fractal and fractional

Article

Robust Stability of Fractional Order Memristive BAM Neural Networks with Mixed and Additive Time Varying Delays

Xiuping Han [1], M. Hymavathi [2], Sumaya Sanober [3,*], Bhawna Dhupia [3] and M. Syed Ali [2]

1 School of Mathematics and Statistics, Shandong Normal University, Jinan 250014, China; xiuphan@sdnu.edu.cn
2 Department of Mathematics, Thiruvalluvar University, Vellore 632115, Tamil Nadu, India; vmhymavathi.1995@gmail.com (M.H.); syedgru@gmail.com (M.S.A.)
3 Department of Computer Science, College of Arts and Science, Prince Sattam Bin Abdulaziz University, Wadi Aldawaser 11991, Saudi Arabia; ndstvu@gmail.com
* Correspondence: sumayacsdept@gmail.com

Abstract: This paper is concerned with the problem of the robust stability of fractional-order memristive bidirectional associative memory (BAM) neural networks. Based on Lyapunov theory, fractional-order differential inequalities and linear matrix inequalities (LMI) are applied to obtain a robust asymptotical stability. Finally, numerical examples are presented.

Keywords: robust stability; memristive BAM neural networks; fractional order; mixed and additive time varying delays

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1. Introduction

Fractional calculus has a long history of more than three hundred years. Fractional calculus can be considered as the generalization for traditional calculus, from the integer order to the arbitrary order [1], and it relates to the calculus of the integrals and derivatives of orders that may be real or complex. Very recently, a study concerning FNNs with mixed and additive time-varying delays has made this an active research topic. They discussed applications in various fields, such as viscoelastic systems, diffusion waves, quantitative finance, etc. [2–9].

The BAM neural network is a two-layer neural network which can generalize not only auto-associative memory, but also hetero-associative memory [10–16]. This has been widely applied in many fields, such as image processing, pattern recognition, automatic control, and optimization problems. With the development and application of memristors, BAM memristive neural networks have become an active area of research [17–20]. MBAMNNs are the combination of memristors and BAMNNs. By adjusting the connection weight matrice, they can simulate human brains better than traditional BAMNNs.

On the other hand, we know that signals transmitted from one point to another may pass through two sections of networks, and due to the changes in network transmission conditions, time delays have different features; therefore, there exists another kind of delay, named additive (successive) delay. Recently, a new model for neural networks with pair additive time-varying delays should be considered in [21]. By constructing some advanced techniques, a new asymptotic stability criterion for neural networks with two successive delay components is derived in [22]. Inspired by [23,24] we first consider systems with two additive delay components. As we all know, in the process of the realization of neural network circuits, time delay is inevitable, and the existence of time delay often causes system stability or deteriorates its performance [25–29] .

In practice, the stability of a well-designed neural network system may often be destroyed by its unavoidable uncertainty, due to the existence of modeling error, external disturbance, and parameter fluctuation during implementation. Most results are related to the dynamical analysis of FONNs concentrated on robust stability [30–35]. One instinctive plan to manage this issue is to access the disturbance or effect of the disturbances from quantifiable variables, and thereafter, a control move can be made, in context of the disturbance estimate, to compensate for the effect of the disturbances. This essential idea can be ordinarily stretched out to manage uncertainties or unmodelled dynamics that could be considered as a part of the disturbance. When analyzing the stability of NNs, not only delay, but also parameter uncertainty, should be considered because of the existence of disturbances in the environment [36–39].

Motivated by the above discussions, we try to investigate the robust stability of fractional-order memristive BAM neural networks.

(1) The proposed memristive BAM neural networks model contains mixed and additive time-varying delays.
(2) The proposed main proofs are proved with the some effective analytical techniques.
(3) A new sufficient criterion is derived in terms of LMI, which can be effectively solved in the LMI MATLAB toolbox.
(4) Finally, we provide a numerical example.

2. Preliminaries

Definition 1 ([40])**.** *The Caputo derivative of the fractional order γ of the function $\mathbb{h}(\mathtt{t})$ is given*

$$ {}_{0}^{C}\mathscr{D}_{\mathtt{t}}^{\gamma}\mathbb{h}(\mathtt{t}) = \frac{1}{\Gamma(m-\gamma)}\int_0^{\mathtt{t}} (\mathtt{t}-\mathtt{s})^{m-\gamma-1}\mathbb{h}^{(\mathtt{m})}(\mathtt{s})\mathrm{d}\mathtt{s}. $$

in which $\mathtt{m}-1<\gamma<\mathtt{m}$.

Lemma 1 ([41])**.** *If $\mathscr{U}>0$ is a constant and $\mathscr{P}$, $\mathscr{Q}$ are real matrices, then*

$$ \mathscr{P}^T\mathscr{Q}+\mathscr{Q}^T\mathscr{P}\leq\mathscr{U}\mathscr{P}^T\mathscr{P}+\mathscr{U}^{-1}\mathscr{Q}^T\mathscr{Q}. $$

Lemma 2 ([41])**.** *Given that $\mathscr{P},\mathscr{Q},\mathscr{R}$ are constant matrices, where $\mathscr{P}=\mathscr{P}^T$, $\mathscr{Q}=\mathscr{Q}^T$, then*

$$ \begin{bmatrix} \mathscr{P} & \mathscr{R} \\ \mathscr{R}^T & -\mathscr{Q} \end{bmatrix} < 0. $$

iff $\mathscr{Q}>0$ and $\mathscr{P}+\mathscr{R}\mathscr{Q}^{-1}\mathscr{R}^T<0$.

Lemma 3 ([42])**.** *Let $\mathscr{T}_{\mathtt{i}}\in\mathscr{R}^{\mathtt{m}\times\mathtt{m}}$, $(\mathtt{i}=0,1,....,\mathtt{p})$ be symmetric matrices. The conditions on $\mathscr{T}_{\mathtt{i}}(\mathtt{i}=0,1,...,\mathtt{p})$, $\xi^T\mathscr{T}_{\mathtt{i}}\xi>0$, for all $\xi\neq 0$, such that $\xi^T\mathscr{T}_0\xi>0(\mathtt{i}=1,2,..,\mathtt{p})$ hold if there exist $\tau_{\mathtt{i}}\geq 0$, $(\mathtt{i}=1,2,...,\mathtt{p})$ such that*

$$ \mathscr{T}_0-\sum_{\mathtt{i}=1}^{\mathtt{p}}\tau_{\mathtt{i}}\mathscr{T}_{\mathtt{i}}>0. $$

Lemma 4 ([43])**.** *Let $\mu(\wp):[\mathtt{a},\mathbb{b}]\to\mathscr{R}^{\mathtt{m}}$ be a scalar function with scalars $\mathtt{a}<\mathbb{b}$ and $\mathscr{S}$ is matrix then*

$$ \Big(\int_{\mathtt{a}}^{\mathbb{b}}\mu^T(\wp)\mathrm{d}\wp\Big)\mathscr{S}\Big(\int_{\mathtt{a}}^{\mathbb{b}}\mu(\wp)\mathrm{d}\wp\Big)\leq(\mathbb{b}-\mathtt{a})\int_{\mathtt{a}}^{\mathbb{b}}\mu^T(\wp)\mathscr{S}\mu(\wp)\mathrm{d}\wp. $$

Hypothesis 1. *The functions $\mathbb{h}_{\mathtt{j}}(\cdot)$ and $\mathtt{g}_{\mathtt{i}}(\cdot)$ satisfy*

$$ |\mathbb{h}_{\mathtt{j}}(\mathtt{x})-\mathbb{h}_{\mathtt{j}}(\mathtt{y})|\leq\mathtt{p}_{\mathtt{j}}|\mathtt{x}-\mathtt{y}|,\ \mathtt{j}=1,2,..,\mathtt{n}, $$
$$ |\mathtt{g}_{\mathtt{i}}(\mathtt{x})-\mathtt{g}_{\mathtt{i}}(\mathtt{y})|\leq\mathtt{q}_{\mathtt{i}}|\mathtt{x}-\mathtt{y}|,\ \mathtt{i}=1,2,..,\mathtt{m}. $$

3. Main Results

Consider fractional-order memristive BAM neural networks with additive and mixed time-varying delays,

$$\begin{cases} \mathscr{D}^\alpha \wp_{\mathtt{i}}(\mathtt{t}) & = -\mathtt{r}_{\mathtt{i}}\wp_{\mathtt{i}}(\mathtt{t}) + \sum_{\mathtt{j}=1}^{\mathtt{m}} \mathtt{d}_{\mathtt{ji}}(\wp_{\mathtt{i}}(\mathtt{t}))\mathtt{h}_{\mathtt{j}}(\Im_{\mathtt{j}}(\mathtt{t})) \\ & + \sum_{\mathtt{j}=1}^{\mathtt{m}} \mathtt{b}_{\mathtt{ji}}(\wp_{\mathtt{i}}(\mathtt{t}))\mathtt{h}_{\mathtt{j}}(\Im_{\mathtt{j}}(\mathtt{t}-\sigma_1(\mathtt{t})-\sigma_2(\mathtt{t}))) \\ & + \sum_{\mathtt{j}=1}^{\mathtt{m}} \mathtt{a}_{\mathtt{ji}}(\wp_{\mathtt{i}}(\mathtt{t})) \int_{\mathtt{t}-\tau(\mathtt{t})}^{\mathtt{t}} \mathtt{h}_{\mathtt{j}}(\Im_{\mathtt{j}}(\mathtt{s}))\mathtt{ds} + \mathscr{I}_{\mathtt{i}}(\mathtt{t}), \mathtt{i} = 1, ..., \mathtt{m} \\ \mathscr{D}^\alpha \Im_{\mathtt{j}}(\mathtt{t}) & = -\mathtt{m}_{\mathtt{j}}\Im_{\mathtt{j}}(\mathtt{t}) + \sum_{\mathtt{i}=1}^{\mathtt{m}} \mathtt{n}_{\mathtt{ij}}(\Im_{\mathtt{j}}(\mathtt{t}))\mathtt{g}_{\mathtt{i}}(\wp_{\mathtt{i}}(\mathtt{t})) \\ & + \sum_{\mathtt{i}=1}^{\mathtt{m}} \mathtt{c}_{\mathtt{ij}}(\Im_{\mathtt{j}}(\mathtt{t}))\mathtt{g}_{\mathtt{i}}(\wp_{\mathtt{i}}(\mathtt{t}-\eta_1(\mathtt{t})-\eta_2(\mathtt{t}))) \\ & + \sum_{\mathtt{i}=1}^{\mathtt{m}} \mathtt{e}_{\mathtt{ij}}(\Im_{\mathtt{j}}(\mathtt{t})) \int_{\mathtt{t}-\upsilon(\mathtt{t})}^{\mathtt{t}} \mathtt{g}_{\mathtt{i}}(\wp_{\mathtt{i}}(\mathtt{s}))\mathtt{ds} + \mathscr{J}_{\mathtt{j}}(\mathtt{t}), \mathtt{j} = 1, ..., \mathtt{m} \end{cases} \tag{1}$$

where, $\wp_{\mathtt{i}}(\mathtt{t})$ and $\Im_{\mathtt{j}}(\mathtt{t})$ denote the state variable related to the $\mathtt{i}$th and $\mathtt{j}$th neurons. $(\mathtt{d}_{\mathtt{ji}}(\wp_{\mathtt{i}}(\mathtt{t})))_{\mathtt{m}\times\mathtt{m}}$, $(\mathtt{n}_{\mathtt{ij}}(\Im_{\mathtt{j}}(\mathtt{t})))_{\mathtt{m}\times\mathtt{m}}$ $(\mathtt{b}_{\mathtt{ji}}(\wp_{\mathtt{i}}(\mathtt{t})))_{\mathtt{m}\times\mathtt{m}}$, $(\mathtt{c}_{\mathtt{ij}}(\Im_{\mathtt{j}}(\mathtt{t})))_{\mathtt{m}\times\mathtt{m}}$ $(\mathtt{a}_{\mathtt{ji}}(\wp_{\mathtt{i}}(\mathtt{t})))_{\mathtt{m}\times\mathtt{m}}$, $(\mathtt{e}_{\mathtt{ij}}(\Im_{\mathtt{j}}(\mathtt{t})))_{\mathtt{m}\times\mathtt{m}}$ are connection weight matrices, $\mathtt{r}_{\mathtt{i}} > 0$, $\mathtt{m}_{\mathtt{j}} > 0$, are positive diagonal matrices. The external inputs are $\mathscr{I}_{\mathtt{i}}(\mathtt{t})$ and $\mathscr{J}_{\mathtt{j}}(\mathtt{t})$. $\sigma_1(\mathtt{t})$, $\sigma_2(\mathtt{t})$, $\eta_1(\mathtt{t})$ and $\eta_2(\mathtt{t})$ can be assumed by

$$\begin{cases} 0 \le \sigma_1(\mathtt{t}) \le \sigma_1, \ \dot{\sigma}_1 \le \delta_1, \ 0 \le \sigma_2(\mathtt{t}) \le \sigma_2, \ \dot{\sigma}_2 \le \delta_2, \\ 0 \le \eta_1(\mathtt{t}) \le \alpha_1, \ \dot{\eta}_1 \le \alpha_1, \ 0 \le \eta_2(\mathtt{t}) \le \alpha_2, \ \dot{\eta}_2 \le \alpha_2. \end{cases}$$

By using differential inclusion theory, the above system becomes

$$\begin{cases} \mathscr{D}^\alpha \wp_i(\mathtt{t}) & = -\mathtt{r}_{\mathtt{i}}\wp_{\mathtt{i}}(\mathtt{t}) + \sum_{\mathtt{j}=1}^{\mathtt{m}} co[\mathtt{d}_{\mathtt{ji}}(\wp_{\mathtt{i}}(\mathtt{t}))\mathtt{h}_{\mathtt{j}}(\Im_{\mathtt{j}}(\mathtt{t}))] \\ & + \sum_{\mathtt{j}=1}^{\mathtt{m}} co[\mathtt{b}_{\mathtt{ji}}(\wp_{\mathtt{i}}(\mathtt{t}))\mathtt{h}_{\mathtt{j}}(\Im_{\mathtt{j}}(\mathtt{t}-\sigma_1(\mathtt{t})-\sigma_2(\mathtt{t})))] \\ & + \sum_{\mathtt{j}=1}^{\mathtt{m}} co[\mathtt{a}_{\mathtt{ji}}(\wp_{\mathtt{i}}(\mathtt{t})) \int_{\mathtt{t}-\tau(\mathtt{t})}^{\mathtt{t}} \mathtt{h}_{\mathtt{j}}(\Im_{\mathtt{j}}(\mathtt{s}))\mathtt{ds}] + \mathscr{I}_{\mathtt{i}}(\mathtt{t}), \\ \mathscr{D}^\alpha \Im_j(\mathtt{t}) & -\mathtt{m}_{\mathtt{j}}\Im_{\mathtt{j}}(\mathtt{t}) + \sum_{\mathtt{i}=1}^{\mathtt{m}} co[\mathtt{n}_{\mathtt{ij}}(\Im_{\mathtt{j}}(\mathtt{t}))\mathtt{g}_{\mathtt{i}}(\wp_{\mathtt{i}}(\mathtt{t}))] \\ & + \sum_{\mathtt{i}=1}^{\mathtt{m}} co[\mathtt{c}_{\mathtt{ij}}(\Im_{\mathtt{j}}(\mathtt{t}))\mathtt{g}_{\mathtt{i}}(\wp_{\mathtt{i}}(\mathtt{t}-\eta_1(\mathtt{t})-\eta_2(\mathtt{t})))] \\ & + \sum_{\mathtt{i}=1}^{\mathtt{m}} co[\mathtt{e}_{\mathtt{ij}}(\Im_{\mathtt{j}}(\mathtt{t})) \int_{\mathtt{t}-\upsilon(\mathtt{t})}^{\mathtt{t}} \mathtt{g}_{\mathtt{i}}(\wp_{\mathtt{i}}(\mathtt{s}))\mathtt{ds}] + \mathscr{J}_{\mathtt{j}}(\mathtt{t}) \end{cases} \tag{2}$$

Furthermore, let just for efficiency

$$\begin{aligned} co[\mathtt{d}_{\mathtt{ji}}(\wp_{\mathtt{i}}(\mathtt{t}))]\mathtt{h}_{\mathtt{j}}(\Im_{\mathtt{j}}(\mathtt{t})) &= \mathtt{h}_{\mathtt{d}}(\wp_{\mathtt{i}}(\mathtt{t})), \\ co[\mathtt{b}_{\mathtt{ji}}(\wp_{\mathtt{i}}(\mathtt{t}))]\mathtt{h}_{\mathtt{j}}(\Im_{\mathtt{j}}(\mathtt{t}-\sigma_1(\mathtt{t})-\sigma_2(\mathtt{t}))) &= \mathtt{h}_{\mathtt{b}}(\wp_{\mathtt{i}}(\mathtt{t})), \\ co[\mathtt{a}_{\mathtt{ji}}(\wp_{\mathtt{i}}(\mathtt{t}))] \int_{\mathtt{t}-\tau(\mathtt{t})}^{\mathtt{t}} \mathtt{h}_{\mathtt{j}}(\Im_{\mathtt{j}}(\mathtt{s}))\mathtt{ds} &= \mathtt{h}_{\mathtt{a}}(\wp_{\mathtt{i}}(\mathtt{t})), \\ co[\mathtt{n}_{\mathtt{ij}}(\Im_{\mathtt{j}}(\mathtt{t}))]\mathtt{g}_{\mathtt{i}}(\wp_i(\mathtt{t})) &= \mathtt{g}_{\mathtt{n}}(\Im_{\mathtt{j}}(\mathtt{t})), \\ co[\mathtt{c}_{\mathtt{ij}}(\Im_{\mathtt{j}}(\mathtt{t}))]\mathtt{g}_{\mathtt{i}}(\wp_{\mathtt{i}}(\mathtt{t}-\eta_1(\mathtt{t})-\eta_2(\mathtt{t}))) &= \mathtt{g}_{\mathtt{c}}(\Im_{\mathtt{j}}(\mathtt{t})), \\ co[\mathtt{e}_{\mathtt{ij}}(\Im_{\mathtt{j}}(\mathtt{t}))] \int_{\mathtt{t}-\upsilon(\mathtt{t})}^{\mathtt{t}} \mathtt{g}_{\mathtt{i}}(\wp_{\mathtt{i}}(\mathtt{s}))\mathtt{ds} &= \mathtt{g}_{\mathtt{e}}(\Im_{\mathtt{j}}(\mathtt{t})) \end{aligned}$$

