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Fuzzy Fractional-Order PD Vibration Control of Uncertain Building Structures

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Abstract: A new control strategy is proposed to suppress earthquake-induced vibrations on uncertain building structures. The control strategy embeds fuzzy logic in a fractional-order (FO) proportional derivative (FOPD) controller. A new improved FO particle swarm optimization (IFOPSO) algorithm is derived to adjust the initial parameters of the FOPD controller. An original fuzzy logic-FOPD (FFOPD) controller is then designed by combining the advantages of the fuzzy logic and FOPD control, to deal with large displacements on structures under earthquake excitation. Simulation experiments are carried out on uncertain building structures subjected to the effects of different kinds of seismic signals, illustrating the validity and feasibility of the proposed method.

Keywords: uncertain structural vibration; particle swarm optimization; fractional-order PD control; fuzzy logic control



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1. Introduction

Earthquakes are major natural disasters that seriously threaten human life and property. Finding ways to reduce the influence of seismic waves on building structures is significant and challenging. The vibration control of structures has been shown to be promising and attracted much attention [1]. Many vibration control strategies have been proposed to improve the performance of structures under severe loadings, such as those induced by earthquakes and typhoons [2,3]. Indeed, passive, active and semi-active control methods have been studied to reduce the vibrations on building structures under earthquake excitation [4,5]. Recently, researchers have focused on active and intelligent control strategies due to their superior performance [6]. The most common are based on LQR [7], H_2 and H_∞ [8], sliding mode [9], bang–bang [10], proportional derivative (PD) [11], neural network [12], and optimal and fuzzy logic [13] control.

The PD controller has been widely used in engineering due to its simple structure, easy implementation, and remarkable effectiveness [14,15], and was also adopted to suppress vibrations on building structures under earthquake excitation [16]. However, in applications with high dynamical requirements, the traditional PD may perform poorly. To improve the performance of classical PD algorithms, the FOPD controller was introduced, taking advantage of the tools of the fractional calculus [17,18]. The FOPD controller is an extension of the traditional PD controller [19]. By adding an adjustable fractional order, the FOPD design can be more flexible, and attain better performance in term of improved steady-state error, good disturbance rejection, and superior behavior with nonlinearities [20]. Especially for time delay, nonlinear, and non-minimum phase systems, the FOPD controller can perform better than classical PD. Indeed, the FOPD scheme was successfully applied to motors [21], robots [22], building structures [23], autonomous underwater vehicles [24], and converters [25], to mention a few. Nevertheless, for complex systems and high-control performance requirements, the capacity of the FOPD is still limited.

The appropriate selection of the parameters of the FO controllers is determinant to the performance of the controlled systems. For most FOPD controllers, the parameters are constant and result from off-line optimization procedures. However, in the presence of unknown earthquakes, and uncertain and possibly variable system parameters, the controllers with fixed parameters may perform poorly.

Fuzzy logic controllers can be tuned online without knowing the systems' exact mathematical models [26]. Moreover, they reveal good self-adaptability and robustness [27,28], leading to excellent dynamic performance [29].

In this paper a new FFOPD strategy is proposed to control the vibration of uncertain building structures under unknown earthquake excitation. The FFOPD combines the advantages of the FOPD controller, namely more degrees of freedom for tuning than the PD, and the fuzzy logic control, specifically its intrinsic good self-adaptability. The initial FOPD controller is designed in the discrete time domain, which overcomes the drawbacks associated with the Oustaloup's approximations. The main contributions of this study are: (a) the derivation of a new IFOPSO algorithm for tuning the initial parameters of the FOPD controller; (b) the design of an original FFOPD control strategy for vibration control of uncertain structures; (c) the evaluation of the FFOPD's effectiveness and robustness under different earthquake seismic excitation.

The paper is organized as follows. Section 2 introduces the basic concepts of fractional calculus and the vibration model of an uncertain building structure. Section 3 presents the IFOPSO algorithm and develops the new FFOPD strategy for controlling unwanted displacements of uncertain building structures. Section 4 applies the proposed method to a three-storey frame structure and assesses the controlled system performance. Finally, Section 5 draws the conclusions.

2. Preliminary Concepts and Model Description

2.1. Fractional Calculus

The FO integro-differential operator is a generalization of the classical integer-order one, meaning that we have [30]

$${}_t D_t^\nu = \begin{cases} \frac{d^\nu}{dt^\nu}, & \mathbb{R}(\nu) > 0, \\ 1, & \mathbb{R}(\nu) = 0, \\ \int_{t_0}^t (d\tau)^{-\nu}, & \mathbb{R}(\nu) < 0, \end{cases} \quad (1)$$

where $\nu \in \mathbb{C}$ is the order, and t_0 and t are the limits of the operation. There are several definitions for fractional derivative. The most commonly used are the Grünwald–Letnikov (GL), Riemann–Liouville (RL), and Caputo (C) formulations. The GL derivative of order ν of a function $f(t)$ is given by

$$\begin{aligned} {}_{t_0}^{GL} D_t^\nu f(t) &= \lim_{h \rightarrow 0} f_h^{(\nu)}(t) = \lim_{h \rightarrow 0} h^{-\nu} \sum_{i=0}^{\lfloor \frac{t-t_0}{h} \rfloor} (-1)^i \binom{\nu}{i} f(t - ih) \\ &= \lim_{h \rightarrow 0} \frac{1}{\Gamma(\nu) h^\nu} \sum_{i=0}^{\lfloor \frac{t-t_0}{h} \rfloor} \frac{\Gamma(\nu + i)}{\Gamma(i + 1)} f(t - ih), \end{aligned} \quad (2)$$

where $\lfloor \cdot \rfloor$ means the integer part of the argument, $\Gamma(\cdot)$ is the gamma function:

$$\Gamma(z) = \int_0^\infty e^{-t} t^{z-1} dt = \lim_{n \rightarrow \infty} \frac{n! n^z}{z(z+1)(z+2) \cdots (z+n)} \quad (3)$$

and

$$\binom{\nu}{i} = \frac{\nu(\nu-1)(\nu-2) \cdots (\nu-i+1)}{i!} = \frac{\nu}{i!(\nu-i)!}. \quad (4)$$

In the discrete time, Equation (2) can be approximated by

$${}_{t_0}^{GL}D_t^\nu f(t) = \frac{1}{T^\nu} \sum_{i=0}^r (-1)^i \binom{\nu}{i} f(t - iT), \quad (5)$$

where T is the sampling period and r is the truncation order.

From the above, we verify that the fractional derivative includes information from the past, since it weights previous values of $f(t)$. This reflects the operator ability for capturing memory, which is not present in the integer-order operator.

