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Robust H_∞ Controller Design of Switched Delay Systems with Linear Fractional Perturbations by Synchronous Switching of Rule and Sampling Input

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Abstract: In this paper, we propose synchronous switching of rule and input to achieve H_∞ performance for an uncertain switched delay system with linear fractional perturbations. Our developed simple scheme utilizes the linear matrix inequality optimization problem to provide a feasible solution for the proposed results; if the optimization problem was feasible, our proposed robust H_∞ control could be designed. The feasibility of the optimization problem could be solved using the LMI toolbox of Matlab. In this paper, robust control with sampling is proposed to stabilize uncertain switching with interval time-varying delay and achieve H_∞ performance. Interval time-varying delay and sampling were considered instead of constant delay and pointwise sampling. A full-matrix formulation approach is presented to improve the conservativeness of our proposed results. Some numerical examples are demonstrated to show our main contributions.

Keywords: H_∞ performance; interval time-varying delay; switched delay system; switching rule; synchronous switching; sampling input



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1. Introduction

It is well known that delay is usually encountered and exists in some engineering problems, such as circuit systems, internal combustion engines, long communication systems, and networked control. Delay often causes instability or bad performance in control systems [1–3]. Delayed systems have been applied in aircraft models, engine idle speed control systems, load balancing control systems, networked control systems, power amplifiers in mobile telecommunications, robotized teleoperation, and remote web control of mechanical systems [4–8]. According to [7,9,10], interval time-varying delay would be a more suitable model to present a practical environment for finite speed signal transmission. On the other hand, the dynamic behaviors of switched systems are controlled by several subsystems and use a switching rule to handle switching between those subsystems [11,12]. Switched systems also appear with increasing frequency in aerospace and automotive engineering, biological systems, multi-rate systems, power engineering, robotics, robot mechanical systems, stepper motors, and water quality processing systems [11–14]. According to [13,14], many complex nonlinear dynamics are witnessed in switched systems based on switching among subsystems, such as multiple limit cycles and chaos. Hence, stability and performance analysis for switched systems with interval time-varying delays has received more consideration by many researchers [13–16].

Two interesting facts relating to switched systems are: (1) stability in a switched system can be achieved by selecting a suitable switching rule, even when each subsystem

is unstable [13–15], and (2) the stability of each subsystem cannot guarantee the stability of the entire switched system under an arbitrary switching rule [16]. The average dwell-time method was used to identify the stability of a system under consideration in [17]. In [13–15], the stability and performance of switched systems were guaranteed by proposed different design schemes for switching rules. Based on those specific schemes, a simple design rule was developed for continuous and discrete-time switched systems under consideration [18].

In the past, H_∞ methods were proposed as a means to diminish the effect of disturbance on the output of systems under consideration [19,20]. In [15], stability in switched time-delay systems could be achieved by using real-time switching. The main disadvantage of this real-time switching is the difficulty of implementation. Hence, a proposed rule for synchronous switching at each sampling instant could be a better approach for control than real-time switching [18]. In [18], it was proposed to use aperiodic sampling instead of the periodic sampling in [21]. However, the pointwise sampling period in [18] would constrain the applicability of the proposed results. Hence, this constraint can be evaluated as relaxing to an upper bound for the sampling period in this paper. In the past, linear fractional perturbations were proposed to present more general forms of uncertainties of systems [22,23]. In [24], a novel inequality was developed to improve conservativeness in relation to the Wirtinger-based one. However, this proposed inequality made it more complicated and difficult to develop and design controller schemes. The main advantages and innovations of our proposed scheme can be highlighted:

1. Robust H_∞ control for a switched system with interval time-varying delay and linear fractional perturbations is provided by synchronous switching of rule and input.
2. For simplicity, a full-matrix formulation approach is used to present the developed results. Linear matrix inequality (LMI) optimization results can be performed directly using this approach.
3. The proposed robust controller can confront uncertain switching with interval time-varying delay and sampling. Some upper bounds for sampling and interval time-varying delay can be estimated, as opposed to the pointwise sampling and constant delay in [18].
4. The vector $X(t)$ in the Lyapunov–Krasovskii functional contains possible knowledge regarding our considered system.

The paper is organized as follows. In Section 2, the problem statement for synchronous switching is provided. In Section 3, some numerical examples are provided to show the applicability of the main results. Finally, concluding remarks are in Section 4.

2. Problem Statement

In this paper, we considered the following uncertain switched delay system with linear fractional perturbations:

$$\begin{aligned}\dot{x}(t) &= \bar{A}_{x0\sigma}(t)x(t) + \bar{A}_{x1\sigma}(t)x(t-h(t)) + \bar{B}_{xu\sigma}(t)u(t) + \bar{B}_{xw\sigma}(t)w(t), \quad t \geq 0 \\ z(t) &= \bar{A}_{z0\sigma}(t)x(t) + \bar{A}_{z1\sigma}(t)x(t-h(t)) + \bar{B}_{zu\sigma}(t)u(t) + \bar{B}_{zw\sigma}(t)w(t), \quad t \geq 0 \\ x(t) &= \varphi(t), \quad t \in [-h_M, 0]\end{aligned}\quad (1)$$

where $\bar{A}_{x0\sigma}(t) = A_{x0\sigma} + \Delta A_{x0\sigma}(t)$, $\bar{A}_{x1\sigma}(t) = A_{x1\sigma} + \Delta A_{x1\sigma}(t)$, $\bar{B}_{xu\sigma}(t) = B_{xu\sigma} + \Delta B_{xu\sigma}(t)$, $\bar{B}_{xw\sigma}(t) = B_{xw\sigma} + \Delta B_{xw\sigma}(t)$, $\bar{A}_{z0\sigma}(t) = A_{z0\sigma} + \Delta A_{z0\sigma}(t)$, $\bar{A}_{z1\sigma}(t) = A_{z1\sigma} + \Delta A_{z1\sigma}(t)$, $\bar{B}_{zu\sigma}(t) = B_{zu\sigma} + \Delta B_{zu\sigma}(t)$, $\bar{B}_{zw\sigma}(t) = B_{zw\sigma} + \Delta B_{zw\sigma}(t)$, $x(t) \in \mathbb{R}^n$ is defined as the state variable, $u(t) \in \mathbb{R}^p$ is defined as the control variable, $w(t) \in \mathbb{R}^m$ is defined as the disturbance variable, $z(t) \in \mathbb{R}^q$ is defined as the output variable, $\sigma \in \underline{N}$ is defined as the switching rule and depends on x or t , $h(t) > 0$ is defined as the time-varying delay satisfying $0 < h_m \leq h(t) \leq h_M$, $\dot{h}(t) \leq h_D < 1$, h_m , where h_M and h_D are two given positive constants, and the initial condition is $\varphi \in C_0$, where C_0 is defined to be a set of continuous functions from $[-h_M, 0]$ to $\mathbb{R}^n$. A_{x0i} , $A_{x1i} \in \mathbb{R}^{n \times n}$, $B_{xui} \in \mathbb{R}^{n \times p}$, $B_{xwi} \in \mathbb{R}^{n \times m}$, A_{z0i} , $A_{z1i} \in \mathbb{R}^{q \times n}$, $B_{zui} \in \mathbb{R}^{q \times p}$, and $B_{zwi} \in \mathbb{R}^{q \times m}$, $i \in \underline{N}$, are defined as the given constant matrices. $\Delta A_{x0i}(t)$,

$\Delta A_{x1i}(t)$, $\Delta B_{xui}(t)$, $\Delta B_{xwi}(t)$, $\Delta A_{z0i}(t)$, $\Delta A_{z1i}(t)$, $\Delta B_{zui}(t)$, and $\Delta B_{zwi}(t)$, $i \in \underline{N}$, are defined as the uncertain matrices satisfying the following constraints:

$$\begin{aligned} [\Delta A_{x0i}(t) \Delta A_{x1i}(t) \Delta B_{xui}(t) \Delta B_{xwi}(t)] &= M_{xi} \cdot \Delta_{xi}(t) \cdot [N_{x0i} N_{x1i} N_{xui} N_{xwi}] \\ [\Delta A_{z0i}(t) \Delta A_{z1i}(t) \Delta B_{zui}(t) \Delta B_{zwi}(t)] &= M_{zi} \cdot \Delta_{zi}(t) \cdot [N_{z0i} N_{z1i} N_{zui} N_{zwi}] \\ \Delta_{xi}(t) &= [I - \Gamma_{xi}(t) \Xi_{xi}]^{-1} \Gamma_{xi}(t), \quad \Xi_{xi} \Xi_{xi}^T < T \\ \Delta_{zi}(t) &= [I - \Gamma_{zi}(t) \Xi_{zi}]^{-1} \Gamma_{zi}(t), \quad \Xi_{zi} \Xi_{zi}^T < T \end{aligned} \quad (2)$$

where constant matrices M_{xi} , M_{zi} , N_{x0i} , N_{x1i} , N_{xui} , N_{zwi} , N_{z0i} , N_{z1i} , N_{zui} , N_{zwi} , Ξ_{xi} , and Ξ_{zi} , $i \in \underline{N}$, are provided with appropriate dimensions. Perturbed matrices $\Gamma_{xi}(t)$ and $\Gamma_{zi}(t)$, $\forall i \in \underline{N}$, are unknown and satisfy the following conditions [22,23]:

$$\begin{aligned} \Gamma_{xi}(t)^T \cdot \Gamma_{xi}(t) &\leq I, \quad \forall i \in \underline{N}, t \geq 0, \\ \Gamma_{zi}(t)^T \cdot \Gamma_{zi}(t) &\leq I, \quad \forall i \in \underline{N}, t \geq 0, \end{aligned}$$

Some domains are defined by

$$\Omega_i(Z_i) = \left\{ x \in \mathfrak{R}^n : x^T Z_i x \geq 0 \right\} \quad (3)$$

where matrices $Z_i = Z_i^T \in \mathfrak{R}^{n \times n}$, $i = 1, 2, \dots, N$, can be obtained from the developed results. The switching domains used in this paper can be defined by the following:

$$\overline{\Omega}_1 = \Omega_1, \quad \overline{\Omega}_2 = \Omega_2 / \overline{\Omega}_1, \quad \overline{\Omega}_3 = \Omega_3 / \overline{\Omega}_1 / \overline{\Omega}_2, \quad \dots \quad \text{and} \quad \overline{\Omega}_N = \Omega_N / \overline{\Omega}_1 \dots \dots / \overline{\Omega}_{N-1}$$

We provide the following lemmas to obtain our main results in this paper.

