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Simple Graphical Prediction of Relative Permeability of Unsaturated Soils under Deformations

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Abstract: At present, there are only a few existing models that can be used to predict the relative permeability of unsaturated soil under deformations, and the calculation process is relatively complex. In order to fit the measured value of the relative permeability coefficient of unsaturated soil before deformation, this work employs the simplified unified model of the relative permeability coefficient of unsaturated soil, and it obtains the index λ before deformation of the soil. In addition, the value of index λ remains unchanged before and after deformation. Based on the actual measured value of the soil–water characteristic curve before deformation, the air-entry value prediction model is used to predict the air-entry value of soil with different initial void ratios. The relative permeability coefficient of unsaturated soil is then conveniently predicted using the graphical method in combination with the simplified unified model. The method is validated by using the test data of silt loam, sandy loam, and unconsoildated sand. The results show that the predicted results are consistent with the measured values. The prediction method in this paper is simple and overcomes the limitations associated with the determination of the index λ. It expands the application range of the unsaturated relative permeability coefficient model while improving the accuracy of predictions.

Keywords: deformation conditions; relative permeability coefficient; air-entry value; void ratio

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1. Introduction

At present, there are many studies on the mechanical properties and hydraulic properties of soil [1–3]. Accurate prediction of permeability coefficient is an important prerequisite for studying soil water conservancy characteristics, and it plays an important role in the impact of groundwater on buildings, slope instability caused by rainfall, seepage and diffusion of pollutants in landfills, and seepage prevention of soil dams. Considering the high prevalence with which unsaturated soils are encountered in these projects, the permeability coefficient is a critical parameter for the research of unsaturated soils and has garnered widespread attention from scholars both at home and abroad [4–6]. Unsaturated soil is composed of soil particles, water, and air. Water and air occupy the pore volume in the soil, so its permeability coefficient is easily affected by saturation, even in the near-saturated state (suction range is 0~1 kPa). A small change in suction can cause a great influence in the permeability coefficient [7]. This renders the task of measuring the permeability of unsaturated soil difficult to achieve. In particular, the measurement of the permeability coefficient at low saturation is time demanding. Therefore, it is crucial to propose a method to predict the unsaturated permeability coefficient. The current classic prediction models mainly include the CCG model [8], the Burdine model [9], and the Mualem model [10], among which the Mualem model is widely used in practice. Additionally, Tao and Kong developed a novel methodology for forecasting the permeability coefficient [11]. Tao et al. [12] derived the above four models using fractal theory and analyzed the applicable scope of each model. They then proposed a simplified unified model for predicting the unsaturated permeability coefficient and verified its effectiveness using a large number of measured values. However, these models do not consider the influence of deformation on the variation in permeability coefficient of unsaturated soil.

Mitchell et al. [13] applied the deformable saturated soil permeability theory proposed by Taylor to unsaturated soil and conducted corresponding experiments with compacted clay to evaluate the permeability coefficient of deformable soils. Chang et al. [14] considered the influence of saturation and void ratio on the permeability coefficient and proposed a more reasonable empirical equation. Zhang et al. [15] applied the theories of fluid mechanics and probability to study the change of pore structure parameters under deformation conditions and put forward an expression of the permeability coefficient of unsaturated soil considering pore changes. Hu et al. [16] introduced new parameters to reflect the influence of the change of pore structure under effective saturation and deformation conditions and proposed an unsaturated soil permeability coefficient prediction model considering deformation conditions. Gao et al. [17] used the normalized soil water retention curves (SWRCs) method to investigate the permeability and hydraulic hysteresis behavior of unsaturated soils with varying bulk densities or void ratios and proposed a simple and effective method for simulating the effect of bulk density or void ratio on the relative permeability coefficient of unsaturated soils.

Based on the simplified unified model [12] of relative permeability coefficient of unsaturated soil and air-entry value prediction model [18], a new approach is proposed to readily obtain the relative permeability coefficient of unsaturated soil under deformation conditions by using the graphical technique method. The proposed approach is simple to implement and widely applicable.

2. Prediction Method

2.1. Unified Model

Tao et al. [18] proposed a fractal form of the soil-water characteristic curve (SWCC) expressed in terms of gravimetric water content on the basis of fractal theory. Based on the relationship between volumetric and gravimetric water content and saturation, this is translated into the fractal form of the soil-water characteristic curve expressed in terms of volumetric water content and saturation. Then, these fractal forms are combined with the known CCG model, Burdine model, Mualem model, and Tao–Kong model and their unsaturated permeability coefficient fractal form is deduced. The results found that they have similarities, and then a simplified unified model was proposed [12]. Finally, the effectiveness and applicable scope of the unified model was verified.

This is expressed by the following equation:

$$k_r(\psi) = (a + bS_r) \cdot (\frac{\psi_a}{\psi})^{\lambda} \tag{1}$$

where k_r is the relative permeability coefficient, ψ_a is the air-entry value, ψ is matric suction, S_r is the degree of saturation, a and b are coefficients related to the fractal dimension D, and λ is model parameters. The relationship between a, b, λ, and D is shown in Table 1.

Table 1. The relationship between model coefficients, λ and D.

