



Article

The Passivity of Uncertain Fractional-Order Neural Networks with Time-Varying Delays

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Abstract: In this paper, we study the passive problem of uncertain fractional-order neural networks (UFONNs) with time-varying delays. First, we give a sufficient condition for the asymptotic stability of UFONNs with bounded time-varying delays by using the fractional-order Razumikhin theorem. Secondly, according to the above stability criteria and some properties of fractional-order calculus, a delay-dependent condition that can guarantee the passivity of UFONNs with time-varying delays is given in the form of a linear matrix inequality (LMI) that can be reasonably solved in polynomial time using the LMI Control Toolbox. These conditions are not only delay-dependent but also order-dependent, and less conservative than some existing work. Finally, the rationality of the research results is proved by simulation.

Keywords: passivity analysis; time-varying delays; uncertain fractional-order neural network; asymptotically stable

MSC: 47H09; 34A08



Citation: Xu, S.; Liu, H.; Han, Z.

The Passivity of Uncertain Fractional-Order Neural Networks with Time-Varying Delays. *Fractal Fract.* **2022**, *6*, 375. <https://doi.org/10.3390/fractalfract6070375>

Academic Editor: Gani Stamov

Received: 25 May 2022

Accepted: 28 June 2022

Published: 2 July 2022

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1. Introduction

As neural networks (NNs) play an important role in the field of control engineering and are used for applications such as association [1,2], pattern recognition [3,4], and signal processing [5,6], fractional-order neural networks (FONNs) are becoming more and more popular among scientists. Moreover, since two important properties of fractional-order calculus are memorability and heritability, FONNs can more precisely describe the dynamic behavior of neurons in NNs. Therefore, many problems related to FONNs have been studied, such as finite-time stability [7–9], asymptotic stability [10–13], synchronization analysis [14–17], and guaranteed cost control [18,19].

Passivity theory has been a major concern in the field of control engineering since the 1970s (see references [20–24]). As an effective tool for studying system stability, passivity theory has been applied to a variety of areas, for example, fuzzy control, complexity, power systems, and signal processing. Some researchers have also extended passivity theory to NNs. The passive filtering of NNs with time-varying mixed delays using the Wirtinger integral inequality was studied in [25]. Other research on the passivity of dynamic NNs can be found in the literature [26–30]. Although there are many research results on the passivity of NNs, the research on the passivity of NNs with time-varying delays has not been fully explored. At present, the passivity of integer-order NNs with time-varying delays has been studied using different methods. For example, in [31], Lyapunov–Krasovskii functions were used to analyze the stability and passivity of NNs with time-varying delays. The passivity of integer-order NNs with discrete and distributed time-varying delays was studied in [32]. By means of stochastic analysis and linear matrix inequality (LMI)

techniques, event-triggered passive synchronization for NNs with random gain changes was addressed in [33]. More research on the passivity of neural networks with time delays can be found in [34–36]. In the above-mentioned research results, we find that the methods used to study passivity mainly used LMI techniques and Lyapunov–Krasovskii functions. The passive analysis of an integer-order system can be simplified by taking the integer derivative of a Lyapunov–Krasovskii function. However, constructing fractional-order Lyapunov–Krasovskii functions for FONNs is challenging due to the limitation of integral intervals. In other words, the method mentioned above cannot be extended to the study of the passivity of FONNs with time-varying delays. The question of how to solve this problem is interesting and challenging.

It is well known that fractional derivatives have weak singular kernels and are nonlocal. Therefore, compared with integer-order NNs, the passive research of FONNs is more challenging. Up to now, there have only been very few works that considered passivity [37,38]. In [37], the authors studied the passive problem of FONNs by means of an LMI technique and control theory; only the order-independent case was studied, and time delays were not considered. However, time delays are very common in all kinds of systems, and their existence can lead to system instability and transient responses. When the delay is small, the delay-independent stability condition is less conservative. Very recently, the passivity of FONNs with time-varying delays was studied using an LMI technique and control theory in [38], where system uncertainties were not considered. However, the connection weight of a neuron depends on its capacitance and resistance values, including uncertainties, so it is very important to study the passivity of uncertain fractional-order neural networks (UFONNs).

Based on the above analysis, in this article, our main task is to find a relatively simple order-dependent and delay-dependent condition for the passive problem of UFONNs with time-varying delays. The innovations of this article are as follows. (1) A relatively small conservative order-dependent and delay-dependent stability criterion for UFONNs with time-varying delays is presented using LMI techniques and the fractional-order Razumikhin theorem. (2) Based on fractional-order calculus and existing stability criteria, we solve the passive problem of UFONNs with time-varying delays. (3) Two sufficient conditions for UFONNs with passive time-varying delays are shown using an LMI. Furthermore, these conditions can be reasonably solved.

The rest of this article is divided into the following parts. In Section 2, we detail some assumptions, definitions, and lemmas and describe the research object of this article. In Section 3, the passive study of UFONNs with time-varying delays is presented. A simulation example is used to prove the rationality of the theoretical results in Section 4.

Notations: In this article, \mathbb{R}^n is defined as an n -dimensional Euclidean space, and I is an appropriately dimensional identity matrix. $\mathbb{R}^{m \times n}$ is defined as the set of $m \times n$ matrices, N^T stands for the transpose of the matrix N , and $\text{diag}\{\dots\}$ represents the block-diagonal matrix. $C^1[0, T]$ is defined as the whole function on the interval $[0, T]$, in which the first-order derivative of any function exists and is continuous. $L_2[0, \infty)$ is a space of all 2-integrable functions on $[0, \infty)$. $\zeta_t = \{\zeta(t + \omega) : \omega \in [-\zeta_0, 0]\}$ is a segment of trajectory $\zeta(t)$. For convenience, we use $*$ to replace the symmetric term in the symmetric matrix.

2. Preliminaries

The β -th Caputo fractional-order integral is given as:

$$I_t^\beta f(t) = \frac{1}{\Gamma(\beta)} \int_0^t (t - \varepsilon)^{\beta-1} f(\varepsilon) d\varepsilon, \quad (1)$$

where $\Gamma(\beta) = \int_0^{+\infty} t^{\beta-1} e^{-t} dt$.