Lemma 1. Suppose that $x(t)$ is a differentiable function from $[t - h_2, t - h_1] \rightarrow \mathfrak{R}^n$. Then, for a matrix $R > 0$ and some matrices $N_1, N_2, N_3 \in \mathfrak{R}^{4n \times n}$, the following inequality is satisfied [24]:

$$- \int_{t-h_2}^{t-h_1} \dot{x}^T(s) R \dot{x}(s) ds \leq \delta^T(t) \Omega \delta(t),$$

where

$$\begin{aligned} \delta^T(t) &= \left[x^T(t - h_1) x^T(t - h_2) \frac{1}{h} \int_{t-h_2}^{t-h_1} x^T(s) ds \cdot \frac{2}{h^2} \int_{t-h_2}^{t-h_1} \int_{t-h_2}^s (u) du ds \right] \\ h &= h_2 - h_1 \geq 0 \\ \Omega &= h \cdot \left[N_1 R^{-1} N_1^T + \frac{1}{3} N_2 R^{-1} N_2^T + \frac{1}{5} N_3 R^{-1} N_3^T \right] + \text{Sym}(N_1 \Pi_1 + N_2 \Pi_2 + N_3 \Pi_3) \\ \Pi_1 &= E_{4,1} - E_{4,2}, \quad \Pi_2 = E_{4,1} + E_{4,2} - 2E_{4,3}, \quad \Pi_3 = E_{4,1} - E_{4,2} - 6E_{4,3} + 6E_{4,4}. \end{aligned}$$

Lemma 2: Suppose that $x(t)$ is a differentiable function from $[t - h_2, t - h_1] \rightarrow \mathfrak{R}^n$ with constants $0 \leq h_m \leq h_M$. Then, for two matrices $R > 0$ and S , satisfying [25]

$$\begin{bmatrix} R & S \\ * & R \end{bmatrix} > 0,$$

the following inequality holds:

$$- (h_M - h_m) \cdot \int_{t-h_M}^{t-h_m} \dot{x}^T(s) R \dot{x}(s) ds$$

$$\leq - \begin{bmatrix} x(t-h(t)) - x(t-h_M) \\ x(t-h_m) - x(t-h(t)) \end{bmatrix}^T \begin{bmatrix} R & S \\ * & R \end{bmatrix} \begin{bmatrix} x(t-h(t)) - x(t-h_M) \\ x(t-h_m) - x(t-h(t)) \end{bmatrix} = Z_1^T(t) \Omega_1 Z_1(t),$$

where the time-varying function $h(t) \in [h_m, h_M]$,

$$Z_1^T(t) = [x^T(t-h(t)) \quad x^T(t-h_m) \quad x^T(t-h_M)],$$

$$\Omega_1 = \begin{bmatrix} -2R + S + S^T & R - S & R - S^T \\ * & -R & S^T \\ * & * & -R \end{bmatrix}.$$

Lemma 3. With $0 \leq \alpha_i \leq 1, i \in \underline{N}, \sum_{i=1}^N \alpha_i = 1, Z_i = Z_i^T \in \mathbb{R}^{n \times n}, i \in \underline{N}$, and condition [18]

$$\sum_{i=1}^N \alpha_i \cdot Z_i > 0,$$

We obtain the following results

$$\bigcup_{i=1}^N \bar{\Omega}_i = \mathbb{R}^n \text{ and } \bar{\Omega}_i \cap \bar{\Omega}_j = \Phi, \forall i \neq j,$$

$$x^T(t) Z_i x(t) \geq 0 \text{ and } \sigma(x(t)) = i, \forall x(t) \in \bar{\Omega}_i,$$

where $\bar{\Omega}_i$ is shown in (3b).

Remark 1. With the proposed affine Bessel–Legendre inequality in [26], the used inequality in Lemma 1 of this paper is a special case for $N = 2$. Our proposed results in this paper could be improved by using more complicated Lyapunov–Krasovskii functional in the future. The proposed inequality in Lemma 2 is applied to reduce conservativeness regarding the interval time-varying delay in this paper [26]. It is interesting to note that the computational complexity for the feasibility of LMI conditions (LMIs) is highly dependent on both the dimensions of the LMIs and the number of decision variables. Based on the constrained number of decision variables, it would be important to propose a useful tool to formulate these LMIs and use the LMI toolbox of Matlab for feasibility. Hence, a full-matrix formulation approach is used to serve this purpose.

Remark 2. From Lemma 3 and the definition in (3b), uniqueness and compactness regarding our proposed state feedback control can be achieved to verify our results.

It is well known that the state of a system may be sampled and data obtained by some electronic devices. In this paper, the sampling instants are defined by $0 = T_0 < T_1 < T_2 < \dots$. The following function can be defined from the sampling period:

$$\tau(t) = t - T_k, t \in [T_k, T_{k+1}),$$

where $\tau_k = T_{k+1} - T_k$ and $\tau_M = \max_{k=0}^{k=\infty} \tau_k$. From the above formulation, we can provide the following conditions:

$$0 \leq \tau(t) \leq \tau_M, t \geq 0, \text{ and } T_k = t - \tau(t), t \in [T_k, T_{k+1}).$$

The synchronous sampling switching rule can be provided as follows:

$$\text{when } x(T_k) \in \bar{\Omega}_i, \sigma(t) = i, \forall t \in [T_k, T_{k+1}), \quad (4)$$

and sampling input can be defined as:

$$\text{when } x(T_k) \in \bar{\Omega}_i, u(t) = -K_i x(T_k), \forall t \in [T_k, T_{k+1}), \quad (5)$$

where $\overline{\Omega}_i$ is shown in (3). In (4) and (5), the switching rule and sampling input are switched synchronously at each sampling instant.

Definition 1. With system (1) with (2) and synchronous switching of rule and input in (4), assume:

- (i) With $w(t) = 0$, system (1) with (2) is asymptotically stable due to the synchronous switching of rule and input in (4) and (5).
- (ii) With a zero initial state (i.e., $\varphi(t) = 0$, $-h_M \leq t \leq 0$), $w(t)$ and $z(t)$ are satisfied by the following condition:

$$\int_0^\ell z^T(t)z(t)dt \leq \gamma^2 \cdot \int_0^\ell w^T(t)w(t)dt, \forall w \neq 0,$$

for constants $\ell > 0$ and $\gamma > 0$. If $\ell = \infty$, the disturbance w should be defined by $L_2(0, \infty)$.

Then, we can say that system (1) with (2) stabilizes at H_∞ performance γ according to switching rule in (4) with Z_i and input in (5) with K_i .

The following notations can be defined before the presentation of the main results:

$$\begin{aligned} \Pi_1 &= E_{4,1} - E_{4,2}, \Pi_2 = E_{4,1} + E_{4,2} - 2E_{4,3}, \Pi_3 = E_{4,1} - E_{4,2} - 6E_{4,3} + 6E_{4,4}, \\ \Delta_1 &= [E_{16,1}^T h_m \cdot E_{16,7}^T h_M \cdot E_{16,8}^T (h_M - h_m) \cdot E_{16,9}^T \tau_M \cdot E_{16,10}^T 0.5 \cdot h_m^2 \cdot E_{16,11}^T \\ &\quad 0.5 \cdot h_M^2 \cdot E_{16,12}^T 0.5 \cdot (h_M - h_m)^2 \cdot E_{16,13}^T 0.5 \cdot \tau_M^2 \cdot E_{16,14}^T]^T, \\ \Delta_2 &= [E_{16,16}^T (E_{16,1} - E_{16,3})^T (E_{16,1} - E_{16,4})^T \\ &\quad (E_{16,3} - E_{16,4})^T (E_{16,1} - E_{16,6})^T h_m \cdot (E_{16,7} - E_{16,3})^T \\ &\quad h_M \cdot (E_{16,8} - E_{16,4})^T (h_M - h_m) \cdot (E_{16,9} - E_{16,4})^T \tau_M \cdot (E_{16,10} - E_{16,6})^T]^T, \\ \Lambda_1 &= [E_{16,1}^T \quad E_{16,3}^T \quad E_{16,7}^T \quad E_{16,11}^T]^T, \Lambda_2 = [E_{16,1}^T \quad E_{16,4}^T \quad E_{16,8}^T \quad E_{16,12}^T]^T, \\ \Lambda_3 &= [E_{16,3}^T \quad E_{16,4}^T \quad E_{16,9}^T \quad E_{16,13}^T]^T, \Lambda_4 = [E_{16,1}^T \quad E_{16,6}^T \quad E_{16,10}^T \quad E_{16,14}^T]^T, \\ \Lambda_5 &= [E_{16,2}^T \quad E_{16,3}^T \quad E_{16,4}^T]^T, \Lambda_6 = [E_{16,5}^T \quad E_{16,1}^T \quad E_{16,6}^T]^T. \end{aligned} \quad (6)$$

An LMI optimization result is investigated in order to achieve asymptotic stability and H_∞ performance for the system under consideration according to the synchronous switching of rule and input given in (4) and (5).