Model	*a* Coefficient	*b* Coefficient	Relationship between λ and D
CCG	$a = (8 - 2D)/(3 - D)$	$b = (D - 5)/(3 - D)$	$\lambda = 5 - D$
Mualemm	$a = 1$	$b = 0$	$\lambda = 9.5 - 2.5D$
Burdine	$a = 1$	$b = 0$	$\lambda = 11 - 3D$
Tao–Kong	$a = 1$	$b = 0$	$\lambda = 5 - D$

For different types of soil, the model parameters are different. The reference for the specific parameter selection is related to the fractal dimension D. The CCG model and the Tao–Kong model predict better when the range of D is $2.8 \leq D < 3$. Within the range of $2.6 \leq D < 2.8$, the Tao–Kong model results into good predictions as well. The Burdine model and the Mualem model predict better when $D < 2.6$. The coefficients $a = (8 - 2D)/(3 - D)$ and $b = (D - 5)/(3 - D)$ in the CCG model are more difficult to evaluate, while the Tao–Kong model also yields good prediction for $2.8 \leq D < 3$. The coefficients $a = 1$ and $b = 0$, as applicable to three models (i.e., Mualemm, Burdine, Tao–Kong), are easier to calculate. Therefore, for the sake of computational simplicity, the CCG model is not considered in this paper. Equation (1) can be simplified as follows:

$$k_r = \left(\frac{\psi_a}{\psi}\right)^{\lambda} \tag{2}$$

2.2. Relationship between Parameter λ and Fractal Dimension D

Since the 1980s, many scholars have discovered that the distribution of internal pores and particles for many types of rock and soil have similar characteristics in their natural state. Therefore, the fractal theory has been applied to a variety of soils and achieved promising results [19–22]. Fractal dimension is a physical parameter used to describe the irregularity of fractal geometry, which is closely related to the pore structure of the soil. With different fractal dimensions, the soil shows great difference in mechanical properties and hydraulic properties. Therefore, the fractal dimension is a very important parameter in the study of soil permeability coefficient. Tao et al. [18] analyzed SWCC variation under different starting void ratio conditions and discovered that when the initial void ratio is changed, the air-entry value is the primary component affecting the change in SWCC. In a study [18], the data after the air-entry value were chosen to calculate the fractional dimension, so it could be assumed that the soil fractal dimension does not change significantly at different void ratios. The relationship between the index λ and the fractal dimension D in the simplified unified model of the unsaturated soil permeability coefficient are described in the previous section, shown in Table 1. It can be obtained that the index λ of the unified model also does not change significantly under different initial pore ratios.

However, the relationship between index λ and fractal dimension D in the above four models cannot cover all the cases relevant to practice. In some instances, the relationship between λ and D is beyond the scope of the above expressions. This paper suggested that, in order to predict the unsaturated permeability coefficient under deformation conditions, the unsaturated permeability coefficient of the soil should be measured first, and the λ value obtained by fitting the experimental data of the unsaturated relative coefficient and air-entry value prior to deformation. From this, the λ value after deformation can be obtained.

2.3. Law of Relative Permeability Coefficient of Unsaturated Soil under Deformation Conditions

Tao and Kong [11] established the relationship model between the SWCC and saturated/unsaturated permeability coefficient based on the SWCC, capillary, and fluid mechanics theories from the microscopic pore channel, which has ideal effects. The total pore channels are assumed to have n grades, and now only the channels of grade 1 m are filled with water (m < n). Combined with the soil–water characteristic curve prediction method of deformed soil proposed by Tao et al. [18], a novel expression of unsaturated relative permeability coefficient is proposed as follows:

$$k_r(\theta_{i=\mathrm{m}}) = \sum_{i=1}^{i=\mathrm{m}} \frac{\Delta\theta_i}{{\psi_i}^2} \Big/ \sum_{i=1}^{i=\mathrm{n}} \frac{\Delta\theta_i}{{\psi_i}^2} \tag{3}$$

where the water content in the above formula requires the form of volumetric water content; ψ_i is the matrix suction of the corresponding pore channel; θ_i is the volumetric water content corresponding to the matrix suction ψ_i; n is the total number of pore channels; and m is the number of pore channels filled with water (m < n).

The model is used to predict the unsaturated relative permeability coefficient of Wuhan clay soil under different deformations. It is found that the variation law of the unsaturated relative permeability coefficient with the matrix suction under different deformation conditions can be described by the "brush type distribution" under double logarithmic coordinates. That is: before air-entry value ψ_a, unsaturated relative permeability coefficient is 1; after air-entry value, ψ_a, the unsaturated relative permeability coefficient decreases with the increase in matrix suction, and the corresponding slopes are approximately equal under different initial void ratios, as shown in Figure 1.

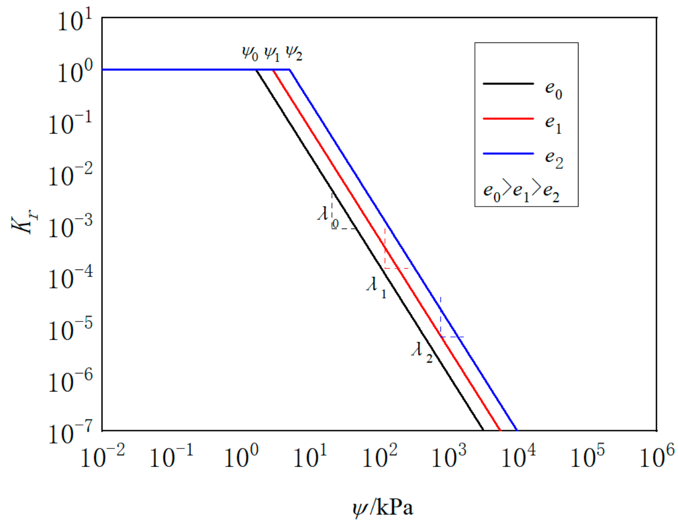

Figure 1. Law of permeability coefficient of unsaturated soil under deformation conditions.