The equilibrium point $\varrho^* = (\varrho_1^*, \varrho_2^*, ..., \varrho_{\mathtt{m}}^*)^T$, $\chi^* = (\chi_1^*, \chi_2^*, ..., \chi_{\mathtt{m}}^*)^T$, of neural networks (2) to the origin. This transformation $\varrho_{\mathtt{i}}(\mathtt{t}) = \wp_{\mathtt{i}}(\mathtt{t}) - \wp_{\mathtt{i}}^*$, $\chi_{\mathtt{j}}(\mathtt{t}) = \Im_{\mathtt{j}}(\mathtt{t}) - \Im_{\mathtt{j}}^*$ put neural networks (2) into

$$\begin{cases} \mathscr{D}^\alpha \varrho(\mathtt{t}) & = -\mathtt{r}_{\mathtt{i}}\varrho_{\mathtt{i}}(\mathtt{t}) + \sum_{\mathtt{j}=1}^{\mathtt{m}} \mathtt{h}_{\mathtt{d}}(\varrho_{\mathtt{i}}(\mathtt{t})) + \sum_{\mathtt{j}=1}^{\mathtt{m}} \mathtt{h}_{\mathtt{b}}(\varrho_{\mathtt{i}}(\mathtt{t})) \\ & + \sum_{\mathtt{j}=1}^{\mathtt{m}} \mathtt{h}_{\mathtt{a}}(\varrho_{\mathtt{i}}(\mathtt{t})), \mathtt{i} = \mathbb{1}, \mathbb{2}, ..., \mathtt{m}, \\ \mathscr{D}^\alpha \chi_{\mathtt{j}}(\mathtt{t}) & = -\mathtt{m}_{\mathtt{j}}\chi_{\mathtt{j}}(\mathtt{t}) + \sum_{\mathtt{i}=1}^{\mathtt{m}} \mathtt{g}_{\mathtt{n}}(\chi_{\mathtt{j}}(\mathtt{t})) + \sum_{\mathtt{i}=1}^{\mathtt{m}} \mathtt{g}_{\mathtt{c}}(\chi_{\mathtt{j}}(\mathtt{t})) \\ & + \sum_{\mathtt{i}=1}^{\mathtt{m}} \mathtt{g}_{\mathtt{e}}(\chi_{\mathtt{j}}(\mathtt{t})), \mathtt{j} = \mathbb{1}, \mathbb{2}, ..., \mathtt{n}. \end{cases} \tag{3}$$

The compact form is

$$\begin{cases} \mathscr{D}^\alpha \varrho(\mathtt{t}) & = -\mathscr{R}\varrho(\mathtt{t}) + \mathscr{H}_{\mathtt{d}}(\varrho(\mathtt{t})) + \mathscr{H}_{\mathtt{b}}(\varrho(\mathtt{t})) + \mathscr{H}_{\mathtt{a}}(\varrho(\mathtt{t})), \\ \mathscr{D}^\alpha \chi(\mathtt{t}) & = -\mathscr{M}\chi(\mathtt{t}) + \mathscr{G}_{\mathtt{m}}(\chi(\mathtt{t})) + \mathscr{G}_{\mathtt{c}}(\chi(\mathtt{t})) + \mathscr{G}_{\mathtt{e}}(\chi(\mathtt{t})), \end{cases} \tag{4}$$

The system (4) initial condition is considered to be

$$\varrho(\mathtt{s}) = \Psi(\mathtt{s}) \in \mathscr{R}^{\mathtt{m}}, \chi(\mathtt{s}) = \phi(\mathtt{s}) \in \mathscr{R}^{\mathtt{m}}. \tag{5}$$

Theorem 1. *The equilibrium of system (4) is globally robust and asymptotically stable if there exist real matrices $\mathscr{P}_1$, $\mathscr{Q}_1$, such that:*

$$\begin{aligned} &-2\mathscr{P}_1\mathscr{R} + \mathscr{P}_1\mathscr{U}_1\mathscr{P}_1 + \mu\mathscr{D}^T\mathscr{U}_1^{-1}\mathscr{D}\mu + \mathscr{P}_1\mathscr{U}_2\mathscr{P}_1 \\ &+ \mu\mathscr{B}^T\mathscr{U}_2^{-1}\mathscr{B}\mu + \mathscr{P}_1\mathscr{U}_3\mathscr{P}_1 + \mu\mathscr{A}^T\mathscr{U}_3^{-1}\mathscr{A}\mu < 0, \end{aligned} \tag{6}$$

$$\begin{aligned} &-2\mathscr{Q}_1\mathscr{M} + \mathscr{Q}_1\mathscr{U}_4\mathscr{Q}_1 + \beta\mathscr{N}^T\mathscr{U}_4^{-1}\mathscr{N}\beta + \mathscr{Q}_1\mathscr{U}_5\mathscr{Q}_1 \\ &+ \beta\mathscr{C}^T\mathscr{U}_5^{-1}\mathscr{C}\beta + \mathscr{Q}_1\mathscr{U}_6\mathscr{Q}_1 + \beta\mathscr{E}^T\mathscr{U}_6^{-1}\mathscr{E}\beta < 0 \end{aligned} \tag{7}$$

Proof. Define a Lyapunov functional

$$\mathscr{V}(\mathtt{t}) = \sum_{\mathtt{k}=1}^{2} \mathscr{V}_{\mathtt{k}}(\mathtt{t}), \tag{8}$$

where,

$$\begin{aligned} \mathscr{V}_1(\mathtt{t}) &= \mathscr{D}^{-(1-\alpha)}[\varrho^T(\mathtt{t})\mathscr{P}_1\varrho(\mathtt{t})], \\ \mathscr{V}_2(\mathtt{t}) &= \mathscr{D}^{-(1-\alpha)}[\chi^T(\mathtt{t})\mathscr{Q}_1\chi(\mathtt{t})], \end{aligned}$$

Taking the time derivative of $\mathscr{V}(\mathtt{t})$ along the trajectories of (4) and using Lemma 2, we obtain

$$\begin{aligned} \dot{\mathscr{V}_1}(\mathtt{t}) &= 2\varrho^T(\mathtt{t})\mathscr{P}_1\{-\mathscr{R}\varrho(\mathtt{t}) + \mathscr{H}_{\mathtt{d}}(\varrho(\mathtt{t})) + \mathscr{H}_{\mathtt{b}}(\varrho(\mathtt{t})) + \mathscr{H}_{\mathtt{a}}(\varrho(\mathtt{t}))\} \\ \dot{\mathscr{V}_2}(\mathtt{t}) &= 2\chi^T(\mathtt{t})\mathscr{P}_2\{-\mathscr{M}\chi(\mathtt{t}) + \mathscr{G}_{\mathtt{m}}(\chi(\mathtt{t})) + \mathscr{G}_{\mathtt{c}}(\chi(\mathtt{t})) + \mathscr{G}_{\mathtt{e}}(\chi(\mathtt{t}))\} \end{aligned}$$

$$\begin{aligned} 2\varrho^T(\mathtt{t})\mathscr{P}_1\mathscr{H}_{\mathtt{d}}(\varrho(\mathtt{t})) &\le \varrho^T(\mathtt{t})\mathscr{P}_1\mathscr{U}_1\mathscr{P}_1\varrho(\mathtt{t}) \\ &+ \varrho^T(\mathtt{t})\mu\mathscr{D}^T\mathscr{U}_1^{-1}\mathscr{D}\mu\varrho(\mathtt{t}), \\ 2\varrho^T(\mathtt{t})\mathscr{P}_1\mathscr{H}_{\mathtt{b}}(\varrho(\mathtt{t})) &\le \varrho^T(\mathtt{t})\mathscr{P}_1\mathscr{U}_2\mathscr{P}_1\varrho(\mathtt{t}) \\ &+ \varrho^T(\mathtt{t})\mu\mathscr{B}^T\mathscr{U}_2^{-1}\mathscr{B}\mu\varrho(\mathtt{t}), \\ 2\varrho^T(\mathtt{t})\mathscr{P}_1\mathscr{H}_{\mathtt{a}}(\varrho(\mathtt{t})) &\le \varrho^T(\mathtt{t})\mathscr{P}_1\mathscr{U}_3\mathscr{P}_1\varrho(\mathtt{t}) \\ &+ \varrho^T(\mathtt{t})\mu\mathscr{A}^T\mathscr{U}_3^{-1}\mathscr{A}\mu\varrho(\mathtt{t}), \\ 2\chi^T(\mathtt{t})\mathscr{Q}_1\mathscr{G}_{\mathtt{m}}(\chi(\mathtt{t})) &\le \chi^T(\mathtt{t})\mathscr{Q}_1\mathscr{U}_4\mathscr{Q}_1\chi(\mathtt{t}) \\ &+ \chi^T(\mathtt{t})\beta\mathscr{N}^T\mathscr{U}_4^{-1}\mathscr{N}\beta\chi(\mathtt{t}), \\ 2\chi^T(\mathtt{t})\mathscr{Q}_1\mathscr{G}_{\mathtt{c}}(\chi(\mathtt{t})) &\le \chi^T(\mathtt{t})\mathscr{Q}_1\mathscr{U}_5\mathscr{Q}_1\chi(\mathtt{t}) \\ &+ \chi^T(\mathtt{t})\beta\mathscr{C}^T\mathscr{U}_5^{-1}\mathscr{C}\beta\chi(\mathtt{t}), \\ 2\chi^T(\mathtt{t})\mathscr{Q}_1\mathscr{G}_{\mathtt{e}}(\chi(\mathtt{t})) &\le \chi^T(\mathtt{t})\mathscr{Q}_1\mathscr{U}_6\mathscr{Q}_1\chi(\mathtt{t}) \\ &+ \chi^T(\mathtt{t})\beta\mathscr{E}^T\mathscr{U}_6^{-1}\mathscr{E}\beta\chi(\mathtt{t}), \end{aligned}$$

Substituting, we obtain

$$\dot{\mathscr{V}}((\mathtt{t})) \leq \varrho^T(\mathtt{t})\bigg\{ -2\mathscr{P}_1\mathscr{R} + \mathscr{P}_1\mathscr{U}_1\mathscr{P}_1 + \mu\mathscr{D}^T\mathscr{U}_1^{-1}\mathscr{D}\mu + \mathscr{P}_1\mathscr{U}_2\mathscr{P}_1 + \mu\mathscr{B}^T\mathscr{U}_2^{-1}\mathscr{B}\mu + \mathscr{P}_1\mathscr{U}_3\mathscr{P}_1 + \mu\mathscr{A}^T\mathscr{U}_3^{-1}\mathscr{A}\mu\bigg\}\varrho(\mathtt{t}) + \chi^T(\mathtt{t})\bigg\{ -2\mathscr{Q}_1\mathscr{M} + \mathscr{Q}_1\mathscr{U}_4\mathscr{Q}_1 + \beta\mathscr{N}^T\mathscr{U}_4^{-1}\mathscr{N}\beta + \mathscr{Q}_1\mathscr{U}_5\mathscr{Q}_1 + \beta\mathscr{C}^T\mathscr{U}_5^{-1}\mathscr{C}\beta + \mathscr{Q}_1\mathscr{U}_6\mathscr{Q}_1 + \beta\mathscr{E}^T\mathscr{U}_6^{-1}\mathscr{E}\beta\bigg\}\chi(\mathtt{t})$$

$\dot{\mathscr{V}}(\mathtt{t}) < 0$. As a result, the equilibrium point of (4) is determined to be globally robustly stable. The proof is now completed. □

Remark 1. *System (1) is the drive system, followed by the response system being*

$$\begin{cases} \mathscr{D}^\alpha\varsigma_{\mathtt{i}}(\mathtt{t}) & = -\mathtt{r}_{\mathtt{i}}\varsigma_{\mathtt{i}}(\mathtt{t}) + \sum_{\mathtt{j}=1}^{\mathtt{m}} \mathtt{d}_{\mathtt{ji}}(\varsigma_{\mathtt{i}}(\mathtt{t}))\mathtt{h}_{\mathtt{j}}(\mathtt{s}_{\mathtt{j}}(\mathtt{t})) \\ & \quad + \sum_{\mathtt{j}=1}^{\mathtt{m}} \mathtt{b}_{\mathtt{ji}}(\varsigma_{\mathtt{i}}(\mathtt{t}))\mathtt{h}_{\mathtt{j}}(\mathtt{s}_{\mathtt{j}}(\mathtt{t}-\sigma_1(\mathtt{t})-\sigma_2(\mathtt{t}))) \\ & \quad + \sum_{\mathtt{j}=1}^{\mathtt{m}} \mathtt{a}_{\mathtt{ji}}(\varsigma_{\mathtt{i}}(\mathtt{t}))\int_{\mathtt{t}-\tau(\mathtt{t})}^{\mathtt{t}} \mathtt{h}_{\mathtt{j}}(\mathtt{s}_{\mathtt{j}}(\mathtt{s}))\mathtt{ds} + \mathscr{I}_{\mathtt{i}}(\mathtt{t}), \\ \mathscr{D}^\alpha\mathtt{s}_{\mathtt{j}}(\mathtt{t}) & = -\mathtt{m}_{\mathtt{j}}\mathtt{s}_{\mathtt{j}}(\mathtt{t}) + \sum_{\mathtt{i}=1}^{\mathtt{m}} \mathtt{n}_{\mathtt{ij}}(\mathtt{s}_{\mathtt{j}}(\mathtt{t}))\mathtt{g}_{\mathtt{i}}(\varsigma_{\mathtt{i}}(\mathtt{t})) \\ & \quad + \sum_{\mathtt{i}=1}^{\mathtt{m}} \mathtt{c}_{\mathtt{ij}}(\mathtt{s}_{\mathtt{j}}(\mathtt{t}))\mathtt{g}_{\mathtt{i}}(\varsigma_{\mathtt{i}}(\mathtt{t}-\eta_1(\mathtt{t})-\eta_2(\mathtt{t}))) \\ & \quad + \sum_{\mathtt{i}=1}^{\mathtt{m}} \mathtt{e}_{\mathtt{ij}}(\mathtt{s}_{\mathtt{j}}(\mathtt{t}))\int_{\mathtt{t}-\upsilon(\mathtt{t})}^{\mathtt{t}} \mathtt{g}_{\mathtt{i}}(\varsigma_{\mathtt{i}}(\mathtt{s}))\mathtt{ds} + \mathscr{J}_{\mathtt{j}}(\mathtt{t}). \end{cases} \quad (9)$$

The memristive connection weights $\mathtt{d}_{\mathtt{ji}}(\wp_{\mathtt{i}}(\mathtt{t}))$, $\mathtt{b}_{\mathtt{ji}}(\wp_{\mathtt{i}}(\mathtt{t}))$, $\mathtt{a}_{\mathtt{ji}}(\wp_{\mathtt{i}}(\mathtt{t}))$, $\mathtt{n}_{\mathtt{ij}}(\Im_{\mathtt{j}}(\mathtt{t}))$, $\mathtt{c}_{\mathtt{ij}}(\Im_{\mathtt{j}}(\mathtt{t}))$, $\mathtt{e}_{\mathtt{ij}}(\Im_{\mathtt{j}}(\mathtt{t}))$, $\mathtt{d}_{\mathtt{ji}}(\varsigma_{\mathtt{i}}(\mathtt{t}))$, $\mathtt{b}_{\mathtt{ji}}(\varsigma_{\mathtt{i}}(\mathtt{t}))$, $\mathtt{a}_{\mathtt{ji}}\varsigma_{\mathtt{i}}(\mathtt{t}))$, $\mathtt{n}_{\mathtt{ij}}(\mathtt{s}_{\mathtt{j}}(\mathtt{t}))$, $\mathtt{c}_{\mathtt{ij}}(\mathtt{s}_{\mathtt{j}}(\mathtt{t}))$, $\mathtt{e}_{\mathtt{ij}}(\mathtt{s}_{\mathtt{j}}(\mathtt{t}))$ *will change with time. Then, we let*

$$\mathtt{d}_{\mathtt{ji}}(\wp_{\mathtt{i}}(\mathtt{t})) = \begin{cases} \hat{\mathtt{d}}_{\mathtt{ji}}, & |\wp_{\mathtt{i}}(\mathtt{t})| \leq \mathscr{T}_{\mathtt{i}}, \\ \check{\mathtt{d}}_{\mathtt{ji}}, & |\wp_{\mathtt{i}}(\mathtt{t})| > \mathscr{T}_{\mathtt{i}}, \end{cases}$$

$$\mathtt{a}_{\mathtt{ji}}(\wp_{\mathtt{i}}(\mathtt{t})) = \begin{cases} \hat{\mathtt{a}}_{\mathtt{ji}}, & |\wp_{\mathtt{i}}(\mathtt{t})| \leq \mathscr{T}_{\mathtt{i}}, \\ \check{\mathtt{a}}_{\mathtt{ji}}, & |\wp_{\mathtt{i}}(\mathtt{t})| > \mathscr{T}_{\mathtt{i}}, \end{cases}$$

$$\mathtt{b}_{\mathtt{ji}}(\wp_{\mathtt{i}}(\mathtt{t})) = \begin{cases} \hat{\mathtt{b}}_{\mathtt{ji}}, & |\wp_{\mathtt{i}}(\mathtt{t})| \leq \mathscr{T}_{\mathtt{i}}, \\ \check{\mathtt{b}}_{\mathtt{ji}}, & |\wp_{\mathtt{i}}(\mathtt{t})| > \mathscr{T}_{\mathtt{i}}, \end{cases}$$

$$\mathtt{d}_{\mathtt{ji}}(\varsigma_{\mathtt{i}}(\mathtt{t})) = \begin{cases} \hat{\mathtt{d}}_{\mathtt{ji}}, & |\varsigma_{\mathtt{i}}(\mathtt{t})| \leq \mathscr{T}_{\mathtt{i}}, \\ \check{\mathtt{d}}_{\mathtt{ji}}, & |\varsigma_{\mathtt{i}}(\mathtt{t})| > \mathscr{T}_{\mathtt{i}}, \end{cases}$$

$$\mathtt{a}_{\mathtt{ji}}(\varsigma_{\mathtt{i}}(\mathtt{t}) = \begin{cases} \hat{\mathtt{a}}_{\mathtt{ji}}, & |\varsigma_{\mathtt{i}}(\mathtt{t})| \leq \mathscr{T}_{\mathtt{i}}, \\ \check{\mathtt{a}}_{\mathtt{ji}}, & |\varsigma_{\mathtt{i}}(\mathtt{t})| > \mathscr{T}_{\mathtt{i}}, \end{cases}$$

$$\mathtt{b}_{\mathtt{ji}}(\varsigma_{\mathtt{i}}(\mathtt{t})) = \begin{cases} \hat{\mathtt{b}}_{\mathtt{ji}}, & |\varsigma_{\mathtt{i}}(\mathtt{t})| \leq \mathscr{T}_{\mathtt{i}}, \\ \check{\mathtt{b}}_{\mathtt{ji}}, & |\varsigma_{\mathtt{i}}(\mathtt{t})| > \mathscr{T}_{\mathtt{i}}, \end{cases}$$

$$\mathtt{n}_{\mathtt{ij}}(\Im_{\mathtt{j}}(\mathtt{t})) = \begin{cases} \hat{\mathtt{n}}_{\mathtt{ij}}, & |\Im_{\mathtt{j}}(\mathtt{t})| \leq \mathscr{W}_{\mathtt{j}}, \\ \check{\mathtt{n}}_{\mathtt{ij}}, & |\Im_{\mathtt{j}}(\mathtt{t})| > \mathscr{W}_{\mathtt{j}}, \end{cases}$$

$$\mathtt{c}_{\mathtt{ij}}(\Im_{\mathtt{j}}(\mathtt{t})) = \begin{cases} \hat{\mathtt{c}}_{\mathtt{ij}}, & |\Im_{\mathtt{j}}(\mathtt{t})| \leq \mathscr{W}_{\mathtt{j}}, \\ \check{\mathtt{c}}_{\mathtt{ij}}, & |\Im_{\mathtt{j}}(\mathtt{t})| > \mathscr{W}_{\mathtt{j}}, \end{cases}$$

$$\mathtt{e}_{\mathtt{ij}}(\Im_{\mathtt{j}}(\mathtt{t})) = \begin{cases} \hat{\mathtt{e}}_{\mathtt{ij}}, & |\Im_{\mathtt{j}}(\mathtt{t})| \leq \mathscr{W}_{\mathtt{j}}, \\ \check{\mathtt{e}}_{\mathtt{ij}}, & |\Im_{\mathtt{j}}(\mathtt{t})| > \mathscr{W}_{\mathtt{j}}, \end{cases}$$

$$\mathtt{n}_{\mathtt{ij}}(\mathtt{s}_{\mathtt{j}}(\mathtt{t})) = \begin{cases} \hat{\mathtt{n}}_{\mathtt{ij}}, & |\mathtt{s}_{\mathtt{j}}(\mathtt{t})| \leq \mathscr{W}_{\mathtt{j}}, \\ \check{\mathtt{n}}_{\mathtt{ij}}, & |\mathtt{s}_{\mathtt{j}}(\mathtt{t})| > \mathscr{W}_{\mathtt{j}}, \end{cases}$$

$$\mathtt{c}_{\mathtt{ij}}(\mathtt{s}_{\mathtt{j}}(\mathtt{t})) = \begin{cases} \hat{\mathtt{c}}_{\mathtt{ij}}, & |\mathtt{s}_{\mathtt{j}}(\mathtt{t})| \leq \mathscr{W}_{\mathtt{j}}, \\ \check{\mathtt{c}}_{\mathtt{ij}}, & |\mathtt{s}_{\mathtt{j}}(\mathtt{t})| > \mathscr{W}_{\mathtt{j}}, \end{cases}$$

$$\mathtt{e}_{\mathtt{ij}}(\mathtt{s}_{\mathtt{j}}(\mathtt{t})) = \begin{cases} \hat{\mathtt{e}}_{\mathtt{ij}}, & |\mathtt{s}_{\mathtt{j}}(\mathtt{t})| \leq \mathscr{W}_{\mathtt{j}}, \\ \check{\mathtt{e}}_{\mathtt{ij}}, & |\mathtt{s}_{\mathtt{j}}(\mathtt{t})| > \mathscr{W}_{\mathtt{j}}. \end{cases}$$