The β -th Caputo fractional differential is given as:

$$D_t^\beta f(t) = \frac{1}{\Gamma(n - \beta)} \int_0^t (t - \varepsilon)^{n-\beta-1} f^{(n)}(\varepsilon) d\varepsilon, \quad (2)$$

where $n - 1 \leq \beta < n$. In addition, for the sake of convenience, we only consider $0 < \beta \leq 1$.

That yields the following UFONNs:

$$\begin{cases} D_t^\beta \boldsymbol{\eta}(t) = -(\mathbf{B}_1 + \Delta\mathbf{B}_1(t))\boldsymbol{\eta}(t) + (\mathbf{B}_2 + \Delta\mathbf{B}_2(t))\mathbf{g}(\boldsymbol{\eta}(t)) \\ \quad + (\mathbf{B}_3 + \Delta\mathbf{B}_3(t))\mathbf{g}(\boldsymbol{\eta}(t - \zeta(t))) + \mathbf{u}(t), \\ \zeta(t) = \mathbf{G}\boldsymbol{\eta}(t), \\ \boldsymbol{\eta}(t) = \boldsymbol{\varphi}(t), t \in [-\zeta_0, 0], \end{cases} \tag{3}$$

where $\boldsymbol{\eta}(t) = [\eta_1(t), \dots, \eta_n(t)]^T \in \mathbb{R}^n$ is the neuron state vector, $\mathbf{B}_1 = \text{diag}(b_{11}, b_{22}, \dots, b_{nn})$ is a positive diagonal matrix, $\mathbf{B}_2, \mathbf{B}_3 \in \mathbb{R}^{n \times n}$ are interconnection weight matrices, $0 \leq \zeta(t) \leq \zeta_0$ is the time delay, and $\mathbf{G} \in \mathbb{R}^{n \times n}$ is a known constant matrix. $\mathbf{u}(t)$ is the input vector; $\Delta\mathbf{B}_1(t), \Delta\mathbf{B}_2(t)$, and $\Delta\mathbf{B}_3(t)$ are unknown matrices representing the uncertainties of time-varying parameters; $\mathbf{g}(\boldsymbol{\eta}(t)) = [g_1(\eta_1(t)), \dots, g_n(\eta_n(t))]^T$ represents the neural activation function; and $\boldsymbol{\zeta}(t)$ is output vector. $\boldsymbol{\varphi}(t)$ is the initial vector value function.

Remark 1. In engineering applications and real-world systems, parameters often fluctuate within certain ranges, which can lead to divergence or instability. Thus, it makes sense to introduce parameter uncertainty into fractional-order neural networks; doing so can reveal more realistic dynamic properties of fractional-order neural networks. The passive theory can maintain the internal stability of the system, which plays an important role in network control theory. It also has a wide range of applications in robotic systems, electromechanical systems, power systems, internal combustion engine engineering, and chemical processes. Therefore, studying the passivity of uncertain fractional-order neural networks provides an important theoretical basis for engineering applications.

The following are the relevant lemmas, definitions, and assumptions.

Assumption 1. The activation function $g_i(\cdot)$ satisfies for any $d, e \in \mathbb{R}, d \neq e$:

$$\kappa_i^-(d - e) \leq g_i(d) - g_i(e) \leq \kappa_i^+(d - e), \quad g_i(0) = 0 (i = 1, \dots, n), \tag{4}$$

where κ_i^+, κ_i^- are known constants.

Remark 2. Note that κ_i^- and κ_i^+ can be any real number, and that is a much broader set of conditions than the Lipschitz-type activation functions.

Assumption 2. $\Delta\mathbf{B}_1(t), \Delta\mathbf{B}_2(t), \Delta\mathbf{B}_3(t)$ are defined by:

$$\Delta\mathbf{B}_1(t) = \mathbf{H}\mathbf{F}(t)\mathbf{E}, \Delta\mathbf{B}_2(t) = \mathbf{H}_1\mathbf{F}_1(t)\mathbf{E}_1, \Delta\mathbf{B}_3(t) = \mathbf{H}_2\mathbf{F}_2(t)\mathbf{E}_2, \tag{5}$$

where $\mathbf{F}(t), \mathbf{F}_1(t)$, and $\mathbf{F}_2(t)$ are unknown time-varying matrices, and $\mathbf{H}, \mathbf{E}, \mathbf{H}_1, \mathbf{E}_1, \mathbf{H}_2$, and \mathbf{E}_2 are known constant matrices with appropriate dimensions and satisfy:

$$\mathbf{F}^T(t)\mathbf{F}(t) \leq \mathbf{I}, \mathbf{F}_1^T(t)\mathbf{F}_1(t) \leq \mathbf{I}, \mathbf{F}_2^T(t)\mathbf{F}_2(t) \leq \mathbf{I}. \tag{6}$$

Lemma 1 ([39]). If $g(t) \in C^1[0, T]$ for some $T > 0$, then for $0 < \beta \leq 1$, we have:

$$I_t^\beta D_t^\beta g(t) = g(t) - g(0), \tag{7}$$

and:

$$D_t^\beta I_t^\beta g(t) = g(t). \tag{8}$$

Lemma 2 ([40]). If $g(t)$ is a continuous function, then:

$$I_t^{\beta_1} (I_t^{\beta_2} g(t)) = I_t^{\beta_1 + \beta_2} (g(t)), \forall t \geq t_0. \tag{9}$$

Lemma 3 ([40]). *Caputo’s fractional-order calculus satisfies:*

$$D_t^\beta(\lambda h_1(t) + \mu h_2(t)) = \lambda D_t^\beta h_1(t) + \mu D_t^\beta h_2(t), \tag{10}$$

where $\lambda, \mu \in \mathbb{R}$.

Lemma 4 ([39]). *If function $\eta(t) \in \mathbb{R}^n$ is differentiable, then there is a symmetric positive definite matrix $P \in \mathbb{R}^{n \times n}$ such that:*

$$D_t^\beta(\eta^T(t)P\eta(t)) \leq 2\eta^T(t)PD_t^\beta\eta(t), \forall t \geq t_0. \tag{11}$$

Lemma 5 ([41]). *If there are three positive constants k_1, k_2 , and k_3 and a quadratic Lyapunov function $V(t, \eta(t))$ that satisfies:*

- (1) $k_1 \|\eta(t)\|^2 \leq V(t, \eta(t)) \leq k_2 \|\eta(t)\|^2, \eta \in \mathbb{R}^n,$
- (2) $D_t^\beta V(t, \eta(t)) \leq -k_3 \|\eta(t)\|^2$ whenever $V(t + \omega, \eta(t + \omega)) < \rho V(t, \eta(t))$, and $\forall \omega \in [-s, 0]$ for some $\rho > 1$, then the zero solution of system $D_t^\beta \eta(t) = g(t, \eta_t)$ is asymptotically stable.

