fractal and fractional

Editorial

Special Issue: Fractal Functions and Applications

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Citation: Navascués, M.A.; Sebastián, M.V. Special Issue: Fractal Functions and Applications. *Fractal Fract.* **2022**, *6*, 411. https://doi.org/10.3390/fractalfract6080411

Received: 21 July 2022
Accepted: 25 July 2022
Published: 26 July 2022

This volume gathers some important advances in the fields of fractional calculus and fractal curves and functions. Fractional derivatives and integrals play an increasingly important role in applied science, and these types of models are ubiquitous in the current scientific literature. The references [1,2] are devoted to fractional calculus and an application of it to a coronavirus spreading model. The first one studies three procedures of inverse Laplace Transforms: A Sinc–Thiele approximation, a Sinc and a Sinc–Gaussian (SG) method. Both Sinc versions are exact methods of inverse Laplace Transforms. The author proves that SG-based transformations present some advantages over the pure Sinc version regarding stability and convergence properties. The convergence is of exponential type. All the methods presented are applied to Mittag-Leffler functions depending on one, two and three parameters, and the author proves that the representation of this kind of functions is very effective. The author concludes that even for variable-order fractional differential or integral equations, the Sinc–Gaussian method is a powerful procedure.

In the reference [2], the authors consider a fractional-order epidemiological model incorporating a spread of coronavirus. They consider different classes of individuals: exposed and both symptomatic and asymptomatic infected. The population evolution system is formulated by means of fractional operators (derivatives). Basic analytical properties and equilibria of the dynamical system are studied and the authors present a numerical simulation in the last section. The model contains an endemic equilibrium, where the disease persists, a disease-free point with only susceptible individuals, and an intermediate state with asymptomatic individuals, but still endemic. The results obtained analytically basically coincide with those obtained by the model using standard derivatives. However, it is observed that the order of the fractional derivative influences the final stage of the populations.

This Special Issue also collects some new insights in the theory of Iterated Function Systems (IFS) and fractal interpolation, that are attracting increasing attention in the scientific and technical community. We summarize the contributions hereafter.

Classical Iterated Function Systems are composed of a set of Banach contractions giving rise to a fractal attractor in a metric space E. In the reference [3], the authors extend this concept in different ways. They consider θ-contractions instead of contractions, considering a broader family of maps. Then they consider generalized θ-contractions, defined on the product E^m, instead of the basic metric space E, thus constructing generalized IFS. Additionally, in the last part of the paper, they consider an infinite collection of maps and multivalued mappings $w_n : E \to \mathcal{K}(E)$, where $\mathcal{K}(E)$ is the Hausdorff space of compact subsets of E. The authors prove that under certain conditions, these IFSs own an attractor.

The Koch curve was proposed by Helge von Koch, who was a disciple of the distinguished professor G. Mittag-Leffler. This curve can be modeled by means of a fractal interpolation function. In the reference [4], the authors go a step further and define an Iterated Function System (IFS) on the Koch curve. They prove that the system owns an attractor, which can be seen as a curve whose domain is the Koch curve. In this way, they define a fractal function whose domain is also fractal.

The article corresponding to the reference [5] is based on the notion of α-fractal function. This is a special type of fractal interpolation function that owns the peculiarity of being a fractal perturbation of any continuous or integrable map. That is to say, near a standard map, we can find a fractal function whose properties can be modified by means of an appropriate Iterated Function System. Let us say that the characteristics of the new function can be customized. These fractal perturbations of a map compose a family that extends and includes the original. The operator mapping the original function in its fractal version can be defined to be linear and bounded. In the paper [5], this operator is used to construct spanning families of functions for the space of square integrable functions on a rectangle. In this way, the authors define Bessel, Riesz and Schauder sequences, frames and bases for maps defined on a rectangle. These spanning systems contain the classical ones as particular cases. This is done in this case via tensor product of maps and operators.

In the reference [6], the authors deepen the concept of multivariate α-fractal function. They define this type of map in several complete functional spaces like Lebesgue $\mathcal{L}^p$, Sobolev $\mathcal{W}^{m,p}$ and Hölder $\mathcal{C}^{m,\sigma}$ spaces, that appear in a large number of problems of applied mathematics. They generalize some properties of the fractal operator quoted above associated to these mappings. They deal also with bounds for the Hausdorff dimension of the graph of the multivariate α-fractal functions, obtained as a consequence of the Hölder continuity of the same, under specific conditions on the parameters. In a second part of the paper, they prove that the Riemann–Liouville fractional integral of the considered multivariate fractal functions is also a fractal function of the same type, assuming some prescribed hypotheses.

Funding: This research received no external funding.

Conflicts of Interest: The authors declare no conflict of interest.

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