Using the differential inclusion theory, the above systems can be rewritten as

$$\begin{cases} \mathscr{D}^\alpha \wp_{\mathtt{i}}(\mathtt{t}) & = -\mathtt{r}_{\mathtt{i}}\wp_{\mathtt{i}}(\mathtt{t}) + \sum_{\mathtt{j}=1}^{\mathtt{m}} co[\mathtt{d}_{\mathtt{ji}}(\wp_{\mathtt{i}}(\mathtt{t}))]\mathtt{h}_{\mathtt{j}}(\Im_{\mathtt{j}}(\mathtt{t})) \\ & + \sum_{\mathtt{j}=1}^{\mathtt{m}} co[\mathtt{b}_{\mathtt{ji}}(\wp_{\mathtt{i}}(\mathtt{t}))]\mathtt{h}_{\mathtt{j}}(\Im_{\mathtt{j}}(\mathtt{t}-\sigma_1(\mathtt{t})-\sigma_2(\mathtt{t}))) \\ & + \sum_{\mathtt{j}=1}^{n} co[\mathtt{a}_{\mathtt{ji}}(\wp_{\mathtt{i}}(\mathtt{t}))]\int_{\mathtt{t}-\tau(\mathtt{t})}^{\mathtt{t}} \mathtt{h}_{\mathtt{j}}(\Im_{\mathtt{j}}(\mathtt{s}))d\mathtt{s} + \mathscr{I}_{\mathtt{i}}(\mathtt{t}), \\ \mathscr{D}^\alpha \Im_{\mathtt{j}}(\mathtt{t}) & = -\mathtt{m}_{\mathtt{j}}\Im_{\mathtt{j}}(\mathtt{t}) + \sum_{\mathtt{i}=1}^{\mathtt{m}} co[\mathtt{n}_{\mathtt{ij}}(\Im_{\mathtt{j}}(\mathtt{t}))]\mathtt{g}_{\mathtt{i}}(\wp_{\mathtt{i}}(\mathtt{t})) \\ & + \sum_{\mathtt{i}=1}^{\mathtt{m}} co[\mathtt{c}_{\mathtt{ij}}(\Im_{\mathtt{j}}(\mathtt{t}))]\mathtt{g}_{\mathtt{i}}(\wp_{\mathtt{i}}(\mathtt{t}-\eta_1(\mathtt{t})-\eta_2(\mathtt{t}))) \\ & + \sum_{i=1}^{\mathtt{m}} co[\mathtt{e}_{\mathtt{ij}}(\Im_{\mathtt{j}}(\mathtt{t}))]\int_{\mathtt{t}-\upsilon(\mathtt{t})}^{\mathtt{t}} \mathtt{g}_{\mathtt{i}}(\wp_{\mathtt{i}}(\mathtt{s}))d\mathtt{s} + \mathscr{J}_{\mathtt{j}}(\mathtt{t}). \end{cases} \quad (10)$$

and

$$\begin{cases} \mathscr{D}^\alpha \varsigma_{\mathtt{i}}(\mathtt{t}) & = -\mathtt{r}_{\mathtt{i}}\varsigma_{\mathtt{i}}(\mathtt{t}) + \sum_{\mathtt{j}=1}^{\mathtt{m}} co[\mathtt{d}_{\mathtt{ji}}\varsigma_{\mathtt{i}}(\mathtt{t}))]\mathtt{h}_{\mathtt{j}}(\mathtt{s}_{\mathtt{j}}(\mathtt{t})) \\ & + \sum_{\mathtt{j}=1}^{\mathtt{m}} co[\mathtt{b}_{\mathtt{ji}}(\varsigma_{\mathtt{i}}(\mathtt{t}))]\mathtt{h}_{\mathtt{j}}(\mathtt{s}_{\mathtt{j}}(\mathtt{t}-\sigma_1(\mathtt{t})-\sigma_2(\mathtt{t}))) \\ & + \sum_{\mathtt{j}=1}^{n} co[\mathtt{a}_{\mathtt{ji}}(\varsigma_{\mathtt{i}}(\mathtt{t}))]\int_{\mathtt{t}-\tau(\mathtt{t})}^{\mathtt{t}} \mathtt{h}_{\mathtt{j}}(\mathtt{s}_{\mathtt{j}}(\mathtt{s}))d\mathtt{s} \\ & + \mathscr{I}_{\mathtt{i}}(\mathtt{t}) + \mathbb{U}_{\mathtt{i}}(\mathtt{t}), \\ \mathscr{D}^\alpha \mathtt{s}_{\mathtt{j}}(\mathtt{t}) & = -\mathtt{m}_{\mathtt{j}}\mathtt{s}_{\mathtt{j}}(\mathtt{t}) + \sum_{\mathtt{i}=1}^{\mathtt{m}} co[\mathtt{n}_{\mathtt{ij}}(\mathtt{s}_{\mathtt{j}}(\mathtt{t}))]\mathtt{g}_{\mathtt{i}}\varsigma_{\mathtt{i}}(\mathtt{t})) \\ & + \sum_{\mathtt{i}=1}^{\mathtt{m}} co[\mathtt{c}_{\mathtt{ij}}(\mathtt{s}_{\mathtt{j}}(\mathtt{t}))]\mathtt{g}_{\mathtt{i}}(\varsigma_{\mathtt{i}}(\mathtt{t}-\eta_1(\mathtt{t})-\eta_2(\mathtt{t}))) \\ & + \sum_{\mathtt{i}=1}^{\mathtt{m}} co[\mathtt{e}_{\mathtt{ij}}(\mathtt{s}_{\mathtt{j}}(\mathtt{t}))]\int_{\mathtt{t}-\upsilon(\mathtt{t})}^{\mathtt{t}} \mathtt{g}_{\mathtt{i}}(\varsigma_{\mathtt{i}}(\mathtt{s}))d\mathtt{s} \\ & + \mathscr{J}_{\mathtt{j}}(\mathtt{t}) + \mathbb{V}_{\mathtt{i}}(\mathtt{t}). \end{cases} \quad (11)$$

with

$$co[\mathtt{d}_{\mathtt{ji}}(\wp_{\mathtt{i}}(\mathtt{t}))] = \begin{cases} \hat{\mathtt{d}}_{\mathtt{ji}}, & |\wp_{\mathtt{i}}(\mathtt{t})| < \mathscr{T}_{\mathtt{i}}, \\ co\{\hat{\mathtt{d}}_{\mathtt{ji}}, \check{\mathtt{d}}_{\mathtt{ji}}\}, & |\wp_{\mathtt{i}}(\mathtt{t})| = \mathscr{T}_{\mathtt{i}}, \\ \check{\mathtt{d}}_{\mathtt{ji}}, & |\wp_{\mathtt{i}}(\mathtt{t})| > \mathscr{T}_{\mathtt{i}}, \end{cases}$$

$$co[\mathtt{a}_{\mathtt{ji}}(\wp_{\mathtt{i}}(\mathtt{t}))] = \begin{cases} \hat{\mathtt{a}}_{\mathtt{ji}}, & |\wp_{\mathtt{i}}(\mathtt{t})| < \mathscr{T}_{\mathtt{i}}, \\ co\{\hat{\mathtt{a}}_{\mathtt{ji}}, \check{\mathtt{a}}_{\mathtt{ji}}\}, & |\wp_{\mathtt{i}}(\mathtt{t})| = \mathscr{T}_{\mathtt{i}}, \\ \check{\mathtt{a}}_{\mathtt{ji}}, & |\wp_{\mathtt{i}}(\mathtt{t})| > \mathscr{T}_{\mathtt{i}}, \end{cases}$$

$$\mathtt{co}[\mathtt{b}_{\mathtt{ji}}(\wp_{\mathtt{i}}(\mathtt{t}))] = \begin{cases} \hat{\mathtt{b}}_{\mathtt{ji}}, & |\wp_{\mathtt{i}}(\mathtt{t})| < \mathscr{T}_{\mathtt{i}}, \\ co\{\hat{\mathtt{b}}_{\mathtt{ji}}, \check{\mathtt{b}}_{\mathtt{ji}}\}, & |\wp_{\mathtt{i}}(\mathtt{t})| = \mathscr{T}_{\mathtt{i}}, \\ \check{\mathtt{b}}_{\mathtt{ji}}, & |\wp_{\mathtt{i}}(\mathtt{t})| > \mathscr{T}_{\mathtt{i}}, \end{cases}$$

$$co[\mathtt{d}_{\mathtt{ji}}(\varsigma_{\mathtt{i}}(\mathtt{t}))] = \begin{cases} \hat{\mathtt{d}}_{\mathtt{ji}}, & |\varsigma_{\mathtt{i}}(\mathtt{t})| < \mathscr{T}_{\mathtt{i}}, \\ co\{\hat{\mathtt{d}}_{\mathtt{ji}}, \check{\mathtt{d}}_{\mathtt{ij}}\}, & |\varsigma_{\mathtt{i}}(\mathtt{t})| = \mathscr{T}_{\mathtt{i}}, \\ \check{\mathtt{d}}_{\mathtt{ji}}, & |\varsigma_{\mathtt{i}}(\mathtt{t})| > \mathscr{T}_{\mathtt{i}}, \end{cases}$$

$$co[\mathtt{a}_{\mathtt{ji}}(\varsigma_{\mathtt{i}}(\mathtt{t}))] = \begin{cases} \hat{\mathtt{a}}_{\mathtt{ji}}, & |\varsigma_{\mathtt{i}}(\mathtt{t})| < \mathscr{T}_{\mathtt{i}}, \\ co\hat{\mathtt{a}}_{\mathtt{ji}}, \check{\mathtt{a}}_{\mathtt{ji}}\}, & |\varsigma_{\mathtt{i}}(\mathtt{t})| = \mathscr{T}_{\mathtt{i}}, \\ \check{\mathtt{a}}_{\mathtt{ji}}, & |\varsigma_{\mathtt{i}}(\mathtt{t})| > \mathscr{T}_{\mathtt{i}}, \end{cases}$$

$$co[\mathtt{b}_{\mathtt{ji}}(\varsigma_{\mathtt{i}}(\mathtt{t}))] = \begin{cases} \hat{\mathtt{b}}_{\mathtt{ji}}, & |\varsigma_{\mathtt{i}}(\mathtt{t})| < \mathscr{T}_{\mathtt{i}}, \\ co\{\hat{\mathtt{b}}_{\mathtt{ji}}, \check{\mathtt{b}}_{\mathtt{ji}}\}, & |\varsigma_{\mathtt{i}}(\mathtt{t})| = \mathscr{T}_{\mathtt{i}}, \\ \check{\mathtt{b}}_{\mathtt{ji}}, & |\varsigma_{\mathtt{i}}(\mathtt{t})| > \mathscr{T}_{\mathtt{i}}, \end{cases}$$

$$co[\mathtt{n}_{\mathtt{ij}}(\Im_{\mathtt{j}}(\mathtt{t}))] = \begin{cases} \hat{\mathtt{n}}_{\mathtt{ij}}, & |\Im_{\mathtt{j}}(\mathtt{t})| < \mathscr{W}_{\mathtt{j}}, \\ co\{\hat{\mathtt{n}}_{\mathtt{ij}}, \check{\mathtt{n}}_{\mathtt{ij}}\}, & |\Im_{\mathtt{j}}(\mathtt{t})| = \mathscr{W}_{\mathtt{j}}, \\ \check{\mathtt{n}}_{\mathtt{ij}}, & |\Im_{\mathtt{i}}(\mathtt{t})| > \mathscr{W}_{\mathtt{j}}, \end{cases}$$

$$co[\mathtt{c}_{\mathtt{ij}}(\Im_{\mathtt{j}}(\mathtt{t}))] = \begin{cases} \hat{\mathtt{c}}_{\mathtt{ij}}, & |\Im_{\mathtt{j}}(\mathtt{t})| < \mathscr{W}_{\mathtt{j}}, \\ co\{\hat{\mathtt{c}}_{\mathtt{ij}}, \check{\mathtt{c}}_{\mathtt{ij}}\}, & |\Im_{\mathtt{j}}(\mathtt{t})| = \mathscr{W}_{\mathtt{j}}, \\ \check{\mathtt{c}}_{\mathtt{ij}}, & |\Im_{\mathtt{j}}(\mathtt{t})| > \mathscr{W}_{\mathtt{j}}, \end{cases}$$

$$co[\mathtt{e}_{\mathtt{ij}}(\Im_{\mathtt{j}}(\mathtt{t}))] = \begin{cases} \hat{\mathtt{e}}_{\mathtt{ij}}, & |\Im_{\mathtt{j}}(\mathtt{t})| < \mathscr{W}_{\mathtt{j}}, \\ co\{\hat{\mathtt{e}}_{\mathtt{ij}}, \check{\mathtt{e}}_{\mathtt{ij}}\}, & |\Im_{\mathtt{j}}(\mathtt{t})| = \mathscr{W}_{\mathtt{j}}, \\ \check{\mathtt{e}}_{\mathtt{ij}}, & |\Im_{\mathtt{j}}(\mathtt{t})| > \mathscr{W}_{\mathtt{j}}, \end{cases}$$

$$co[\mathtt{n}_{\mathtt{ij}}(\mathtt{s}_{\mathtt{j}}(\mathtt{t}))] = \begin{cases} \hat{\mathtt{n}}_{\mathtt{ij}}, & |\mathtt{s}_{\mathtt{j}}(\mathtt{t})| < \mathscr{W}_{\mathtt{j}}, \\ co\{\hat{\mathtt{n}}_{\mathtt{ij}}, \check{\mathtt{n}}_{\mathtt{ij}}\}, & |\mathtt{s}_{\mathtt{j}}(\mathtt{t})| = \mathscr{W}_{\mathtt{j}}, \\ \check{\mathtt{n}}_{\mathtt{ij}}, & |\mathtt{s}_{\mathtt{j}}(\mathtt{t})| > \mathscr{W}_{\mathtt{j}}, \end{cases}$$

$$co[\mathtt{c}_{\mathtt{ij}}(\mathtt{s}_{\mathtt{j}}(\mathtt{t}))] = \begin{cases} \hat{\mathtt{c}}_{\mathtt{ij}}, & |\mathtt{s}_{\mathtt{j}}(\mathtt{t})| < \mathscr{W}_{\mathtt{j}}, \\ co\{\hat{\mathtt{c}}_{\mathtt{ij}}, \check{\mathtt{n}}_{\mathtt{ij}}\}, & |\mathtt{s}_{\mathtt{j}}(\mathtt{t})| = \mathscr{W}_{\mathtt{j}}, \\ \check{\mathtt{n}}_{\mathtt{ij}}, & |\mathtt{s}_{\mathtt{j}}(\mathtt{t})| > \mathscr{W}_{\mathtt{j}}, \end{cases}$$

$$co[\mathtt{e}_{\mathtt{ij}}(\mathtt{s}_{\mathtt{j}}(\mathtt{t}))] = \begin{cases} \hat{\mathtt{e}}_{\mathtt{ij}}, & |\mathtt{s}_{\mathtt{j}}(\mathtt{t})| < \mathscr{W}_{\mathtt{j}}, \\ \mathtt{co}\{\hat{\mathtt{e}}_{\mathtt{ij}}, \check{\mathtt{e}}_{\mathtt{ij}}\}, & |\mathtt{s}_{\mathtt{j}}(\mathtt{t})| = \mathscr{W}_{\mathtt{j}}, \\ \check{\mathtt{e}}_{\mathtt{ij}}, & |\mathtt{s}_{\mathtt{j}}(\mathtt{t})| > \mathscr{W}_{\mathtt{j}}. \end{cases}$$