2.2. Vibration Control System of Uncertain Building Structures

The differential dynamic equation of the building structure system under earthquake excitation is given as

$$M\ddot{X}(t) + C\dot{X}(t) + KX(t) = E_s V(t). \quad (6)$$

Let us consider the general structure with parameter uncertainties and control input. The equation of motion of a n degrees-of-freedom uncertain building structure under earthquake excitation can be described as [9,23]

$$(M + \Delta M)\ddot{X}(t) + (C + \Delta C)\dot{X}(t) + (K + \Delta K)X(t) = B_s U(t) + E_s V(t), \quad (7)$$

where M, C and K are the mass, stiffness and damping matrices, respectively. The matrices $\Delta M = \alpha M$, $\Delta C = \beta C$ and $\Delta K = \gamma K$, where α, β and γ are the maximum rate of change of mass, damping and stiffness, respectively. Moreover, $X(t) \in \mathbb{R}^{n \times 1}$, $\dot{X}(t) \in \mathbb{R}^{n \times 1}$ and $\ddot{X}(t) \in \mathbb{R}^{n \times 1}$ represent the displacement, velocity, and acceleration vectors of the building, respectively. The matrix $B_s \in \mathbb{R}^{n \times p}$ stands for the actuators' locations, vector $U(t) \in \mathbb{R}^{p \times 1}$ denotes the actuators control forces, where p is the number of actuators, matrix $E_s \in \mathbb{R}^{n \times q}$ stands for the external excitation locations, where q is the number of external excitations, $E_s V(t) = -ML\ddot{a}(t)$, where L is a unit column vector, and $\ddot{a}(t)$ denotes the seismic wave acceleration, and vector $F(t) \in \mathbb{R}^{q \times 1}$ denotes the external excitations. The matrices M and K can be written explicitly as:

$$M = \begin{bmatrix} m_1 & 0 & \cdots & 0 & 0 \\ 0 & m_2 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & m_{n-1} & 0 \\ 0 & 0 & \cdots & 0 & m_n \end{bmatrix}_{n \times n},$$

$$K = \begin{bmatrix} k_1 + k_2 & -k_2 & \cdots & 0 & 0 \\ -k_2 & k_2 + k_3 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & k_{n-1} + k_n & -k_n \\ 0 & 0 & \cdots & -k_n & k_n \end{bmatrix}_{n \times n}.$$

The Rayleigh damping matrix is given by $C = \alpha_1 M + \beta_1 K$, where $\alpha_1, \beta_1 \in \mathbb{R}$ are determined by the value of the damping ratio of the two vibration modes, respectively. The calculation formula is as follows:

$$\alpha_1 = \frac{2\omega_1\omega_2(\xi_1\omega_2 - \xi_2\omega_1)}{\omega_2^2 - \omega_1^2},$$

$$\beta_1 = \frac{2(\xi_2\omega_2 - \xi_1\omega_1)}{\omega_2^2 - \omega_1^2},$$

where, the damping ratios $\xi_{i=1,2}$ of all modes are set to 0.05, and ω_1 and ω_2 are the first and second mode circular frequencies of the structure, respectively. The vibration control system

of the uncertain building structure is represented schematically in Figure 1. Displacement deviations are measured by means of sensors, providing input signals for the controller. The controller generates control actions that are sent to the actuators. Finally, the actuators output control forces to suppress earthquake-induced vibrations on the building structure.

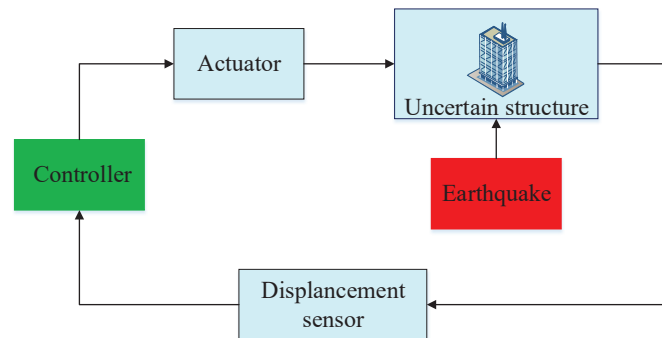


Figure 1. Vibration control system of uncertain building structures.

Let us define the state variables $Z(t) = [X(t) \dot{X}(t)]_{2n \times 1}^T$. The system Equation (7) can be written in the standard state-space form as [23,31],

$$\begin{cases} \dot{Z}(t) = (A + \Delta A)Z(t) + (B + \Delta B)U(t) + EF(t), \\ Y(t) = DZ(t), \end{cases} \quad (8)$$

where $Y(t)$ is the output vector, $D = [I \ 0]_{1 \times 2n}$, and

$$\begin{aligned} A &= \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix}_{2n \times 2n}, B = \begin{bmatrix} 0 \\ M^{-1}B_s \end{bmatrix}_{2n \times p}, E = \begin{bmatrix} 0 \\ -M^{-1}E_s \end{bmatrix}_{2n \times q}, \\ \Delta A &= \begin{bmatrix} 0 & I \\ -\Delta_{MK} & -\Delta_{MC} \end{bmatrix}_{2n \times 2n}, \Delta B = \begin{bmatrix} 0 \\ \Delta_1 M B_s \end{bmatrix}_{2n \times p}, \\ \Delta_1 M &= \frac{-\alpha}{1+\alpha} M^{-1}, \Delta_{MK} = \Delta_1 M(K + \Delta K) + M^{-1} \Delta K, \\ \Delta_{MC} &= \Delta_1 M(C + \Delta C) + M^{-1} \Delta C. \end{aligned}$$

The time-varying uncertain matrices ΔA and ΔB have appropriate dimensions and obey the condition

$$[\Delta A \ \Delta B] = GH(t)[L_1 \ L_2],$$

where G and L are known constant real matrices, and $H(t)$ is an unknown time-varying matrix, $H^T(t)H(t) \leq I$. The matrices G and L are

$$G = \begin{bmatrix} 0 & 0 \\ M^{-1} & M^{-1} \end{bmatrix}_{2n \times 2n}, L_1 = \begin{bmatrix} \frac{\gamma-\alpha}{1+\alpha} K & 0 \\ 0 & \frac{\beta-\alpha}{1+\alpha} C \end{bmatrix}_{2n \times 2n}, L_2 = \begin{bmatrix} 0 \\ \frac{-\alpha}{1+\alpha} B_s \end{bmatrix}_{2n \times 2n},$$

and $H(t) = \delta_t [I]_{2n \times 2n}$, with $|\delta_t| \leq 1$ standing for an uncertain real scalar. The state space model (8) can be rewritten in the standard form

$$\begin{cases} \dot{Z}(t) = \tilde{A}Z(t) + \tilde{B}U_d(t), \\ Y(t) = \tilde{C}Z(t), \end{cases} \quad (9)$$

where $U_d(t) = -L\ddot{a}(t) + (M^{-1} + \Delta_1 M)P f_d$, the variable P is a column vector representing the positions where the forces act, f_d is the force output generated by the actuator, and

$$\tilde{A} = A + \Delta A, \tilde{B} = [0 \quad I]_{2n \times 2n}^T, \tilde{C} = [I \quad 0]_{1 \times 2n}.$$

The actuator adopts the Active Bracing System (ABS) [31,32]. Vibration of buildings may be mitigated by means of ABS, which is placed between the ground and the first floor, or between two successive floors of the building [33]. Herein, we just consider the horizontal vibration of the building structure under earthquake excitation, and only address the design of the controller. The n -DOF shear frame equipped with an ABS between the ground and the first floor is shown in Figure 2.