Theorem 1. Suppose there exist constants η , $0 \leq \alpha_i \leq 1$, $i \in \underline{N}$, with $\sum_{i=1}^N \alpha_i = 1$, such that the LMI optimization problem:

$$\text{minimize } \bar{\gamma}, \quad (7)$$

subject to

$$\sum_{i=1}^N \alpha_i \cdot \hat{Z}_i > 0, \quad (8)$$

$$\begin{bmatrix} \hat{R}_5 & \hat{S}_1 \\ * & \hat{R}_5 \end{bmatrix} > 0, \quad (9)$$

$$\begin{bmatrix} \hat{R}_6 & \hat{S}_2 \\ * & \hat{R}_6 \end{bmatrix} > 0, \quad (10)$$

$$\hat{\Psi}_i = \begin{bmatrix} \hat{\Psi}_{11i} & \hat{\Psi}_{12i} & \hat{\Psi}_{13i} & 0 & \hat{\Psi}_{15i} & \hat{\Psi}_{16i} & \hat{\Psi}_{17} \\ * & -I & 0 & \varepsilon_i \cdot M_{zi} & 0 & 0 & 0 \\ * & * & -\varepsilon_i \cdot I & 0 & \varepsilon_i \cdot \Xi_{xi} & 0 & 0 \\ * & * & * & -\varepsilon_i \cdot I & 0 & \varepsilon_i \cdot \Xi_{zi} & 0 \\ * & * & * & * & -\varepsilon_i \cdot I & 0 & 0 \\ * & * & * & * & * & -\varepsilon_i \cdot I & 0 \\ * & * & * & * & * & * & \hat{\Psi}_{77} \end{bmatrix} < 0, i \in \underline{N}, \quad (11)$$

where

$$\begin{aligned} \hat{\Psi}_{11i} &= \text{Sym}(\Delta_2^T \hat{P} \Delta_1 + (\eta \cdot E_{16,1}^T + E_{16,16}^T) \hat{F}_{1i}) + \hat{\Omega}_1 + \hat{\Omega}_2 + \hat{\Omega}_3, \\ \hat{\Psi}_{12i} &= \hat{F}_{2i}^T, \hat{\Psi}_{13i} = \varepsilon_i \cdot (\eta \cdot E_{16,1}^T + E_{16,16}^T) M_{xi}, \\ \hat{\Psi}_{15i}^T &= [N_{x0i} \hat{U}^T \ N_{x1i} \hat{U}^T \ 0 \ 0 - N_{xui} \hat{K}_i \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ N_{xwi} \ 0], \\ \hat{\Psi}_{16i}^T &= [N_{z0i} \hat{U}^T \ N_{z1i} \hat{U}^T \ 0 \ 0 - N_{zui} \hat{K}_i \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ N_{zwi} \ 0], \\ \hat{\Psi}_{17} &= [\sqrt{h_m} \cdot \Lambda_1^T \hat{N}_{11} \ \sqrt{h_m} \cdot \Lambda_1^T \hat{N}_{12} \ \sqrt{h_m} \cdot \Lambda_1^T \hat{N}_{13} \ \sqrt{h_m} \cdot \Lambda_2^T \hat{N}_{21} \ \sqrt{h_m} \cdot \Lambda_2^T \hat{N}_{22} \\ &\quad \sqrt{h_m} \cdot \Lambda_2^T \hat{N}_{23} \ \sqrt{h_m - h_m} \cdot \Lambda_3^T \hat{N}_{31} \ \sqrt{h_m - h_m} \cdot \Lambda_3^T \hat{N}_{32} \ \sqrt{h_m - h_m} \cdot \Lambda_3^T \hat{N}_{33} \\ &\quad \sqrt{\tau_m} \cdot \Lambda_4^T \hat{N}_{41} \ \sqrt{\tau_m} \cdot \Lambda_4^T \hat{N}_{42} \ \sqrt{\tau_m} \cdot \Lambda_4^T \hat{N}_{43}], \\ \hat{\Psi}_{77} &= -\text{diag}[\hat{R}_1 \ 3\hat{R}_1 \ 5\hat{R}_1 \ \hat{R}_2 \ 3\hat{R}_2 \ 5\hat{R}_2 \ \hat{R}_3 \ 3\hat{R}_3 \ 5\hat{R}_3 \ \hat{R}_4 \ 3\hat{R}_4 \ 5\hat{R}_4], \\ \hat{\Omega}_1 &= \sum_{i=1}^3 \text{Sym}(\Lambda_1^T \hat{N}_{1i} \Pi_i \Lambda_1 + \Lambda_2^T \hat{N}_{2i} \Pi_i \Lambda_2 + \Lambda_3^T \hat{N}_{3i} \Pi_i \Lambda_3 + \Lambda_4^T \hat{N}_{4i} \Pi_i \Lambda_4), \\ \hat{\Omega}_2 &= \Lambda_5^T \begin{bmatrix} -2\hat{R}_5 + \hat{S}_1 + \hat{S}_1^T & \hat{R}_5 - \hat{S}_1 & \hat{R}_5 - \hat{S}_1^T \\ * & -\hat{R}_5 & \hat{S}_1^T \\ * & * & -\hat{R}_5 \end{bmatrix} \Lambda_5 \\ &\quad + \Lambda_6^T \begin{bmatrix} -2\hat{R}_6 + \hat{S}_2 + \hat{S}_2^T & \hat{R}_6 - \hat{S}_2 & \hat{R}_6 - \hat{S}_2^T \\ * & -\hat{R}_6 & \hat{S}_2^T \\ * & * & -\hat{R}_6 \end{bmatrix} \Lambda_6, \\ \hat{\Omega}_3 &= E_{16,1}^T \hat{Q}_0 E_{16,1} - (1 - h_D) E_{16,2}^T \hat{Q}_4 E_{16,2} - E_{16,3}^T (\hat{Q}_1 - \hat{Q}_3) E_{16,3} - E_{16,4}^T (\hat{Q}_2 + \hat{Q}_3) E_{16,4} \\ &\quad - E_{16,6}^T \hat{Q}_5 E_{16,6} + E_{16,5}^T \hat{Z}_i E_{16,5} - \bar{\gamma} \cdot E_{16,15}^T E_{16,15} - \eta \cdot \text{Sym}(E_{16,1}^T \hat{U}^T E_{16,16}) \\ &\quad - E_{16,16}^T [\hat{U} + \hat{U}^T - \hat{R}_0] E_{16,16}, \\ \hat{Q}_0 &= \hat{Q}_1 + \hat{Q}_2 + \hat{Q}_4 + \hat{Q}_5, \\ \hat{R}_0 &= h_m \cdot \hat{R}_1 + h_M \cdot \hat{R}_2 + (h_M - h_m) \cdot (\hat{R}_3 + \hat{R}_5) + \tau_M \cdot (\hat{R}_4 + \hat{R}_6), \\ \hat{F}_{1i} &= [A_{x0i} \hat{U}^T \ A_{x1i} \hat{U}^T \ 0 \ 0 - B_{xui} \hat{K}_i \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ B_{xwi} \ 0], \\ \hat{F}_{2i} &= [A_{z0i} \hat{U}^T \ A_{z1i} \hat{U}^T \ 0 \ 0 - B_{zui} \hat{K}_i \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ B_{zwi} \ 0], \end{aligned}$$

is feasible with constants $\bar{\gamma} > 0, \varepsilon_i > 0, i \in \underline{N}$, a nonsingular matrix $\hat{U} \in \mathbb{R}^{n \times n}$, $\hat{P} > 0 \in \mathbb{R}^{9n \times 9n}$, $\hat{Q}_i > 0, \hat{R}_j > 0 \in \mathbb{R}^{n \times n}, i \in \underline{5}, j \in \underline{6}$, matrices $\hat{Z}_i = \hat{Z}_i^T \in \mathbb{R}^{n \times n}, i \in \underline{N}$, and $\hat{S}_1, \hat{S}_2 \in \mathbb{R}^{n \times n}$, $\hat{N}_{ij} \in \mathbb{R}^{4n \times n}, i \in \underline{4}, j \in \underline{3}, \hat{K}_i \in \mathbb{R}^{p \times n}, i \in \underline{N}$. Then, system (1) with (2) stabilizes with H_∞ performance $\gamma = \sqrt{\bar{\gamma}}$ via switching rule in (4) with $Z_i = \hat{U}^{-1} \hat{Z}_i \hat{U}^{-T}$ and input in (5) with $K_i = \hat{K}_i \hat{U}^{-T}$.