2.4. Simple Graphing Method

According to the variation law for the relative permeability coefficient of unsaturated soil under deformation conditions, this article presents a simple graphing method for predicting the relative permeability coefficient of unsaturated soil with varying initial void ratios. This method is based on the measured value of the unsaturated permeability coefficient and the air-entry value prior to deformation; the relevant model is not confined, unlike the Tao–Kong, Burdine, or Mualem models, and is applicable to a broader range of situations.

2.4.1. Measure the SWCC and Unsaturated Relative Permeability Coefficient before Deformation

The unsaturated permeability coefficient is calculated by multiplying the saturated permeability coefficient by the unsaturated relative permeability coefficient. After determining the soil's unsaturated and saturated permeability coefficients, the measured value of the soil's unsaturated relative permeability coefficient can be computed by dividing the unsaturated permeability coefficient by the saturated permeability coefficient. Currently, the primary methods for determining the SWCC are the salt solution method, the filter paper method, and the tensiometer approach.

2.4.2. Solve the Fractal Dimension

The literature [18] uses the fractal theory to express the fractal form of the SWCC by gravimetric water content:

$$\begin{cases} w = \frac{e}{G_s}\left(\frac{\psi_a}{\psi}\right)3 - D & \psi \geq \psi_a \\ w = \frac{e}{G_s} & \psi < \psi_a \end{cases} \quad (4)$$

where w is the gravimetric water content, e is the void ratio, ψ is matric suction, Gs is the relative density, ψ_a is air-entry value, and D is the fractal dimension.

Equation (4) can be transformed into the fractal form of the SWCC concerning saturation, as follows:

$$\begin{cases} S_{\mathrm{r}} = \left(\frac{\psi_{\mathrm{a}}}{\psi}\right)3 - D & \psi \geq \psi_{\mathrm{a}} \\ S_{\mathrm{r}} = 1 & \psi < \psi_{\mathrm{a}} \end{cases} \quad (5)$$

where S_{r} is the degree of saturation. The method of solving the fractal dimension D is as follows: Using—$ln\psi$ as the horizontal coordinate and $ln\ w$, $ln\ \theta$, or $ln\ S_r$ as the vertical coordinate, a scatter plot is drawn and a straight line fit is then made; assuming a slope of k, the number of fractional dimensions $D = 3 - k$.

2.4.3. Predict the Air-Entry Value after Deformation

A previous study [18] analyzed the fractal dimension and the change law of air-entry value under different initial void ratio conditions, and then gave the following formula for predicting air-entry value under deformation conditions:

$$\psi = \frac{\psi_{a0}}{\left(\frac{e_1}{e_0}\right)^{1/(3-D_0)}}, \quad (6)$$

where ψ_{a0} is the air-entry value corresponding to the maximum initial void ratio e_0, which can be obtained through the fitting of Equation (4), and D_0 is the fractal dimension, which is almost unchanged under deformation conditions. The fractal dimension D_0 can be calculated from the test results of the SWCC at e_0. According to Equation (6), the air-entry value under any initial void ratio can be predicted.

2.4.4. Determine λ According to the Measured Value of the Unsaturated Relative Permeability Coefficient before Deformation

Based on the relative permeability coefficient of the unsaturated soil before deformation, the air-entry value ψ_{a0} of the soil before deformation is obtained by fitting Equation (4) against experimental data, and the two are substituted into the simplified unified model Equation (2) to obtain the index λ before deformation.

2.4.5. Determine the Deformed λ

As shown in Figure 1, if the index is λ_0 at the maximum initial porosity ratio e_0, and after obtaining the index λ_0 at e_0 before deformation, it is known that λ will not change significantly under deformation conditions, then $\lambda_0 = \lambda_1 = \lambda_2$ for e_1, e_2 ($e_0 > e_1 > e_2$) for any deformation condition can be taken.

2.4.6. Simple Drawing

As shown in Figure 1, based on the relative permeability coefficient and air-entry value of unsaturated soil at e_0 before deformation mentioned above, the parameter λ can be obtained by fitting with Equation (2) and then drawing the relationship between the measured relative permeability coefficient and matrix suction at e_0 before deformation. The air-entry value under void ratio e_1 and e_2 after deformation can be predicted using Equation (6). Then, starting with the air-entry value, a straight line can be drawn to the right. The slope of the straight line and the initial void ratio e_0 after the air-entry value are the same. The slope of the straight line is the same, so the relationship between the relative coefficient of the unsaturated soil and the matrix suction after deformation can be quickly obtained, and the purpose of predicting the relative coefficient of the unsaturated soil under arbitrary deformation conditions can be achieved.

3. Method Verification

To verify the rationality of the above method, this paper uses the soil–water characteristic curve test data and the measured values of the unsaturated relative permeability coefficients of silt loam, sandy loam, and unconsoildated sand in the literature [23] to verify the rationality of the proposed method.

3.1. Calculation of Fractal Dimension

To predict the air-entry value of the soil after deformation by Equation (6), firstly, it is required to calculate the fractal dimension D_0 of the soil before deformation. The solution method is as mentioned above. Figure 2 shows the fractal dimensions of three types of soil.