Definition 1 ([42]). *System (3) is passive if it satisfies the following conditions:*

- (1) *System (3) is asymptotically stable when both the output vector and the external input vector of the system are zero.*
- (2) *When the initial condition is $\varphi(t) = \mathbf{0}$, and $\forall t \in [-\zeta_0, 0]$ for System (3), then there exists a constant $\rho > 0$ such that:*

$$2 \int_0^{t_p} \zeta^T(t)u(t)dt \geq -\rho \int_0^{t_p} u^T(t)u(t)dt, \forall t_p \geq 0, \tag{12}$$

where $u(t) \in L_2[0, \infty)$ is external input of the system, and $\zeta(t)$ is the output of the system.

Remark 3. *In fact, there is a connection between passivity and stability. If the system is passive, then we find that the system is asymptotically stable with zero output vectors and zero external input vectors, as can be seen in [30]. In addition, our Definition 1 is equivalent to an expansion of the definition in [42]. If the system satisfies both (1) and (2) in Definition 1, then the system must be passive. $\zeta(t)$ and $u(t)$ in Definition 1 are consistent with those given by System (3).*

3. Main Results

3.1. Stability Study of the System

In this part, the stability of System (3) with $\zeta(t) = \mathbf{0}$ and $u(t) = \mathbf{0}$ is analyzed. First, let us emphasize that for the sake of our calculations, in Theorems 1 and 2, the following substitutions are made: ζ to represent $\zeta(t)$, u to represent $u(t)$, $\eta = [\eta_1, \dots, \eta_n]^T$ to represent $\eta(t) = [\eta_1(t), \dots, \eta_n(t)]^T$, $g(\eta) = [g_1(\eta_1), \dots, g_n(\eta_n)]^T$ means $g(\eta(t)) = [g_1(\eta_1(t)), \dots, g_n(\eta_n(t))]^T$, and $g(\eta(\zeta)) = [g_1(\eta_1(\zeta)), \dots, g_n(\eta_n(\zeta))]^T$ represents $g(\eta(t - \zeta(t))) = [g_1(\eta_1(t - \zeta(t))), \dots, g_n(\eta_n(t - \zeta(t)))]^T$.

Theorem 1. *Assuming that Assumptions 1 and 2 are true, System (3) with $\zeta = \mathbf{0}$ and $u = \mathbf{0}$ is asymptotically stable if there exist symmetric positive definite matrices P and $Q \in \mathbb{R}^{n \times n}$, a symmetric semi-positive definite matrix $W \in \mathbb{R}^{8n \times 8n}$, and three positive diagonal matrices $\Theta_j = \text{diag}\{\mu_{j1}, \mu_{j2}, \dots, \mu_{jn}\} \in \mathbb{R}^{n \times n} (j = 1, 2, 3)$ such that:*

$$\Phi = \zeta_0^\beta \beta^{-1} W + \Lambda < \mathbf{0}, \tag{13}$$

where:

$$\Lambda = \begin{bmatrix} \Lambda_{11} & \Lambda_{12} & \Lambda_{13} & \Lambda_{14} & -QH & QH_1 & QH_2 & \Lambda_{18} \\ * & \Lambda_{22} & -\Xi_2\Theta_3 - \Xi_1\Theta_3 & \Lambda_{24} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & * & \Lambda_{33} & 2\Theta_3 & \mathbf{0} & \mathbf{0} & \mathbf{0} & B_2^T Q \\ * & * & * & \Lambda_{44} & \mathbf{0} & \mathbf{0} & \mathbf{0} & B_3^T Q \\ * & * & * & * & -mI & \mathbf{0} & \mathbf{0} & -H^T Q \\ * & * & * & * & * & -m_1 I & \mathbf{0} & H_1^T Q \\ * & * & * & * & * & * & -m_2 I & H_2^T Q \\ * & * & * & * & * & * & * & -2Q \end{bmatrix},$$

$$\Lambda_{11} = -QB_1 - B_1^T Q - 2\Xi_1\Theta_1\Xi_2 - 2\Xi_1\Theta_3\Xi_2 + mE^T E + P,$$

$$\Lambda_{12} = \Xi_1\Theta_3\Xi_2 + \Xi_2\Theta_3\Xi_1,$$

$$\Lambda_{13} = QB_2 + \Xi_2\Theta_1 + \Xi_1\Theta_1 + \Xi_2\Theta_3 + \Xi_1\Theta_3,$$

$$\Lambda_{14} = QB_3 - \Xi_2\Theta_3 - \Xi_1\Theta_3,$$

$$\Lambda_{18} = P - Q - B_1^T Q,$$

$$\Lambda_{22} = -2\Xi_1\Theta_2\Xi_2 - 2\Xi_1\Theta_3\Xi_2 - P,$$

$$\Lambda_{24} = \Xi_2\Theta_2 + \Xi_1\Theta_2 + \Xi_2\Theta_3 + \Xi_1\Theta_3,$$

$$\Lambda_{33} = -2\Theta_1 - 2\Theta_3 + m_1 E_1^T E_1,$$

$$\Lambda_{44} = -2\Theta_2 - 2\Theta_3 + m_2 E_2^T E_2,$$

$$\Xi_1 = \text{diag}\{\kappa_1^-, \dots, \kappa_n^-\}, \Xi_2 = \text{diag}\{\kappa_1^+, \dots, \kappa_n^+\}.$$

Proof. Let us choose the Lyapunov function:

$$V(t, \eta) = \eta^T P \eta, \tag{14}$$

and it satisfies:

$$\lambda_{\min}(P) \|\eta\|^2 \leq V(t, \eta) \leq \lambda_{\max}(P) \|\eta\|^2. \tag{15}$$

