



# Article Design of Fractional-Order Lead Compensator for a Car Suspension System Based on Curve-Fitting Approximation

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Abstract: An alternative procedure for the implementation of fractional-order compensators is presented in this work. The employment of a curve-fitting-based approximation technique for the approximation of the compensator transfer function offers improved accuracy compared to the Oustaloup and Padé methods. As a design example, a lead compensator intended for usage in car suspension systems is realized. The open-loop and closed-loop behavior of the system is evaluated by post-layout simulation results obtained using the Cadence IC design suite and the Metal Oxide Semiconductor (MOS) transistor models provided by the Austria Mikro Systeme 0.35  $\mu$ m Complementary Metal Oxide Semiconductor (CMOS) process. The derived results verify the efficient performance of the introduced implementation.

**Keywords:** fractional-order controllers; motion control systems; Oustaloup approximation; Padé approximation; CMOS analog integrated circuits; operational transconductance amplifiers

## 1. Introduction

The objective of a car suspension system is to provide both road holding/handling and ride quality, which are at odds with each other. It is important for the suspension to keep the wheel in contact with the road surface as much as possible, because all the road or ground forces, acting on the vehicle, do so through the contact patches of the tires. Therefore, the suspension system is responsible for maintaining stability between vehicle position shifts during driving in order to achieve good ride control. The suspension also protects the vehicle itself and any cargo or luggage from damage and wear. According to the particular requirements of cars, the suspension system can be active or passive [1-10]. A passive vehicle suspension system with fixed parameters that cannot be adjusted after they are determined offers a simple structure and reliable performance at low cost. The passive system is generally composed of a stiffness-damping system and can only achieve either good ride comfort or good road holding, as the parameters cannot be changed with the external excitation—a fact that limits the vehicle's performance. An active suspension system possesses the ability to reduce the acceleration of the sprung mass continuously as well as to minimize suspension deflection, which results in an improvement of the tire grip with the road surface. Some performance requirements are offered by advanced suspension systems, which prevent the road disturbances from affecting passenger comfort while increasing riding capabilities and a delivering smoother driving experience; increasing the ride comfort results in a larger suspension stroke and smaller damping in the wheel hop mode. The advantage of a system with continuous adaptability comes at the expense of a higher cost. The selection of appropriate controller parameters in order to achieve the good performance of the suspension system is a very important task in the development of



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**Copyright:** © 2021 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (https:// creativecommons.org/licenses/by/ 4.0/). a top-class vehicle that simultaneously offers comfort and safety. In this work, an active car suspension system is considered.

A lead-lag compensator can help the suspension system to meet the demanded robustness requirements, as it is able to stabilize unstable systems and obtain the desired performance specifications. The classical integer-order lead-lag compensator is described by the following transfer function:

$$C_{IO}(s) = K \cdot \frac{\tau s + 1}{x \tau s + 1}, \qquad (1)$$

where *K* is the low-frequency gain and  $\tau$  is a time constant associated with the pole and zero according to the formulas  $\omega_z = \frac{1}{\tau}$  and  $\omega_p = \frac{1}{x \cdot \tau}$ , respectively. The kind of compensator is determined by the range of values of the variable *x*, which describes the scaling of the time constant. In the case that 0 < x < 1, then  $\omega_z < \omega_p$  and the resulting compensator is called the lead compensator. Lead compensators [11] push the open-loop poles to the left, and thus they give a more stable system with a fast response. In addition, they increase the phase margin and, thanks to the existence of the pole, high frequencies which are mostly corrupted by noise are less amplified. Lag compensators [12] (i.e., x > 1 and  $\omega_z > \omega_p$ ) decrease the bandwidth and the speed of response, which is preferable if the model does not have good performance at high frequencies, in order to reduce the impact of (mostly high-frequency) noise. By decreasing the magnitude, the gain crossover frequency is shifted to a frequency with a larger phase margin. In contrast, the open-loop gain at low frequencies is increased, reducing the static error, and the transient response becomes slower. Under such conditions, a phase lag angle is added to the gain crossover frequency; this design also allows gain to be added at low frequencies (which improves the steadystate error). As the achieved stability is an important feature for the car suspension system, a fractional-order lead-lag compensator can be a more advantageous option for the control stage, due to the additional degree of freedom it provides. The transfer function of such a compensator is described by an enhanced version of (1), which is given by (2):

$$C_{FO}(s) = K \cdot \left(\frac{\tau s + 1}{x\tau s + 1}\right)^q.$$
(2)

where q > 0 is an extra degree of freedom [11,13–15]. Setting  $s = j\omega$  in both (1) and (2), we canderive that

$$|C_{FO}(j\omega)| = K^{1-q} \cdot |C_{IO}(j\omega)|^q, \qquad (3)$$

$$\angle C_{FO}(j\omega) = \angle K + q \cdot \angle C_{IO}(j\omega).$$
(4)

Inspecting (3)–(4), it is readily obtained that the factor q affects both the magnitude and phase responses, while the pole and zero both remain unaffected. With regards to its integer-order counterpart, it provides a scaling of phase, which can be used for the better stabilization of the system through the phase-margin parameter, by optimizing the time constant and/or its associated scaling factor [16,17]. The result is a more precise control of the suspension system's characteristics, which brings the system's performance closer to the ideally predicted behavior.