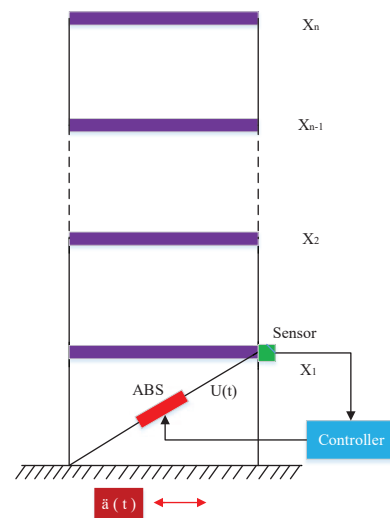


Figure 2. The n -DOF shear frame equipped with an ABS between the ground and the first floor.

3. Control Strategy

3.1. Improved Discrete FOPD Control

In this subsection, a discrete FOPD controller is designed. For practical computational implementation, we consider the FOPD with memory length truncation given by [30]:

$$U(k) = K_p e(k) + \sum_{j=0}^M K_d h^{-\mu} d_j e(k-j). \quad (10)$$

However, memory truncation leads to system oscillation that must be mitigated. Herein, we propose the discrete FOPD:

$$U(k) = K_p e(k) + \sum_{j=0}^M K_d h^{-\mu} d_j e(k-j) + \sum_{j=M+1}^{k-1} K_d h^{-\mu} d_M e(k-j), \quad k > M, \quad (11)$$

where $d_0 = 1$ and $d_j = (1 - (1 - \mu)/j)d_{j-1}$. The parameters K_p and K_d represent the proportional and differential gains, respectively, μ is the differential order, and $e(k)$ stands for the input signal to the controller at the k -th time instant. Hereafter, we adopt $M = 500$.

In the follow-up, an IFOPSO algorithm is proposed to optimize the initial values of the parameters K_p , K_d , and μ of the FOPD.

3.2. Improved FO Particle Swarm Optimization

The PSO is an intelligent optimization algorithm inspired in the swarm behavior of animals when searching for food and protection. The PSO was proposed by Kennedy and

Eberhart [34]. The velocity and position of each particle are updated at the same time as follows:

$$v_{ij}(t+1) = \omega v_{ij}(t) + c_1 \phi_1 [Pb_{ij}(t) - \chi_{ij}(t)] + c_2 \phi_2 [Gb_{gj}(t) - \chi_{ij}(t)], \quad (12)$$

$$\chi_{ij}(t+1) = \chi_{ij}(t) + v_{ij}(t+1), \quad (13)$$

where $Pb_{ij}(t)$ is the best position found so far for each particle, $Gb_{gj}(t)$ is the best position of the swarm, and c_1 and c_2 denote the coefficients of the particles accelerations. Variables ϕ_1 and ϕ_2 are randomly generated real numbers between 0 and 1, and ω is the velocity inertia weight. After calculating the velocity of each particle, the new position can be updated.

The PSO can be generalized in the light of the tools of fractional calculus, yielding the fractional PSO (FOPSO). Therefore, considering Equation (5) and $\omega = 1$, Equation (12) can be written as [35,36]:

$$D^\alpha v_{ij}(t+1) = c_1 \phi_1 [Pb_{ij}(t) - \chi_{ij}(t)] + c_2 \phi_2 [Gb_{gj}(t) - \chi_{ij}(t)]. \quad (14)$$

Taking the first four terms of the differential derivative given by Equation (5), then Equation (14) can be rewritten as:

$$\begin{aligned} v_{ij}(t+1) &= \alpha v_{ij}(t) + \frac{1}{2} \alpha (1 - \alpha) v_{ij}(t-1) \\ &+ \frac{1}{6} \alpha (1 - \alpha) (2 - \alpha) v_{ij}(t-2) \\ &+ \frac{1}{24} \alpha (1 - \alpha) (2 - \alpha) (3 - \alpha) v_{ij}(t-3) \\ &+ c_1 \phi_1 [Pb_{ij}(t) - \chi_{ij}(t)] + c_2 \phi_2 [Gb_{gj}(t) - \chi_{ij}(t)]. \end{aligned} \quad (15)$$

From (15), one can see that the PSO is a particular case of the FOPSO when $\alpha = 1$. In order to enhance the FOPSO algorithm so that it can better balance the local and global optimal solutions and increase the convergence speed, an improved FOPSO algorithm is proposed, yielding the IFOPSO.

Firstly, Equation (15) can be rewritten as:

$$\begin{aligned} v_{ij}(t+1) &= \alpha_1 v_{ij}(t) + \frac{1}{2} \alpha_2 (1 - \alpha_2) v_{ij}(t-1) \\ &+ \frac{1}{6} \alpha_3 (1 - \alpha_3) (2 - \alpha_3) v_{ij}(t-2) \\ &+ \frac{1}{24} \alpha_4 (1 - \alpha_4) (2 - \alpha_4) (3 - \alpha_4) v_{ij}(t-3) \\ &+ c_1 \phi_1 [Pb_{ij}(t) - \chi_{ij}(t)] + c_2 \phi_2 [Gb_{gj}(t) - \chi_{ij}(t)] \\ &= \omega_1 v_{ij}(t) + \omega_2 v_{ij}(t-1) \\ &+ \omega_3 v_{ij}(t-2) + \omega_4 v_{ij}(t-3) + D^\alpha v_{ij}(t+1), \end{aligned} \quad (16)$$

with ω_i ($i = 1, 2, 3, 4$) denoting velocity weights, given by:

$$\begin{cases} \omega_1 = \alpha_1 = \lambda_1 ev + \lambda_2 gd + 0.85, \lambda_1 < 0, \lambda_2 > 0, \omega_1 = \alpha_1 \in (0.5, 1), \\ \omega_2 = \frac{1}{2} \alpha_2 (1 - \alpha_2), & d_{\omega_2} < 0 \text{ if } \alpha_2 \in (0.5, 1), \\ \omega_3 = \frac{1}{6} \alpha_3 (1 - \alpha_3) (2 - \alpha_3), & d_{\omega_3} < 0 \text{ if } \alpha_3 \in (0.42, 1), \\ \omega_4 = \frac{1}{24} \alpha_4 (1 - \alpha_4) (2 - \alpha_4) (3 - \alpha_4), & d_{\omega_4} < 0 \text{ if } \alpha_4 \in (0, 1), \end{cases} \quad (17)$$

where ev and gd are the evolution speed and aggregation degree factors of the swarm, respectively. Their expressions are:

$$ev = \frac{f(Pbest(t))}{f(Pbest(t-1))}, \quad ev \in (0, 1), \quad (18)$$

$$gd(t+1) = \frac{f(Gbest(t))}{\frac{1}{N} \sum_{i=1}^N f(Pbest_i(t))}, \quad gd \in (0, 1). \quad (19)$$

The weight ω_1 is formulated as a function of ev and gd according to their impact on the search performance of the swarm. We know that ω_1 is negatively correlated with ev and positively correlation with gd . We set $\lambda_1 = -0.35$ and $\lambda_2 = 0.15$, so that ω_1 is an adaptive variable related to the evolutionary state to ensure the global search ability. The α_i ($i = 2, 3, 4$) at the T -th iteration is calculated as:

$$\alpha_i = \alpha_{min} + \frac{\alpha_{max} - \alpha_{min}}{T_{max}} T, \quad \alpha_i \in [0.5, 1], \quad (20)$$

where T and T_{max} are the current iteration and the maximum number of iterations, respectively. As the number of iterations increases, α_i ($i = 2, 3, 4$) also increases, but the velocity weight ω_i ($i = 2, 3, 4$) decreases to ensure the local search ability. Through the above analysis, the proposed algorithm not only improves the convergence rate, but also searches the global optimal solution more effectively in the search domain.