For the sake of convenience, let

$$\begin{aligned}
co[\mathtt{d}_{\mathtt{ji}}(\wp_{\mathtt{i}}(\mathtt{t}))]\mathbb{h}_{\mathtt{j}}(\Im_{\mathtt{j}}(\mathtt{t})) &= \mathbb{h}_{\mathtt{d}}(\wp_{\mathtt{i}}(\mathtt{t})), \\
co[\mathtt{b}_{\mathtt{ji}}(\wp_{\mathtt{i}}(\mathtt{t}))]\mathbb{h}_{\mathtt{j}}(\Im_{\mathtt{j}}(\mathtt{t}-\sigma_1(\mathtt{t})-\sigma_2(\mathtt{t}))) &= \mathbb{h}_{\mathtt{b}}(\wp_{\mathtt{i}}(\mathtt{t})), \\
co[\mathtt{a}_{\mathtt{ji}}(\wp_{\mathtt{i}}(\mathtt{t}))]\int_{\mathtt{t}-\tau(\mathtt{t})}^{\mathtt{t}} \mathbb{h}_{\mathtt{j}}(\Im_{\mathtt{j}}(\mathtt{s}))\mathtt{ds} &= \mathbb{h}_{\mathtt{a}}(\wp_{\mathtt{i}}(\mathtt{t})), \\
co[\mathtt{d}_{\mathtt{ji}}(\varsigma_{\mathtt{i}}(\mathtt{t}))]\mathbb{h}_{\mathtt{j}}(\mathtt{s}_{\mathtt{j}}(\mathtt{t})) &= \mathbb{h}_{\mathtt{d}}(\varsigma_{\mathtt{i}}(\mathtt{t})), \\
co[\mathtt{b}_{\mathtt{ji}}(\varsigma_{\mathtt{i}}(\mathtt{t}))]\mathbb{h}_{\mathtt{j}}(\mathtt{s}_{\mathtt{j}}(\mathtt{t}-\sigma_1(\mathtt{t}))-\sigma_2(\mathtt{t}))) &= \mathbb{h}_{\mathtt{b}}(\varsigma_{\mathtt{i}}(\mathtt{t})), \\
co[\mathtt{a}_{\mathtt{ji}}(\varsigma_{\mathtt{i}}(\mathtt{t}))]\int_{\mathtt{t}-\tau(\mathtt{t})}^{\mathtt{t}} \mathbb{h}_{\mathtt{j}}(\mathtt{s}_{\mathtt{j}}(\mathtt{s}))\mathtt{ds} &= \mathbb{h}_{\mathtt{a}}(\varsigma_{\mathtt{i}}(\mathtt{t})), \\
co[\mathtt{n}_{\mathtt{ij}}(\Im_{\mathtt{j}}(\mathtt{t}))]\mathtt{g}_{\mathtt{i}}(\varsigma_{\mathtt{i}}(\mathtt{t})) &= \mathtt{g}_{\mathtt{n}}(\Im_{\mathtt{j}}(\mathtt{t})), \\
co[\mathtt{c}_{\mathtt{ij}}(\Im_{\mathtt{j}}(\mathtt{t}))]\mathtt{g}_{\mathtt{i}}(\wp_{\mathtt{i}}(\mathtt{t}-\eta_1(\mathtt{t})-\eta_2(\mathtt{t}))) &= \mathtt{g}_{\mathtt{c}}(\Im_{\mathtt{j}}(\mathtt{t})), \\
co[\mathtt{e}_{\mathtt{ij}}(\Im_{\mathtt{j}}(\mathtt{t}))]\int_{\mathtt{t}-\upsilon(\mathtt{t})}^{\mathtt{t}} \mathtt{g}_{\mathtt{i}}(\wp_{\mathtt{i}}(\mathtt{s}))\mathtt{ds} &= \mathtt{g}_{\mathtt{e}}(\Im_{\mathtt{j}}(\mathtt{t})), \\
co[\mathtt{n}_{\mathtt{ij}}(\mathtt{s}_{\mathtt{j}}(\mathtt{t}))]\mathtt{g}_{\mathtt{i}}(\varsigma_{\mathtt{i}}(\mathtt{t})) &= \mathtt{g}_{\mathtt{n}}(\mathtt{s}_{\mathtt{j}}(\mathtt{t})), \\
co[\mathtt{c}_{\mathtt{ij}}(\mathtt{s}_{\mathtt{j}}(\mathtt{t}))]\mathtt{g}_{\mathtt{i}}(\varsigma_{\mathtt{i}}(\mathtt{t}-\eta_1(\mathtt{t})-\eta_2(\mathtt{t}))) &= \mathtt{g}_{\mathtt{c}}(\mathtt{s}_{\mathtt{j}}(\mathtt{t})), \\
co[\mathtt{e}_{\mathtt{ij}}(\mathtt{s}_{\mathtt{j}}(\mathtt{t}))]\int_{\mathtt{t}-\upsilon(\mathtt{t})}^{\mathtt{t}} \mathtt{g}_{\mathtt{i}}(\mathtt{r}_{\mathtt{i}}(\mathtt{s}))\mathtt{ds} &= \mathtt{g}_{\mathtt{e}}(\mathtt{s}_{\mathtt{j}}(\mathtt{t})).
\end{aligned}$$

Before processing our main results, we set $\varsigma_{\mathtt{i}}(\mathtt{t}) - \wp_{\mathtt{i}}(\mathtt{t}) = \varpi_{\mathtt{i}}(\mathtt{t})$ $\mathtt{s}_{\mathtt{j}}(\mathtt{t}) - \Im_{\mathtt{j}}(\mathtt{t}) = \psi_{\mathtt{j}}(\mathtt{t})$, *then, the synchronization error system is*

$$\begin{cases} \mathscr{D}^{\alpha}\varpi_{\mathtt{i}}(\mathtt{t}) & = -\mathtt{r}_{\mathtt{i}}\varpi_{\mathtt{i}}(\mathtt{t}) + \sum_{\mathtt{j}=1}^{\mathtt{m}} \mathbb{h}_{\mathtt{d}}(\varsigma_{\mathtt{i}}(\mathtt{t})) - \sum_{\mathtt{j}=1}^{\mathtt{m}} \mathbb{h}_{\mathtt{d}}(\wp_{\mathtt{i}}(\mathtt{t})) + \sum_{\mathtt{j}=1}^{\mathtt{m}} \mathbb{h}_{\mathtt{b}}(\varsigma_{\mathtt{i}}(\mathtt{t})) \\ & \quad - \sum_{\mathtt{j}=1}^{\mathtt{m}} \mathbb{h}_{\mathtt{b}}(\wp_{\mathtt{i}}(\mathtt{t})) + \sum_{\mathtt{j}=1}^{\mathtt{m}} \mathbb{h}_{\mathtt{a}}(\varsigma_{\mathtt{i}}(\mathtt{t})) - \sum_{\mathtt{j}=1}^{\mathtt{m}} \mathbb{h}_{\mathtt{a}}(\wp_{\mathtt{i}}(\mathtt{t})) + \mathbb{U}(\mathtt{t}), \\ \mathscr{D}^{\alpha}\psi_{\mathtt{j}}(\mathtt{t}) & = -\mathtt{m}_{\mathtt{j}}\psi_{\mathtt{j}}(\mathtt{t}) + \sum_{\mathtt{i}=1}^{\mathtt{m}} \mathtt{g}_{\mathtt{n}}(\mathtt{s}_{\mathtt{j}}(\mathtt{t})) - \sum_{\mathtt{i}=1}^{\mathtt{m}} \mathtt{g}_{\mathtt{n}}(\Im_{\mathtt{j}}(\mathtt{t})) + \sum_{\mathtt{i}=1}^{\mathtt{m}} \mathtt{g}_{\mathtt{c}}(\mathtt{s}_{\mathtt{j}}(\mathtt{t})) \\ & \quad - \sum_{\mathtt{j}=1}^{\mathtt{m}} \mathtt{g}_{\mathtt{c}}(\Im_{\mathtt{j}}(\mathtt{t})) + \sum_{\mathtt{i}=1}^{\mathtt{m}} \mathtt{g}_{\mathtt{e}}(\mathtt{s}_{\mathtt{j}}(\mathtt{t})) - \sum_{\mathtt{i}=1}^{\mathtt{m}} \mathtt{g}_{\mathtt{e}}(\Im_{\mathtt{j}}(\mathtt{t})) + \mathbb{V}(\mathtt{t}), \end{cases} \tag{12}$$

By using Hypothesis 1, we have,

$$\begin{aligned}
|\mathbb{h}_{\mathtt{d}}(\varsigma_{\mathtt{i}}(\mathtt{t})) - \mathbb{h}_{\mathtt{d}}(\wp_i(\mathtt{t}))| &\leq \mathtt{d}_{\mathtt{ji}}\mathtt{p}_{\mathtt{j}}|\mathtt{s}_{\mathtt{j}}(\mathtt{t}) - \Im_{\mathtt{j}}(\mathtt{t})|, \\
&\leq \mathtt{d}_{\mathtt{ji}}\mathtt{p}_{\mathtt{j}}|\psi_{\mathtt{j}}(\mathtt{t})|, \\
|\mathbb{h}_{\mathtt{b}}(\varsigma_{\mathtt{i}}(\mathtt{t})) - \mathbb{h}_{\mathtt{b}}(\wp_{\mathtt{i}}(\mathtt{t}))| &\leq \mathtt{b}_{\mathtt{ji}}\mathtt{p}_{\mathtt{j}}|\mathtt{s}_{\mathtt{j}}(\mathtt{t}-\sigma_1(\mathtt{t})-\sigma_2(\mathtt{t})) - \Im_{\mathtt{j}}(\mathtt{t}-\sigma_1(\mathtt{t})-\sigma_2(\mathtt{t}))|, \\
&\leq \mathtt{b}_{\mathtt{ji}}\mathtt{p}_{\mathtt{j}}|\psi_{\mathtt{j}}(\mathtt{t}-\sigma_1(\mathtt{t})-\sigma_2(\mathtt{t}))|, \\
|\mathbb{h}_{\mathtt{a}}(\varsigma_{\mathtt{i}}(\mathtt{t})) - \mathbb{h}_{\mathtt{a}}(\wp_{\mathtt{i}}(\mathtt{t}))| &\leq \mathtt{a}_{\mathtt{ji}}\mathtt{p}_{\mathtt{j}}\int_{\mathtt{t}-\tau(\mathtt{t})}^{\mathtt{t}} |\mathtt{s}_{\mathtt{j}}(\mathtt{s})\mathtt{ds} - \Im_{\mathtt{j}}(\mathtt{s})\mathtt{ds}|, \\
&\leq \mathtt{a}_{\mathtt{ji}}\mathtt{p}_{\mathtt{j}}\int_{\mathtt{t}-\tau(\mathtt{t})}^{\mathtt{t}} |\psi_{\mathtt{j}}(\mathtt{s})|\mathtt{ds}, \\
|\mathtt{g}_{\mathtt{n}}(\mathtt{s}_{\mathtt{j}}(\mathtt{t})) - \mathtt{g}_{\mathtt{n}}(\Im_{\mathtt{j}}(\mathtt{t}))| &\leq \mathtt{n}_{\mathtt{ij}}\mathtt{q}_{\mathtt{i}}|\varsigma_{\mathtt{i}}(\mathtt{t}) - \wp_{\mathtt{i}}(\mathtt{t})|, \\
&\leq \mathtt{n}_{\mathtt{ij}}\mathtt{q}_{\mathtt{i}}|\varpi_{\mathtt{i}}(\mathtt{t})|,
\end{aligned}$$

$$
\begin{aligned}
|\mathtt{g_c}(\mathtt{s_j}(\mathtt{t}))-\mathtt{g_c}(\Im_j(\mathtt{t}))| &\leq \mathtt{c_{ij}q_i}|\varsigma_{\mathtt{i}}(\mathtt{t}-\eta_1(\mathtt{t})-\eta_2(\mathtt{t}))-\wp_{\mathtt{i}}(\mathtt{t}-\eta_1(\mathtt{t})-\eta_2(\mathtt{t}))|,\\
&\leq \mathtt{c_{ij}q_i}|\varpi_{\mathtt{i}}(\mathtt{t}-\eta_1(\mathtt{t})-\eta_2(\mathtt{t}))|,\\
|\mathtt{g_e}(\mathtt{s}_j(\mathtt{t}))-\mathtt{g_e}(\Im_{\mathtt{j}}(\mathtt{t}))| &\leq \mathtt{e_{ij}q_i}\int_{\mathtt{t}-\upsilon(\mathtt{t})}^{\mathtt{t}}|\varsigma_{\mathtt{i}}(\mathtt{s})-\wp_{\mathtt{i}}((\mathtt{s}))|\mathtt{ds},\\
&\leq \mathtt{e_{ij}q_i}\int_{\mathtt{t}-\upsilon(\mathtt{t})}^{\mathtt{t}}|\varpi_{\mathtt{i}}(\mathtt{s})|\mathtt{ds},
\end{aligned}
$$

The compact form of (12) is

$$
\begin{cases}
\mathscr{D}^{\alpha}\varpi(\mathtt{t}) &= -\mathscr{R}\varpi(\mathtt{t})+\mathscr{H}_{\mathtt{d}}(\psi_{\mathtt{j}}(\mathtt{t}))+\mathscr{H}_{\mathtt{b}}(\psi_{\mathtt{j}}(\mathtt{t}-\sigma_1(\mathtt{t})-\sigma_2(\mathtt{t})))\\
&\quad+\int_{\mathtt{t}-\tau(\mathtt{t})}^{\mathtt{t}}\mathscr{H}_{\mathtt{a}}(\psi_{\mathtt{j}}(\mathtt{s}))\mathtt{ds}+\mathbb{U}(\mathtt{t}),\\
\mathscr{D}^{\alpha}\psi(\mathtt{t}) &= -\mathscr{M}\psi(\mathtt{t})+\mathscr{G}_{\mathtt{m}}(\varpi_{\mathtt{i}}(\mathtt{t}))+\mathscr{G}_{\mathtt{c}}(\varpi_{\mathtt{i}}(\mathtt{t}-\eta_1(\mathtt{t})-\eta_2(\mathtt{t})))\\
&\quad+\int_{\mathtt{t}-\upsilon(\mathtt{t})}^{\mathtt{t}}\mathscr{G}_{\mathtt{e}}(\varpi_{\mathtt{i}}(\mathtt{s}))\mathtt{ds}+\mathbb{V}(\mathtt{t}).
\end{cases}
$$

When $\mathbb{U}(\mathtt{t}),\mathbb{V}(\mathtt{t})=0$, *then the system decreases to*

$$
\begin{cases}
\mathscr{D}^{\alpha}\varpi(\mathtt{t}) &= -\mathscr{R}\varpi(\mathtt{t})+\mathscr{H}_{\mathtt{d}}(\psi_{\mathtt{j}}(\mathtt{t}))+\mathscr{H}_{\mathtt{b}}(\psi_{\mathtt{j}}(\mathtt{t}-\sigma_1(\mathtt{t})-\sigma_2(\mathtt{t})))\\
&\quad+\int_{\mathtt{t}-\tau(\mathtt{t})}^{\mathtt{t}}\mathscr{H}_{\mathtt{a}}(\psi_{\mathtt{j}}(\mathtt{s}))\mathtt{ds},\\
\mathscr{D}^{\alpha}\psi(\mathtt{t}) &= -\mathscr{M}\psi(\mathtt{t})+\mathscr{G}_{\mathtt{m}}(\varpi_{\mathtt{i}}(\mathtt{t}))+\mathscr{G}_{\mathtt{c}}(\varpi_{\mathtt{i}}(\mathtt{t}-\eta_1(\mathtt{t})-\eta_2(\mathtt{t})))\\
&\quad+\int_{\mathtt{t}-\upsilon(\mathtt{t})}^{\mathtt{t}}\mathscr{G}_{\mathtt{e}}(\varpi_{\mathtt{i}}(\mathtt{s}))\mathtt{ds}.
\end{cases}
\tag{13}
$$

Theorem 2. *Under Hypothesis 1, the real matrices are* $\mathscr{R}_1$, $\mathscr{R}_2$, $\mathscr{R}_3$, $\mathscr{R}_4$, $\mathscr{Z}_1$, $\mathscr{Z}_2$, $\mathscr{T}_1$, $\mathscr{T}_2$, $\mathscr{T}_3$, $\mathscr{T}_4$, $\mathscr{T}_5$, $\mathscr{T}_6$, $\mathscr{W}_1$, $\mathscr{W}_2$, $\mathscr{W}_3$, $\mathscr{W}_4$, $\mathscr{W}_5$, $\mathscr{W}_6$, $\mathscr{W}_7$, $\mathscr{W}_8$, *such that the following LMIs holds*

$$\Phi = diag(\Phi_{1,1},....,\Phi_{11,11})<0, \tag{14}$$

$$\Psi = diag(\Psi_{1,1},....,\Psi_{11,11})<0, \tag{15}$$

where

$$
\begin{aligned}
\Phi_{1,1} &= -2\mathscr{R}\mathscr{R}_1+\mathscr{R}_1\mathscr{U}_1\mathscr{R}_1+\mathscr{R}_1\mathscr{U}_2\mathscr{R}_1+\mathscr{R}_1\mathscr{U}_3\mathscr{R}_1+\beta\mathscr{N}^T\mathscr{U}_4^{-1}\mathscr{N}\beta\\
&\quad+\sigma_1^2\mathscr{R}_3+\upsilon^2\mathscr{W}_7+\mathscr{Q}_1+\sigma_2^2\mathscr{R}_4+\mathscr{T}_1+\mathscr{T}_2+\mathscr{T}_3+\mathscr{T}_4+\mathscr{T}_5+\mathscr{T}_6,\\
\Phi_{2,2} &= \beta\mathscr{C}^T\mathscr{U}_5^{-1}\mathscr{C}\beta-\mathscr{Q}_1, \Phi_{3,3}=-\mathscr{T}_1(1-\alpha_1-\alpha_2),\\
\Phi_{4,4} &= -\mathscr{T}_2(1-\alpha_1), \Phi_{5,5}=-\mathscr{T}_3(1-\alpha_2), \Phi_{6,6}=-\mathscr{T}_4,\\
\Phi_{7,7} &= -\mathscr{T}_5, \Phi_{8,8}=-\mathscr{T}_6, \Phi_{9,9}=\beta\mathscr{E}^{\mathscr{T}}\mathscr{U}_6^{-1}\mathscr{E}\beta-\mathscr{W}_7,\\
\Phi_{10,10} &= -\mathscr{R}_3, \Phi_{11,11}=-\mathscr{R}_4.
\end{aligned}
$$

and

$$
\begin{aligned}
\Psi_{1,1} &= -2\mathscr{M}\mathscr{R}_2+\mathscr{R}_2\mathscr{U}_4\mathscr{R}_2+\mathscr{R}_2\mathscr{U}_5\mathscr{R}_2+\mathscr{R}_2\mathscr{U}_6\mathscr{R}_2+\mu\mathscr{D}^T\mathscr{U}_1^{-1}\mathscr{D}\mu\\
&\quad+\eta_1^2\mathscr{Z}_1+\tau^2\mathscr{W}_8+\mathscr{Q}_2+\eta_2^2\mathscr{Z}_2+\mathscr{W}_1+\mathscr{W}_2+\mathscr{W}_3+\mathscr{W}_4+\mathscr{W}_5+\mathscr{W}_6,\\
\Psi_{2,2} &= \mu\mathscr{B}^T\mathscr{U}_2^{-1}\mathscr{B}\mu-\mathscr{Q}_2, \Psi_{3,3}=-\mathscr{W}_1(1-\delta_1-\delta_2),\\
\Psi_{4,4} &= -\mathscr{W}_2(1-\delta_1), \Psi_{5,5}=-\mathscr{W}_3(1-\delta_2), \Psi_{6,6}=-\mathscr{W}_4, \Psi_{7,7}=-\mathscr{W}_5,\\
\Psi_{8,8} &= -\mathscr{W}_6, \Psi_{9,9}=\mu\mathscr{A}^T\mathscr{U}_3^{-1}\mathscr{A}\mu-\mathscr{W}_8, \Psi_{10,10}=-\mathscr{Z}_1, \Psi_{11,11}=-\mathscr{Z}_2.
\end{aligned}
$$

then the system (13) is globally asymptotically stable.