Therefore, condition (1) of Lemma 5 is satisfied. Then, by taking the β -th fractional derivative of the above $V(t, \eta(t))$, we have:

$$D_t^\beta V(t, \eta) \leq 2\eta^T P D_t^\beta \eta. \tag{16}$$

For any symmetric positive definite matrix $Q \in \mathbb{R}^{n \times n}$ and any symmetric semi-positive definite matrix $W \in \mathbb{R}^{8n \times 8n}$, the following formulae can be obtained:

$$[2\eta^T + 2(D_t^\beta \eta)^T] Q [-D_t^\beta \eta - (B_1 + \Delta B_1(t))\eta + (B_2 + \Delta B_2(t))g(\eta) + (B_3 + \Delta B_3(t))g(\eta(\zeta))] = 0, \tag{17}$$

and:

$$\begin{aligned} & \zeta_0^\beta \beta^{-1} e^T(t) W \epsilon(t) - \int_{t-\zeta(t)}^t (t-\omega)^{\beta-1} e^T(t) W \epsilon(t) d\omega \\ & = \zeta_0^\beta \beta^{-1} e^T(t) W \epsilon(t) - \zeta^\beta(t) \beta^{-1} e^T(t) W \epsilon(t) \\ & \geq 0, \end{aligned} \tag{18}$$

where $\epsilon(t) = [\eta^T, \eta^T(\zeta), g^T(\eta), g^T(\eta(\zeta)), (F(t)E\eta)^T, (F_1(t)E_1g(\eta))^T, (F_2(t)E_2g(\eta(\zeta)))^T, (D_t^\beta \eta)^T]^T$.

For Assumption 1, we know that for any $\mu_{ji} > 0 (i = 1, \dots, n; j = 1, 2, 3)$, we obtain:

$$\begin{aligned} & 2(g_i(\eta_i) - \kappa_i^- \eta_i) \mu_{1i} (\kappa_i^+ \eta_i - g_i(\eta_i)) \geq 0, \\ & 2(g_i(\eta_i(\zeta)) - \kappa_i^- \eta_i(\zeta)) \mu_{2i} (\kappa_i^+ \eta_i(\zeta) - g_i(\eta_i(\zeta))) \geq 0, \\ & 2(g_i(\eta_i) - g_i(\eta_i(\zeta)) - \kappa_i^- (\eta_i - \eta_i(\zeta))) \mu_{3i} (\kappa_i^+ (\eta_i - \eta_i(\zeta)) - (g_i(\eta_i) - g_i(\eta_i(\zeta)))) \geq 0. \end{aligned} \tag{19}$$

It is equivalent to:

$$\begin{aligned} & 2(g(\eta) - \Xi_1 \eta)^T \Theta_1 (\Xi_2 \eta - g(\eta)) \geq 0, \\ & 2(g(\eta(\zeta)) - \Xi_1 \eta(\zeta))^T \Theta_2 (\Xi_2 \eta(\zeta) - g(\eta(\zeta))) \geq 0, \\ & 2(g(\eta) - g(\eta(\zeta)) - \Xi_1 (\eta - \eta(\zeta)))^T \Theta_3 (\Xi_2 (\eta - \eta(\zeta)) - (g(\eta) - g(\eta(\zeta)))) \geq 0. \end{aligned} \tag{20}$$

For Assumption 2, we derive that for any $m > 0, m_1 > 0, m_2 > 0$, one obtains:

$$\begin{aligned}
 & m\eta^T E^T E\eta - m(F(t)E\eta)^T (F(t)E\eta) \geq 0, \\
 & m_1 g^T(\eta) E_1^T E_1 g(\eta) - m_1 (F_1(t)E_1 g(\eta))^T (F_1(t)E_1 g(\eta)) \geq 0, \\
 & m_2 g^T(\eta(\zeta)) E_2^T E_2 g(\eta(\zeta)) - m_2 (F_2(t)E_2 g(\eta(\zeta)))^T (F_2(t)E_2 g(\eta(\zeta))) \geq 0.
 \end{aligned}
 \tag{21}$$

For Lemma 5, we assume that $\rho > 1$ yields:

$$V(t + \omega, \eta(t + \omega)) < \rho V(t, \eta), \forall \omega \in [-\zeta_0, 0],
 \tag{22}$$

which implies:

$$\rho \eta^T P \eta - \eta^T(\zeta) P \eta(\zeta) > 0.
 \tag{23}$$

Because $\rho > 1$ is any parameter that is greater than 1, we take $\rho \rightarrow 1^+$ for (23), and we have:

$$\eta^T P \eta - \eta^T(\zeta) P \eta(\zeta) \geq 0.
 \tag{24}$$

From (16)–(18), (20), (21), and (24), we obtain:

$$\begin{aligned}
 D_t^\beta V(t, \eta) & \leq 2\eta^T P D_t^\beta \eta + [2\eta^T + 2(D_t^\beta \eta)^T] Q [-D_t^\beta \eta - (B_1 + \Delta B_1(t))\eta \\
 & \quad + (B_2 + \Delta B_2(t))g(\eta) + (B_3 + \Delta B_3(t))g(\eta(\zeta))] + \zeta_0^\beta \beta^{-1} \epsilon^T(t) W \epsilon(t) \\
 & \quad - \int_{t-\zeta(t)}^t (t-\omega)^{\beta-1} \epsilon^T(t) W \epsilon(t) d\omega + 2(g(\eta) - \Xi_1 \eta)^T \Theta_1 (\Xi_2 \eta - g(\eta)) \\
 & \quad + 2(g(\eta(\zeta)) - \Xi_1 \eta(\zeta))^T \Theta_2 (\Xi_2 \eta(\zeta) - g(\eta(\zeta))) + 2(g(\eta) - g(\eta(\zeta))) \\
 & \quad - \Xi_1 (\eta - \eta(\zeta)))^T \Theta_3 (\Xi_2 (\eta - \eta(\zeta)) - (g(\eta) - g(\eta(\zeta)))) \\
 & \quad + m\eta^T E^T E\eta - m(F(t)E\eta)^T (F(t)E\eta) + m_1 g^T(\eta) E_1^T E_1 g(\eta) \\
 & \quad - m_1 (F_1(t)E_1 g(\eta))^T (F_1(t)E_1 g(\eta)) + m_2 g^T(\eta(\zeta)) E_2^T E_2 g(\eta(\zeta)) \\
 & \quad - m_2 (F_2(t)E_2 g(\eta(\zeta)))^T (F_2(t)E_2 g(\eta(\zeta))) + \eta^T P \eta - \eta^T(\zeta) P \eta(\zeta).
 \end{aligned}
 \tag{25}$$