The approximation of (2) can be performed using the Oustaloup or Padé approximation tools, which are one-step processes requiring only one function. The main advantage of the Padé approximation is the convergence acceleration, which leads to an efficient approximation even outside a power series expansion's radius of convergence, but it suffers from an absence of control of the range, where a specific degree of accuracy is achieved. The Oustaloup approximation suffers from reduced accuracy at the limits of the frequency bandwidth of interest [18]. Other possible solutions, based on the approximation of both magnitude and phase frequency characteristics of (2), are the employment of the CRONE control toolbox [19], as well as the built-in functions of MATLAB software. The main contribution of this work is the employment of a curve-fitting based procedure for implementing a FO lead-lag compensator. Simply for demonstration purposes, the tool offered in MATLAB for curve fitting approximation is employed, and the comparison with the Oustaloup and Padé approximations shows that an improved accuracy is achieved.

The paper is organized as follows: the procedure for approximating the transfer function of FO compensator is presented in Section 2, while the possible implementations are compared in Section 3. The performance of the system is evaluated in Section 4, where post-layout simulation results, obtained using the Cadence software and the Design Kit provided by the Austria Mikro Systeme (AMS) CMOS 0.35  $\mu$ m technology, are presented.

#### 2. Curve-Fitting-Based Approximation of the Compensator Transfer Function

The approximation procedure is based on MATLAB software (ManthWorks, Natick, MA, USA) and the built-in functions it provides. Following the floating chart in Figure 1, a clear view of the approximation process is obtained. The starting point is the derivation of the frequency response data (both magnitude and phase) of the operator  $(\tau s + 1)/(x\tau s + 1)$  using the built-in function *freqresp*. Then, these data are raised to the fractional power *q* and multiplied by the low-frequency gain factor *K*; after that, the frequency response data model is formed utilizing the built-in function *frd*. The approximation process is performed by applying the *fitfrd* function to the frequency response data model—a procedure that is based on the Sanathanan–Koerner (SK) least square iterative method and fits the frequency response data into a state-space model [20–24]. A corresponding rational integer-order transfer function is derived using the *tf* command and has the form of a ratio of integer-order polynomials given by (5)

$$C_{fitfrd}(s) = \frac{B_n s^n + B_{n-1} s^{n-1} + \dots + B_1 s + B_0}{A_n s^n + A_{n-1} s^{n-1} + \dots + A_1 s + A_0},$$
(5)

where  $A_i$  and  $B_i$  ( $i = 0 \dots n$ ) are positive, real coefficients.

$$freqresp\left(\frac{\tau s+1}{x\tau s+1}\right) \xrightarrow{1} C = K \cdot \left[freqresp\left(\frac{\tau s+1}{x\tau s+1}\right)\right]^{q}$$

$$\downarrow 2$$

$$C_{approx} = fitfrd(C) \xrightarrow{3} C_{data} = frd(C)$$

$$\downarrow 4$$

$$C_{fitfrd} = tf(C_{approx}) = \frac{B_{n}s^{n}+B_{n,l}s^{n-l}+...+B_{l}s+B_{0}}{A_{n}s^{n}+A_{n,l}s^{n-l}+...+A_{l}s+A_{0}}$$

**Figure 1.** Floating chart description of the curve-fitting-based approximation procedure for approximating the transfer function in (2).

The performance of the approximation, performed through the aforementioned method, will be compared with that offered by the Padé approximation. For this purpose, let us consider a car suspension system, which is described by the transfer function

$$P(s) = \frac{1}{M \cdot s^2},\tag{6}$$

where *M* is the mass supported by each wheel, which is assumed to be equal to the fourth of total vehicle weight, lying in the range [100,900] kg [25]. Setting the gain crossover frequency for the typical mass of  $M_o = 300$  kg equal to  $\omega_{cg} = 10$  rad/s to maximize passenger comfort and restricting the step response overshoot equal to 20% for all values of *M*, the resulting transfer function of the required FO lead compensator is

$$C(s) = 4260 \cdot \left(\frac{1 + \frac{s}{0.5}}{1 + \frac{s}{200}}\right)^{0.65}.$$
(7)

Comparing (2) with (7), it is readily obtained that K = 4260,  $\omega_p = 20 \cdot \omega_{cg} = 200$  rad/s and  $\omega_z = \omega_{cg}/20 = 0.5$  rad/s.