Appendix A contains the detailed proof of Theorem 8.

Remark 2. *When $\sigma_1(\mathtt{t}) = \sigma_2(\mathtt{t}) = \sigma(\mathtt{t})$, system (13) correspondingly decreases to*

$$\begin{cases} \mathscr{D}^\alpha \varpi(\mathtt{t}) & = -\mathscr{R}\varpi(\mathtt{t}) + \mathscr{H}_{\mathtt{d}}(\psi_{\mathtt{j}}(\mathtt{t})) \\ & + \mathscr{H}_{\mathtt{b}}(\psi_{\mathtt{j}}(\mathtt{t} - \sigma(\mathtt{t}))) + \int_{\mathtt{t}-\tau(\mathtt{t})}^{\mathtt{t}} \mathscr{H}_{\mathtt{a}}(\psi_{\mathtt{j}}(\mathtt{s}))\mathtt{ds}, \\ \mathscr{D}^\alpha \psi(\mathtt{t}) & = -\mathscr{M}\psi(\mathtt{t}) + \mathscr{G}_{\mathtt{m}}(\varpi_{\mathtt{i}}(\mathtt{t})) \\ & + \mathscr{G}_{\mathtt{c}}(\varpi_{\mathtt{i}}(\mathtt{t} - \eta(\mathtt{t}))) + \int_{\mathtt{t}-\upsilon(\mathtt{t})}^{\mathtt{t}} \mathscr{G}_{\mathtt{e}}(\varpi_{\mathtt{i}}(\mathtt{s}))\mathtt{ds}. \end{cases} \tag{16}$$

Theorem 3. *Under Hypothesis 1, the real matrices are $\mathscr{Z}_1$, $\mathscr{Z}_2$ $\mathscr{R}_3$, $\mathscr{R}_4$, $\mathscr{Q}_1$, $\mathscr{Y}_1$, $\mathscr{T}_7$, $\mathscr{W}_7$, such that the following LMIs hold,*

$$\Lambda = diag(\Lambda_{1,1}, ..., \Lambda_{5,5}) < 0, \tag{17}$$

$$\Upsilon = diag(\Upsilon_{1,1}, ..., \Upsilon_{5,5}) < 0, \tag{18}$$

$$\begin{aligned} \Lambda_{1,1} = & -2\mathscr{R}\mathscr{R}_3 + \mathscr{R}_3\mathscr{U}_7\mathscr{R}_3 + \mathscr{R}_3\mathscr{U}_8\mathscr{R}_3 + \mathscr{R}_3\mathscr{U}_9\mathscr{R}_3 + \beta\mathscr{N}^T\mathscr{U}_{10}^{-1}\mathscr{N}\beta + \sigma^2\mathscr{Q}_1 \\ & + \mathscr{T}_7 + \upsilon^2\mathscr{Z}_1 + \mathscr{R}_1, \Lambda_{2,2} = \beta\mathscr{C}^T\mathscr{U}_{11}^{-1}\mathscr{C}\beta - \mathscr{R}_1, \Lambda_{3,3} = \beta\mathscr{E}^T\mathscr{U}_{12}^{-1}\mathscr{E}\beta - \mathscr{Z}_1, \\ \Lambda_{4,4} = & -\mathscr{Q}_1, \Lambda_{5,5} = -\mathscr{T}_7. \end{aligned}$$

$$\begin{aligned} \Upsilon_{1,1} = & -2\mathscr{M}\mathscr{R}_4 + \mathscr{R}_4\mathscr{U}_{10}\mathscr{R}_4 + \mathscr{R}_4\mathscr{U}_{11}\mathscr{R}_4 + \mathscr{R}_4\mathscr{U}_{12}\mathscr{R}_4 + \mu\mathscr{D}^T\mathscr{U}_7^{-1}\mathscr{D}\mu + \eta^2\mathscr{Y}_1 + \mathscr{W}_7 \\ & + \tau^2\mathscr{Z}_2 + \mathscr{R}_2, \Upsilon_{2,2} = \mu\mathscr{B}^T\mathscr{U}_8^{-1}\mathscr{B}\mu - \mathscr{R}_2, \Upsilon_{3,3} = \mu\mathscr{A}^T\mathscr{U}_9^{-1}\mathscr{A}\mu - \mathscr{Z}_2, \\ \Upsilon_{4,4} = & -\mathscr{Y}_1, \Upsilon_{5,5} = -\mathscr{W}_7. \end{aligned}$$

then, system (16) is globally asymptotically stable.

Appendix B contains the detailed proof of Theorem 10.

Remark 3. *When $\mathscr{H}_{\mathtt{a}} = \mathscr{G}_{\mathtt{e}} = 0$, the system (13) correspondingly decreases to*

$$\begin{cases} \mathscr{D}^\alpha \varpi(\mathtt{t}) & = -\mathscr{R}\varpi(\mathtt{t}) + \mathscr{H}_{\mathtt{d}}(\psi_{\mathtt{j}}(\mathtt{t})) + \mathscr{H}_{\mathtt{b}}(\psi_{\mathtt{j}}(\mathtt{t} - \sigma_1(\mathtt{t}) - \sigma_2(\mathtt{t}))), \\ \mathscr{D}^\alpha \psi(\mathtt{t}) & = -\mathscr{M}\psi(\mathtt{t}) + \mathscr{G}_{\mathtt{m}}(\varpi_{\mathtt{i}}(\mathtt{t})) + \mathscr{G}_{\mathtt{c}}(\varpi_{\mathtt{i}}(\mathtt{t} - \eta_1(\mathtt{t}) - \eta_2(\mathtt{t})). \end{cases} \tag{19}$$

Theorem 4. *Under Hypothesis 1, the real matrices are $\mathscr{R}_1$, $\mathscr{R}_2$, $\mathscr{T}_1$, $\mathscr{T}_2$, $\mathscr{T}_3$, $\mathscr{T}_4$, $\mathscr{T}_5$, $\mathscr{T}_6$, $\mathscr{W}_1$, $\mathscr{W}_2$, $\mathscr{W}_3$, $\mathscr{W}_4$, $\mathscr{W}_5$, $\mathscr{W}_6$ such that the following LMIs hold*

$$\xi = diag(\xi_{1,1}, ..., \xi_{8,8}) < 0, \tag{20}$$

$$\pi = diag(\pi_{1,1}, ..., \pi_{8,8}) < 0, \tag{21}$$

$$\begin{aligned} \xi_{1,1} = & -2\mathscr{R}\mathscr{R}_1 + \mathscr{R}_1\mathscr{U}_1\mathscr{R}_1 + \mathscr{R}_1\mathscr{U}_2\mathscr{R}_1 + \mathscr{T}_1 + \mathscr{T}_2 + \mathscr{T}_3 + \mathscr{T}_4 + \mathscr{T}_5 + \mathscr{T}_6 + \mathscr{Q}_1 \\ & + \beta\mathscr{N}^T\mathscr{U}_4^{-1}\mathscr{N}\beta, \xi_{2,2} = \beta\mathscr{C}^T\mathscr{U}_5^{-1}C\beta - \mathscr{Q}_1, \xi_{3,3} = -\mathscr{T}_1(1 - \alpha_1 - \alpha_2), \\ \xi_{4,4} = & -\mathscr{T}_2(1 - \alpha_1), \xi_{5,5} = -\mathscr{T}_3(1 - \alpha_2), \xi_{6,6} = -\mathscr{T}_4, \xi_{7,7} = -\mathscr{T}_5, \xi_{8,8} = -\mathscr{T}_6. \end{aligned}$$

and

$$\begin{aligned} \pi_{1,1} = & -2\mathscr{M}\mathscr{R}_2 + \mathscr{R}_2\mathscr{U}_4\mathscr{R}_2 + \mathscr{R}_2\varepsilon_5\mathscr{R}_2 + \mathscr{W}_1 + \mathscr{W}_2 + \mathscr{W}_3 + \mathscr{W}_4 + \mathscr{W}_5 + \mathscr{W}_6 + \mathscr{Q}_2 \\ & + \mu\mathscr{D}^T\mathscr{U}_1^{-1}\mathscr{D}\mu, \pi_{2,2} = \mu\mathscr{B}^T\mathscr{U}_2^{-1}\mathscr{B}\mu - \mathscr{Q}_2, \pi_{3,3} = -\mathscr{W}_1(1 - \delta_1 - \delta_2), \\ \pi_{4,4} = & -\mathscr{W}_2(1 - \delta_1), \pi_{5,5} = -\mathscr{W}_3(1 - \delta_2), \pi_{6,6} = -\mathscr{W}_4, \pi_{7,7} = -\mathscr{W}_5, \pi_{8,8} = -\mathscr{W}_6. \end{aligned}$$

then system (19) is globally asymptotically stable.

Appendix C contains the detailed proof of Theorem 11.

4. Illustrative Example

Example 1. *Consider the fact that system (13), as with the parameters, is*

$$\mathtt{d}_{11}(\wp_1(\mathtt{t})) = \begin{cases} -0.3, & |\wp_1(\mathtt{t})| \leq \mathscr{T}_1, \\ -0.1, & |\wp_1(\mathtt{t})| > \mathscr{T}_1, \end{cases}$$

$$\mathtt{d}_{12}(\wp_2(\mathtt{t})) = \begin{cases} -4.05, & |\wp_2(\mathtt{t})| \leq \mathscr{T}_2, \\ -3.95, & |\wp_2(\mathtt{t})| > \mathscr{T}_2, \end{cases}$$

$$\mathtt{d}_{21}(\wp_1(\mathtt{t})) = \begin{cases} 0.15, & |\wp_1(\mathtt{t})| \leq \mathscr{T}_1, \\ 0.05, & |\wp_1(\mathtt{t})| > \mathscr{T}_1, \end{cases}$$

$$\mathtt{d}_{22}(\wp_2(\mathtt{t})) = \begin{cases} -0.35, & |\wp_2(\mathtt{t})| \leq \mathscr{T}_2, \\ -0.25, & |\wp_2(\mathtt{t})| > \mathscr{T}_2, \end{cases}$$

$$\mathtt{a}_{11}(\wp_1(\mathtt{t})) = \begin{cases} -0.2, & |\wp_1(\mathtt{t})| \leq \mathscr{T}_1, \\ 0, & |\wp_1(\mathtt{t})| > \mathscr{T}_1, \end{cases}$$

$$\mathtt{a}_{12}(\wp_2(\mathtt{t})) = \begin{cases} -5.05, & |\wp_2(\mathtt{t})| \leq \mathscr{T}_2, \\ -4.95, & |\wp_2(\mathtt{t})| > \mathscr{T}_2, \end{cases}$$

$$\mathtt{a}_{21}(\wp_1(\mathtt{t})) = \begin{cases} 0.05, & |\wp_1(\mathtt{t})| \leq \mathscr{T}_1, \\ -0.05, & |\wp_1(\mathtt{t})| > \mathscr{T}_1, \end{cases}$$

$$\mathtt{a}_{22}(\wp_2(\mathtt{t})) = \begin{cases} -0.35, & |\wp_2(\mathtt{t})| \leq \mathscr{T}_2, \\ -0.25, & |\wp_2(\mathtt{t})| > \mathscr{T}_2, \end{cases}$$

$$\mathtt{b}_{11}(\wp_1(\mathtt{t})) = \begin{cases} 0.1, & |\wp_1(\mathtt{t})| \leq \mathscr{T}_1, \\ 0, & |\wp_1(\mathtt{t})| > \mathscr{T}_1, \end{cases}$$

$$\mathtt{b}_{12}(\wp_2(\mathtt{t})) = \begin{cases} 0.05, & |\wp_2(\mathtt{t})| \leq \mathscr{T}_2, \\ -0.25, & |\wp_2(\mathtt{t})| > \mathscr{T}_2, \end{cases}$$

$$\mathtt{b}_{21}(\wp_1(\mathtt{t})) = \begin{cases} 0.05, & |\wp_1(\mathtt{t})| \leq \mathscr{T}_1, \\ -0.36, & |\wp_1(\mathtt{t})| > \mathscr{T}_1, \end{cases}$$

$$\mathtt{b}_{22}(\wp_2(\mathtt{t})) = \begin{cases} -0.05, & |\wp_2(\mathtt{t})| \leq \mathscr{T}_2, \\ -0.11, & |\wp_2(\mathtt{t})| > \mathscr{T}_2, \end{cases}$$

$$\mathtt{m}_{11}(\varsigma_1(\mathtt{t})) = \begin{cases} 0.1, & |\varsigma_1(\mathtt{t})| \leq \mathscr{W}_1, \\ -0.4, & |\varsigma_1(\mathtt{t})| > \mathscr{W}_1, \end{cases}$$

$$\mathtt{m}_{12}(\varsigma_2(\mathtt{t})) = \begin{cases} 0.06, & |\varsigma_2(\mathtt{t})| \leq \mathscr{W}_2, \\ -0.1, & |\varsigma_2(\mathtt{t})| > \mathscr{W}_2, \end{cases}$$

$$\mathtt{m}_{21}(\varsigma_1(\mathtt{t})) = \begin{cases} 0.06, & |\varsigma_1(\mathtt{t})| \leq \mathscr{W}_1, \\ -0.5, & |\varsigma_1(\mathtt{t})| > \mathscr{W}_1, \end{cases}$$

$$\mathtt{m}_{22}(\varsigma_2(\mathtt{t})) = \begin{cases} 0.06, & |\varsigma_2(\mathtt{t})| \leq \mathscr{W}_2, \\ -0.3, & |\varsigma_2(\mathtt{t})| > \mathscr{W}_2, \end{cases}$$

$$\mathtt{c}_{11}(\varsigma_1(\mathtt{t})) = \begin{cases} 0.4, & |\varsigma_1(\mathtt{t})| \leq \mathscr{W}_1, \\ -0.38, & |\varsigma_1(\mathtt{t})| > \mathscr{W}_1, \end{cases}$$

$$\mathtt{c}_{12}(\varsigma_2(\mathtt{t})) = \begin{cases} 0.2, & |\varsigma_2(\mathtt{t})| \leq \mathscr{W}_2, \\ -0.25, & |\varsigma_2(\mathtt{t})| > \mathscr{W}_2, \end{cases}$$

$$\mathtt{c}_{21}(\varsigma_1(\mathtt{t})) = \begin{cases} 0.87, & |\varsigma_1(\mathtt{t})| \leq \mathscr{W}_1, \\ -0.5, & |\varsigma_1(\mathtt{t})| > \mathscr{W}_1, \end{cases}$$

$$\mathtt{c}_{22}(\varsigma_2(\mathtt{t})) = \begin{cases} -0.5, & |\varsigma_2(\mathtt{t})| \leq \mathscr{W}_2, \\ 0.38, & |\varsigma_2(\mathtt{t})| > \mathscr{W}_2, \end{cases}$$

$$\mathtt{e}_{11}(\varsigma_1(\mathtt{t})) = \begin{cases} 1.3, & |\varsigma_1(\mathtt{t})| \leq \mathscr{W}_1, \\ -0.2, & |\varsigma_1(\mathtt{t})| > \mathscr{W}_1, \end{cases}$$

$$\mathtt{e}_{12}(\varsigma_2(\mathtt{t})) = \begin{cases} 0.28, & |\varsigma_2(\mathtt{t})| \leq \mathscr{W}_2, \\ -0.1, & |\varsigma_2(\mathtt{t})| > \mathscr{W}_2, \end{cases}$$
$$\mathtt{e}_{21}(\varsigma_1(\mathtt{t})) = \begin{cases} -1.1, & |\varsigma_1(\mathtt{t})| \leq \mathscr{W}_1, \\ -0.4, & |\varsigma_1(\mathtt{t})| > \mathscr{W}_1, \end{cases}$$
$$\mathtt{e}_{22}(\varsigma_2(\mathtt{t})) = \begin{cases} -0.88, & |\varsigma_2(\mathtt{t})| \leq \mathscr{W}_2, \\ -0.5, & |\varsigma_2(\mathtt{t})| > \mathscr{W}_2. \end{cases}$$

Here, we take an activation function as $\mathtt{h}(\mathtt{s}) = \mathtt{g}(\mathtt{s}) = tanh(\mathtt{s})$. *Then, let* $\sigma_1 = 0.87$, $\sigma_2 = 0.88$, $\alpha_1 = 0.27$, $\alpha_2 = 0.36, \beta = 0.75$, $\tau = 0.87$, $\eta_1 = 0.78, \eta_2 = 0.76$, $\mu = 0.65$, $\delta_1 = 0.78$, $\delta_2 = 0.76, v = 0.76$. *Then, there exist the matrices as follows:*

$$\mathscr{R} = \begin{bmatrix} 4 & 0 \\ 0 & 6 \end{bmatrix}, \mathscr{M} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix}, \mathscr{D} = \begin{bmatrix} -0.2 & -4 \\ 0.1 & -0.3 \end{bmatrix},$$
$$\mathscr{A} = \begin{bmatrix} -0.1 & -5 \\ 0 & -0.3 \end{bmatrix}, \mathscr{B} = \begin{bmatrix} 0.1 & 0.05 \\ 0.05 & 0.05 \end{bmatrix}, \mathscr{C} = \begin{bmatrix} 0.4 & 0.25 \\ 0.87 & -0.5 \end{bmatrix},$$
$$\mathscr{N} = \begin{bmatrix} 0.1 & 0.06 \\ 0.06 & 0.06 \end{bmatrix}, \mathscr{E} = \begin{bmatrix} 0.3 & 0.28 \\ -1.1 & -0.88 \end{bmatrix}.$$

The following feasible solutions are obtained by solving LMIs (14), (15).