The fitness function is the optimization goal of the algorithm. Herein, the integral of time multiplied by the absolute value of the error (ITAE) is used

$$J = \int t |e(t)| dt, \quad (21)$$

where $e(t) = r(t) - y(t)$ is the difference between expected value and top displacement.

The algorithm comprises the following steps:

- Step 1. Initialize the particle swarm. Randomly generate χ_{ij} and v_{ij} of the particles, and determine $Pbest$ of each particle and $Gbest$ of the swarm;
- Step 2. Calculate the fitness value of each particle. If it is good, then update $Pbest$ and $Gbest$;
- Step 3. Use Equations (12) and (16) to update the velocity and location of the particles;
- Step 4. Use Equations (18)–(20) to calculate the fractional-order α_i ;
- Step 5. Use Equation (17) to update the weight ω_i ;
- Step 6. Check stop conditions. If they are satisfied, then stop the search and output the results; Otherwise, return back to Step 2.

The flowchart of the IFOPSO algorithm is shown in Figure 3.

Table 1 summarizes the values of the FOPD parameters obtained with the PSO, FOPSO and IFOPSO algorithms. For the PSO and FOPSO we set $\omega = 0.85$ and decrease it linearly.

In order to verify the effectiveness of the IFOPSO, the FOPD controller was applied to the vibration control of a three-level building structure with model parameters shown in Table 2, and El-Centro earthquake seismic wave with maximum amplitude adjusted to 4 m/s², as depicted in Figure 4. Figure 5 represents the vibration displacements versus time at the top of the building and Figure 6 shows the maximum displacement values. The displacements obtained with the FOPD-PSO, FOPD-FOPSO, and FOPD-IFOPSO are 0.0442 m, 0.0401 m, and 0.0375 m, respectively. This means that the maximum displacement for the FOPD-IFOPSO is reduced by 17.9% and 7.0%, when compared with the FOPD-PSO and FOPD-FOPSO, respectively, which reveals the superiority of the FOPD-IFOPSO.

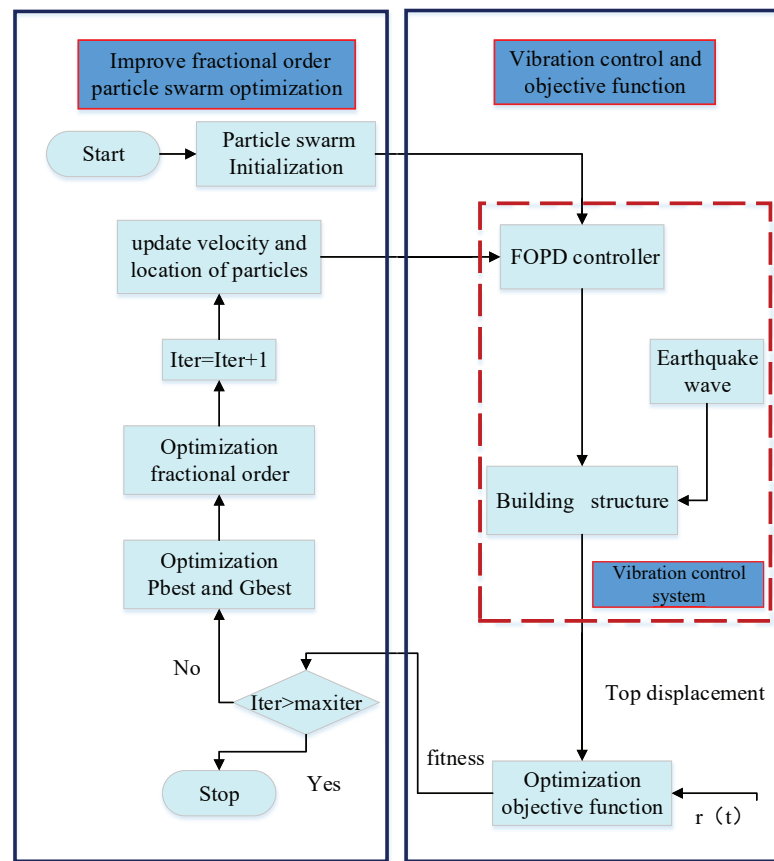


Figure 3. Flowchart of the IFOPSO algorithm for determining the parameters of the FOPD controller.

Table 1. The FOPD parameters generated by the PSO, FOPSO, and IFOPSO algorithms.

Range	$[K_p, K_d, \mu] = [0 \sim 20, 0 \sim 5, 0 \sim 2]$
Name of the Optimization Algorithm	Control System Parameter Value
FOPD-PSO	$[K_p, K_d, \mu] = [8.43, 3.89, 1.61]$
FOPD-FOPSO	$[K_p, K_d, \mu] = [14.92, 4.51, 1.65]$
FOPD-IFOPSO	$[K_p, K_d, \mu] = [19.48, 4.93, 1.63]$

Table 2. Structural parameters of the uncertain building.

Floor	1	2	3
Height (m)	3.2	3.0	3.0
Quality (kg)	2762	2760	2300
Rigidity (1×10^5 N/m)	2.485	1.921	1.522
Name of Various	Parameter Values		
uncertain parameters	$\alpha = 0.1$	$\beta = 0.15$	$\gamma = 0.15$
various of δ_t	$\delta_t = 0$	$\delta_t = -1$	$\delta_t = 1$
various of $H(t)$	$H^T(t)H(t) \leq I$		

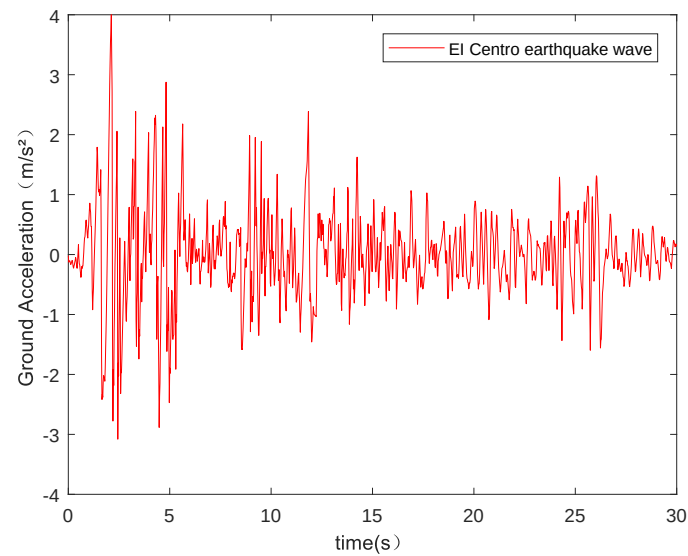


Figure 4. Earthquake seismic wave with maximum amplitude adjusted to 4 m/s^2 .