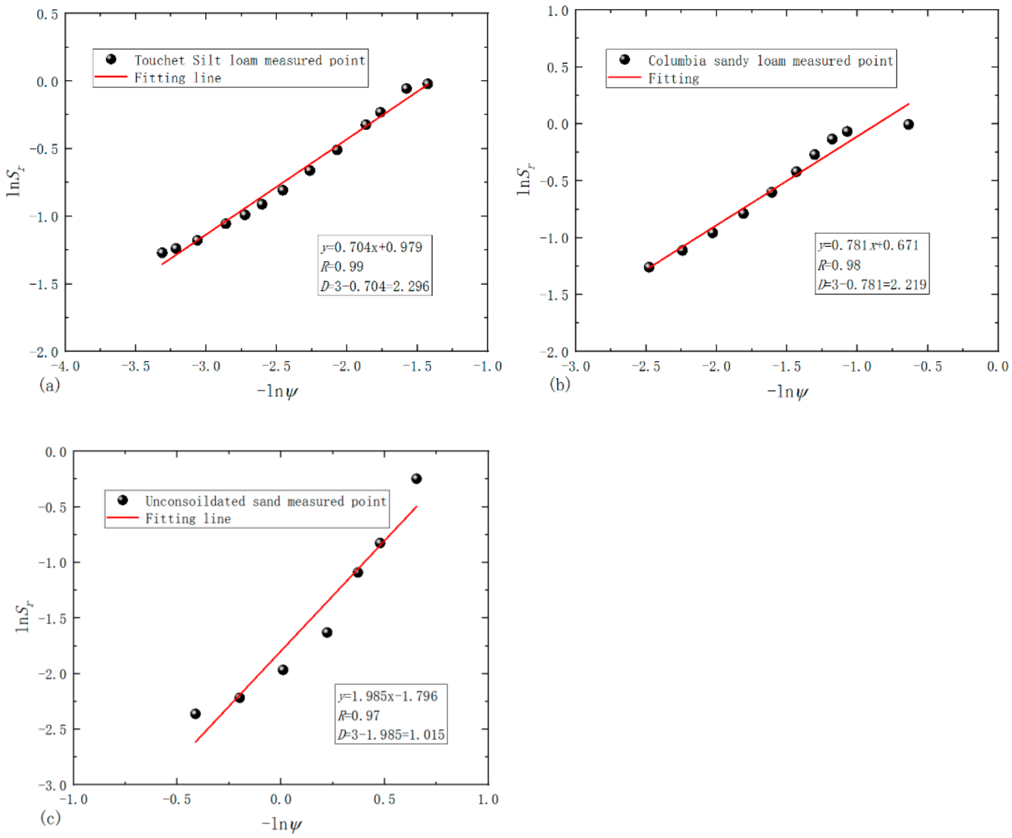

Figure 2. Fitting to obtain the fractal dimension of three soil types: (**a**) Touchet Silt loam; (**b**) Columbia sandy loam; (**c**) unconsoildated sand.

In Figure 2a–c, it is shown that the fitting correlation coefficient R is above 0.97, indicating that the fractal characteristic of soil is obvious. For different types of soils, this method has better applicability when solving fractal dimensions. This demonstrates the significance of the proposed approach.

3.2. Prediction of Air-Entry Value under Deformation Conditions

Based on the obtained fractal dimensions of the soil and the air-entry value ψ_{a0} of the maximum initial void ratio of the three soils obtained by Equation (4), substituting it into Equation (6), the air-entry values after deformation are predicted and listed in Table 2.

It can be seen from Table 2 that the predicted values of air-entry value under different void ratios are all very close to the measured values, which indicates that the prediction effect is good.

Table 2. Air-entry value predicted value.

Soil Type	Void Ratio	Air-Entry Value/kPa Measured	Prediction
Touchet silt loam [23]	1.012	4.13	4.13
	0.916	5.07	4.76
	0.815	6.35	5.62
	0.733	7.56	6.53
	0.653	8.95	7.70
Columbia sandy loam [23]	1.268	2.65	2.65
	1.114	3.34	3.13
	0.942	4.52	3.88
	0.890	5.07	4.17
	0.815	5.87	4.67
Unconsoildated sand [23]	0.852	0.49	0.49
	0.825	0.52	0.50
	0.799	0.54	0.51
	0.767	0.56	0.52
	0.715	0.59	0.54

3.3. Determine λ According to the Measured Value of the Unsaturated Relative Permeability Coefficient before Deformation

Based on the measured values of the relative permeability coefficients of the three kinds of soils before deformation, the air-entry value ψ_{a0} of the soil before deformation is obtained by Equation (4). Substituting the two parameters into the simplified unified model Equation (2), the model indices λ_0 before deformation are obtained, which are 4.96, 4.83, and 12.87 (the data before the air-entry value is discarded during fitting).

In Figure 3a–c, the correlation coefficient R^2 is higher than 0.96, which indicates that the λ index obtained by the fitting process has high accuracy. As illustrated in the figure, with the increase in matrix suction, its fitting correlation increases, which indicates the high accuracy of the model. It shows that, for different types of soil, the method of obtaining the parameter λ by using the unified model to fit the measured values and air-entry values of the unsaturated relative permeability coefficients of the soils before deformation has good applicability.

3.4. Determine the Deformed λ

It is apparent from the relationship between parameter λ and fractal dimension D that the index λ does not change significantly under deformation conditions. The λ of Touchet silt loam, Columbia sandy loam, and unconsoildated sand are the same irrespective of deformation, and are 4.96, 4.83, and 12.87, respectively.