Proof. In this paper, we define the Lyapunov–Krasovskii functional as follows:

$$\begin{aligned} V(x(t)) = & X^T(t)PX(t) + \int_{t-h_m}^t x^T(s)Q_1x(s)ds + \int_{t-h_M}^t x^T(s)Q_2x(s)ds \\ & + \int_{t-h_m}^{t-h_M} x^T(s)Q_3x(s)ds + \int_{t-h(t)}^t x^T(s)Q_4x(s)ds \\ & + \int_{t-\tau_M}^t x^T(s)Q_5x(s)ds + \int_{-h_m}^0 \int_{t+s}^t \dot{x}^T(u)R_1\dot{x}(u)duds \\ & + \int_{-h_M}^0 \int_{t+s}^t \dot{x}^T(u)R_2\dot{x}(u)duds + \int_{-h_m}^0 \int_{t+s}^t \dot{x}^T(u)(R_3 + R_5)\dot{x}(u)duds \\ & + \int_{-\tau_M}^0 \int_{t+s}^t \dot{x}^T(u)(R_4 + R_6)\dot{x}(u)duds, \end{aligned} \quad (12)$$

where

$$\begin{aligned} P = \hat{\hat{U}}\hat{\hat{P}}\hat{\hat{U}}^T > 0, \hat{\hat{U}} = \text{diag}[U \ U \ U \ U \ U \ U \ U \ U \ U], \ U = \hat{U}^{-1}, \\ Q_i = U\hat{Q}_iU^T > 0, \ R_j = U\hat{R}_jU^T > 0, \ i \in \underline{5}, \ j \in \underline{6}, \end{aligned}$$

$$\begin{aligned} X(t) = & [x^T(t) \int_{t-h_m}^t x^T(s)ds \int_{t-h_M}^t x^T(s)ds \int_{t-h_m}^{t-h_M} x^T(s)ds \int_{t-\tau_M}^t x^T(s)ds \int_{t-h_m}^t \int_{t-h_m}^s x^T(u)duds \\ & \int_{t-h_M}^t \int_{t-h_M}^s x^T(u)duds \int_{t-h_m}^{t-h_M} \int_{t-h_M}^s x^T(u)duds \int_{t-\tau_M}^t \int_{t-\tau_M}^s x^T(u)duds]^T. \end{aligned}$$

The derivatives of $V(x_t)$ for system (1) with (2)–(4) have

$$\begin{aligned} \dot{V}(x(t)) = & \dot{X}^T(t)PX(t) + X^T(t)P\dot{X}(t) + x^T(t)Q_0x(t) - (1 \\ & - \dot{h}(t))x^T(t-h(t))Q_4x(t-h(t)) \\ & - x^T(t-h_m)(Q_1 - Q_3)x(t-h_m) - x^T(t-h_M)(Q_2 + Q_3)x(t-h_M) \\ & - x^T(t-\tau_M)Q_5x(t-\tau_M) + \dot{x}^T(t)R_0\dot{x}(t) \\ & - \left[\nabla(t) + \int_{t-h_M}^{t-h_m} \dot{x}^T(s)R_5\dot{x}(s)ds + \int_{t-\tau_M}^t \dot{x}^T(s)R_6\dot{x}(s)ds \right], \end{aligned} \quad (13)$$

where

$$\begin{aligned} Q_0 = & Q_1 + Q_2 + Q_4 + Q_5, \\ R_0 = & h_m \cdot R_1 + h_M \cdot R_2 + (h_M - h_m) \cdot (R_3 + R_5) + \tau_M \cdot (R_4 + R_6), \\ \nabla(t) = & \int_{t-h_m}^t \dot{x}^T(s)R_1\dot{x}(s)ds + \int_{t-h_M}^t \dot{x}^T(s)R_2\dot{x}(s)ds \\ & + \int_{t-h_M}^{t-h_m} \dot{x}^T(s)R_3\dot{x}(s)ds + \int_{t-\tau_M}^t \dot{x}^T(s)R_4\dot{x}(s)ds. \end{aligned}$$

Defining

$$\begin{aligned} Y(t) = & \left[x^T(t) \ x^T(t-h(t)) \ x^T(t-h_m) \ x^T(t-h_M) \ x^T(t-\tau(t)) \ x^T(t-\tau_M) \right. \\ & \frac{1}{h_m} \int_{t-h_m}^t x^T(s)ds \ \frac{1}{h_M} \int_{t-h_M}^t x^T(s)ds \ \frac{1}{h_M-h_m} \int_{t-h_M}^{t-h_m} x^T(s)ds \ \frac{1}{\tau_M} \int_{t-\tau_M}^t x^T(s)ds \\ & \frac{2}{h_m^2} \int_{t-h_m}^t \int_{t-h_m}^s x^T(u)duds \ \frac{2}{h_M^2} \int_{t-h_M}^t \int_{t-h_M}^s x^T(u)duds \\ & \left. \frac{2}{(h_M-h_m)^2} \int_{t-h_M}^{t-h_m} \int_{t-h_M}^s x^T(u)duds \ \frac{2}{\tau_M^2} \int_{t-\tau_M}^t \int_{t-\tau_M}^s x^T(u)duds \ w^T(t) \ \dot{x}^T(t) \right]^T, \\ \Gamma_{1i}(t) = & [\bar{A}_{x0i}(t) \ \bar{A}_{x1i}(t) \ 0 \ 0 \ -\bar{B}_{xui}(t)K_i \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ \bar{B}_{xwi}(t) \ 0], \\ \Gamma_{2i}(t) = & [\bar{A}_{z0i}(t) \ \bar{A}_{z1i}(t) \ 0 \ 0 \ -\bar{B}_{zui}(t)K_i \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ \bar{B}_{zwi}(t) \ 0], \end{aligned}$$

we have

$$X(t) = \Delta_1 Y(t),$$

$$\dot{X}(t) = \Delta_2 Y(t),$$

where Δ_1 and Δ_2 were shown in (6), respectively. By using Lemma 1 and $\nabla(t)$ given in (13), we can provide the following condition:

$$-\nabla(t) \leq Y^T(t) \Omega_1 Y(t), \quad (14)$$

where

$$\Omega_1 = h_m \cdot \Lambda_1^T \Xi_1 \Lambda_1 + h_M \cdot \Lambda_2^T \Xi_2 \Lambda_2 + (h_M - h_m) \cdot \Lambda_3^T \Xi_3 \Lambda_3 + \tau_M \cdot \Lambda_4^T \Xi_4 \Lambda_4 + \bar{\Omega}_1,$$

$$\Xi_i = N_{i1} R_i^{-1} N_{i1}^T + \frac{1}{3} N_{i2} R_i^{-1} N_{i2}^T + \frac{1}{5} N_{i3} R_i^{-1} N_{i3}^T, \quad i \in \underline{4},$$

$$\bar{\Omega}_1 = \sum_{i=1}^3 \text{Sym} \left(\Lambda_1^T N_{1i} \Pi_i \Lambda_1 + \Lambda_2^T N_{2i} \Pi_i \Lambda_2 + \Lambda_3^T N_{3i} \Pi_i \Lambda_3 + \Lambda_4^T N_{4i} \Pi_i \Lambda_4 \right),$$

$N_{ij} \in \mathbb{R}^{4n \times n}$, $i \in \underline{4}$, $j \in \underline{3}$, will be chosen, matrices Λ_i and Π_j , $i \in \underline{4}$, $j \in \underline{3}$, are shown in (6). By using Lemma 2, we obtain the following inequality:

$$-\int_{t-h_M}^{t-h_m} \dot{x}^T(s) R_5 \dot{x}(s) ds + \int_{t-\tau_M}^t \dot{x}^T(s) R_6 \dot{x}(s) ds \leq Y^T(t) \Omega_2 Y(t), \quad (15)$$

where

$$\begin{aligned} \Omega_2 = & \Lambda_5^T \begin{bmatrix} -2R_5 + S_1 + S_1^T & R_5 - S_1 & R_4 - S_1^T \\ * & -R_5 & S_1^T \\ * & * & -R_5 \end{bmatrix} \Lambda_5 \\ & + \Lambda_6^T \begin{bmatrix} -2R_6 + S_2 + S_2^T & R_6 - S_2 & R_5 - S_2^T \\ * & -R_6 & S_2^T \\ * & * & -R_6 \end{bmatrix} \Lambda_6, \end{aligned}$$

Λ_5 and Λ_6 are shown in (6). By the system (1) with conditions (2)–(5), and $\sigma(x(t)) = i$, $\forall t \in [T_k, T_{k+1})$, we have

$$\text{Sym} \left((-\dot{x}(t) + \Gamma_{1i}(t) Y(t))^T U^T (\eta x(t) + \dot{x}(t)) \right) = 0. \quad (16)$$