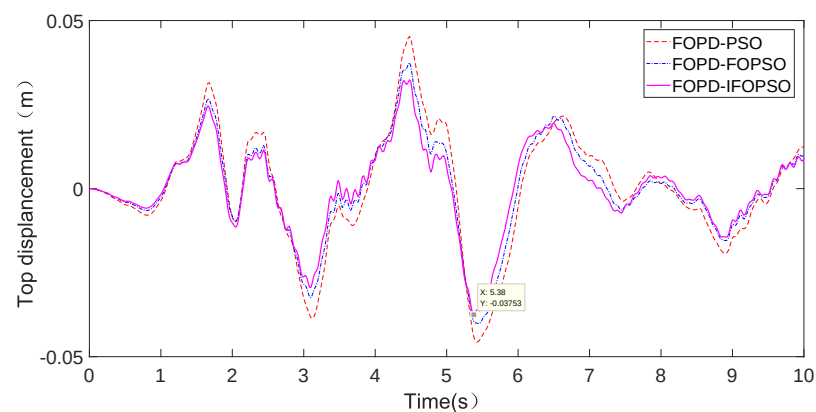


Figure 5. Top displacement versus time for the three algorithms under earthquake seismic wave excitation.

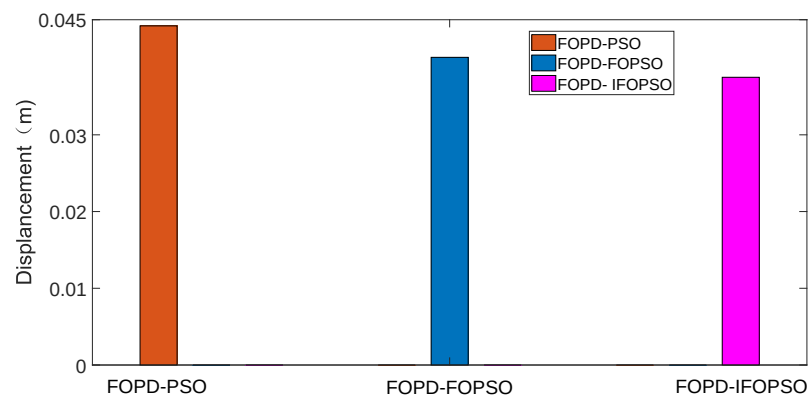


Figure 6. Maximum displacement for the three algorithms under earthquake seismic wave excitation.

3.3. FFOPD Controller

The parameters of the building structure, as well as the earthquake excitation, are often uncertain. Therefore, optimizing the parameters of the controller off-line, without taking into consideration system model and excitation uncertainty, results in poor control performance.

Fuzzy controllers can be used without an explicit mathematical model of the controlled system, and possess intrinsic adaptability to nonlinear, time-varying, and uncertain effects present in practical systems. To combine the advantages of the FOPD and the fuzzy logic control, we propose a new FFOPD controller given by

$$U(k) = K_p(k)r_1(k)e(k) + \sum_{j=0}^M K_d(k)r_2(k)h^{-\mu(k)}d_{\mu(k),j}e(k-j) + \sum_{j=M+1}^{k-1} K_d(k)r_2(k)h^{-\mu(k)}d_{\mu(k),M}e(k-j), \quad k > M, \quad (22)$$

where K_p , K_d , μ , r_1 and r_2 are arrays with length m . Setting the running time of the system t , then we have $m = t/h$. The two variables r_1 and r_2 regulate the fractional behavior of the whole algorithm.

Let $e(k) = r(t) - y(t)$ be the system error vector. The change rate of the system error at the τ -th time instant, $ec(k)$, can be expressed as

$$ec(t) = (e(t) - e(t-1))/\Delta t. \quad (23)$$

Both e and ec are chosen as input variables. The FOPD controller parameters ΔK_p , ΔK_d , $\Delta\mu$, Δr_1 , Δr_2 are the output variables of the fuzzy controller. The parameters' self-tuning formula is

$$\begin{cases} K_p(k) = K_p + \Delta K_p(k), \\ K_d(k) = K_d + \Delta K_d(k), \\ \mu(k) = \mu + \Delta\mu(k), \\ r_1(k) = \Delta r_1(k), \\ r_2(k) = \Delta r_2(k). \end{cases} \quad (24)$$

The fundamental structure of the new FFOPD scheme with five tuning parameters k_p, k_d, μ, r_1, r_2 changing with the uncertain building structure response is showed in Figure 7. The fuzzy algorithm for the new controller involves four stages: fuzzy rule base, inference mechanism, fuzzification interface, and defuzzification interface.

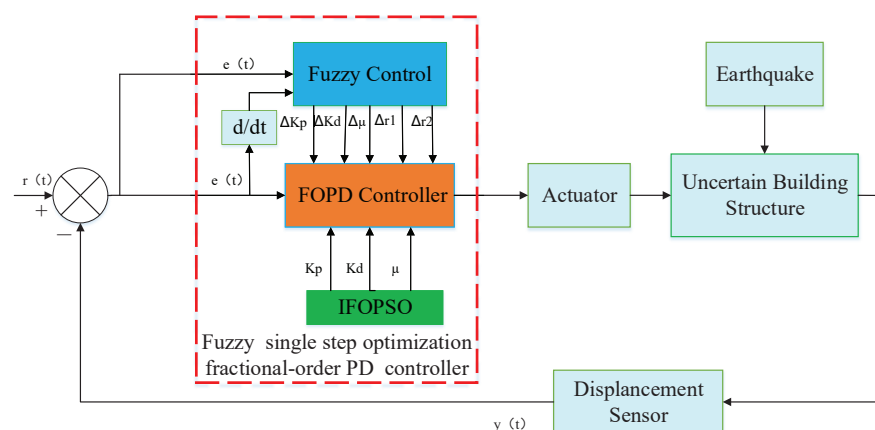


Figure 7. Control system using the FFOPD under uncertain building structure.

It follows from the displacement response of the structure under FOPD control with initial parameters that the domain of the fuzzy controller parameters is as presented in Table 3.

Table 3. Fuzzy logic control parameters.

Variables of Fuzzy Algorithm	Fuzzy Logic Parameter Value
Fuzzy domain variables (input)	$e \in [-4, 4]; ec \in [-3, 3]$
Fuzzy domain variables (output)	$\Delta K_p \in [-4, 4]; \Delta K_d \in [-3, 3]$ $\Delta \mu \in [-0.2, 0]; \Delta r_1 \in [1.1, 1.4]; \Delta r_2 \in [-1, 1.3]$
Input quantization factors	$K_e = 100; K_{ec} = 10$
Output proportion factors	$K = 1$

There are 7 fuzzy linguistic variables: negative big value (NB), negative medium value (NM), negative small value (NS), zero value (Z), positive small value (PS), positive medium value (PM), and positive big value (PB), as well as 49 rules in the rule base. The membership functions and fuzzy control rules of K_p and K_d are kept almost the same as for the conventional fuzzy PD controllers [37,38]. The fuzzy control rules of μ, r_1 and r_2 are designed according to the influence of them into the control system, which are tested repeatedly. A set of fuzzy rules that relates the input and output variables is established in Tables 4 and 5. Figures 8–11 depict the membership functions.