$$\mathscr{R}_1 = \begin{bmatrix} 1.8987 & -0.0564 \\ -0.0564 & 1.2887 \end{bmatrix}, \mathscr{R}_2 = \begin{bmatrix} 98.0141 - 3.4251 \\ -3.4251 - 0.2475 \end{bmatrix},$$
$$\mathscr{R}_3 = \begin{bmatrix} 1.5071 & 0.0000 \\ 0.0000 & 1.5071 \end{bmatrix}, \mathscr{R}_4 = \begin{bmatrix} 1.5071 & 0.0000 \\ 0.0000 & 1.5071 \end{bmatrix},$$
$$\mathscr{T}_1 = \begin{bmatrix} 4.0678 & 0.0000 \\ 0.0000 & 4.0680 \end{bmatrix}, \mathscr{T}_2 = \begin{bmatrix} 2.0641 & 0.0000 \\ 0.0000 & 2.0641 \end{bmatrix},$$
$$\mathscr{T}_3 = \begin{bmatrix} 2.3541 & 0.0000 \\ 0.0000 & 2.3541 \end{bmatrix}, \mathscr{T}_4 = \begin{bmatrix} 1.4091 & 0.0011 \\ 0.0011 & 1.5072 \end{bmatrix},$$
$$\mathscr{T}_5 = \begin{bmatrix} 1.5071 & 0.0000 \\ 0.0000 & 1.5071 \end{bmatrix}, \mathscr{T}_6 = \begin{bmatrix} 1.5071 & 0.0000 \\ 0.0000 & 1.5071 \end{bmatrix},$$
$$\mathscr{Z}_1 = \begin{bmatrix} 1.4475 & 0.0007 \\ 0.0007 & 1.5072 \end{bmatrix}, \mathscr{Z}_2 = \begin{bmatrix} 1.4505 & 0.0007 \\ 0.0007 & 1.5072 \end{bmatrix},$$
$$\mathscr{W}_1 = \begin{bmatrix} -3.1255 & 0.0039 \\ 0.0039 & -2.7892 \end{bmatrix}, \mathscr{W}_2 = \begin{bmatrix} 4.8069 & 0.0235 \\ 0.0235 & 6.8262 \end{bmatrix},$$
$$\mathscr{W}_3 = \begin{bmatrix} 4.5634 & 0.0198 \\ 0.0198 & 6.2613 \end{bmatrix}, \mathscr{W}_4 = \begin{bmatrix} 1.4091 & 0.0011 \\ 0.0011 & 1.5072 \end{bmatrix},$$
$$\mathscr{W}_5 = \begin{bmatrix} 1.4091 & 0.0011 \\ 0.0011 & 1.5072 \end{bmatrix}, \mathscr{W}_6 = \begin{bmatrix} 1.3111 & 0.0023 \\ 0.0023 & 1.5074 \end{bmatrix},$$
$$\mathscr{W}_7 = \begin{bmatrix} 0.6275 & -0.7118 \\ -0.7118 & 0.9301 \end{bmatrix}, \mathscr{W}_8 = \begin{bmatrix} 1.4294 & -0.1632 \\ -0.1632 & -6.7240 \end{bmatrix},$$
$$\mathscr{Q}_1 = \begin{bmatrix} 1.5456 & 0.0540 \\ 0.0540 & 1.4589 \end{bmatrix}, \mathscr{Q}_2 = \begin{bmatrix} 1.2582 & -0.0307 \\ -0.0307 & 1.4745 \end{bmatrix}.$$

$$\mathscr{U}_1 = -2.7767, \mathscr{U}_2 = -0.3447, \mathscr{U}_3 = -0.5048, \mathscr{U}_4 = -0.0022, \mathscr{U}_5 = 0.0643, \mathscr{U}_6 = -0.9023.$$

Therefore, it follows from Theorem 8 that the memristive BAM neural network with given parameters is globally asymptotically stable.

5. Conclusions

There is a new sufficient condition for guaranteeing the globally robust asymptotically stability of the equilibrium point for fractional-order memristive BAM neural networks, with mixed and additive time varying delays. Using the Lyapunov functional and LMI approaches, robust stability results can be obtained. At last, a numerical example is presented. Future research topics are stochastic BAM neural networks, and stochastic recurrent neural networks.

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Appendix A

Proof of Theorem 8. Define a Lyapunov functional

$$\mathscr{V}(\mathtt{t}) = \sum_{\mathbb{k}=1}^{8} \mathscr{V}_{\mathbb{k}}(\mathtt{t}),$$

where

$$\begin{aligned}
\mathscr{V}_1(\mathtt{t}) &= \mathscr{D}^{-(1-\alpha)}[\varpi^T(\mathtt{t})\mathscr{R}_1\varpi(\mathtt{t})],\\
\mathscr{V}_2(\mathtt{t}) &= \mathscr{D}^{-(1-\alpha)}[\psi^T(\mathtt{t})\mathscr{R}_2\psi(\mathtt{t})],\\
\mathscr{V}_3(\mathtt{t}) &= \sigma_1\int_{-\sigma_1}^{0}\int_{\mathtt{t}+\theta}^{\mathtt{t}}\varpi^T(\mathbb{s})\mathscr{R}_3\varpi(s)\mathrm{d}\mathbb{s}\mathrm{d}\theta + \sigma_2\int_{-\sigma_2}^{0}\int_{\mathtt{t}+\theta}^{\mathtt{t}}\varpi^T(s)\mathscr{R}_4\varpi(\mathbb{s})\mathrm{d}\mathbb{s}\mathrm{d}\theta,\\
\mathscr{V}_4(\mathtt{t}) &= \eta_1\int_{-\eta_1}^{0}\int_{\mathtt{t}+\theta}^{\mathtt{t}}\psi^T(\mathbb{s})\mathscr{Z}_1\psi(\mathbb{s})\mathrm{d}\mathbb{s}\mathrm{d}\theta + \eta_2\int_{-\eta_2}^{0}\int_{\mathtt{t}+\theta}^{\mathtt{t}}\psi^T(\mathbb{s})\mathscr{Z}_2\psi(\mathbb{s})\mathrm{d}\mathbb{s}\mathrm{d}\theta,\\
\mathscr{V}_5(\mathtt{t}) &= \int_{\mathtt{t}-\eta(\mathtt{t})}^{\mathtt{t}}\varpi^T(\mathbb{s})\mathscr{T}_1\varpi(\mathbb{s})\mathrm{d}\mathbb{s} + \int_{\mathtt{t}-\eta_1(\mathtt{t})}^{\mathtt{t}}\varpi^T(\mathbb{s})\mathscr{T}_2\varpi(\mathbb{s})\mathrm{d}\mathbb{s}\\
&\quad + \int_{\mathtt{t}-\eta_2(\mathtt{t})}^{\mathtt{t}}\varpi^T(\mathbb{s})\mathscr{T}_3\varpi(\mathbb{s})\mathrm{d}\mathbb{s} + \int_{\mathtt{t}-\eta}^{\mathtt{t}}\varpi^T(\mathbb{s})\mathscr{T}_4\varpi(\mathbb{s})\mathrm{d}\mathbb{s}\\
&\quad + \int_{\mathtt{t}-\eta_1}^{\mathtt{t}}\varpi^T(\mathbb{s})\mathscr{T}_5\varpi(\mathbb{s})\mathrm{d}\mathbb{s} + \int_{\mathtt{t}-\eta_2}^{\mathtt{t}}\varpi^T(\mathbb{s})\mathscr{T}_6\varpi(\mathbb{s})\mathrm{d}\mathbb{s},\\
\mathscr{V}_6(\mathtt{t}) &= \int_{\mathtt{t}-\sigma(\mathtt{t})}^{\mathtt{t}}\psi^T(\mathbb{s})\mathscr{W}_1\psi(\mathbb{s})\mathrm{d}\mathbb{s} + \int_{\mathtt{t}-\sigma_1(\mathtt{t})}^{\mathtt{t}}\psi^T(\mathbb{s})\mathscr{W}_2\psi(\mathbb{s})\mathrm{d}\mathbb{s}\\
&\quad + \int_{\mathtt{t}-\sigma_2(\mathtt{t})}^{\mathtt{t}}\psi^T(\mathbb{s})\mathscr{W}_3\psi(\mathbb{s})\mathrm{d}\mathbb{s} + \int_{\mathtt{t}-\sigma}^{\mathtt{t}}\psi^T(\mathbb{s})\mathscr{W}_4\psi(\mathbb{s})\mathrm{d}\mathbb{s}\\
&\quad + \int_{\mathtt{t}-\sigma_1}^{\mathtt{t}}\psi^T(\mathbb{s})\mathscr{W}_5\psi(\mathbb{s})\mathrm{d}\mathbb{s} + \int_{\mathtt{t}-\sigma_2}^{\mathtt{t}}\psi^T(\mathbb{s})\mathscr{W}_6\psi(\mathbb{s})\mathrm{d}\mathbb{s},\\
\mathscr{V}_7(\mathtt{t}) &= \upsilon(\mathtt{t})\int_{-\upsilon(\mathtt{t})}^{0}\int_{\mathtt{t}+\theta}^{\mathtt{t}}\varpi^T(\mathbb{s})\mathscr{W}_7\varpi(\mathbb{s})\mathrm{d}\mathbb{s}\mathrm{d}\theta,\\
\mathscr{V}_8(\mathtt{t}) &= \tau(\mathtt{t})\int_{-\tau(\mathtt{t})}^{0}\int_{\mathtt{t}+\theta}^{\mathtt{t}}\psi^T(\mathbb{s})\mathscr{W}_8\psi(\mathbb{s})\mathrm{d}\mathbb{s}\mathrm{d}\theta.
\end{aligned}$$

The time derivatives of $\mathscr{V}(\mathtt{t})$ as follows:

$$
\begin{aligned}
\dot{\mathscr{V}_1}(\mathtt{t}) &= 2\varpi^T(\mathtt{t})\mathscr{R}_1\{-\mathscr{R}\varpi(\mathtt{t}) + \mathscr{H}_{\mathtt{d}}(\psi_{\mathtt{j}}(\mathtt{t})) + \mathscr{H}_{\mathtt{b}}(\psi_{\mathtt{j}}(\mathtt{t}-\sigma_1(\mathtt{t})-\sigma_2(\mathtt{t}))) \\
&\quad + \int_{\mathtt{t}-\tau(\mathtt{t})}^{\mathtt{t}} \mathscr{H}_{\mathtt{a}}(\psi_{\mathtt{j}}(\mathtt{s}))d\mathtt{s}\} \\
\dot{\mathscr{V}_2}(\mathtt{t}) &= 2\psi^T(\mathtt{t})\mathscr{R}_2\{-\mathscr{M}\psi(\mathtt{t}) + \mathscr{G}_{\mathtt{m}}(\varpi_{\mathtt{i}}(\mathtt{t})) + \mathscr{G}_{\mathtt{c}}(\varpi_{\mathtt{i}}(\mathtt{t}-\eta_1(\mathtt{t})-\eta_2(\mathtt{t}))) \\
&\quad + \int_{\mathtt{t}-\upsilon(\mathtt{t})}^{\mathtt{t}} \mathscr{G}_{\mathtt{e}}(\varpi_{\mathtt{i}}(\mathtt{s}))d\mathtt{s}\}
\end{aligned}
$$

By applying Lemma 2 and Hypothesis 1, we obtain

$$
\begin{aligned}
2\varpi^T(\mathtt{t})\mathscr{R}_1\mathscr{H}_{\mathtt{d}}(\psi_{\mathtt{j}}(\mathtt{t})) &\le \varpi^T(\mathtt{t})\mathscr{R}_1\mathscr{U}_1\mathscr{R}_1\varpi(\mathtt{t}) \\
&\quad + \psi^T(\mathtt{t})\mu\mathscr{D}^T\mathscr{U}_1^{-1}\mathscr{D}\mu\psi(\mathtt{t}), \\
2\varpi^T(\mathtt{t})\mathscr{R}_1\mathscr{H}_b(\psi_{\mathtt{j}}(\mathtt{t}-\sigma_1(\mathtt{t})-\sigma_2(\mathtt{t}))) &\le \varpi^T(\mathtt{t})\mathscr{R}_1\mathscr{U}_2\mathscr{R}_1\varpi(\mathtt{t}) \\
&\quad + \psi^T(\mathtt{t}-\sigma_1(\mathtt{t})-\sigma_2(\mathtt{t}))\mu\mathscr{B}^T\mathscr{U}_2^{-1}\mathscr{B}\mu \\
&\quad \psi(\mathtt{t}-\sigma_1(\mathtt{t})-\sigma_2(\mathtt{t})), \\
2\varpi^T(\mathtt{t})\mathscr{R}_1[\int_{\mathtt{t}-\tau(\mathtt{t})}^{\mathtt{t}} \mathscr{H}_{\mathtt{a}}(\psi_{\mathtt{j}}(\mathtt{s}))d\mathtt{s}] &\le \varpi^T(\mathtt{t})\mathscr{R}_1\mathscr{U}_3\mathscr{R}_1\theta(\mathtt{t}) \\
&\quad + [\int_{\mathtt{t}-\tau(\mathtt{t})}^{\mathtt{t}} \psi(\mathtt{s})d\mathtt{s}]^T[\mu\mathscr{A}^T\mathscr{U}_3^{-1}\mathscr{A}\mu] \\
&\quad [\int_{\mathtt{t}-\tau(\mathtt{t})}^{\mathtt{t}} \psi(\mathtt{s})d\mathtt{s}], \\
2\psi^T(\mathtt{t})\mathscr{R}_2\mathscr{G}_{\mathtt{m}}(\varpi_{\mathtt{i}}(\mathtt{t})) &\le \psi^T(\mathtt{t})\mathscr{R}_2\mathscr{U}_4\mathscr{R}_2\psi(\mathtt{t}) \\
&\quad + \varpi^T(\mathtt{t})\beta\mathscr{N}^T\mathscr{U}_4^{-1}\mathscr{N}\beta\varpi(\mathtt{t}), \\
2\psi^T(\mathtt{t})\mathscr{R}_2\mathscr{G}_{\mathtt{c}}(\varpi_{\mathtt{i}}(\mathtt{t}-\eta_1(\mathtt{t})-\eta_2(\mathtt{t}))) &\le \psi^T(\mathtt{t})\mathscr{R}_2\mathscr{U}_5\mathscr{R}_2\psi(\mathtt{t}) \\
&\quad + \varpi^T(\mathtt{t}-\eta_1(\mathtt{t})-\eta_2(\mathtt{t}))\beta\mathscr{C}^T\mathscr{U}_5^{-1}\mathscr{C}\beta \\
&\quad \varpi(\mathtt{t}-\eta_1(\mathtt{t})-\eta_2(\mathtt{t})), \\
2\psi^T(\mathtt{t})\mathscr{R}_2[\int_{\mathtt{t}-\upsilon(\mathtt{t})}^{\mathtt{t}} \mathscr{G}_{\mathtt{e}}(\varpi_{\mathtt{i}}(\mathtt{s}))d\mathtt{s}] &\le \psi^T(\mathtt{t})\mathscr{R}_2\mathscr{U}_6\mathscr{R}_2\psi(\mathtt{t}) \\
&\quad + [\int_{\mathtt{t}-\upsilon(\mathtt{t})}^{\mathtt{t}} \varpi(\mathtt{s})d\mathtt{s}]^T[\beta\mathscr{E}^T\mathscr{U}_6^{-1}\mathscr{E}\beta] \\
&\quad [\int_{\mathtt{t}-\upsilon(\mathtt{t})}^{\mathtt{t}} \varpi(\mathtt{s})d\mathtt{s}].
\end{aligned}
$$

then, one has

$$
\begin{aligned}
\dot{\mathscr{V}_1}(\mathtt{t}) &\le \varpi^T(\mathtt{t})[-2\mathscr{R}\mathscr{R}_1]\varpi(\mathtt{t}) + \varpi^T(\mathtt{t})[\mathscr{R}_1\mathscr{U}_1\mathscr{R}_1]\varpi(\mathtt{t}) \\
&\quad + \psi^T(\mathtt{t})[\mu\mathscr{D}^T\mathscr{U}_1^{-1}\mathscr{D}\mu]\psi(\mathtt{t}) + \varpi^T(\mathtt{t})[\mathscr{R}_1\mathscr{U}_2\mathscr{R}_1]\varpi(\mathtt{t}) \\
&\quad + \psi^T(\mathtt{t}-\sigma_1(\mathtt{t})-\sigma_2(\mathtt{t}))[\mu\mathscr{B}^T\mathscr{U}_2^{-1}\mathscr{B}\mu]\psi(\mathtt{t}-\sigma_1(\mathtt{t})-\sigma_2(\mathtt{t})) \\
&\quad + \varpi^T(\mathtt{t})[\mathscr{R}_1\mathscr{U}_3\mathscr{R}_1]\varpi(\mathtt{t}) + \Big[\int_{\mathtt{t}-\tau(\mathtt{t})}^{\mathtt{t}} \psi(\mathtt{s})d\mathtt{s}\Big]^T[\mu\mathscr{A}^T\mathscr{U}_3^{-1}\mathscr{A}\mu]\Big[\int_{\mathtt{t}-\tau(\mathtt{t})}^{\mathtt{t}} \psi(\mathtt{s})d\mathtt{s}\Big], \\
\dot{\mathscr{V}_2}(\mathtt{t}) &\le \psi^T(\mathtt{t})[-2\mathscr{R}_2\mathscr{M}]\psi\mathtt{t}) + \psi^T(\mathtt{t})[\mathscr{R}_2\mathscr{U}_4\mathscr{R}_2]\psi(\mathtt{t}) \\
&\quad + \varpi^T(\mathtt{t})[\beta N^T\mathscr{U}_4^{-1}\mathscr{N}\beta]\varpi(\mathtt{t}) + \psi^T(\mathtt{t})[\mathscr{R}_2\mathscr{U}_5\mathscr{R}_2]\psi(\mathtt{t}) \\
&\quad + \varpi^T(\mathtt{t}-\eta_1(\mathtt{t})-\eta_2(\mathtt{t}))[\beta\mathscr{C}^T\mathscr{U}_5^{-1}\mathscr{C}\beta](\mathtt{t}-\eta_1(\mathtt{t})-\eta_2(\mathtt{t})) \\
&\quad + \psi^T(\mathtt{t})[\mathscr{R}_2\mathscr{U}_6\mathscr{R}_2]\psi(\mathtt{t}) + \Big[\int_{\mathtt{t}-\upsilon(\mathtt{t})}^{\mathtt{t}} \theta(\mathtt{s})d\mathtt{s}\Big]^T[\beta\mathscr{E}^T\mathscr{U}_6^{-1}\mathscr{E}\beta]\Big[\int_{\mathtt{t}-\upsilon(\mathtt{t})}^{\mathtt{t}} \varpi(\mathtt{s})d\mathtt{s}\Big],
\end{aligned}
$$