So (25) is equivalent to:

$$\begin{aligned}
 D_t^\beta V(t, \eta) & \leq \zeta_0^\beta \beta^{-1} \epsilon^T(t) W \epsilon(t) - \int_{t-\zeta(t)}^t (t-\omega)^{\beta-1} \epsilon^T(t) W \epsilon(t) d\omega \\
 & \quad + \eta^T [-QB_1 - B_1^T Q - 2\Xi_1 \Theta_1 \Xi_2 - 2\Xi_1 \Theta_3 \Xi_2 + mE^T E + P] \eta \\
 & \quad + \eta^T [2\Xi_1 \Theta_3 \Xi_2 + 2\Xi_2 \Theta_3 \Xi_1] \eta(\zeta) + \eta^T [2QB_2 + 2\Xi_2 \Theta_1 + 2\Xi_1 \Theta_1 \\
 & \quad + 2\Xi_2 \Theta_3 + 2\Xi_1 \Theta_3] g(\eta) + \eta^T [2QB_3 - 2\Xi_2 \Theta_3 - 2\Xi_1 \Theta_3] g(\eta(\zeta)) \\
 & \quad + \eta^T [-2QH] (F(t)E\eta) + \eta^T [2QH_1] (F_1(t)E_1 g(\eta)) \\
 & \quad + \eta^T [2QH_2] (F_2(t)E_2 g(\eta(\zeta))) + \eta^T [2P - 2Q - 2B_1^T Q] D_t^\beta \eta \\
 & \quad + \eta^T(\zeta) [-2\Xi_1 \Theta_2 \Xi_2 - 2\Xi_1 \Theta_3 \Xi_2 - P] \eta(\zeta) \\
 & \quad + \eta^T(\zeta) [-2\Xi_2 \Theta_3 - 2\Xi_1 \Theta_3] g(\eta) + \eta^T(\zeta) [2\Xi_2 \Theta_2 + 2\Xi_1 \Theta_2 \\
 & \quad + 2\Xi_2 \Theta_3 + 2\Xi_1 \Theta_3] g(\eta(\zeta)) + g^T(\eta) [-2\Theta_1 - 2\Theta_3 \\
 & \quad + m_1 E_1^T E_1] g(\eta) + g^T(\eta) [4\Theta_3] g(\eta(\zeta)) + g^T(\eta) [2B_2^T Q] D_t^\beta \eta \\
 & \quad + g^T(\eta(\zeta)) [-2\Theta_2 - 2\Theta_3 + m_2 E_2^T E_2] g(\eta(\zeta)) + g^T(\eta(\zeta)) \\
 & \quad \times [2B_3^T Q] D_t^\beta \eta - m(F(t)E\eta)^T (F(t)E\eta) + (F(t)E\eta)^T \\
 & \quad \times [-2H^T Q] D_t^\beta \eta - m_1 (F_1(t)E_1 g(\eta))^T (F_1(t)E_1 g(\eta)) \\
 & \quad + (F_1(t)E_1 g(\eta))^T [2H_1^T Q] D_t^\beta \eta - m_2 (F_2(t)E_2 g(\eta(\zeta)))^T \\
 & \quad \times (F_2(t)E_2 g(\eta(\zeta))) + (F_2(t)E_2 g(\eta(\zeta)))^T [2H_2^T Q] D_t^\beta \eta \\
 & \quad + (D_t^\beta \eta)^T [-2Q] D_t^\beta \eta \\
 & \leq \zeta_0^\beta \beta^{-1} \epsilon^T(t) W \epsilon(t) + \epsilon^T(t) \Lambda \epsilon(t) - \int_{t-\zeta(t)}^t (t-\omega)^{\beta-1} \epsilon^T(t) W \epsilon(t) d\omega.
 \end{aligned}
 \tag{26}$$

Since $\int_{t-\zeta(t)}^t (t-\omega)^{\beta-1} e^T(t) W \epsilon(t) d\omega \geq 0$, we obtain:

$$D_t^\beta V(t, \eta) \leq \epsilon^T(t) \Phi \epsilon(t), \tag{27}$$

where $\Phi = \zeta_0^\beta \beta^{-1} W + \Lambda$.

Therefore, when $\Phi < 0$, it implies that $D_t^\beta V(t, \eta) < 0$. So, this satisfies condition (2) of Lemma 5. Finally, System (3) is asymptotically stable when both the output and the external input are zero. \square

Remark 4. At present, the LMI technique is widely used to research the stability of various FONNs with time-varying delays. Its greatest advantage is that we can use convex algorithms to solve it reasonably in polynomial time.

3.2. Passivity Analysis of the System

In the following, the passivity of System (3) is analyzed.

Theorem 2. Assuming that Assumptions 1 and 2 are true, if there exist symmetric positive definite matrices P and $Q \in \mathbb{R}^{n \times n}$, a symmetric semi-positive definite matrix $D \in \mathbb{R}^{9n \times 9n}$, three positive diagonal matrices $\Theta_j = \text{diag}\{\mu_{j1}, \mu_{j2}, \dots, \mu_{jn}\} \in \mathbb{R}^{n \times n} (j = 1, 2, 3)$, and $\rho > 0$ such that:

$$\Psi = \zeta_0^\beta \beta^{-1} D + \Sigma < 0, \tag{28}$$

where

$$\Sigma = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} & \Sigma_{13} & \Sigma_{14} & -QH & QH_1 & QH_2 & \Sigma_{18} & Q - G^T \\ * & \Sigma_{22} & -\Xi_2 \Theta_3 - \Xi_1 \Theta_3 & \Sigma_{24} & 0 & 0 & 0 & 0 & 0 \\ * & * & \Sigma_{33} & 2\Theta_3 & 0 & 0 & 0 & B_2^T Q & 0 \\ * & * & * & \Sigma_{44} & 0 & 0 & 0 & B_3^T Q & 0 \\ * & * & * & * & -mI & 0 & 0 & -H^T Q & 0 \\ * & * & * & * & * & -m_1 I & 0 & H_1^T Q & 0 \\ * & * & * & * & * & * & -m_2 I & H_2^T Q & 0 \\ * & * & * & * & * & * & * & -2Q & Q \\ * & * & * & * & * & * & * & * & -\rho I \end{bmatrix},$$

$$\Sigma_{11} = -QB_1 - B_1^T Q - 2\Xi_1 \Theta_1 \Xi_2 - 2\Xi_1 \Theta_3 \Xi_2 + mE^T E + P,$$