The control process of the FFOPD can be summarized as:

- Based on the established fuzzy rules, the outputs of the adjusting values are obtained;
- The actual real-time control parameters are calculated by the Equation (24);
- The discrete FOPD controller and the fuzzy single step optimization control force $U(k)$ at the k -th time instant are calculated by Equation (22).

Table 4. Fuzzy control rule of $(\Delta K_p, r_1)/(\Delta K_d, r_2)$.

ec							
e	NB	NM	NS	Z	PS	PM	PB
NB	PB/PS	PB/NS	PM/NB	PM/NB	PS/NB	Z/NM	Z/PS
NM	PB/PS	PB/NS	PM/NB	PS/NM	PS/NM	Z/NS	NS/Z
NS	PM/Z	PM/PS	PM/PM	PS/PM	Z/PS	NS/PS	NS/Z
Z	PM/Z	PM/NS	PS/NS	Z/NS	NS/NS	NM/NS	NM/Z
PS	PS/Z	PS/Z	Z/Z	NS/Z	NS/Z	NM/Z	NM/Z
PM	PS/PB	Z/PS	NS/PS	NM/PS	NM/PS	NM/PS	NB/PB
PB	Z/PB	Z/PM	NM/PM	NM/PM	NM/PS	NB/PS	NB/PB

Table 5. Fuzzy control rule of $\Delta \mu$.

ec							
e	NB	NM	NS	Z	PS	PM	PB
NB	NS	PS	PB	PB	PB	PM	NS
NM	PS	PS	PB	PM	PM	PS	Z
NS	Z	NS	NM	NM	NS	NS	Z
Z	Z	PS	PS	PS	PS	PS	Z
PS	Z	Z	Z	Z	Z	Z	Z
PM	NB	NS	NS	NS	NS	NS	NM
PB	NB	NM	NM	NM	NS	NS	NB

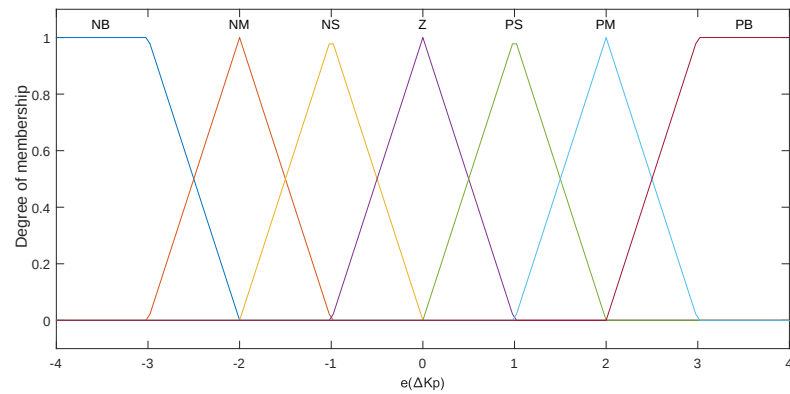


Figure 8. Membership function curve of various e and ΔK_p .

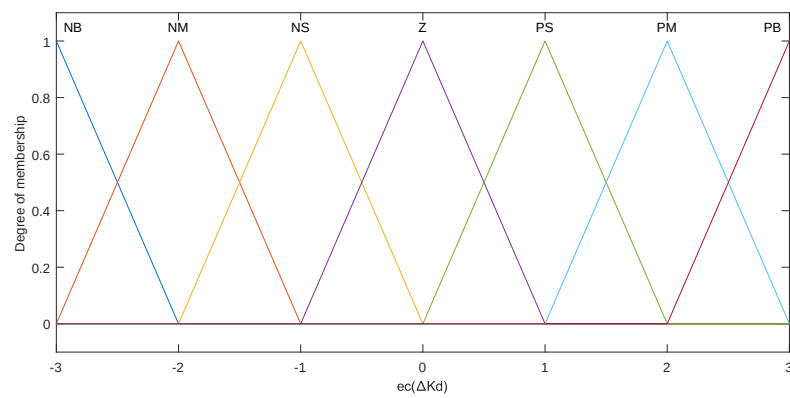


Figure 9. Membership function curve of various ec and ΔK_d .

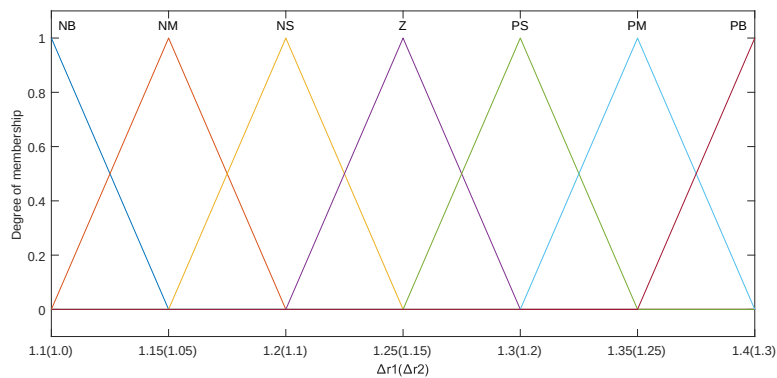


Figure 10. Membership function curve of various Δr_1 and Δr_2 .

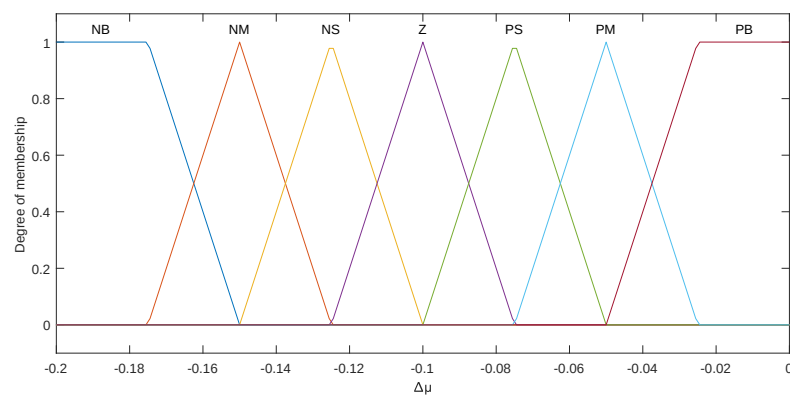


Figure 11. Membership function curve of various $\Delta \mu$.

4. Simulation Results and Analysis

In this section, a three-storey building structure with actuators on each story is considered, as illustrated in Figure 12. The parameters of the uncertain building structure are given in Table 2.

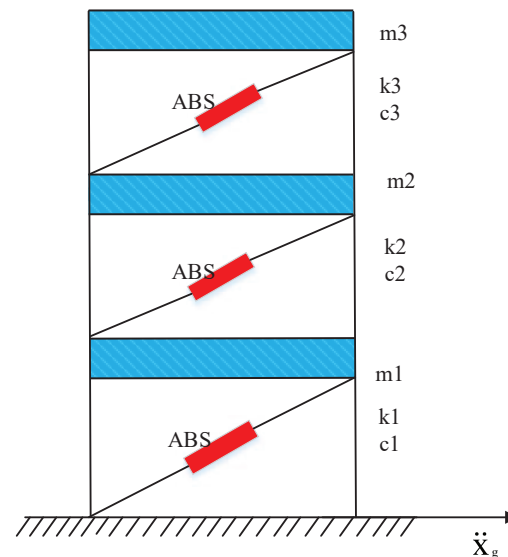


Figure 12. A three-storey building structure with actuators.