Via the conditions in (13)–(16), the following results can be achieved with $\bar{\gamma} = \gamma^2$:

$$\dot{V}(x(t)) + z^T(t) z(t) - \gamma^2 \cdot w^T(t) w(t) \leq Y^T(t) \Pi_i(t) Y(t), \quad (17)$$

where

$$\begin{aligned} \Pi_i(t) = & \Delta_2^T P \Delta_1 + \Delta_1^T P \Delta_2 + \Omega_1 + \Omega_2 + \Omega_3 + \text{Sym} \left(\left(\eta \cdot E_{16,1}^T + E_{16,16}^T \right) U \Gamma_{1i}(t) \right) \\ & + \Gamma_{2i}^T(t) \Gamma_{2i}(t), \end{aligned} \quad (18)$$

$$\begin{aligned} \Omega_3 = & E_{16,1}^T Q_0 E_{16,1} - (1 - h_D) E_{16,2}^T Q_4 E_{16,2} - E_{16,3}^T (Q_1 - Q_3) E_{16,3} \\ & - E_{16,4}^T (Q_2 + Q_3) E_{16,4} - E_{16,6}^T Q_5 E_{16,6} - \gamma^2 \cdot E_{16,15}^T E_{16,15} \\ & - \eta \cdot \text{Sym} \left(E_{16,1}^T U E_{16,16} \right) - E_{16,16}^T \left[U + U^T - R_0 \right] E_{16,16}. \end{aligned}$$

By the following definition

$$\Sigma_i(t) = \begin{bmatrix} \Sigma_{1i}(t) & \Gamma_{2i}^T(t) \\ * & -I \end{bmatrix} = \begin{bmatrix} \bar{\Sigma}_{1i} & \bar{\Gamma}_{2i}^T \\ * & -I \end{bmatrix} + \Delta_{xzi}(t), \quad (19)$$

$$\begin{aligned}
\Sigma_{1i}(t) &= \Delta_2^T P \Delta_1 + \Delta_1^T P \Delta_2 + \Omega_1 + \Omega_2 + \Omega_3 + \text{Sym} \left(\left(\eta \cdot E_{16,1}^T + E_{16,16}^T \right) U \Gamma_{1i}(t) \right), \\
\bar{\Sigma}_{1i} &= \Delta_2^T P \Delta_1 + \Delta_1^T P \Delta_2 + \Omega_1 + \Omega_2 + \Omega_3 + \text{Sym} \left(\left(\eta \cdot E_{16,1}^T + E_{16,16}^T \right) U \bar{\Gamma}_{1i} \right), \\
\bar{\Gamma}_{1i} &= [A_{0i} \ A_{1i} \ 0 \ 0 \ -B_{ui} K_i \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ B_{wi} \ 0], \\
\bar{\Gamma}_{2i} &= [A_{z0i} \ A_{z1i} \ 0 \ 0 \ -B_{zui} K_i \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ B_{zwi} \ 0], \\
\Delta_{xzi}(t) &= \text{Sym} \left(\begin{bmatrix} \left(\eta \cdot E_{16,1}^T + E_{16,16}^T \right) U M_{xi} & 0 \\ 0 & M_{zi} \end{bmatrix} \begin{bmatrix} \Delta_{xi}(t) & 0 \\ 0 & \Delta_{zi}(t) \end{bmatrix} N_{xzi} \right), \\
N_{xzi} &= \begin{bmatrix} N_{x0i} & N_{x1i} & 0 & 0 & -N_{xui} K_i & 0 & 0 & 0 & 0 & 0 & 0 & 0 & N_{xwi} & 0 \\ N_{z0i} & N_{z1i} & 0 & 0 & -N_{zui} K_i & 0 & 0 & 0 & 0 & 0 & 0 & 0 & N_{zwi} & 0 \end{bmatrix},
\end{aligned}$$

we can obtain the following matrices:

$$\Psi_i = \begin{bmatrix} \bar{\Sigma}_{1i} & \bar{\Gamma}_{2i}^T & \varepsilon_i \cdot \left(\eta \cdot E_{16,1}^T + E_{16,16}^T \right) U M_{xi} & 0 & \Psi_{15i} & \Psi_{16i} \\ * & -I & 0 & \varepsilon_i \cdot M_{yi} & 0 & 0 \\ * & * & -\varepsilon_i \cdot I & 0 & \varepsilon_i \cdot \Xi_{xi} & 0 \\ * & * & * & -\varepsilon_i \cdot I & 0 & \varepsilon_i \cdot \Xi_{yi} \\ * & * & * & * & -\varepsilon_i \cdot I & 0 \\ * & * & * & * & * & -\varepsilon_i \cdot I \end{bmatrix}, \quad (20)$$

where

$$\begin{aligned}
\Psi_{15i}^T &= [N_{x0i} \ N_{x1i} \ 0 \ 0 - N_{xui} K_i \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ N_{xwi} \ 0], \\
\Psi_{16i}^T &= [N_{z0i} \ N_{z1i} \ 0 \ 0 - N_{zui} K_i \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ N_{zwi} \ 0].
\end{aligned}$$

By using the switching rule in (4) with (8) and $\hat{Z}_i = \hat{U} Z_i \hat{U}^T$, we have

$$\sum_{i=1}^N \alpha_i \cdot Z_i > 0. \quad (21)$$

When $x(T_k) \in \bar{\Omega}_i$ with (21) and Lemma 3, we have

$$x^T(T_k) Z_i x(T_k) = x^T(t - \tau(t)) Z_i x(t - \tau(t)) \geq 0, \quad \sigma(x(t)) = i, \quad \forall t \in [T_k, T_{k+1}). \quad (22)$$

With (10), (22), and

$$\hat{U} = \text{diag} \ U \ U \ U \ U \ U \ U \ U \ U \ U \ U \ U \ U \ U \ I \ U \ I \ I \ I \ I \ I,$$

$$\hat{U} = U^{-1}, \quad \hat{\hat{U}} = \text{diag}[U \ U \ U \ U \ U \ U \ U \ U \ U], \quad \hat{P} = \hat{\hat{U}}^{-1} P \hat{\hat{U}}^{-T} > 0,$$

$$\hat{S}_i = U^{-1} S_i U^{-T}, \quad \hat{Q}_i = U^{-1} Q_i U^{-T} > 0, \quad \hat{R}_i = U^{-1} R_i U^{-T} > 0, \quad \hat{Z}_i = U^{-1} Z_i U^{-T},$$

We can show the following condition:

$$\hat{\hat{U}}^{-1} \Psi_i \hat{\hat{U}}^{-T} < 0.$$

The above condition also implies $\Psi_i < 0$ in (20). According to the perturbation decomposition in [22,23] and the Schur complement in [27] with condition (20), we can derive the following condition from (18) and (19):

$$\Sigma_i(t) < 0, \quad \Pi_i(t) < 0.$$

With $w(t) = 0$, (17), and $\Pi_i(t) < 0$, we have

$$\dot{V}(x(t)) < 0, \quad \forall x(t) \neq 0, \quad \forall t \in [T_k, T_{k+1}).$$

The system (1) with (2)–(5) and $w(t) = 0$ is asymptotically stable. Using condition (17), $\Pi_i(t) < 0$, and a time integral from 0 to $\ell > 0$, we have

$$V(x(\ell)) - V(\varphi) + \int_0^\ell z^T(t)z(t)dt \leq \gamma^2 \cdot \int_0^\ell w^T(t)w(t)dt.$$

With a zero initial state (i.e., $\varphi(t) = 0$, $-h_M \leq t \leq 0$), we can obtain the following result:

$$V(\varphi(0)) = 0, V(x(\ell)) \geq 0.$$

The following condition can be achieved:

$$\int_0^\ell z^T(t)z(t)dt \leq \gamma^2 \cdot \int_0^\ell w^T(t)w(t)dt, \forall w \neq 0,$$

for $\ell > 0$ and $\gamma > 0$. The proof is completed. $\square$

For an arbitrary signal $\sigma(t)$ with capturing at each sampling instant, the state feedback sampling input can be provided:

$$\text{when } \sigma(T_k) = i, u(t) = -K_i x(T_k), \forall t \in [T_k, T_{k+1}). \quad (23)$$

With sampling input in (23), delay-dependent LMI optimization can be developed.