$$
\begin{aligned}
\dot{\mathscr{V}}_3(\mathtt{t}) &= \sigma_1^2\varpi^T(\mathtt{t})\mathscr{R}_3\varpi(\mathtt{t}) - \sigma_1\int_{\mathtt{t}-\sigma_1}^{\mathtt{t}}\varpi^T(\mathtt{s})\mathscr{R}_3\varpi(\mathtt{s})\mathtt{ds} + \sigma_2^2\varpi^T(\mathtt{t})\mathscr{R}_4\varpi(\mathtt{t}) \\
&\quad - \sigma_2\int_{\mathtt{t}-\sigma_2}^{\mathtt{t}}\varpi^T(\mathtt{s})\mathscr{R}_4\varpi(\mathtt{s})\mathtt{ds}, \\
\dot{\mathscr{V}}_4(\mathtt{t}) &= \eta_1^2\psi^T(\mathtt{t})\mathscr{Z}_1\psi(\mathtt{t}) - \eta_1\int_{\mathtt{t}-\eta_1}^{\mathtt{t}}\psi^T(\mathtt{s})\mathscr{Z}_1\psi(\mathtt{s})\mathtt{ds} + \eta_2^2\psi^T(\mathtt{t})\mathscr{Z}_2\psi(\mathtt{t}) \\
&\quad - \eta_2\int_{\mathtt{t}-\eta_2}^{\mathtt{t}}\psi^T(\mathtt{s})\mathscr{Z}_2\psi(\mathtt{s})\mathtt{ds}, \\
\dot{\mathscr{V}}_5(\mathtt{t}) &\leq [\varpi^T(\mathtt{t})\mathscr{T}_1\varpi(\mathtt{t}) - \varpi^T(\mathtt{t}-\eta(\mathtt{t}))\mathscr{T}_1\varpi(\mathtt{t}-\eta(\mathtt{t}))(1-\alpha_1-\alpha_2)] \\
&\quad + [\varpi^T(\mathtt{t})\mathscr{T}_2\varpi(\mathtt{t}) - \varpi^T(\mathtt{t}-\eta_1(\mathtt{t}))\mathscr{T}_2\theta(\mathtt{t}-\eta_1(\mathtt{t}))(1-\alpha_1) \\
&\quad + [\varpi^T(\mathtt{t})\mathscr{T}_3\varpi(\mathtt{t}) - \varpi^T(\mathtt{t}-\eta_2(\mathtt{t}))\mathscr{T}_3\varpi(\mathtt{t}-\eta_2(\mathtt{t}))(1-\alpha_2)] \\
&\quad + [\varpi^T(\mathtt{t})\mathscr{T}_4\varpi(\mathtt{t}) - \varpi^T(\mathtt{t}-\eta)\mathscr{T}_4\theta(\mathtt{t}-\eta)] \\
&\quad + [\varpi^T(\mathtt{t})\mathscr{T}_5\varpi(\mathtt{t}) - \varpi^T(\mathtt{t}-\eta_1)\mathscr{T}_5\varpi(\mathtt{t}-\eta_1)] \\
&\quad + [\varpi^T(\mathtt{t})\mathscr{T}_6\varpi(\mathtt{t}) - \varpi^T(\mathtt{t}-\eta_2)\mathscr{T}_6\varpi(\mathtt{t}-\eta_2)], \\
\dot{\mathscr{V}}_6(\mathtt{t}) &\leq [\psi^T(\mathtt{t})\mathscr{W}_1\psi(\mathtt{t}) - \psi^T(\mathtt{t}-\sigma(\mathtt{t}))\mathscr{W}_1\psi(\mathtt{t}-\sigma(\mathtt{t}))(1-\delta_1-\delta_2)] \\
&\quad + [\psi^T(\mathtt{t})\mathscr{W}_2\psi(\mathtt{t}) - \psi^T(\mathtt{t}-\sigma_1(\mathtt{t}))\mathscr{W}_2\psi(\mathtt{t}-\sigma_1(\mathtt{t}))(1-\delta_1) \\
&\quad + [\psi^T(\mathtt{t})\mathscr{W}_3\gamma(\mathtt{t}) - \psi^T(\mathtt{t}-\sigma_2(\mathtt{t}))\mathscr{W}_3\psi(\mathtt{t}-\sigma_2(\mathtt{t}))(1-\delta_2)] \\
&\quad + [\psi^T(\mathtt{t})\mathscr{W}_4\psi(\mathtt{t}) - \psi^T(\mathtt{t}-\sigma)\mathscr{W}_4\psi(\mathtt{t}-\sigma)] \\
&\quad + [\psi^T(\mathtt{t})\mathscr{W}_5\psi(\mathtt{t}) - \psi^T(\mathtt{t}-\sigma_1)\mathscr{W}_5\psi(\mathtt{t}-\sigma_1)] \\
&\quad + [\psi^T(\mathtt{t})\mathscr{W}_6\psi(\mathtt{t}) - \psi^T(\mathtt{t}-\sigma_2)\mathscr{W}_6\psi(\mathtt{t}-\sigma_2)], \\
\dot{\mathscr{V}}_7(\mathtt{t}) &= \upsilon^2(\mathtt{t})\varpi^T(\mathtt{t})\mathscr{W}_7\varpi(\mathtt{t}) - \upsilon(\mathtt{t})\int_{\mathtt{t}-\upsilon(\mathtt{t})}^{\mathtt{t}}\varpi^T(\mathtt{s})\mathscr{W}_7\varpi(\mathtt{s})\mathtt{ds}, \\
\dot{\mathscr{V}}_8(\mathtt{t}) &= \tau^2(\mathtt{t})\psi^T(\mathtt{t})\mathscr{W}_8\psi(\mathtt{t}) - \tau(\mathtt{t})\int_{\mathtt{t}-\tau(\mathtt{t})}^{\mathtt{t}}\psi^T(\mathtt{s})\mathscr{W}_8\psi(\mathtt{s})\mathtt{ds},
\end{aligned}
$$

From Hypothesis 1, we have

$$
\begin{aligned}
0 &\leq \varpi^T(\mathtt{t})\mathscr{Q}_1\varpi(\mathtt{t}) - \varpi^T(\mathtt{t}-\eta_1(\mathtt{t})-\eta_2(\mathtt{t}))\mathscr{Q}_1\varpi(\mathtt{t}-\eta_1(\mathtt{t})-\eta_2(\mathtt{t})), \\
0 &\leq \psi^T(\mathtt{t})\mathscr{Q}_2\psi(\mathtt{t}) - \psi^T(\mathtt{t}-\sigma_1(\mathtt{t})-\sigma_2(\mathtt{t}))\mathscr{Q}_2\psi(\mathtt{t}-\sigma_1(\mathtt{t})-\sigma_2(\mathtt{t})).
\end{aligned}
$$

$$
\dot{\mathscr{V}}(\mathtt{t}) \leq \xi^T(\mathtt{t})\Phi\xi(\mathtt{t}) + \zeta^T(\mathtt{t})\Psi\zeta(\mathtt{t}).
$$

where

$$
\begin{aligned}
\xi^T(\mathtt{t}) = [&\varpi^T(\mathtt{t})\ \ \varpi^T(\mathtt{t}-\eta_1(\mathtt{t})-\eta_2(\mathtt{t}),\ \ \varpi^T(\mathtt{t}-\eta(\mathtt{t})), \varpi^T(\mathtt{t}-\eta_1(\mathtt{t})), \\
&\varpi^T(\mathtt{t}-\eta_2(\mathtt{t})), \varpi^T(\mathtt{t}-\eta),\ \varpi^T(\mathtt{t}-\eta_1),\ \ \varpi^T(\mathtt{t}-\eta_2),\ (\int_{\mathtt{t}-\upsilon(\mathtt{t})}^{\mathtt{t}}\varpi(\mathtt{s})\mathtt{ds})^T, \\
&(\int_{\mathtt{t}-\sigma_1}^{\mathtt{t}}\varpi(\mathtt{s})\mathtt{ds})^T, (\int_{\mathtt{t}-\sigma_2}^{\mathtt{t}}\varpi(\mathtt{s})\mathtt{ds})^T.]
\end{aligned}
$$

$$
\begin{aligned}
\zeta^T(\mathtt{t}) = [&\psi^T(\mathtt{t})\ \ \psi^T(\mathtt{t}-\sigma_1(\mathtt{t})-\sigma_2(\mathtt{t}),\ \ \psi^T(\mathtt{t}-\sigma(\mathtt{t})), \psi^T(\mathtt{t}-\sigma_1(\mathtt{t})), \\
&\psi^T(\mathtt{t}-\sigma_2(\mathtt{t})),\ \ \psi^T(\mathtt{t}-\sigma),\ \psi^T(\mathtt{t}-\sigma_1),\ \ \psi^T(\mathtt{t}-\sigma_2),\ (\int_{\mathtt{t}-\tau(\mathtt{t})}^{\mathtt{t}}\psi(s)\mathtt{ds})^T, \\
&(\int_{\mathtt{t}-\eta_1}^{\mathtt{t}}\psi(\mathtt{s})\mathtt{ds})^T, (\int_{\mathtt{t}-\eta_2}^{\mathtt{t}}\psi(\mathtt{s})\mathtt{ds})^T.]
\end{aligned}
$$

Hence, $\dot{\mathscr{V}}(\mathtt{t}) \leq 0$, based on Lyapunov theory (13), is globally asymptotically stable. Hence, the proof is completed. □

Appendix B

Proof of Theorem 10. Define a Lyapunov functional

$$\mathscr{V}(\mathtt{t}) = \sum_{\mathbb{k}=1}^{8} \mathscr{V}_{\mathbb{k}}(\mathtt{t}),$$

where

$$\begin{aligned}
\mathscr{V}_1(\mathtt{t}) &= \mathscr{D}^{-(1-\alpha)}[\varpi^T(\mathtt{t})\mathscr{R}_3\varpi(\mathtt{t})],\\
\mathscr{V}_2(\mathtt{t}) &= \mathscr{D}^{-(1-\alpha)}[\psi^T(\mathtt{t})\mathscr{R}_4\psi(\mathtt{t})],\\
\mathscr{V}_3(\mathtt{t}) &= \sigma\int_{-\sigma}^{0}\int_{\mathtt{t}+\theta}^{\mathtt{t}} \varpi^T(s)\mathscr{Q}_1\varpi(s)\mathrm{d}\mathtt{s}\mathrm{d}\theta,\\
\mathscr{V}_4(\mathtt{t}) &= \eta\int_{-\eta}^{0}\int_{\mathtt{t}+\theta}^{\mathtt{t}} \psi^T(\mathtt{s})\mathscr{Y}_1\psi(\mathtt{s})\mathrm{d}\mathtt{s}\mathrm{d}\theta,\\
\mathscr{V}_5(\mathtt{t}) &= \int_{\mathtt{t}-\eta}^{\mathtt{t}} \varpi^T(\mathtt{s})\mathscr{T}_7\varpi(\mathtt{s})\mathrm{d}\mathtt{s},\\
\mathscr{V}_6(\mathtt{t}) &= \int_{\mathtt{t}-\sigma}^{\mathtt{t}} \psi^T(\mathtt{s})\mathscr{W}_7\psi(\mathtt{s})\mathrm{d}\mathtt{s},\\
\mathscr{V}_7(\mathtt{t}) &= \upsilon(\mathtt{t})\int_{-\upsilon(\mathtt{t})}^{0}\int_{\mathtt{t}+\theta}^{\mathtt{t}} \varpi^T(\mathtt{s})\mathscr{Z}_1\varpi(\mathtt{s})\mathrm{d}\mathtt{s}\mathrm{d},\\
\mathscr{V}_8(\mathtt{t}) &= \tau(\mathtt{t})\int_{-\tau(\mathtt{t})}^{0}\int_{\mathtt{t}+\theta}^{\mathtt{t}} \psi^T(\mathtt{s})\mathscr{Z}_2\psi(\mathtt{s})\mathrm{d}\mathtt{s}\mathrm{d}.
\end{aligned}$$

By applying the Lemma 2, we have,

$$\begin{aligned}
2\varpi^T(\mathtt{t})\mathscr{R}_3\mathscr{H}_d(\psi_{\mathbb{j}}(\mathtt{t})) &\le \varpi^T(\mathtt{t})\mathscr{R}_3\mathscr{U}_7\mathscr{R}_3\varpi(\mathtt{t})\\
&\quad + \psi^T(\mathtt{t})\mu\mathscr{D}^T\mathscr{U}_7^{-1}\mathscr{D}\mu\psi(\mathtt{t}),\\
2\varpi^T(\mathtt{t})\mathscr{R}_3\mathscr{H}_{\mathtt{b}}(\psi_{\mathbb{j}}(\mathtt{t}-\sigma(\mathtt{t}))) &\le \varpi^T(\mathtt{t})\mathscr{R}_3\mathscr{U}_8\mathscr{R}_3\varpi(\mathtt{t})\\
&\quad + \psi^T(\mathtt{t}-\sigma(\mathtt{t})))\mu\mathscr{B}^T\mathscr{U}_8^{-1}\mathscr{B}\mu\psi(\mathtt{t}-\sigma(\mathtt{t})),\\
2\varpi^T(\mathtt{t})\mathscr{R}_3\mathscr{H}_{\mathtt{a}}[\int_{\mathtt{t}-\tau(\mathtt{t})}^{\mathtt{t}}\psi_{\mathbb{j}}(\mathtt{s})\mathrm{d}\mathtt{s}] &\le \varpi^T(\mathtt{t})\mathscr{R}_3\mathscr{U}_9\mathscr{R}_3\varpi(\mathtt{t})\\
&\quad + [\int_{\mathtt{t}-\tau(\mathtt{t})}^{\mathtt{t}}\psi(\mathtt{s})\mathrm{d}\mathtt{s}]^T[\mu\mathscr{A}^T\mathscr{U}_9^{-1}\mathscr{A}\mu]\\
&\quad [\int_{\mathtt{t}-\tau(\mathtt{t})}^{\mathtt{t}}\psi(\mathtt{s})\mathrm{d}\mathtt{s}],\\
2\psi^T(\mathtt{t})\mathscr{R}_4\mathscr{G}_{\mathtt{m}}(\varpi_{\mathtt{i}}(\mathtt{t})) &\le \psi^T(\mathtt{t})\mathscr{R}_4\mathscr{U}_{10}\mathscr{R}_4\psi(\mathtt{t})\\
&\quad + \varpi^T(\mathtt{t})\beta\mathscr{N}^T\mathscr{U}_{10}^{-1}\mathscr{N}\beta\varpi(\mathtt{t}),\\
2\psi^T(\mathtt{t})\mathscr{R}_4\mathscr{G}_{\mathtt{c}}(\varpi_{\mathtt{i}}(\mathtt{t}-\eta(\mathtt{t})) &\le \psi^T(\mathtt{t})\mathscr{R}_4\mathscr{U}_{11}\mathscr{R}_4\psi(\mathtt{t})\\
&\quad + \varpi^T(\mathtt{t}-\eta(\mathtt{t}))\beta\mathscr{C}^T\mathscr{U}_{11}^{-1}\mathscr{C}\beta\varpi(\mathtt{t}-\eta(\mathtt{t})),\\
2\psi^T(\mathtt{t})\mathscr{R}_4\mathscr{G}_{\mathtt{e}}[\int_{\mathtt{t}-\upsilon(\mathtt{t})}^{\mathtt{t}}\varpi_{\mathtt{i}}(\mathtt{s})\mathrm{d}\mathtt{s}] &\le \psi^T(\mathtt{t})\mathscr{R}_4\mathscr{U}_{12}\mathscr{R}_4\psi(\mathtt{t})\\
&\quad + [\int_{\mathtt{t}-\upsilon(\mathtt{t})}^{\mathtt{t}}\varpi(\mathtt{s})ds]^T[\beta\mathscr{E}^T\mathscr{U}_{12}^{-1}\mathscr{E}\beta]\\
&\quad [\int_{\mathtt{t}-\upsilon(\mathtt{t})}^{\mathtt{t}}\varpi(\mathtt{s})\mathrm{d}\mathtt{s}].
\end{aligned}$$

The time derivatives of $\mathscr{V}(\mathtt{t})$ are as follows:

$$\begin{aligned}
\dot{\mathscr{V}}_1(\mathtt{t}) \leq\ & \varpi^T(\mathtt{t})[-2\mathscr{R}\mathscr{R}_3]\varpi(\mathtt{t}) + \varpi^T(\mathtt{t})[\mathscr{R}_3\mathscr{U}_7\mathscr{R}_3]\varpi(\mathtt{t}) \\
& + \psi^T(\mathtt{t})[\mu\mathscr{D}^T\mathscr{U}_7^{-1}\mathscr{D}\mu]\psi(\mathtt{t}) + \varpi^T(\mathtt{t})[\mathscr{R}_3\mathscr{U}_8\mathscr{R}_3]\varpi(\mathtt{t}) \\
& + \psi^T(\mathtt{t}-\sigma(\mathtt{t}))[\mu\mathscr{B}^T\mathscr{U}_8^{-1}\mathscr{B}\mu]\psi(\mathtt{t}-\sigma(\mathtt{t})) + \varpi^T(\mathtt{t})[\mathscr{R}_3\mathscr{U}_9\mathscr{U}_3]\varpi(\mathtt{t}) \\
& + \Big[\int_{\mathtt{t}-\tau(\mathtt{t})}^{\mathtt{t}}\psi(\mathtt{s})d\mathtt{s}\Big]^T[\mu\mathscr{A}^T\mathscr{U}_9^{-1}\mathscr{A}\mu]\Big[\int_{\mathtt{t}-\tau(\mathtt{t})}^{\mathtt{t}}\psi(\mathtt{s})d\mathtt{s}\Big], \\
\dot{\mathscr{V}}_2(\mathtt{t}) \leq\ & \psi^T(\mathtt{t})[-2\mathscr{R}_4\mathscr{M}]\psi(\mathtt{t}) + \psi^T(\mathtt{t})[\mathscr{R}_4\mathscr{U}_{10}\mathscr{R}_4]\psi(\mathtt{t}) \\
& + \varpi^T(\mathtt{t})[\beta\mathscr{N}^T\mathscr{U}_{10}^{-1}\mathscr{N}\beta]\varpi(\mathtt{t} + \psi^T(\mathtt{t})[\mathscr{R}_4\mathscr{U}_{11}\mathscr{R}_4]\psi(\mathtt{t})) \\
& + \varpi^T(\mathtt{t}-\eta(\mathtt{t}))[\beta\mathscr{C}^T\mathscr{U}_{11}^{-1}\mathscr{C}\beta]\varpi(\mathtt{t}-\eta(\mathtt{t})) + \psi^T(\mathtt{t})[\mathscr{R}_4\mathscr{U}_{12}\mathscr{R}_4]\psi(\mathtt{t}) \\
& + \Big[\int_{\mathtt{t}-\upsilon(\mathtt{t})}^{\mathtt{t}}\varpi(\mathtt{s})d\mathtt{s}\Big]^T[\beta\mathscr{E}^T\mathscr{U}_{12}^{-1}\mathscr{E}\beta]\Big[\int_{\mathtt{t}-\upsilon(\mathtt{t})}^{\mathtt{t}}\varpi(\mathtt{s})d\mathtt{s}\Big], \\
\dot{\mathscr{V}}_3(\mathtt{t}) =\ & \sigma^2\varpi^T(\mathtt{t})\mathscr{Q}_1\varpi(\mathtt{t}) - \sigma\int_{\mathtt{t}-\sigma}^{\mathtt{t}}\varpi^T(\mathtt{s})\mathscr{Q}_1\varpi(\mathtt{s})d\mathtt{s}, \\
\dot{\mathscr{V}}_4(\mathtt{t}) =\ & \eta^2\psi^T(\mathtt{t})\mathscr{Y}_1\psi(\mathtt{t}) - \eta\int_{\mathtt{t}-\eta}^{\mathtt{t}}\psi^T(\mathtt{s})\mathscr{Y}_1\psi(\mathtt{s})d\mathtt{s}, \\
\dot{\mathscr{V}}_5(\mathtt{t}) =\ & [\varpi^T(\mathtt{t})\mathscr{T}_7\varpi(\mathtt{t}) - \varpi^T(\mathtt{t}-\eta)\mathscr{T}_7\varpi(\mathtt{t}-\eta)], \\
\dot{\mathscr{V}}_6(\mathtt{t}) =\ & [\psi^T(\mathtt{t})\mathscr{W}_7\psi(\mathtt{t}) - \psi^T(\mathtt{t}-\sigma)\mathscr{W}_7\psi(\mathtt{t}-\sigma), \\
\dot{\mathscr{V}}_7(\mathtt{t}) =\ & \upsilon^2(\mathtt{t})\varpi^T(\mathtt{t})\mathscr{Z}_1\varpi(\mathtt{t}) - \upsilon(\mathtt{t})\int_{\mathtt{t}-\upsilon(\mathtt{t})}^{\mathtt{t}}\varpi^T(\mathtt{s})\mathscr{Z}_1\varpi(\mathtt{s})d\mathtt{s}, \\
\dot{\mathscr{V}}_8(\mathtt{t}) =\ & \tau^2(\mathtt{t})\psi^T(\mathtt{t})\mathscr{Z}_2\psi(\mathtt{t}) - \tau(\mathtt{t})\int_{\mathtt{t}-\tau(\mathtt{t})}^{\mathtt{t}}\psi^T(\mathtt{s})\mathscr{Z}_2\psi(\mathtt{s})d\mathtt{s},
\end{aligned}$$