$$\Sigma_{12} = \Xi_1 \Theta_3 \Xi_2 + \Xi_2 \Theta_3 \Xi_1,$$

$$\Sigma_{13} = QB_2 + \Xi_2 \Theta_1 + \Xi_1 \Theta_1 + \Xi_2 \Theta_3 + \Xi_1 \Theta_3,$$

$$\Sigma_{14} = QB_3 - \Xi_2 \Theta_3 - \Xi_1 \Theta_3,$$

$$\Sigma_{18} = P - Q - B_1^T Q,$$

$$\Sigma_{22} = -2\Xi_1 \Theta_2 \Xi_2 - 2\Xi_1 \Theta_3 \Xi_2 - P,$$

$$\Sigma_{24} = \Xi_2 \Theta_2 + \Xi_1 \Theta_2 + \Xi_2 \Theta_3 + \Xi_1 \Theta_3,$$

$$\Sigma_{33} = -2\Theta_1 - 2\Theta_3 + m_1 E_1^T E_1,$$

$$\Sigma_{44} = -2\Theta_2 - 2\Theta_3 + m_2 E_2^T E_2,$$

$$\Xi_1 = \text{diag}\{\kappa_1^-, \dots, \kappa_n^-\}, \Xi_2 = \text{diag}\{\kappa_1^+, \dots, \kappa_n^+\},$$

then System (3) is passive.

Proof. To facilitate the treatment of the passivity of System (3), we use $V(t, \eta)$ in Theorem 1. By using the same method that we used to prove Theorem 1, we obtain an estimate:

$$D_t^\beta V(t) - 2\zeta^T u - \rho u^T u \leq \zeta^T(t) \Psi \zeta(t), \tag{29}$$

in which $\zeta(t) = [\eta^T, \eta^T(\zeta), g^T(\eta), g^T(\eta(\zeta)), (F(t)E\eta)^T, (F_1(t)E_1g(\eta))^T, (F_2(t)E_2g(\eta(\zeta)))^T, (D_t^\beta \eta)^T, u^T]^T$. From (28), we have:

$$D_t^\beta V(t) - 2\zeta^T u - \rho u^T u < 0. \tag{30}$$

The integral of Inequality (30) is given as:

$$\begin{aligned}
 & I_{t_p}^1 D_{t_p}^\beta V(t_p) - 2 \int_0^{t_p} \zeta^T \mathbf{u} dt - \rho \int_0^{t_p} \mathbf{u}^T \mathbf{u} dt, \\
 & = I_{t_p}^{1-\beta} I_{t_p}^\beta D_{t_p}^\beta V(t_p) - 2 \int_0^{t_p} \zeta^T \mathbf{u} dt - \rho \int_0^{t_p} \mathbf{u}^T \mathbf{u} dt, \\
 & = I_{t_p}^{1-\beta} (V(t_p) - V(0)) - 2 \int_0^{t_p} \zeta^T \mathbf{u} dt - \rho \int_0^{t_p} \mathbf{u}^T \mathbf{u} dt, \\
 & = I_{t_p}^{1-\beta} V(t_p) - I_{t_p}^{1-\beta} V(0) - 2 \int_0^{t_p} \zeta^T \mathbf{u} dt - \rho \int_0^{t_p} \mathbf{u}^T \mathbf{u} dt, \\
 & = I_{t_p}^{1-\beta} V(t_p) - 2 \int_0^{t_p} \zeta^T \mathbf{u} dt - \rho \int_0^{t_p} \mathbf{u}^T \mathbf{u} dt, \\
 & < 0.
 \end{aligned} \tag{31}$$

Because of $I_{t_p}^{1-\beta} V(t_p) = \frac{1}{\Gamma(1-\beta)} \int_0^{t_p} (t_p - t)^{-\beta} \boldsymbol{\eta}^T \mathbf{P} \boldsymbol{\eta} dt \geq 0, \forall t_p \geq 0$, thus:

$$2 \int_0^{t_p} \zeta^T \mathbf{u} dt > -\rho \int_0^{t_p} \mathbf{u}^T \mathbf{u} dt. \tag{32}$$

Therefore, System (3) is passive and can be obtained using Definition 1 and Theorem 1. \square

Remark 5. It is well known that the time-delay dependent condition is less conservative than the time-delay independent condition because it takes into account time delay information. In [43], the authors derived a time-delay dependent stability criterion, but it is worth noting that this sufficient condition considers systems with constant time delay. In [37], the authors obtained an order-independent passive analysis criterion, whereas we obtained an order-dependent and delay-dependent passive analysis criterion; thus, the results we obtained in Theorems 1 and 2 are less conservative than those in [37,43,44].

Remark 6. From Definition 1, we can see that if we want to prove that System (3) is passive, then first we need to show that System (3) is asymptotically stable with $\zeta(t) = 0$ and $\mathbf{u}(t) = 0$, then prove that System (3) satisfies the definition of passivity with zero initial conditions. Therefore, from Definition 1 combined with the proofs of Theorems 1 and 2 in this paper, we know that Theorem 2 is based on the establishment of Theorem 1; thus, we see that matrix Λ in Theorem 1 is a submatrix of matrix Σ in Theorem 2.

Remark 7. The passivity of NNs has been the focus of many research papers. For example, in [29], the robust finite-time passivity of UFONNs was studied, but the passivity of the system when time delays existed was not considered. In [38], the passivity of FONNs with time-varying delays was studied, but the uncertainties were not considered. In [45], the passivity of FONNs with time-varying parameter uncertainties was studied. However, the author used a sufficient condition that guaranteed the system was passive, independent of the order of the system. As far as we know, there exists no report on the passivity of UFONNs with time-varying delays. For this article, we use the method of [38] to deduce the order-dependent and delay-dependent passivity conditions for making the system passive by means of the definition of passivity and the Razumikhin theorem. Compared with the method of constructing Lyapunov–Krasovskii functions used in the existing literature, our method is simpler and less conservative.

4. Simulation Examples

Below, the correctness of the theoretical results is confirmed by simulation experiments.