Considering a fourth-order approximation, the corresponding transfer functions (scaled by the factor *K*) of the approximated compensator, derived using the curve-fitting-based procedure presented here, are given by

$$C_{fitfrd}(s) = \frac{49.040s^4 + 5157s^3 + 9.802 \cdot 10^4 s^2 + 3.539 \cdot 10^5 s + 1.974 \cdot 10^5}{s^4 + 233.3s^3 + 1.14 \cdot 10^4 s^2 + 1.16 \cdot 10^5 s + 1.941 \cdot 10^5}.$$
 (8)

The corresponding transfer function, based on the Padé approximation method, is also obtained using MATLAB and its built-in function *pade* and is given by

$$C_{pade}(s) = \frac{47.540s^4 + 3146s^3 + 4.162 \cdot 10^4 s^2 + 1.273 \cdot 10^5 s + 7.009 \cdot 10^4}{s^4 + 181.6s^3 + 5938s^2 + 4.448 \cdot 10^4 s + 6.783 \cdot 10^4}.$$
 (9)

The integer-order transfer function derived through the Oustaloup filter approximation is

$$C_{oust}(s) = \frac{49.130s^4 + 3669s^3 + 4.963 \cdot 10^4 s^2 + 1.386 \cdot 10^5 s + 7.009 \cdot 10^4}{s^4 + 197.7s^3 + 7081s^2 + 5.234 \cdot 10^4 s + 7.009 \cdot 10^4}.$$
 (10)

A comparison between the two methods can be performed considering the gain and phase frequency responses in Figure 2a, where the ideal responses described by the transfer function in (7) are also shown by dashes. The associated error plots are given in Figure 2b, where it is concluded that the curve-fitting approximation method provides a more accurate approximation of the compensator transfer function than that offered by the Oustaloup and Padé approximations.



**Figure 2.** (a) Gain and phase frequency responses of the compensator derived from *fitfrd* (8), Padé (9), and Oustaloup (10) approximation methods along with the ideal responses derived from (7) and (b) the relative errors for each method.

#### 3. Implementation of the Approximated Compensator

The transfer function in (8) can be implemented using a multi-feedback topology, as presented in [26], or the partial fraction expansion method introduced in [27]. The last approach can be rewritten in the form of

$$C_{fitfrd-partial}(s) = 49.044 - \frac{33.780}{0.0059s + 1} - \frac{9.840}{0.0201s + 1} - \frac{3.328}{0.0910s + 1} - \frac{1.074}{0.4809s + 1}.$$
 (11)

The corresponding Functional Block Diagram (FBD) is demonstrated in Figure 3, where the implemented transfer function is as follows:

$$H(s) = K_0 - \frac{K_1}{1 + \tau_1 s} - \frac{K_2}{1 + \tau_2 s} - \frac{K_3}{1 + \tau_3 s} - \frac{K_4}{1 + \tau_4 s}.$$
 (12)

Comparing the coefficients of (11) and (12), the derived values of scaling factors and time constants are summarized in Table 1.



Figure 3. Functional Block Diagram implementing the transfer function in (11).

Table 1.	Values of scaling	factors and	time constants of the FBD	in Figure 3
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Scaling	Factors	Time Constants		
Variable	Value	Variable	Value	
K <sub>0</sub>	49.044			
$K_1$	33.780	$ au_1(ms)$	5.9	
<i>K</i> <sub>2</sub>	9.840	$\tau_2(ms)$	20.1	
$K_3$	3.328	$\tau_3(ms)$	91	
$K_4$	1.074	$ au_4(\mathrm{ms})$	480.9	

The implementation of the FBD in Figure 3 is performed using Operational Transconductance Amplifiers (OTAs) as active elements because of the offered flexibility for performing the required inversion of the transfer functions originating from their differential input. The corresponding OTA-C implementation is demonstrated in Figure 4, while the OTA structure employed in the simulations is depicted in Figure 5 [28,29]. The expression of the transconductance of the OTA is given in (13):

$$g_m = \frac{5}{9} \cdot \frac{I_B}{n \cdot V_T} \,, \tag{13}$$

where *n* is the slope factor of a MOS transistor in the sub-threshold region (1 < n < 2),  $V_T$  is the thermal voltage (26 mV at 27 °C), and  $I_B$  is the associated DC bias current. The

time constants of integrators are electronically controlled through the bias current, and the required scaling factors are implemented through an appropriate scaling of the DC bias currents of the associated transconductances.



Figure 4. OTA-C implementation of the approximated compensator.