From Hypothesis 1, we have

$$\begin{aligned}
0 &\leq \varpi^T(\mathtt{t})\mathscr{R}_1\varpi(\mathtt{t}) - \varpi^T(\mathtt{t}-\eta(\mathtt{t}))\mathscr{R}_1\varpi(\mathtt{t}-\eta(\mathtt{t})), \\
0 &\leq \psi^T(\mathtt{t})\mathscr{R}_2\psi(\mathtt{t}) - \psi^T(\mathtt{t}-\sigma(\mathtt{t}))\mathscr{R}_2\psi(\mathtt{t}-\sigma(\mathtt{t})).
\end{aligned}$$

According to Λ and Υ, this implies that

$$\dot{\mathscr{V}}(\mathtt{t}) \leq \vartheta^T(\mathtt{t})\Lambda\vartheta(\mathtt{t}) + \Omega^T(\mathtt{t})\Upsilon\Omega(\mathtt{t}).$$

$$\vartheta^T(\mathtt{t}) = [\varpi^T(\mathtt{t})\ \ \varpi^T(\mathtt{t}-\eta(\mathtt{t})), (\int_{\mathtt{t}-\upsilon(\mathtt{t})}^{\mathtt{t}}\varpi(\mathtt{s})d\mathtt{s})^T,\ \ (\int_{\mathtt{t}-\sigma}^{\mathtt{t}}\varpi(\mathtt{s})d\mathtt{s})^T, (\varpi(\mathtt{t}-\eta))^T],$$

$$\Omega^T(\mathtt{t}) = [\psi^T(\mathtt{t})\ \ \psi^T(\mathtt{t}-\sigma(\mathtt{t})), (\int_{\mathtt{t}-\tau(\mathtt{t})}^{\mathtt{t}}\psi(\mathtt{s})d\mathtt{s})^T,\ \ (\int_{\mathtt{t}-\eta}^{\mathtt{t}}\psi(\mathtt{s})d\mathtt{s})^T, (\psi(\mathtt{t}-\sigma))^T].$$

Hence, $\dot{V}(\mathtt{t}) \leq 0$, based on Lyapunov theory (16), is globally asymptotically stable. Hence, the proof is completed. □

Appendix C

Proof of Theorem 11. Define a Lyapunov functional

$$\mathscr{V}(\mathtt{t}) = \sum_{\mathbb{k}=1}^{4}\mathscr{V}_{\mathbb{k}}(\mathtt{t}),$$

where

$$
\begin{aligned}
\mathscr{V}_1(\mathtt{t}) &= \mathscr{D}^{-(1-\alpha)}[\varpi^T(\mathtt{t})\mathscr{R}_1\varpi(\mathtt{t})],\\
\mathscr{V}_2(\mathtt{t}) &= \mathscr{D}^{-(1-\alpha)}[\psi^T(\mathtt{t})\mathscr{R}_2\psi(\mathtt{t})],\\
\mathscr{V}_3(\mathtt{t}) &= \int_{\mathtt{t}-\eta(\mathtt{t})}^{\mathtt{t}} \varpi^T(\mathtt{s})\mathscr{T}_1\varpi(\mathtt{s})d\mathtt{s} + \int_{\mathtt{t}-\eta_1(\mathtt{t})}^{\mathtt{t}} \varpi^T(\mathtt{s})\mathscr{T}_2\varpi(\mathtt{s})d\mathtt{s}\\
&\quad + \int_{\mathtt{t}-\eta_2(\mathtt{t})}^{\mathtt{t}} \varpi^T(\mathtt{s})\mathscr{T}_3\varpi(\mathtt{s})d\mathtt{s} + \int_{\mathtt{t}-\eta}^{\mathtt{t}} \varpi^T(\mathtt{s})T_4\varpi(\mathtt{s})d\mathtt{s}\\
&\quad + \int_{\mathtt{t}-\eta_1}^{\mathtt{t}} \varpi^T(\mathtt{s})\mathscr{T}_5\varpi(\mathtt{s})d\mathtt{s} + \int_{\mathtt{t}-\eta_2}^{\mathtt{t}} \varpi^T(\mathtt{s})\mathscr{T}_6\varpi(\mathtt{s})d\mathtt{s},\\
\mathscr{V}_4(\mathtt{t}) &= \int_{\mathtt{t}-\sigma(\mathtt{t})}^{\mathtt{t}} \psi^T(\mathtt{s})\mathscr{W}_1\psi(\mathtt{s})d\mathtt{s} + \int_{\mathtt{t}-\sigma_1(\mathtt{t})}^{\mathtt{t}} \psi^T(\mathtt{s})\mathscr{W}_2\psi(\mathtt{s})d\mathtt{s}\\
&\quad + \int_{\mathtt{t}-\sigma_2(\mathtt{t})}^{\mathtt{t}} \psi^T(\mathtt{s})\mathscr{W}_3\psi(\mathtt{s})d\mathtt{s} + \int_{\mathtt{t}-\sigma}^{\mathtt{t}} \psi^T(\mathtt{s})\mathscr{W}_4\psi(\mathtt{s})d\mathtt{s}\\
&\quad + \int_{\mathtt{t}-\sigma_1}^{\mathtt{t}} \psi^T(\mathtt{s})\mathscr{W}_5\psi(\mathtt{s})d\mathtt{s} + \int_{\mathtt{t}-\sigma_2}^{\mathtt{t}} \psi^T(\mathtt{s})\mathscr{W}_6\psi(\mathtt{s})d\mathtt{s}.
\end{aligned}
$$

By applying Lemma 2, we have

$$
\begin{aligned}
2\varpi^T(\mathtt{t})\mathscr{R}_1\mathscr{H}_{\mathtt{d}}(\psi_{\mathtt{j}}(\mathtt{t})) &\le \varpi^T(\mathtt{t})\mathscr{R}_1\mathscr{U}_1\mathscr{R}_1\varpi(\mathtt{t})\\
&\quad + \psi^T(\mathtt{t})\mu\mathscr{D}^T\mathscr{U}_1^{-1}\mathscr{D}\mu\psi(\mathtt{t}),\\
2\varpi^T(\mathtt{t})\mathscr{R}_1\mathscr{H}_{\mathtt{b}}(\psi_{\mathtt{j}}(\mathtt{t}-\sigma_1(\mathtt{t})-\sigma_2(\mathtt{t}))) &\le \varpi^T(\mathtt{t})\mathscr{R}_1\mathscr{U}_2\mathscr{R}_1\varpi(\mathtt{t})\\
&\quad + \psi^T(\mathtt{t}-\sigma_1(\mathtt{t})-\sigma_2(\mathtt{t}))\mu\mathscr{B}^T\mathscr{U}_2^{-1}\\
&\quad \mathscr{B}\mu\psi(\mathtt{t}-\sigma_1(\mathtt{t})-\sigma_2(\mathtt{t})),\\
2\psi^T(\mathtt{t})\mathscr{R}_2\mathscr{G}_{\mathtt{m}}(\varpi_{\mathtt{i}}(\mathtt{t})) &\le \psi^T(\mathtt{t})\mathscr{R}_2\mathscr{U}_4\mathscr{R}_2\psi(\mathtt{t})\\
&\quad + \varpi^T(\mathtt{t})\beta\mathscr{N}^T\mathscr{U}_4^{-1}\mathscr{N}\beta\varpi(\mathtt{t}),\\
2\psi^T(\mathtt{t})\mathscr{R}_2\mathscr{G}_{\mathtt{c}}(\varpi_{\mathtt{i}}(\mathtt{t}-\eta_1(\mathtt{t})-\eta_2(\mathtt{t}))) &\le \psi^T(\mathtt{t})\mathscr{R}_2\mathscr{U}_5\mathscr{R}_2\psi(\mathtt{t})\\
&\quad + \varpi^T(\mathtt{t}-\eta_1(\mathtt{t})-\eta_2(\mathtt{t}))\beta\mathscr{C}^T\mathscr{U}_5^{-1}\mathscr{C}\\
&\quad \beta\varpi(\mathtt{t}-\eta_1(\mathtt{t})-\eta_2(\mathtt{t})),
\end{aligned}
$$

The time derivatives of $\mathscr{V}(\mathtt{t})$ are as follows:

$$
\begin{aligned}
\dot{\mathscr{V}_1}(\mathtt{t}) &\le \varpi^T(\mathtt{t})[-2\mathscr{R}\mathscr{R}_1]\varpi(\mathtt{t}) + \varpi^T(\mathtt{t})[\mathscr{R}_1\mathscr{U}_1\mathscr{R}_1]\varpi(\mathtt{t})\\
&\quad + \psi^T(\mathtt{t})[\mu\mathscr{D}^T\mathscr{U}_1^{-1}\mathscr{D}\mu]\psi(\mathtt{t}) + \varpi^T(\mathtt{t})[\mathscr{R}_1\mathscr{U}_2\mathscr{R}_1]\varpi(\mathtt{t})\\
&\quad + \psi^T(\mathtt{t}-\sigma_1(\mathtt{t})-\sigma_2(\mathtt{t}))[\mu\mathscr{B}^T\mathscr{U}_2^{-1}\mathscr{B}\mu]\psi(\mathtt{t}-\sigma_1(\mathtt{t})-\sigma_2(\mathtt{t}))\\
&\quad + \varpi^T(\mathtt{t})[\mathscr{R}_1\mathscr{U}_3\mathscr{R}_1]\varpi(\mathtt{t})\\
&\quad + \Big[\int_{\mathtt{t}-\tau(\mathtt{t})}^{\mathtt{t}} \psi(\mathtt{s})d\mathtt{s}\Big]^T[\mu\mathscr{A}^T\mathscr{U}_3^{-1}\mathscr{A}\mu]\Big[\int_{\mathtt{t}-\tau(\mathtt{t})}^{\mathtt{t}} \psi(\mathtt{s})d\mathtt{s}\Big],\\
\dot{\mathscr{V}_2}(\mathtt{t}) &\le \psi^T(\mathtt{t})[-2\mathscr{R}_2\mathscr{M}]\psi(\mathtt{t}) + \psi^T(\mathtt{t})[\mathscr{R}_2\mathscr{U}_4\mathscr{R}_2]\psi(\mathtt{t})\\
&\quad + \varpi^T(\mathtt{t})[\beta\mathscr{N}^T\mathscr{U}_4^{-1}\mathscr{N}\beta]\varpi(\mathtt{t}) + \psi^T(\mathtt{t})[\mathscr{R}_2\mathscr{U}_5\mathscr{R}_2]\psi(\mathtt{t})\\
&\quad + \varpi^T(\mathtt{t}-\eta_1(\mathtt{t})-\eta_2(\mathtt{t}))[\beta\mathscr{C}^T\mathscr{U}_5^{-1}\mathscr{C}\beta]\varpi(\mathtt{t}-\eta_1(\mathtt{t})-\eta_2(\mathtt{t}))\\
&\quad + \psi^T(\mathtt{t})[\mathscr{R}_2\mathscr{U}_6\mathscr{R}_2]\psi(\mathtt{t})\\
&\quad + \Big[\int_{\mathtt{t}-\upsilon(\mathtt{t})}^{\mathtt{t}} \varpi(\mathtt{s})d\mathtt{s}\Big]^T[\beta\mathscr{E}^T\mathscr{U}_6^{-1}\mathscr{E}\beta]\Big[\int_{\mathtt{t}-\vartheta(\mathtt{t})}^{\mathtt{t}} \varpi(\mathtt{s})d\mathtt{s}\Big],
\end{aligned}
$$

$$\begin{aligned}
\dot{\mathscr{V}}_3(\mathtt{t}) \leq {} & [\varpi^T(\mathtt{t})\mathscr{T}_1\varpi(\mathtt{t}) - \varpi^T(\mathtt{t}-\eta(\mathtt{t}))\mathscr{T}_1\varpi(\mathtt{t}-\eta(\mathtt{t}))(1-\alpha_1-\alpha_2)] \\
& + [\varpi^T(\mathtt{t})\mathscr{T}_2\varpi(\mathtt{t}) - \varpi^T(\mathtt{t}-\eta_1(\mathtt{t}))\mathscr{T}_2\varpi(\mathtt{t}-\eta_1(\mathtt{t}))(1-\alpha_1) \\
& + [\varpi^T(\mathtt{t})\mathscr{T}_3\varpi(\mathtt{t}) - \varpi^T(\mathtt{t}-\eta_2(\mathtt{t}))\mathscr{T}_3\varpi(\mathtt{t}-\eta_2(\mathtt{t}))(1-\alpha_2)] \\
& + [\varpi^T(\mathtt{t})\mathscr{T}_4\varpi(\mathtt{t}) - \varpi^T(\mathtt{t}-\eta)\mathscr{T}_4\varpi(\mathtt{t}-\eta)] \\
& + [\varpi^T(\mathtt{t})\mathscr{T}_5\varpi(\mathtt{t}) - \varpi^T(\mathtt{t}-\eta_1)\mathscr{T}_5\varpi(\mathtt{t}-\eta_1)] \\
& + [\varpi^T(\mathtt{t})\mathscr{T}_6\varpi(\mathtt{t}) - \varpi^T(\mathtt{t}-\eta_2)\mathscr{T}_6\varpi(\mathtt{t}-\eta_2)], \\
\dot{V}_4(\mathtt{t}) \leq {} & [\psi^T(\mathtt{t})\mathscr{W}_1\psi(\mathtt{t}) - \psi^T(\mathtt{t}-\sigma(\mathtt{t}))\mathscr{W}_1\psi(\mathtt{t}-\sigma(\mathtt{t}))(1-\delta_1-\delta_2)] \\
& + [\psi^T(\mathtt{t})\mathscr{W}_2\psi(\mathtt{t}) - \psi^T(\mathtt{t}-\sigma_1(\mathtt{t}))\mathscr{W}_2\gamma(\mathtt{t}-\sigma_1(\mathtt{t}))(1-\delta_1) \\
& + [\psi^T(\mathtt{t})\mathscr{W}_3\gamma(\mathtt{t}) - \psi^T(\mathtt{t}-\sigma_2(\mathtt{t}))\mathscr{W}_3\psi(\mathtt{t}-\sigma_2(\mathtt{t}))(1-\delta_2)] \\
& + [\psi^T(\mathtt{t})\mathscr{W}_4\psi(\mathtt{t}) - \psi^T(\mathtt{t}-\sigma)\mathscr{W}_4\psi(\mathtt{t}-\sigma)] \\
& + [\psi^T(\mathtt{t})\mathscr{W}_5\psi(\mathtt{t}) - \psi^T(\mathtt{t}-\sigma_1)\mathscr{W}_5\psi(\mathtt{t}-\sigma_1)] \\
& + [\psi^T(\mathtt{t})\mathscr{W}_6\psi(\mathtt{t}) - \psi^T(\mathtt{t}-\sigma_2)\mathscr{W}_6\psi(\mathtt{t}-\sigma_2)]
\end{aligned}$$

From Hypothesis 1, we have

$$\begin{aligned}
0 &\leq \varpi^T(\mathtt{t})\mathscr{Q}_1\varpi(\mathtt{t}) - \varpi^T(\mathtt{t}-\eta_1(\mathtt{t})-\eta_2(\mathtt{t}))\mathscr{Q}_1\varpi(\mathtt{t}-\eta(\mathtt{t})), \\
0 &\leq \psi^T(\mathtt{t})\mathscr{Q}_2\psi(\mathtt{t}) - \psi^T(\mathtt{t}-\sigma_1(\mathtt{t})-\sigma_2(\mathtt{t}))\mathscr{Q}_2\psi(\mathtt{t}-\sigma(\mathtt{t})).
\end{aligned}$$

Adding to the above, we obtain

$$\dot{\mathscr{V}}(\mathtt{t}) \leq \mathbb{h}^T(\mathtt{t})\xi\mathbb{h}(\mathtt{t}) + \ell^T(\mathtt{t})\pi\ell(\mathtt{t}) < 0,$$

we have,

$$\dot{\mathscr{V}}(\mathtt{t}) \leq 0$$

$$\begin{aligned}
\mathbb{h}(\mathtt{t}) = {} & [\varpi^T(\mathtt{t}) \;\; \varpi^T(\mathtt{t}-\eta_1(\mathtt{t})-\eta_2(\mathtt{t}), \;\; \varpi^T(\mathtt{t}-\eta(\mathtt{t})), \varpi^T(\mathtt{t}-\eta_1(\mathtt{t})), \\
& \varpi^T(\mathtt{t}-\eta_2(\mathtt{t})), \;\; \varpi^T(\mathtt{t}-\eta), \;\; \varpi^T(\mathtt{t}-\eta_1), \;\; \varpi^T(\mathtt{t}-\eta_2)],
\end{aligned}$$

$$\begin{aligned}
\ell(\mathtt{t}) = {} & [\psi^T(\mathtt{t}) \;\; \psi^T(\mathtt{t}-\sigma_1(\mathtt{t})-\sigma_2(\mathtt{t}), \;\; \psi^T(\mathtt{t}-\sigma(\mathtt{t})), \psi^T(\mathtt{t}-\sigma_1(\mathtt{t})), \\
& \psi^T(\mathtt{t}-\sigma_2(\mathtt{t})), \;\; \psi^T(\mathtt{t}-\sigma), \;\; \psi^T(\mathtt{t}-\sigma_1), \;\; \psi^T(\mathtt{t}-\sigma_2)].
\end{aligned}$$

Hence $\dot{V}(\mathtt{t}) \leq 0$ Based on Lyapunov theory, (19) is a globally asymptotically stable. Hence, the proof is completed. □

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