Example 1. Given the following UFONNs:

$$\begin{cases} D_t^{0.95}\eta(t) = -(\mathbf{B}_1 + \Delta\mathbf{B}_1(t))\eta(t) + (\mathbf{B}_2 + \Delta\mathbf{B}_2(t))g(\eta(t)) \\ \quad + (\mathbf{B}_3 + \Delta\mathbf{B}_3(t))g(\eta(t - \zeta(t))) + \mathbf{u}(t), \\ \zeta(t) = \mathbf{G}\eta(t), \end{cases} \tag{33}$$

where $\eta(t) = [\eta_1(t), \eta_2(t)]^T \in \mathbb{R}^2$ is the state variable of the NNs, $\zeta(t) \in \mathbb{R}^2$ is the output, and $\mathbf{u}(t) \in \mathbb{R}^2$ is the external input of System (33). The parameter matrices are $\mathbf{B}_1 = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$, $\mathbf{B}_2 = \begin{bmatrix} -4 & -0.1 \\ 0.13 & -4 \end{bmatrix}$, $\mathbf{B}_3 = \begin{bmatrix} -1 & 0.1 \\ 0.15 & -2 \end{bmatrix}$, and $\mathbf{G} = \begin{bmatrix} -0.3 & 0.4 \\ 0.5 & 0.5 \end{bmatrix}$. Define the uncertainties as follows: $\mathbf{H} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.1 \end{bmatrix}$, $\mathbf{H}_1 = \mathbf{H}_2 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$, $\mathbf{E} = \mathbf{E}_1 = \mathbf{E}_2 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}$, $\mathbf{F}(t) = \begin{bmatrix} \sin t & 0 \\ 0 & \sin t \end{bmatrix}$, $\mathbf{F}_1(t) = \begin{bmatrix} \cos t & 0 \\ 0 & \cos t \end{bmatrix}$, and $\mathbf{F}_2(t) = \begin{bmatrix} \tanh t & 0 \\ 0 & \tanh t \end{bmatrix}$.

Obviously, these uncertainties satisfy Assumption 2. The time delay is $\zeta(t) = 2 \tanh^2 t$. The activation function is $g(\eta(t)) = (\tanh \eta_1(t), \tanh \eta_2(t))^T$. Based on Assumption 1, we obtain $\Xi_1 = \text{diag}\{-1, -1\}$, and $\Xi_2 = \text{diag}\{1, 1\}$. Using Theorem 1, the feasible solutions for the LMI (13) are $\mathbf{P} = \begin{bmatrix} 19.9153 & -0.3176 \\ -0.3176 & 13.0896 \end{bmatrix}$, $\mathbf{Q} = \begin{bmatrix} 14.3144 & -0.3621 \\ -0.3621 & 18.7749 \end{bmatrix}$, $\Theta_1 = \begin{bmatrix} 6.6745 & 0 \\ 0 & 4.8305 \end{bmatrix}$,

$$\Theta_2 = \begin{bmatrix} 3.2573 & 0 \\ 0 & 2.5367 \end{bmatrix}, \Theta_3 = \begin{bmatrix} 3.0676 & 0 \\ 0 & 1.9969 \end{bmatrix},$$

$$\mathbf{D} = \begin{bmatrix} D_{11} & D_{12} & D_{13} & D_{14} & D_{15} & D_{16} & D_{17} & D_{18} & D_{19} \\ * & D_{22} & \mathbf{0} \\ * & * & D_{33} & D_{34} & D_{35} & \mathbf{0} & \mathbf{0} & D_{38} & D_{39} \\ * & * & * & D_{44} & D_{45} & \mathbf{0} & \mathbf{0} & D_{48} & D_{49} \\ * & * & * & * & D_{55} & \mathbf{0} & \mathbf{0} & D_{58} & D_{59} \\ * & * & * & * & * & D_{66} & \mathbf{0} & D_{68} & \mathbf{0} \\ * & * & * & * & * & * & D_{77} & D_{78} & \mathbf{0} \\ * & * & * & * & * & * & * & D_{88} & D_{89} \\ * & * & * & * & * & * & * & * & D_{99} \end{bmatrix}$$

where

$$D_{11} = \begin{bmatrix} 5.1242 & -0.2706 \\ -0.2706 & 2.6972 \end{bmatrix}, D_{12} = \begin{bmatrix} 3.0170 & 0 \\ 0 & 1.9639 \end{bmatrix},$$

$$D_{13} = \begin{bmatrix} 28.1408 & -1.0046 \\ -1.0046 & 36.9199 \end{bmatrix}, D_{14} = \begin{bmatrix} 7.0577 & -1.3339 \\ -1.3339 & 18.4845 \end{bmatrix},$$

$$D_{15} = \begin{bmatrix} 1.4075 & -0.0278 \\ -0.0278 & 0.9233 \end{bmatrix}, D_{16} = \begin{bmatrix} -0.7039 & 0.0178 \\ 0.0178 & -0.9233 \end{bmatrix},$$

$$D_{17} = \begin{bmatrix} -0.7039 & 0.0178 \\ 0.0178 & -0.9233 \end{bmatrix}, D_{18} = \begin{bmatrix} 11.0972 & -0.2670 \\ -0.2670 & 11.8014 \end{bmatrix},$$

$$D_{19} = \begin{bmatrix} -7.1789 & 0.4219 \\ 0.4219 & -8.9944 \end{bmatrix}, D_{22} = \begin{bmatrix} 1.9244 & -0.1074 \\ -0.1074 & 0.8257 \end{bmatrix},$$

$$D_{33} = \begin{bmatrix} 5.5778 & -0.0044 \\ -0.0044 & 3.5309 \end{bmatrix}, D_{34} = \begin{bmatrix} -3.1146 & -0.0013 \\ -0.0013 & -2.0479 \end{bmatrix},$$

$$D_{35} = \begin{bmatrix} -0.0033 & 0 \\ 0 & -0.0031 \end{bmatrix}, D_{38} = \begin{bmatrix} 28.1837 & -0.8039 \\ -0.8039 & 36.9411 \end{bmatrix},$$

$$D_{39} = \begin{bmatrix} 0.0481 & 0.0016 \\ 0.0016 & 0.0537 \end{bmatrix}, D_{44} = \begin{bmatrix} 3.4042 & -0.0002 \\ -0.0002 & 2.1390 \end{bmatrix},$$

$$D_{45} = \begin{bmatrix} -0.0007 & 0 \\ 0 & -0.0007 \end{bmatrix}, D_{48} = \begin{bmatrix} 7.0665 & -1.2861 \\ -1.2861 & 18.4899 \end{bmatrix},$$

