further, we examined the Caputo sense of the fractional order derivative on the model of

the diabetic population, regardless of remission parameters. Moreover, the existence and uniqueness of the solution of the diabetes mellitus model are discussed. Non-negativity and boundedness are also examined. Subsequently, the stability analysis for the model's equilibrium points uses the Jacobian matrix concept. Eventually, an approximation of the solutions was carried out by using the Daftardar-Gejji and Jafari method. The outcomes of the fractional order model (3) are positive and evenly bounded. Numerical simulations are then conducted to demonstrate the research findings. The graphs show the impact of the fractional order population model without the remission elements. They show that when the fractional order of derivatives decreases, the populations corresponding to D, I, and C drop, while A increases. Also, when remission parameters are considered, it can be observed that the diabetic patient's number decreases as remission rises. Eventually, it is concluded from the displayed graphs that upon incorporating remission parameters, the population of persons leading unhealthy lives decreases more quickly and diabetic complications are quickly cured, showing a clear dependence on the remission parameters and fractional order derivative.

In the future, we will examine our fractional order diabetic population model using alternative approaches, such as treatment functions, and compare the outcomes to acquire new knowledge and insight.

Author Contributions: P.G. led the study and organized the required literature. K.M. wrote the manuscript, interpreted the results, conducted all the numerical calculations, and made the graphs. K.S.A. and B.S.T.A. summarized the data for tables, created the study site map, and formatted the final document. All authors have read and agreed to the published version of the manuscript.

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