*fractal and fractional*

*Editorial*

# Fractional Order Systems and Their Applications

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Fractional calculus (FC) generalizes the concepts of derivative and integral to non-integer orders. It was introduced by Leibniz (1646–1716), but remained a purely mathematical exercise for a long time, despite the original contributions of important mathematicians, physicists, and engineers. FC experienced rapid development during the last few decades both in mathematics and applied sciences, being recognized as an excellent tool to describe complex dynamics. From this perspective, several models governing physical phenomena in the area of science and engineering have been reformulated in light of FC for better reflecting their non-local, frequency- and history-dependent properties. Applications of FC include modeling of diffusion, viscoelasticity, and relaxation processes in fluid mechanics, dynamics of mechanical, electronic and biological systems, signal processing, control, and others.

The Special Issue "Fractional Order Systems and Their Applications" focuses on original and new research results on modeling and control of fractional order systems with applications in science and engineering. It includes 13 manuscripts addressing novel issues and specific topics that illustrate the richness and applicability of fractional calculus. In the follow-up the selected manuscripts are presented in alphabetic order of their titles.

In the paper "A modified Leslie-Gower model incorporating Beddington-DeAngelis functional response, double allee effect and memory effect" [1] the authors propose a modified Leslie–Gower predator-prey model with Beddington–DeAngelis functional response and double Allee effect in the growth rate of a predator population. To consider memory effects the Caputo fractional-order derivative is used. The dynamic behavior of the model for both strong and weak Allee effect is investigated.

The manuscript "A study of coupled systems of $\psi$-Hilfer type sequential fractional differential equations with integro-multipoint boundary conditions" [2] investigates the existence and uniqueness of solutions for a coupled system of $\psi$-Hilfer type sequential fractional differential equations supplemented with nonlocal integro-multi-point boundary conditions. The results are obtained via the classical Banach and Krasnosel'skii's fixed point theorems and the Leray–Schauder alternative.

In "Asymptotic stabilization of delayed linear fractional-order systems subject to state and control constraints" [3] the asymptotic stabilization of delayed linear fractional-order systems (DLFS) subject to state and control constraints is investigated. The existence conditions for feedback controllers of DLFS subject to both state and control constraints are given. A sufficient condition for invariance of polyhedron set is established by using invariant set theory. A new Lyapunov function is constructed on the basis of the constraints, and some sufficient conditions for the asymptotic stability of DLFS are obtained. Feedback controller and the corresponding solution algorithms are also given.

In their work "Chaos control for a fractional-order jerk system via time delay feedback controller and mixed controller" [4] the authors propose a novel fractional-order jerk system. They show that, under some suitable parameters, the fractional-order jerk system displays a chaotic phenomenon. To suppress the chaotic behavior two control strategies are proposed: a time delay feedback controller; and a mixed controller, which includes a time delay

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feedback controller and a fractional-order PD$^{\sigma}$ controller. A sufficient condition ensuring the stability and the creation of Hopf bifurcation for the fractional-order controlled jerk system is derived.

The paper "Control and robust stabilization at unstable equilibrium by fractional controller for magnetic levitation systems" [5] study a method to control and stabilize a levitation system in the presence of disturbance and parameter variations. The stabilization and disturbance rejection are achieved by fractional order PID, fractional order sliding mode, and fractional order Fuzzy control approaches. To design the controllers a tuning hybrid method based on GWO–PSO algorithms is applied with different performance criteria.

In the work "Controllability for fuzzy fractional evolution equations in credibility space" [6] the authors address the exact controllability for Caputo fuzzy fractional evolution equations in the credibility space from the perspective of the Liu process. As a result, the study's theoretical result can be used to create stochastic extensions in credibility space.

In "Dynamics of fractional-order digital manufacturing supply chain system and its control and synchronization" [7] a fractional-order digital manufacturing supply chain system is proposed and solved by the Adomian decomposition method. Dynamical characteristics of the system are studied by using phase portrait, bifurcation diagram, and maximum Lyapunov exponent diagram. The complexity of the system is also investigated by means of complexity measures. The importance of the fractional-order derivative in the modeling of the system is shown. Feedback controllers to control the chaotic supply chain system and synchronize two supply chain systems are proposed.

The manuscript "Existence, uniqueness, and $E_q$-Ulam-type stability of fuzzy fractional differential equation" [8] concerns with the existence and uniqueness of the Cauchy problem for a system of fuzzy fractional differential equation with Caputo derivative of order $q \in (1, 2]$. By using direct analytic methods, the $E_q$–Ulam-type results are also presented.

In "Fractals Parrondo's paradox in alternated superior complex system" [9] a kind of fractals Parrondo's paradoxial phenomenon "deiconnected + diconnected = connected" in an alternated superior complex system $z_{n+1} = \beta(z_2^n + c_i) + (1 - \beta)z_n$, $i = 1, 2$ is addressed. The connectivity variation in superior Julia sets is explored by analyzing the connectivity loci. The position relation between the superior Mandelbrot set and the connectivity loci is graphically investigated. Moreover, graphical examples obtained by the use of the escape-time algorithm and the derived criteria are presented.

In their paper "Fractional integral inequalities for exponentially nonconvex functions and their applications" [10] the authors define a new generic class of functions involving a certain modified Fox–Wright function. A useful identity using fractional integrals and the modified Fox–Wright function with two parameters is found. Some Hermite–Hadamard-type integral inequalities are established.

In the paper "Guaranteed cost leaderless consensus protocol design for fractional-order uncertain multi-agent systems with state and input delays" [11] addresses the guaranteed cost leaderless consensus of delayed fractional-order multi-agent systems (FOMASs) with nonlinearities and uncertainties. A guaranteed cost function for FOMAS is proposed to simultaneously consider consensus performance and energy consumption. By employing the linear matrix inequality approach and the fractional-order Razumikhin theorem, a delay-dependent and order-dependent consensus protocol is formulated for FOMASs with input delay.

In "Jacobi spectral collocation technique for time-fractional inverse heat equations" [12] a numerical solution for time-fractional inverse heat equations is proposed. The authors focus on obtaining the unknown source term along with the unknown temperature function based on an additional condition given in an integral form. The proposed scheme is based on a spectral collocation approach to obtain the two independent variables.

The manuscript "State of charge estimation of lithium-ion batteries based on fuzzy fractional-order unscented Kalman filter" [13] proposes a method to estimate the state of charge of lithium-ion batteries. The algorithm combines fuzzy inference with fractional-order unscented Kalman filter to infer the measurement noise in real time and take advantage of fractional calculus in describing the dynamic behavior of the lithium batteries.

To sum up, the guest editors hope that the selected papers will help scholars and researchers to push forward the progress in fractional calculus and its applications, namely in modeling and control of nonlinear and complex systems.

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