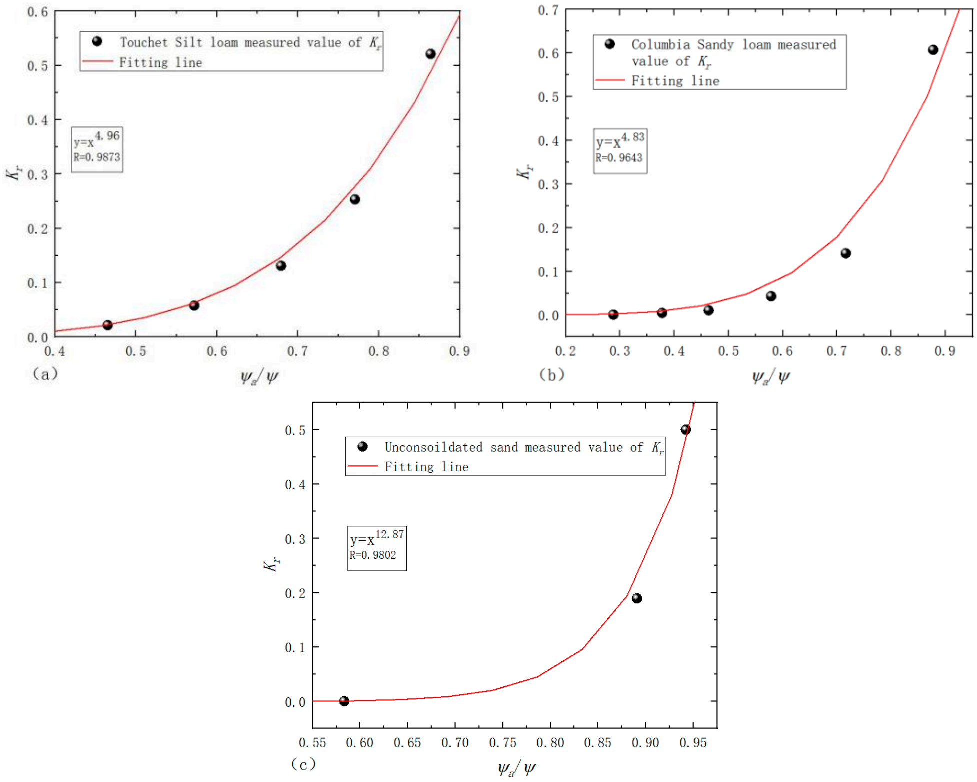

Figure 3. Fitting values of λ for three types of soil: (**a**) Touchet silt loam; (**b**) Columbia sandy loam; (**c**) unconsoildated sand.

3.5. Simple Drawing

The relationship between the measured value of the relative permeability coefficient of the unsaturated soil at the maximum initial void ratio and the matrix suction data is obtained. Then, the air-entry value of each void ratio after deformation is predicted using Equation (6). Then, starting with the air-entry value, a straight line is drawn to the right. When the suction value is after the air-entry value, the slope of the straight line at each void ratio after deformation is the same as the slope of the straight line at the initial void ratio. Therefore, the relationship between the relative coefficient of the unsaturated soil and the matrix suction after deformation can be quickly obtained, as shown in Figures 4–6.

In Figures 4–6, the graphs marked (a) depict the relationship between the unsaturated relative permeability coefficient and the matrix suction at the maximum initial void ratio. The graphs marked (b) to (e) present the relationships between the unsaturated relative permeability coefficient and the matrix suction under different void ratios. It is apparent that the predicted value of the three types of soil unsaturated relative permeability coefficient and the measured value coincide, indicating that the prediction effect is better. It is worth noting that, using the prediction method in this paper, unsaturated relative permeability coefficient of soil under arbitrary deformation conditions can be effectively and quickly predicted.