Corollary 1. Suppose there exists a constant η , such that the following LMI optimization problem:

$$\text{minimize } \bar{\gamma},$$

subject to conditions (6c)–(6d)

$$\hat{\Psi}_{ij} = \begin{bmatrix} \hat{\Psi}_{11ij} & \hat{\Psi}_{12ij} & \hat{\Psi}_{13i} & 0 & \hat{\Psi}_{15ij} & \hat{\Psi}_{16ij} & \hat{\Psi}_{17} \\ * & -I & 0 & \varepsilon_i \cdot M_{zi} & 0 & 0 & 0 \\ * & * & -\varepsilon_i \cdot I & 0 & \varepsilon_i \cdot \Xi_{xi} & 0 & 0 \\ * & * & * & -\varepsilon_i \cdot I & 0 & \varepsilon_i \cdot \Xi_{zi} & 0 \\ * & * & * & * & -\varepsilon_i \cdot I & 0 & 0 \\ * & * & * & * & * & -\varepsilon_i \cdot I & 0 \\ * & * & * & * & * & * & \hat{\Psi}_{77} \end{bmatrix} < 0, i \in \underline{N}, j \in \underline{N},$$

where

$$\hat{\Psi}_{11ij} = \text{Sym}(\Delta_2^T \hat{P} \Delta_1 + (\eta \cdot E_{16,1}^T + E_{16,16}^T) \hat{F}_{1ij}) + \hat{\Omega}_1 + \hat{\Omega}_2 + \hat{\Omega}_3, \hat{\Psi}_{12ij} = \hat{F}_{2ij}^T,$$

$$\hat{\Psi}_{15ij}^T = [N_{x0i} \hat{U}^T - N_{xui} \hat{K}_j N_{x1i} \hat{U}^T \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ N_{xwi} \ 0],$$

$$\hat{\Psi}_{16ij}^T = [N_{z0i} \hat{U}^T - N_{zui} \hat{K}_j N_{z1i} \hat{U}^T \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ N_{zwi} \ 0],$$

$$\begin{aligned} \hat{\Omega}_3 = & E_{16,1}^T \hat{Q}_0 E_{16,1} - (1 - h_D) E_{16,2}^T \hat{Q}_4 E_{16,2} - E_{16,3}^T (\hat{Q}_1 - \hat{Q}_3) E_{16,3} - E_{16,4}^T (\hat{Q}_2 + \hat{Q}_3) E_{16,4} \\ & - E_{16,6}^T \hat{Q}_5 E_{16,6} - \bar{\gamma} \cdot E_{16,15}^T E_{16,15} - \eta \cdot \text{Sym}(E_{16,1}^T \hat{U}^T E_{16,16}) - E_{16,16}^T [\hat{U} + \hat{U}^T - \hat{R}_0] E_{16,16}, \end{aligned}$$

$\hat{\Psi}_{13i}$, $\hat{\Psi}_{17}$, $\hat{\Psi}_{77}$, $\hat{\Omega}_1$, $\hat{\Omega}_2$, $\hat{Q}_0$, and $\hat{R}_0$ defined in the Theorem 1,

$$\hat{F}_{1ij} = [A_{x0i} \hat{U}^T \ A_{x1i} \hat{U}^T \ 0 \ 0 \ -B_{xui} \hat{K}_j \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ B_{xwi} \ 0],$$

$$\hat{F}_{2ij} = [A_{z0i} \hat{U}^T \ A_{z1i} \hat{U}^T \ 0 \ 0 \ -B_{zui} \hat{K}_j \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ B_{zwi} \ 0],$$

is feasible with constants $\bar{\gamma} > 0$, $\varepsilon_i > 0$, $i \in \underline{N}$, a nonsingular matrix $\hat{U} \in \mathbb{R}^{n \times n}$, $\hat{P} > 0 \in \mathbb{R}^{9n \times 9n}$, $\hat{Q}_i > 0$, $\hat{R}_j > 0 \in \mathbb{R}^{n \times n}$, $i \in \underline{5}$, $j \in \underline{6}$, and matrices $\hat{S}_1, \hat{S}_2 \in \mathbb{R}^{n \times n}$, $\hat{N}_{ij} \in \mathbb{R}^{4n \times n}$, $i \in \underline{4}$, $j \in \underline{3}$, $\hat{K}_i \in \mathbb{R}^{p \times n}$, $i \in \underline{N}$. Then, system (1) with (2) stabilizes with H_∞ performance $\gamma = \sqrt{\bar{\gamma}}$ via the sampling input (23) with $K_i = \hat{K}_i \hat{U}^{-T}$.

Proof. This proof can be completed by selecting the matrices $\hat{Z}_i = 0$, $i \in \underline{N}$ in Theorem 1. $\square$

Remark 3. For $Q_4 = 0$ ($\hat{Q}_4 = 0$) in the Lyapunov–Krasovskii functional (12), the proposed conditions in Theorem 1 and Corollary 1 are independent of h_D . Hence, if the parameter h_D is larger than 1 or unknown, the results developed in this paper are valid for $\hat{Q}_4 = 0$.

Remark 4. In [18], aperiodic sampling with a pointwise sampling period was investigated to obtain the H_∞ performance of a switched delay system. In this paper, we consider the H_∞ performance of a switched delay system with an interval sampling period. More flexible and practical results can be provided to guarantee the performance of the considered system [18].

Remark 5. In this paper, bigger values of τ_M , h_M , $h_M - h_m$ can provide better results with reduced conservativeness regarding our developed results. A smaller value for γ ($\bar{\gamma}$) can provide a better result regarding disturbance attenuation in the switched delay system.

Remark 6. The uncertainties presented in (2) are said linear fractional perturbations [22,23]. These perturbations were confronted in many practical systems, such as fuzzy T-S systems, neural networks, neutral systems and switched systems. The proposed robust H_∞ control can be used to achieve the performance requirements of a system with linear fractional perturbations in (2).

3. Some Numerical Examples

To show the efficiency of our proposed results, two examples from [18] can be used to demonstrate the main contributions in this paper.

Example 1. A switched delay system (1) with (2) and the parameters given is considered:

$$\begin{aligned}
 N &= 2, h_m = 1, h_M = 1.2, \tau_M = 0.3, \\
 A_{x01} &= \begin{bmatrix} -1.1 & 0.1 \\ 0.2 & -1 \end{bmatrix}, A_{x02} = \begin{bmatrix} -1.2 & 0 \\ -0.1 & -1.1 \end{bmatrix}, A_{x11} = \begin{bmatrix} -0.9 & 0.05 \\ -0.2 & -1 \end{bmatrix}, \\
 A_{x12} &= \begin{bmatrix} -1 & 0.1 \\ -0.25 & -1 \end{bmatrix}, B_{xw1} = \begin{bmatrix} 0.2 \\ 0.01 \end{bmatrix}, B_{xw2} = \begin{bmatrix} 0.2 \\ 0.02 \end{bmatrix}, B_{xu1} = \begin{bmatrix} 1 \\ -0.1 \end{bmatrix}, B_{xu2} = \begin{bmatrix} 1 \\ 0.1 \end{bmatrix}, \\
 A_{z01} &= [1 \ 0], A_{z02} = [0.8 \ -0.1], A_{z11} = [-0.8 \ 0.6], A_{z12} = [-0.2 \ 1], \\
 B_{zw1} &= B_{zw2} = -0.5, B_{zu1} = B_{zu2} = 0.3, \\
 M_{xi} &= \begin{bmatrix} 0.01 & 0 \\ 0 & 0.02 \end{bmatrix}, M_{zi} = 0.01, N_{x0i} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0 \end{bmatrix}, N_{x1i} = \begin{bmatrix} 0 & 0 \\ 0 & 0.1 \end{bmatrix}, \\
 N_{xwi} &= N_{xui} = 0, N_{zwi} = N_{zui} = 0, \Xi_{xi} = 0.1 \cdot I, \Xi_{zi} = 0.1, \\
 N_{z0i} &= N_{z1i} = [0.1 \ 0], i = 1, 2.
 \end{aligned} \tag{24}$$

By using the proposed synchronous switching (4) of rule and input with $1 \leq h(t) \leq 1.2$ and $\tau_k \leq 0.3, \forall k$, the LMI optimization problem in (7) of Theorem 1 with $\eta = 1$ and $\alpha_1 = \alpha_2 = 0.5$ is feasible with

$$\begin{aligned}
 \bar{\gamma} &= 0.268, \hat{K}_1 = [11.9239 \ 4.6429], \hat{K}_2 = [16.9141 \ 6.7406], \\
 \hat{U} &= \begin{bmatrix} 5.3986 & 1.9342 \\ 0.6831 & 2.3326 \end{bmatrix}, \hat{Z}_1 = \begin{bmatrix} -8.1649 & -3.4668 \\ -3.4668 & -0.6176 \end{bmatrix}, \hat{Z}_2 = \begin{bmatrix} 12.3651 & 5.0374 \\ 5.0374 & 2.0327 \end{bmatrix}.
 \end{aligned}$$

The switched delay system in (1) with (2) and (24) is asymptotically stabilizable by the following switching rule:

$$\sigma(x(t)) = \begin{cases} 1, & x(T_k) \in \overline{\Omega}_1, \\ 2, & x(T_k) \in \overline{\Omega}_2, \end{cases} \forall t \in [T_k, T_{k+1}), \quad (25)$$

where

$$\overline{\Omega}_1 = \Omega_1, \overline{\Omega}_2 = \Omega_2 \setminus \overline{\Omega}_1 = \mathbb{R}^2 \setminus \overline{\Omega}_1, \\ Z_i = \hat{U}^{-1} \hat{Z}_i \hat{U}^{-T}, i = 1, 2,$$

with

$$\Omega_1 = \left\{ x = \begin{bmatrix} x_1 & x_2 \end{bmatrix}^T \in \mathbb{R}^2 : x^T Z_1 x = -0.1216x_1^2 - 0.453x_1x_2 + 0.0296x_2^2 \geq 0 \right\}, \\ \Omega_2 = \left\{ x = \begin{bmatrix} x_1 & x_2 \end{bmatrix}^T \in \mathbb{R}^2 : x^T Z_2 x = 0.2316x_1^2 + 0.459x_1x_2 + 0.2193x_2^2 \geq 0 \right\},$$

and control input given as follows:

$$u(t) = \begin{cases} -K_1 x(T_k), & x(T_k) \in \overline{\Omega}_1, \\ -K_2 x(T_k), & x(T_k) \in \overline{\Omega}_2, \end{cases} \forall t \in [T_k, T_{k+1}), \quad (26)$$

with

$$K_1 = \hat{K}_1 \hat{U}^{-T} = [1.6709 \ 1.5011], K_2 = \hat{K}_2 \hat{U}^{-T} = [2.3436 \ 2.2034]. \quad (27)$$

We conclude that system (1) with (2) and (24) stabilizes with H_∞ performance $\gamma = \sqrt{\bar{\gamma}} = 0.5177$ using the switching rule in (25) and sampling input in (26) with $K_i = \hat{K}_i \hat{U}^{-T}$ in (27).