The ground acceleration $\ddot{x}_g$ is assumed as the seismic records of the 1940 El-Centro and 1952 Taft earthquakes, with wave modulated with 0.4 g and 0.2 g amplitude for 10 s separately. The measured signal is applied to the control algorithm to generate a control signal for the control device. The displacement of the top level floor is used as the real time measured feedback. The duration of the control of the system overlaps with the period of excitation of the structure. The seismic wave is used as the excitation signal for the cases of uncontrolled system and PD, FOPD, and FFOPD control. The IFOPSO is used to optimize the controller initial parameters. The IFOPSO algorithm parameters used in this paper and the values of the initial parameters obtained through optimization are listed in Table 6.

Table 6. Control system parameters.

Name of the IFOPSO Parameters	Value of the Parameters
Particle number	$N = 50$
Number of iterations/Repeated experiments	$T = 200/E = 40$
Scaling factors	$c_1 = c_2 = 2, v_{min} = -1, v_{max} = 1, \omega_i = \sim$
Range	$[K_p, K_d, \mu] = [0 \sim 20, 0 \sim 5, 0 \sim 2]$
Name of the Control System	Control System Parameter Value
PD	$[K_p, K_d] = [0.02, 3.75]$
FOPD	$[K_p, K_d, \mu] = [19.48, 4.93, 1.63]$
FFOPD	$[K_p, K_d, \mu, r_1, r_2] = [\sim, \sim, \sim, \sim, \sim]$

Note: the symbol $\sim$ means that the value changes with time.

With different control cases, namely uncontrolled, PD, FOPD, and FFOPD control, the top floor displacement of the three-storey building under the seismic El-Centro and Taft earthquakes are shown in Figures 13 and 14, respectively. The top displacement curve using the FOPD and FFOPD controller under the El-Centro and Taft earthquakes are depicted in Figures 15 and 16, respectively. The maximum displacement of each floor under the El-Centro earthquake and Taft earthquake are shown in Figures 17 and 18, respectively.

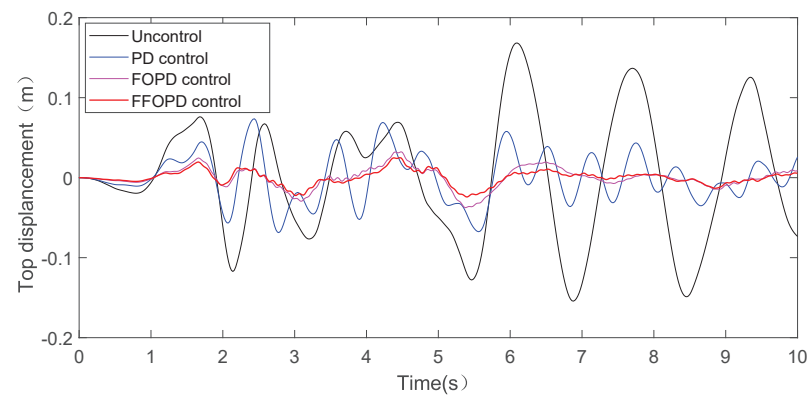


Figure 13. Top floor displacement curve under El-Centro earthquake.

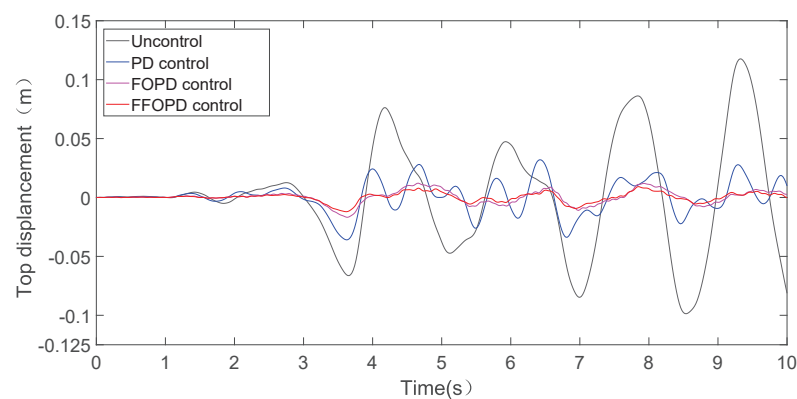


Figure 14. Top floor displacement curve under Taft earthquake.

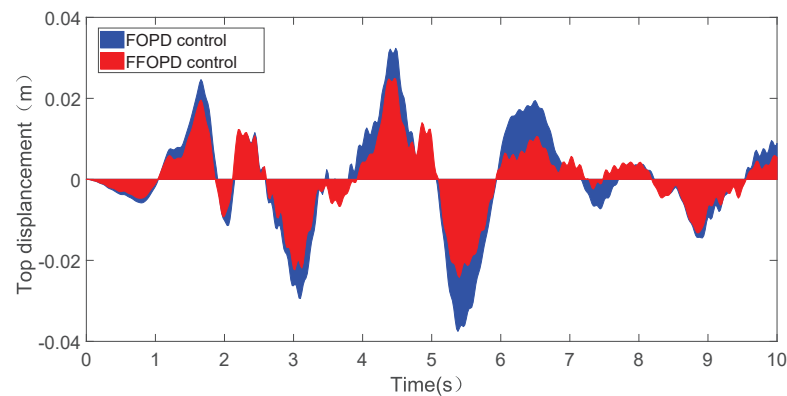


Figure 15. Top displacement curve for FOPD and FFOPD under El-Centro earthquake.

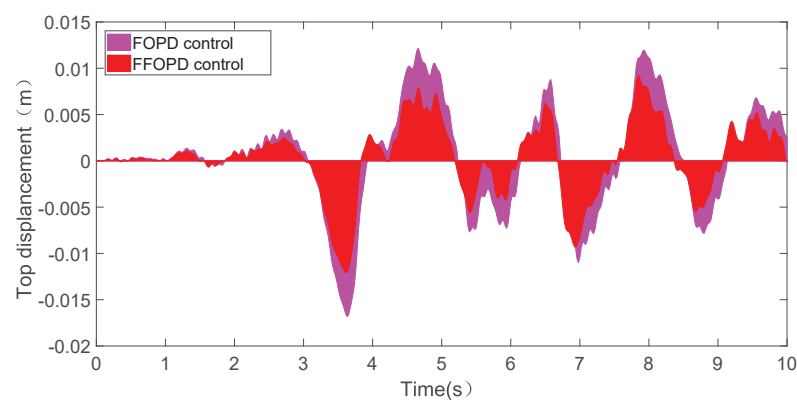


Figure 16. Top displacement curve for FOPD and FFOPD under Taft earthquake.