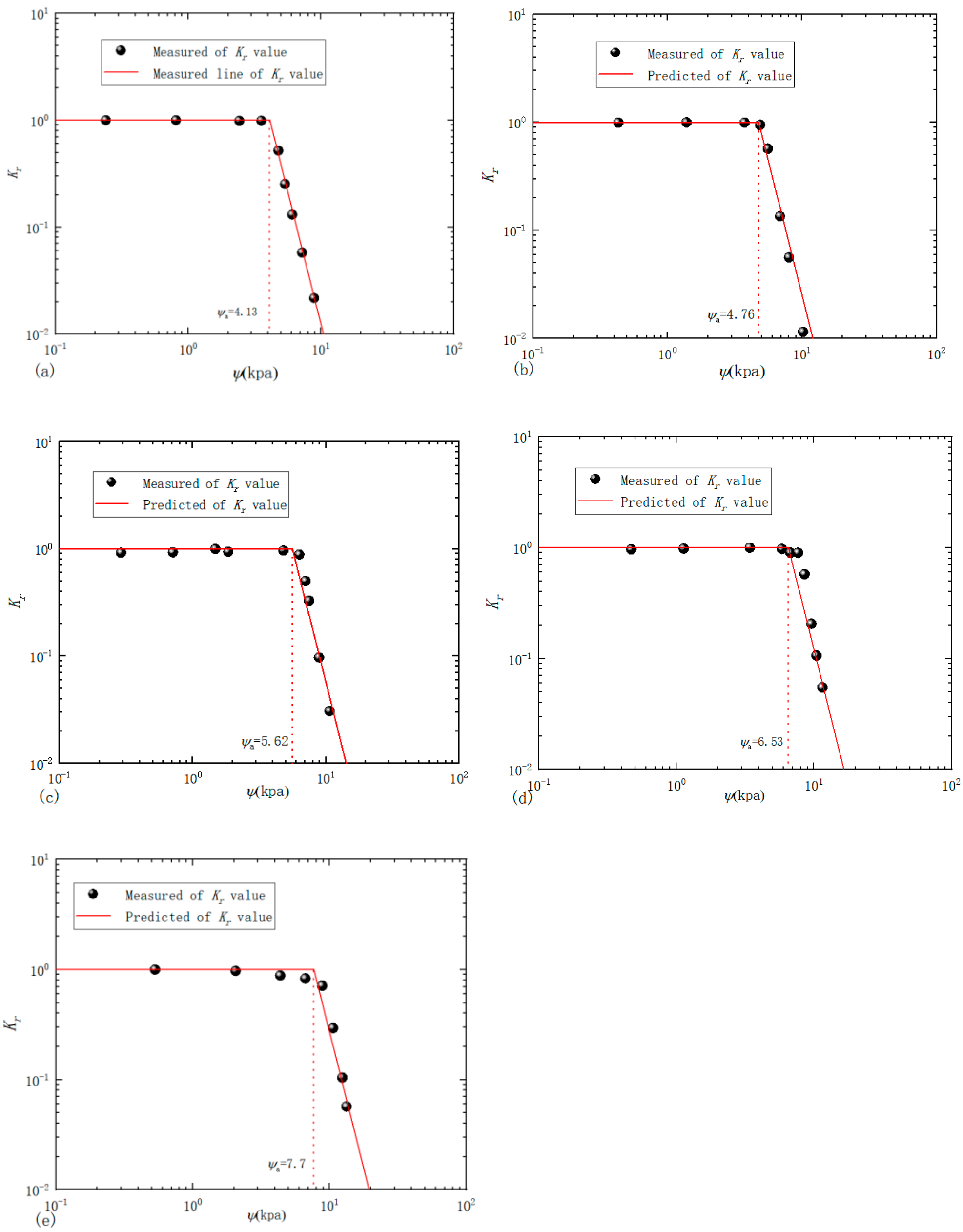

Figure 4. Touchet silt loam's different porosity ratio unsaturated relative permeability coefficient prediction results: (**a**) e = 1.012; (**b**) e = 0.916; (**c**) e = 0.815; (**d**) e = 0.733; (**e**) e = 0.653.

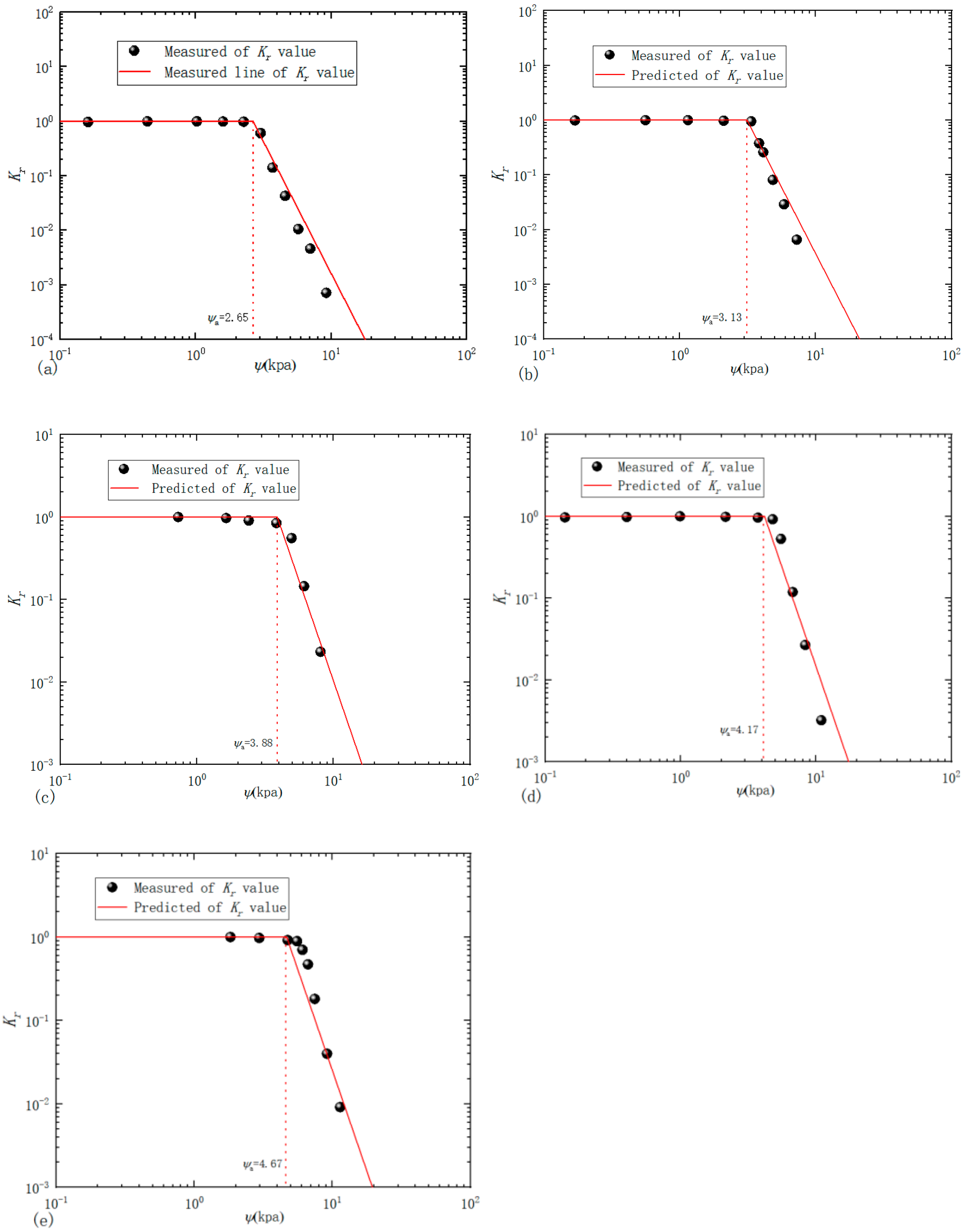

Figure 5. Prediction results of the unsaturated relative permeability coefficients of Columbia sandy loam with different void ratios: (**a**) e = 1.268; (**b**) e = 1.114; (**c**) e = 0.942; (**d**) e = 0.89; (**e**) e = 0.815.