Some comparisons are made in Table 1 to show the improvement using the proposed results. A switched system with interval time-varying delay $1 \leq h(t) \leq 1.2$, instead of constant delay $h(t) = h = 1$ as in [18], can be investigated. Interval sampling of $0 < \tau_i \leq 0.3$, instead of pointwise sampling $\tau_i = 0.2$ and $\tau_j = 0.3$ for some values i and j as in [18], can be guaranteed. The reduced conservativeness and easy implementation of the proposed synchronous switching of switching rule and sampling input can be observed from this numerical example.

Table 1. Comparison of the obtained results with published ones.

Results	Delay $h(t)$	Sampling Period	Disturbance Attenuation
[18]	$h(t) = h = 0.2$ constant delay	$\tau = 0.2$ $\tau = 0.25$	$\gamma = 0.7873$ $\gamma = 0.7918$
[18]	$h(t) = h = 1$ constant delay	$\tau_i = 0.2, \tau_j = 0.3$ for some values in i and j pointwise sampling	$\gamma = 0.632$
Results of Theorem 1 in this paper: synchronous switching of rule and sampling input	$1 \leq h(t) \leq 1.2, h_D$ unknown	$0 < \tau_i \leq 0.3$	$\gamma = 0.5177$

Example 2. Consider a switched delay system (1) with (2) and parameters as follows:

$$N = 2, h_m = 3, h_M = 3.5, \tau_M = 0.1,$$

$$A_{x01} = \begin{bmatrix} -1.1 & 0 \\ -0.2 & 0.25 \end{bmatrix}, A_{x02} = \begin{bmatrix} 0.35 & -0.1 \\ 0 & -1.1 \end{bmatrix}, A_{x11} = \begin{bmatrix} -0.6 & -0.1 \\ 0.2 & -0.2 \end{bmatrix}, \\ A_{x12} = \begin{bmatrix} -0.3 & 0.1 \\ 0.1 & -0.6 \end{bmatrix}, A_{z01} = \begin{bmatrix} -0.1 & 0.1 \\ 0.1 & 0.1 \end{bmatrix}, A_{z02} = \begin{bmatrix} 0.1 & -0.1 \\ -0.1 & -0.1 \end{bmatrix}, \\ A_{z11} = \begin{bmatrix} -0.1 & -0.1 \\ 0.1 & -0.1 \end{bmatrix}, A_{z12} = \begin{bmatrix} 0.1 & 0.1 \\ -0.1 & -0.1 \end{bmatrix}, B_{xw1} = B_{xw2} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.3 \end{bmatrix},$$

$$\begin{aligned}
B_{xu1} &= B_{xu2} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.3 \end{bmatrix}, B_{zw1} = B_{zw2} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.3 \end{bmatrix}, \\
M_{xi} &= N_{x0i} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, N_{x1i} = \begin{bmatrix} 0 & 0.1 \\ 0.1 & 0 \end{bmatrix}, \\
N_{xwi} &= \begin{bmatrix} 0.1 & 0.1 \\ 0 & 0 \end{bmatrix}, N_{xu1} = N_{xu2} = \begin{bmatrix} 0 & 0 \\ 0.1 & 0.1 \end{bmatrix}, \\
B_{zui} &= M_{zi} = N_{z0i} = N_{z1i} = \Xi_{zi} = 0, \Xi_{xi} = 0.01, i = 1, 2.
\end{aligned} \tag{28}$$

Case 1. By using the proposed synchronous switching (4) of rule and input with $3 \leq h(t) \leq 3.5$ and $\tau_k \leq 0.1, \forall k$, the LMI optimization problem in (6) of Theorem 1 with $\eta = 1$ and $\alpha_1 = \alpha_2 = 0.5$ is feasible with

$$\begin{aligned}
\bar{\gamma} &= 0.0967, \hat{K}_1 = \begin{bmatrix} -7.4486 & 4.3961 \\ 5.1354 & -2.6515 \end{bmatrix}, \hat{K}_2 = \begin{bmatrix} 8.4781 & 0.2349 \\ -4.9547 & -12.7028 \end{bmatrix}, \\
\hat{U} &= \begin{bmatrix} 8.0691 & 0.4708 \\ 0.0674 & 19.5961 \end{bmatrix}, \hat{Z}_1 = \begin{bmatrix} 11.3692 & 2.8855 \\ 2.8855 & -29.8478 \end{bmatrix}, \\
\hat{Z}_2 &= \begin{bmatrix} -10.6848 & -2.7967 \\ -2.7967 & 30.615 \end{bmatrix}.
\end{aligned}$$

The switched delay system in (1) with (2) and (28) is asymptotically stabilizable using the following switching rule:

$$\sigma(x(t)) = \begin{cases} 1, & x(T_k) \in \bar{\Omega}_1, \\ 2, & x(T_k) \in \bar{\Omega}_2, \end{cases} \forall t \in [T_k, T_{k+1}), \tag{29}$$

where

$$\begin{aligned}
\bar{\Omega}_1 &= \Omega_1, \bar{\Omega}_2 = \Omega_2 \setminus \bar{\Omega}_1 = \mathbb{R}^2 \setminus \bar{\Omega}_1, \\
Z_i &= \hat{U}^{-1} \hat{Z}_i \hat{U}^{-T}, i = 1, 2, \text{ with} \\
\Omega_1 &= \left\{ x = [x_1 \ x_2]^T \in \mathbb{R}^2 : x^T Z_1 x = 0.1723x_1^2 + 0.0444x_1x_2 - 0.0779x_2^2 \geq 0 \right\}, \\
\Omega_2 &= \left\{ x = [x_1 \ x_2]^T \in \mathbb{R}^2 : x^T Z_2 x = -0.1618x_1^2 - 0.0436x_1x_2 + 0.0799x_2^2 \geq 0 \right\},
\end{aligned}$$

and control input is given as follows:

$$u(t) = \begin{cases} -K_1 x(T_k), & x(T_k) \in \bar{\Omega}_1, \\ -K_2 x(T_k), & x(T_k) \in \bar{\Omega}_2, \end{cases} \forall t \in [T_k, T_{k+1}), \tag{30}$$

with

$$K_1 = \hat{K}_1 \hat{U}^{-T} = \begin{bmatrix} -0.9364 & 0.2276 \\ 0.6445 & -0.1375 \end{bmatrix}, K_2 = \hat{K}_2 \hat{U}^{-T} = \begin{bmatrix} 1.0502 & 0.0084 \\ -0.5763 & -0.6462 \end{bmatrix}. \tag{31}$$

We conclude that system (1) with (2) and (28) stabilizes with H_∞ performance $\gamma = \sqrt{\bar{\gamma}} = 0.311$ using switching rule in (29) and sampling input in (30) with $K_i = \hat{K}_i \hat{U}^{-T}$ in (31).

Case 2. In this case, we assume that $\sigma(t)$ is arbitrary and captured at each sampling instant. Using the above statement in Theorem 1 with Corollary 1, the system (1) with (2) and (28) is stabilized by the following switching input:

$$u(t) = \begin{cases} -K_1 x(T_k), & \sigma(T_k) = 1, \\ -K_2 x(T_k), & \sigma(T_k) = 2, \end{cases} \forall t \in [T_k, T_{k+1}), \tag{32}$$

with $\hat{\gamma} = 0.3819$,

$$K_1 = \hat{K}_1 \hat{U}^{-T} = \begin{bmatrix} 2.1037 & -2.4419 \\ -1.0909 & 5.51345 \end{bmatrix}, K_2 = \hat{K}_2 \hat{U}^{-T} = \begin{bmatrix} 2.033 & -2.4308 \\ -1.0993 & 5.5156 \end{bmatrix}. \quad (33)$$

We conclude that system (1) with (2) and (28) stabilizes with H_∞ performance $\gamma = \sqrt{\hat{\gamma}} = 0.618$ using sampling input in (32) with $K_i = \hat{K}_i \hat{U}^{-T}$ in (33).

Some comparisons are made in Table 2 to show the improvement using the proposed results. A switched system with interval time-varying delay $3 \leq h(t) \leq 3.5$, instead of constant delay $h(t) = h = 3.4$ as in [18], can be investigated. Interval sampling $0 < \tau_i \leq 0.1$, instead of pointwise sampling $\tau_i = 0.03$ and $\tau_j = 0.05$ for some values i and j as in [18], can be guaranteed.