Figure 5. MOS circuitry of a high-linearity OTA, as employed in simulations.

The implemented time constants are given by (14):

$$\tau_i = \frac{C_i}{g_{mi}} \,. \tag{14}$$

Setting the DC bias current of OTAs equal to 30 pA, the calculated values of capacitors, obtained using (13) and (14) and the results in Table 1, are the following:  $C_1 = 3.13 \text{ pF}$ ,  $C_2 = 10.67 \text{ pF}$ ,  $C_3 = 48.51 \text{ pF}$ , and  $C_4 = 256.18 \text{ pF}$ . Utilizing the OTA-C structure in Figure 4 and considering the expression in (13), the compensator can be controlled using the DC currents.

Using the MOS transistor parameters provided by the AMS 0.35  $\mu$ m CMOS Design Kit and considering power supply voltages  $V_{DD} = -V_{SS} = 0.75$  V, the aspect ratios of the MOS transistors of the circuit in Figure 5 for ensuring operation in the subthreshold region are as follows:  $Mp1 - Mp2 = 5 \,\mu\text{m}/15 \,\mu\text{m}$ ,  $Mn1 - Mn2 = 2 \,\mu\text{m}/5 \,\mu\text{m}$ ,  $Mn3 - Mn4 = 10 \,\mu\text{m}/5 \,\mu\text{m}$  and  $Mb1 - Mb3 = 5 \,\mu\text{m}/5 \,\mu\text{m}$ .

#### 4. Simulation Results

The layout design of the compensator, performed using the Virtuoso Layout Editor of the Cadence IC design suite, is depicted in Figure 6 with dimensions of 190.65  $\mu$ m × 156.55  $\mu$ m.



**Figure 6.** Layout design of the OTA-C structure in Figure 4 (the pink framed part corresponds to the integration stage, while the blue framed part represents the summation stage).

The open-loop post-layout gain and phase responses of the system compensatorplant, obtained through the Virtuoso Analog Design Environment of Cadence software, are demonstrated in Figure 7a, along with the theoretically predicted responses given by dashes. The gain crossover frequency  $\omega_{cg}$  was 10 rad/s, as theoretically expected, and the phase margin was equal to 52.6°, with the theoretically predicted value being 55°. The corresponding closed-loop responses are given in Figure 7b. The time-domain behavior is evaluated through the stimulation of the system by a 200 mV step signal, and the output waveform is plotted in Figure 8. The theoretical settling time is 572.5 ms and the overshoot is 22.2%, while the post-layout settling time is 625.8 ms and the overshoot is 25%.



Figure 7. Post-layout (a) open-loop and (b) closed-loop gain and phase frequency responses of the system compensator-plant.



**Figure 8.** Post-layout step response of the closed-loop system stimulated by an input voltage of 200 mV.

The robustness of the step response of the system is evaluated through PVT corner analysis, offered by the Analog Design Environment, considering temperatures of 0 °C, 27 °C and 60 °C and  $\pm$ 5% changes in the power supply voltages. The worst case waveform corresponds to the MOS transistors' "worst-zero" models, and it is plotted in Figure 9. The measured settling time and overshot values were 534.1 ms and 24.8%, respectively, and these results confirm that the system has reasonable sensitivity characteristics.



Figure 9. Worst-case step response of the system obtained through PVT corner analysis.

#### 5. Conclusions

The utilization of a curve-fitting-based approximation procedure for implementing a lead compensator for a car suspension system offers a more efficient approximation of the original fractional-order transfer function compared to the corresponding values achieved through the Oustaloup and Padé approximations. The proposed procedure offers design versatility, in the sense that it can be applied for implementing compensators of any type (i.e., lead and lag) and any order. Another attractive feature is that the implementation of the derived rational integer-order transfer function can be performed using any of the already known design techniques, including multi-feedback structures or a cascade connection of intermediate filter functions [30–33]. In addition, there is no restriction regarding the choice of the active elements; for example, operational amplifiers (op-amps), second generation current conveyors (CCIIs), Current Feedback Operational Amplifiers (CFOAs) or Field-Programmable Analog Arrays (FPAAs) [34] could be utilized for this purpose. Drawbacks of the proposed procedure are the requirement of multiple steps as well as its association with the MATLAB software, in contrast to the one-step Oustaloup and Padé approximation tools, which are not exclusively oriented to this software.

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#### Abbreviations

The following abbreviations are used in this manuscript:

AMS	Austria Mikro Systeme
CMOS	Complementary metal oxide semiconductor
CCII	Second generation current conveyor
FO	Fractional-order
FBD	Functional block diagram
IC	Integrated circuit
IO	Integer-order
MOS	Metal oxide semiconductor
OTA	Operational transconductance amplifier
PFE	Partial fraction expansion

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