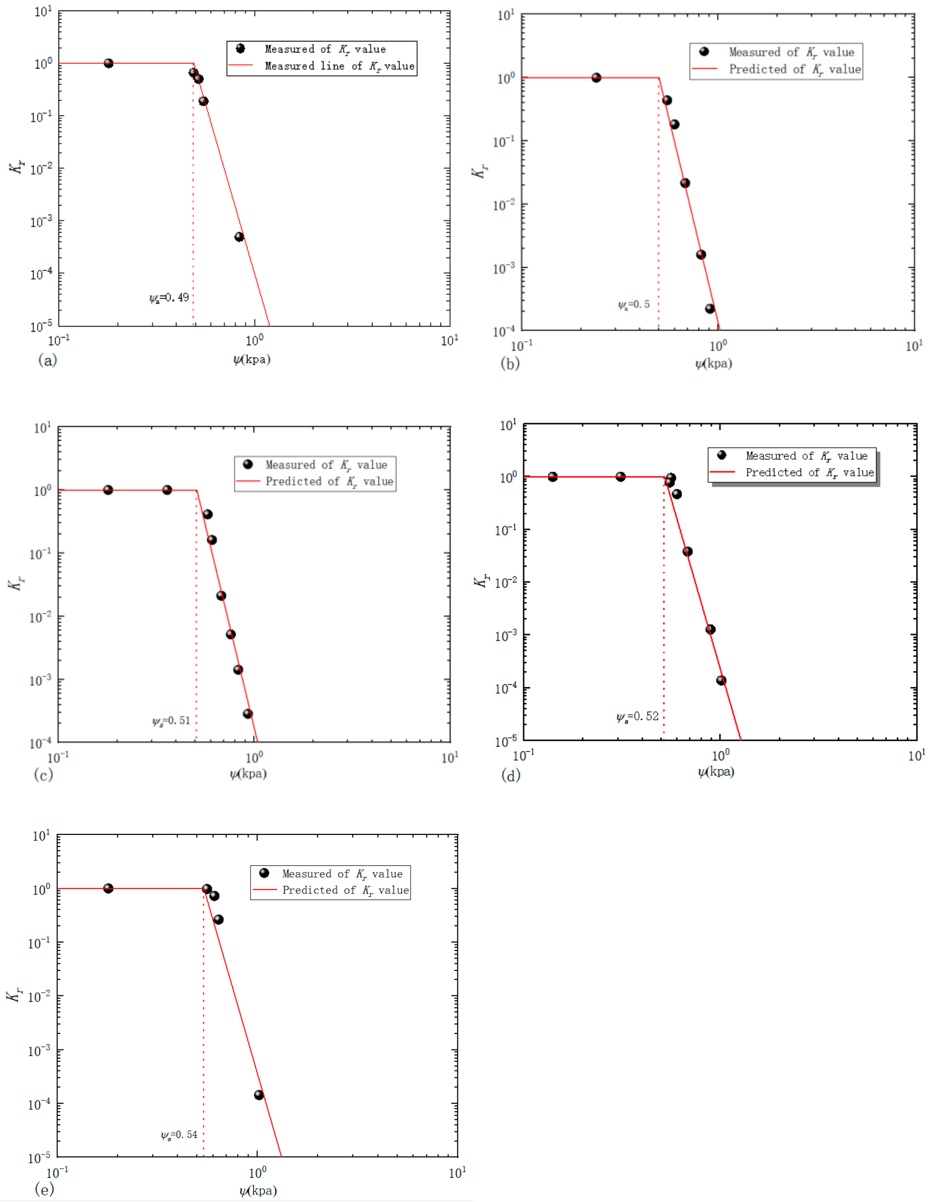

Figure 6. The prediction results of the unsaturated relative permeability coefficients of unconsoildated sand with different porosity ratios: (**a**) e = 0.852; (**b**) e = 0.825; (**c**) e = 0.799; (**d**) e = 0.767; (**e**) e = 0.715.

4. Discussion

The deformation of soil will cause changes in the pore ratio, pore size distribution, and pore shape of soil, for example, large pores will be compressed into small pores [24]. Tao et al. conducted experiments on Wuhan clay through mercury intrusion test, nuclear magnetic resonance test, and scanning electron microscope (SEM) test, and found that the cumulative pore volume distribution per unit particle mass showed a "broom-type" distribution under different compression deformation conditions. The soil–water characteristic curve represented by the mass moisture content also presents a similar distribution, the change law of the two is relatively consistent. Therefore, it is reasonable to use simple mapping method to predict the permeability coefficient of unsaturated soil. Tao and Zhu [25] derive a fractal model of the relative permeability coefficient of unsaturated soil based on fractal theory and seepage theory. The expression is as follows:

$$k_r(\psi) = \left(\frac{\psi_a}{\psi}\right)^{5-D} \tag{7}$$

By combining Equations (6) and (7), we can obtain the air-entry value for various void ratios and, eventually, the unsaturated relative permeability coefficients for soils under deformation conditions. The model index is $5 - D$, which may not apply to some soils. Although the fractal forms of the CCG model, Burdine model, and Mualem model derived in the simplified unified model give other possible forms of the relationship between λ and D, as illustrated in Table 1, these relationships cannot cover all cases. In some instances, λ and D cannot be related using the any of above expressions. Therefore, this paper recommends first measuring the unsaturated relative permeability coefficient and SWCC of the soil before deformation and fitting the value of λ based on the measured data before deformation, that is the λ value after deformation. This paper takes the Touchet silt loam in the literature [23] as an example to compare and analyze the difference between the prediction method proposed by Tao and Zhu [25], and the predicted results are shown in Figure 7.