Table 2. Comparison of the obtained results with published ones.

Results	Sampling Period	Disturbance Attenuation	Approach and Delay
[18]	$\tau_i = 0.03, \tau_j = 0.05$ for some values in i and j pointwise sampling	$\gamma = 0.8355$	Synchronous switching of rule and sampling input with constant delay $h(t) = h = 3.4$.
		$\gamma = 1.0951$	Arbitrary switching with sampling input and constant delay $h(t) = h = 3.4$.
Results in Theorem 1	$0 < \tau_i \leq 0.1$	$\gamma = 0.3523$	Using only switching rule with $3 \leq h(t) \leq 3.5$
		$\gamma = 0.311$	Synchronous switching of rule and sampling input with $3 \leq h(t) \leq 3.5$.
Results in Corollary 1		$\gamma = 0.618$	$\sigma(t)$ is arbitrary and captured at each sampling instant with $3 \leq h(t) \leq 3.5$.

With no perturbations or disturbance, $\varphi(t) = [-2 \ 2]^T$, $-3.5 \leq t \leq 0$, and $h(t) = 3.25 - 0.25\sin(0.1t)$, the state trajectories of system (1) using the synchronous switching signal in (29) and sampling input in (30) are shown in Figure 1. With disturbance $w(t) = \begin{bmatrix} e^{-0.5t} \sin(5t) \\ -e^{-0.5t} \cos(5t) \end{bmatrix}$ depicted in Figure 2, the output $z(t) \in \mathbb{R}^2$ with a zero initial state is shown in Figure 3. According to the simulation results, the variation in state response would be affected by feedback from the time-varying delay. The proposed switching signal in (29) and sampling input in (30) are shown to be effective for stabilization and attenuation of disturbance in our considered system.

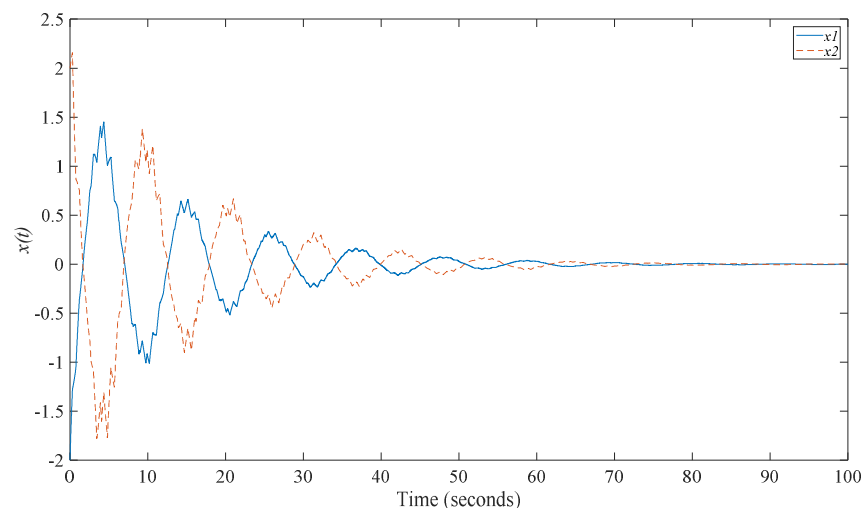


Figure 1. State trajectories of system (1) for the synchronous signal in (29) and sampling input in (30).

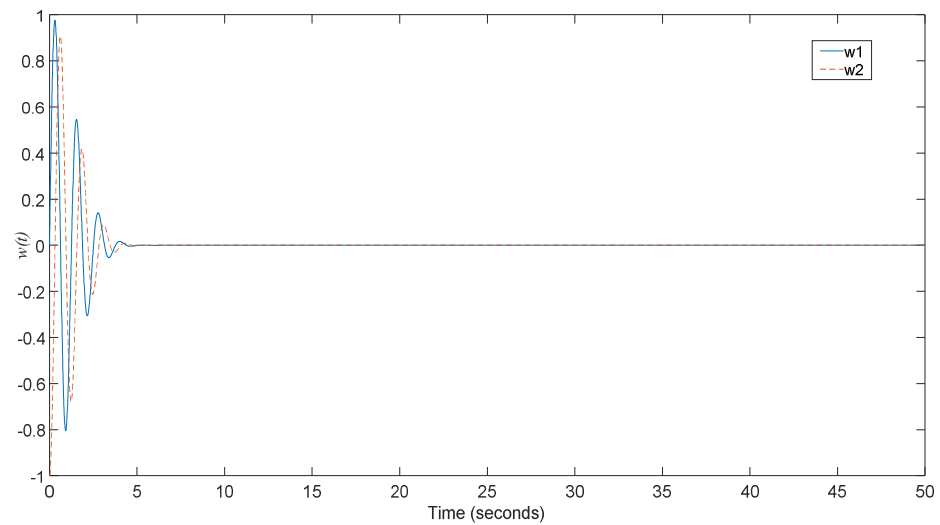


Figure 2. Disturbance input.

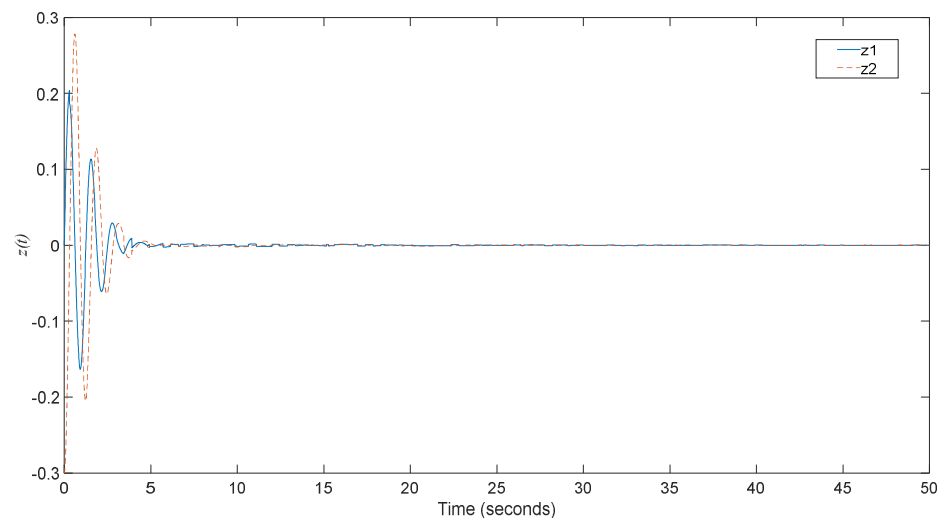


Figure 3. Output of system (1) for the synchronous signal in (29) and sampling input in (30).

4. Conclusions

In this paper, synchronous switching of rule and sampling input was proposed to guarantee the H_∞ performance of switched delay systems with linear fractional perturbations. We developed a full-matrix formulation approach to show the main contribution of this paper. Some useful inequalities and Lyapunov–Krasovskii functionals were investigated to improve the conservativeness of the proposed results. Finally, we illustrated some numerical examples to show the use of our main results. Robust control with sampling input was developed in order to stabilize uncertain switching with interval time-varying delay and achieve H_∞ performance. Interval time-varying delay and sampling were investigated, instead of constant delay and pointwise sampling in [18]. It is interesting to note that fractional-order systems with time delay [28,29] and sampling input for uncertain systems with actuator saturation [30] will be challenging research topics in our future work. Synchronous switching of rule and input to reach mixed H_2 , H_∞ , and passive performances of uncertain fractional-order switched delay systems can be investigated in the future [25].

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Nomenclature

A^T	The transposition of a matrix A
$A > 0$	Matrix A is symmetric positive definite
$A < 0$	Matrix A is symmetric negative definite
$A \leq B$	For any two matrices A and B, the matrix $B - A$ is symmetric positive semi-definite
$\text{Sym}(S)$	For any matrix S, $\text{Sym}(S)$ is defined as the matrix $S + S^T$
$\begin{bmatrix} A & B \\ * & C \end{bmatrix}$	The sign * is defined as the matrix
I	Identity matrix with appropriate dimensions
0	Zero matrix with appropriate dimensions
$0_{i \times j}$	Zero matrix with i rows and j columns
Φ	Empty set
$A \cap B$	Intersection of sets A and B
$A \cup B$	Union of sets A and B
$\mathbb{R}$	Set of real numbers
$E_{q,i}$	$\begin{bmatrix} 0_{n \times (i-1)n} & I & 0_{n \times (q-i)n} \end{bmatrix} \in \mathbb{R}^{n \times qn}, i = 1, 2, \dots, q,$ $q = 2, 3, \dots, 16$
$A \setminus B$	For any two sets A and B, the set $A \setminus B$ is defined to be $\{x x \in A \text{ and } x \notin B\}$
$L_2(0, \infty)$	For a vector valued function $w \in \mathbb{R}^m$, the set $L_2(0, \infty)$ is defined to be $\{w \in \mathbb{R}^m \int_0^\infty w^T(t)w(t)dt < \infty\}$
$\underline{N}$	For a natural number N, the set $\underline{N}$ is defined to be $\{1, 2, \dots, N\}$

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