$$D_{49} = \begin{bmatrix} 0.0143 & 0.0004 \\ 0.0004 & 0.0172 \end{bmatrix}, D_{55} = \begin{bmatrix} 7.1468 & 0 \\ 0 & 7.1468 \end{bmatrix},$$

$$\begin{aligned}
 D_{58} &= \begin{bmatrix} 1.4078 & -0.0262 \\ -0.0262 & 0.9235 \end{bmatrix}, D_{59} = \begin{bmatrix} 0.0004 & 0 \\ 0 & 0.0006 \end{bmatrix}, \\
 D_{66} &= \begin{bmatrix} 7.2343 & 0 \\ 0 & 7.2343 \end{bmatrix}, D_{68} = \begin{bmatrix} -0.7039 & 0.0178 \\ 0.0178 & -0.9233 \end{bmatrix}, \\
 D_{77} &= \begin{bmatrix} 7.0593 & 0 \\ 0 & 7.0593 \end{bmatrix}, D_{78} = \begin{bmatrix} -0.7039 & 0.0178 \\ 0.0178 & -0.9233 \end{bmatrix}, \\
 D_{88} &= \begin{bmatrix} 9.6950 & -0.2972 \\ -0.2972 & 13.2748 \end{bmatrix}, D_{89} = \begin{bmatrix} -7.0432 & 0.1783 \\ 0.1783 & -9.2395 \end{bmatrix}, \\
 D_{99} &= \begin{bmatrix} 7.4195 & -0.0004 \\ -0.0004 & 7.4194 \end{bmatrix},
 \end{aligned}$$

$m = 21.8165, m_1 = 22.0388, \text{ and } m_2 = 21.5946$, which means that when $u(t) = \mathbf{0}$ and $\zeta(t) = \mathbf{0}$, the UFONNs with time-varying delays are asymptotically stable in the sense of Definition 1 and Theorem 1. From Theorem 2, we find that $\rho = 22.4761$, so System (33) is passive. The passive parameter ρ obtained in the simulation example is only the value when the order of System (33) is 0.95. In addition, we take the different order β of System (33) to analyze its relationship with the passive parameter ρ in Table 1.

Table 1. The relationship between the order of the system and the passive parameter in Theorem 2.

The Order β	The Passive Parameter ρ
0.1	1.6504×10^3
0.2	180.3959
0.3	248.7765
0.4	334.8487
0.5	357.8057
0.6	1.6010
0.7	20.5273
0.8	20.3400
0.9	22.4595
1	22.4947

In order to better observe the simulation effect, we set the initial value of System (33) as $\varphi(t) = [0.5, -0.1]^T$ and $\forall t \in [-2, 0]$. The state dynamics of System (33) when $u(t) = [1 - \sin t, 1.5 + \cos t]^T$ are presented in Figure 1. When $u(t) = \mathbf{0}$ and $\zeta(t) = \mathbf{0}$, the state variables of System (33) are asymptotically stable, which can be seen intuitively in Figure 2.

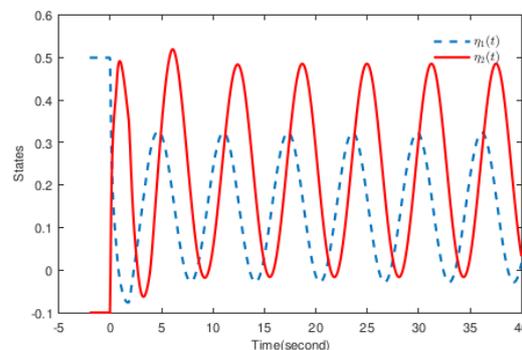


Figure 1. State dynamics of UFONNs with external input.

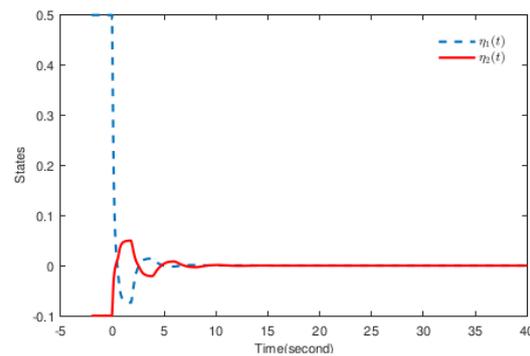


Figure 2. State dynamics of UFONNs.

Remark 8. In the simulation example, the time delay of System (33) is constant (i.e., $\zeta(t) = 2$). By conducting simulation experiments on it, we find that when $\zeta(t) = \mathbf{0}$ and $\mathbf{u}(t) = \mathbf{0}$, System (33) is asymptotically stable. See Figure 3.

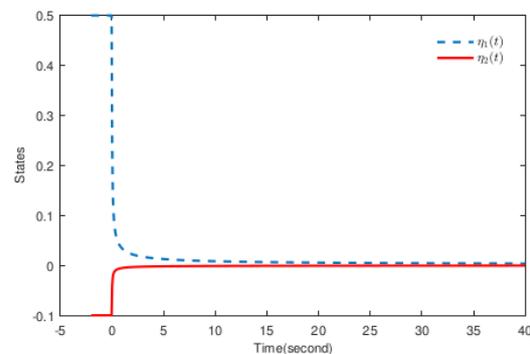


Figure 3. State dynamics of UFONNs.

5. Conclusions

In this paper, by means of knowledge of fractional-order calculus and the fractional-order Razumikhin theorem, the passivity of UFONNs with time-varying delays was studied. It was proved that UFONNs with time-varying delays are asymptotically stable when both the output and the external input are zero, and when the delay-dependent and order-dependent LMI is satisfied. In addition, based on the proof of asymptotic stability, UFONNs with time-varying delays are passive when they contain a nonzero external input vector and output vector. In the future, we will study the passive problem of UFONNs, including unbounded time-varying time delays and multiple time delays.

Author Contributions: S.X.: investigation, writing—review and editing; H.L.: conceptualization, supervision; Z.H.: methodology, writing—original draft. All authors have read and agreed to the published version of the manuscript.

Funding: This work was supported by the Key Research Projects of Humanities and Social Sciences in Anhui Universities (SK2021A0346) and the Anhui Jianzhu University Research Startup Project (2019QDZ61).

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Informed consent was obtained from all subjects involved in the study.

Data Availability Statement: All data included in this study are available upon request via contact with the corresponding author.

Conflicts of Interest: The authors declare no conflict of interest.

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