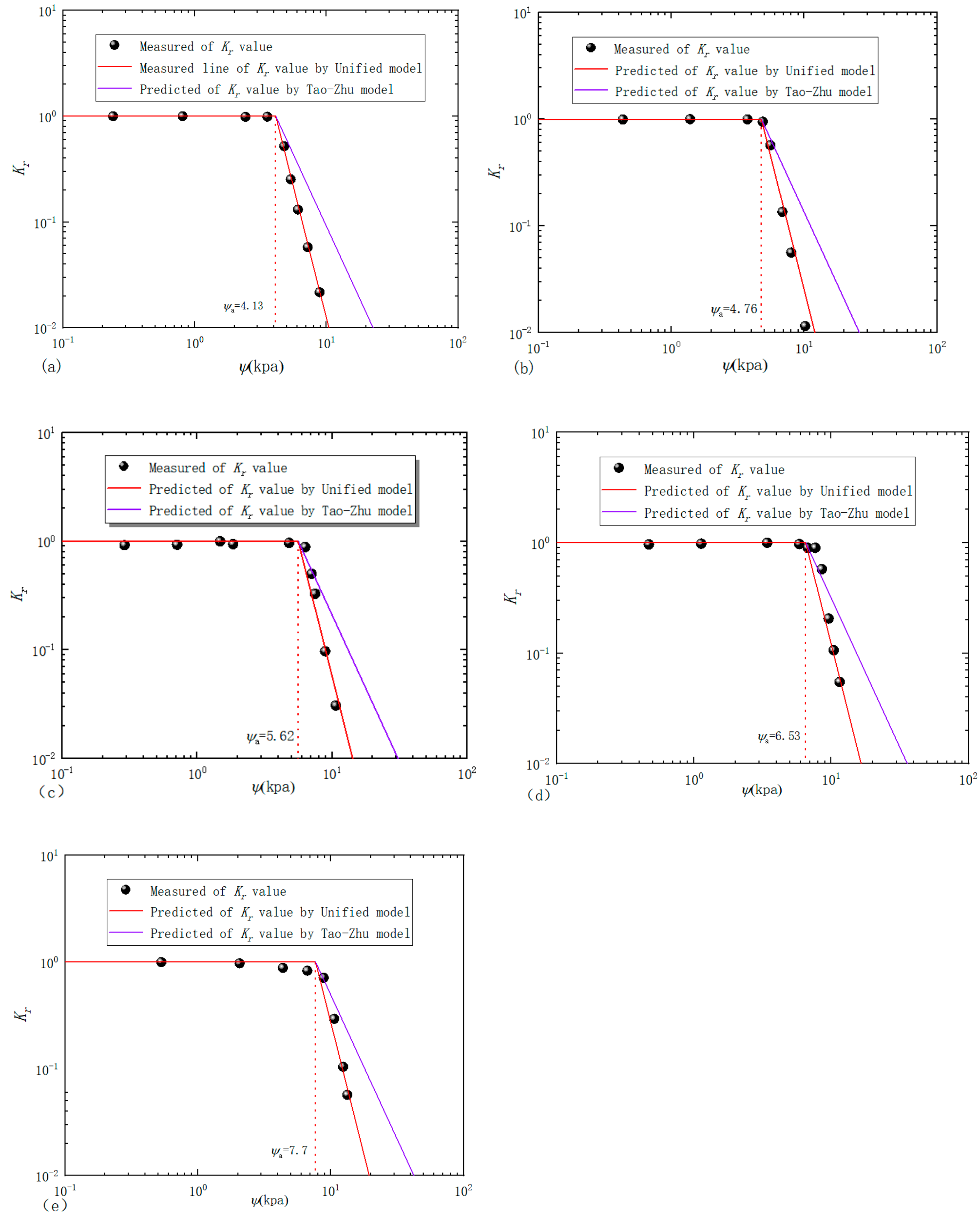

Figure 7. Comparison of Tao and Zhu models with different void ratios: (**a**) e = 1.012; (**b**) e = 0.916; (**c**) e = 0.815; (**d**) e = 0.733; (**e**) e = 0.653.

As illustrated in Figure 7, the predicted values of the unsaturated relative permeability coefficient obtained using the simplified unified model are better to those obtained using the Tao and Zhu model and are more in line with the measured value. It demonstrates that the simple graphing method used to predict the relative permeability coefficient of unsaturated soil under deformations has a broader range of applicability.

5. Conclusions

This study proposed a unified model to determine index λ based on the laboratory measured value of the unsaturated relative permeability coefficient and the air-entry value before deformation. Additionally, the value of index λ remains unchanged before and after deformation. Based on the SWCC obtained before deformation, the fractal dimension and air-entry value could be obtained, which then could be used to predict the air-entry value after deformation. Combined with the simplified unified model, the relationship between the relative permeability coefficient and matrix suction under arbitrary deformations conditions can be readily predicted using the proposed graphing method. The graphical procedure is simple and easy to implement. The method is based on the measured values of the unsaturated permeability coefficient before deformation and overcomes the defect that calculating the index λ through a specific relationship in the unsaturated relative permeability fractal model. The proposed model does not have limitations like the Tao–Kong model, the Burdine model, and the Mualem model, and it can be adopted for a wider range with improved accuracy.

Author Contributions: G.T. and Q.W. contributed to the conceptualization and methodology; Q.C. and Y.P. conducted the analyses; Q.W., S.N. and H.D. wrote and edited the manuscript. All authors discussed the results. All authors have read and agreed to the published version of the manuscript.

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