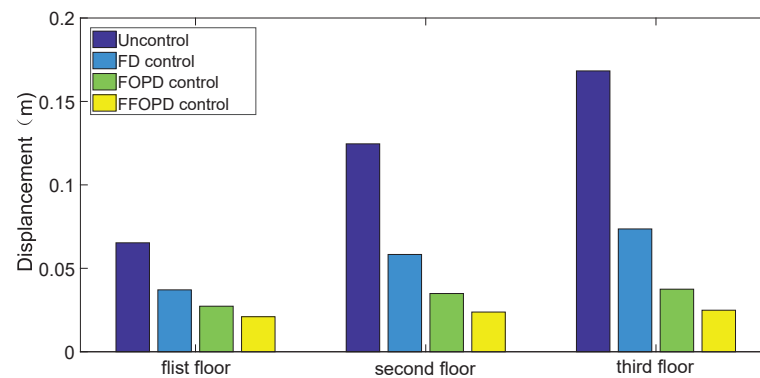


Figure 17. Maximum displacement of each floor under El-Centro earthquake.

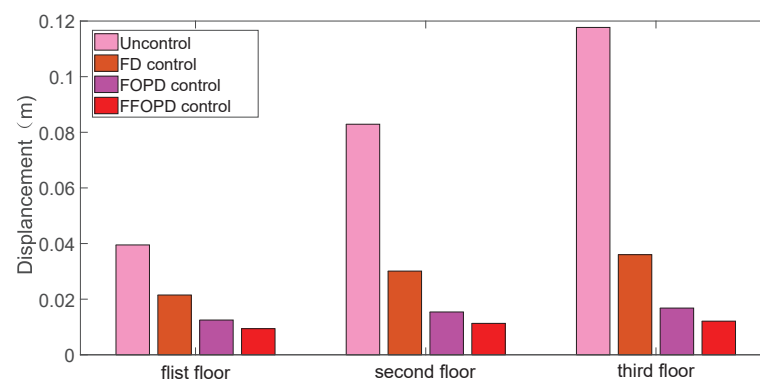


Figure 18. Maximum displacement of each floor under Taft earthquake.

From Figures 13 and 14, we verify that the FOPD and FFOPD yield excellent performance under the two different seismic waves. Compared with the PD, the FOPD controller has one more adjustable parameter, namely the fractional derivative order, μ , that can be selected arbitrarily. This widens the setting range of the controller parameters and leads to greater flexibility and better performance.

Indeed, from Figures 15–18, the top floor maximum displacements obtained with the FOPD are 0.0375 m and 0.0168 m, and the average displacements are 0.0098 m and 0.0042 m. For the FFOPD, the maximum displacements are 0.0249 m and 0.0121 m, and the average displacements are 0.0069 m and 0.0028 m. The top floor maximum displacements for the FFOPD, when compared to the FOPD, are reduced by 50.6% and 38.8%, while the averaged displacements are reduced by 42.1% and 50%, respectively. It is obvious that the performance of FFOPD is superior to that of the FOPD.

To show the robustness of the FFOPD controller, some comparative performance tests are carried out. When the uncertain parameter δ_t is selected as $-1, 1$, the displacement responses of the third floor for the uncertain building structure are depicted in Figures 19 and 20, for the El-Centro and Taft earthquake, respectively. We verify that the maximum displacement changed little, which is almost the same as what occurred in the deterministic parameters case. Figure 21 depicts the maximum displacements on the top floor, confirming the observations. Therefore, we can conclude that the FFOPD has superior performance and good robustness to uncertain structures.

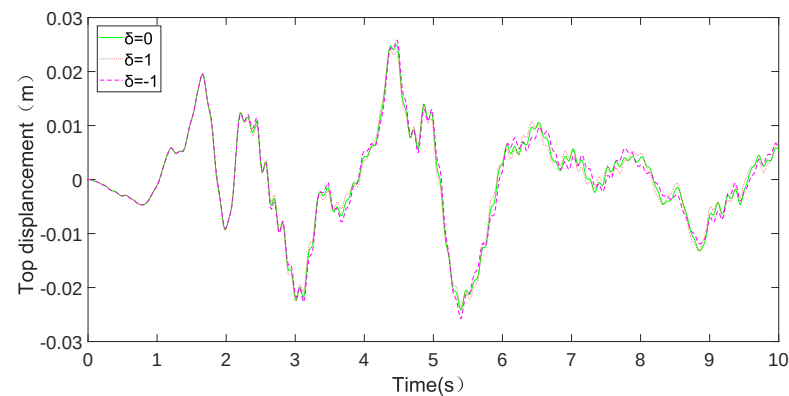


Figure 19. Top floor displacement curve of the uncertain structure under the El-Centro wave.

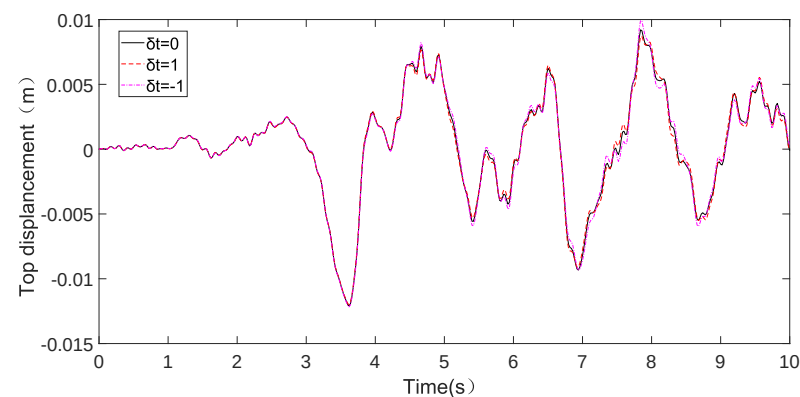


Figure 20. Top floor displacement curve of the uncertain structure under the Taft wave.

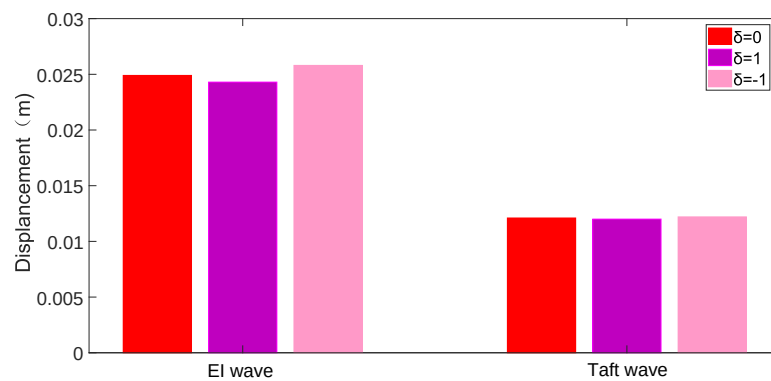


Figure 21. Maximum displacement from top floor of uncertain structure.

5. Conclusions

A new vibration control strategy for uncertain building structures was proposed in this paper. Firstly, a new IFOPSO algorithm was introduced for the optimization of the initial parameters of a FOPD controller. Then, a novel FFOPD controller was designed to adapt the FOPD controller parameters on-line, and to ensure that they are always optimal. Finally, some simulations were carried out for the uncertain system and unknown seismic waves. The results proved that the proposed controller can obtain excellent robustness and superior effectiveness for structural vibration control.

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