

Article

Enhancing Hydraulic Turbine Flexibility Through a Modified Radial-Axial Water Jet

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Abstract

In industrialized countries, existing regulations generally require the use of renewable energy to the greatest feasible extent. A major difficulty with renewable sources is the inherent fluctuation in their power output due to the main source character. By now, one of the best technologies capable of providing rapid compensation for these fluctuations is hydroelectric power. Hydropower plants, those equipped with hydraulic turbines with fixed blades (e.g., Francis, propeller) are typically designed to operate close to their best efficiency point (BEP) with acceptable load limits in the vicinity due to vibrations and pressure pulsations. Usually, the swirling flow exiting the runner is tailored for peak overall efficiency, which minimizes energy losses in the draft tube cone. When operating away from the design point, draft tube cone losses increase abruptly, and pronounce flow instabilities arise (e.g., vortex rope). This study proposes a new method to control such instabilities that inject a radial-axial water jet into the draft tube cone. Compared with conventional axial water jet injection, the radial-axial jet requires a lower additional flow rate while still effectively suppressing hydraulic instabilities in the draft tube cone. The carried-out analysis was done numerically by using Ansys Fluent 2023 R2. The performed 3D unsteady numerical simulations were carried out to examine the internal flow behavior and evaluate the effect of the radial-axial water jet injection on the unsteady behavior of the flow unsteadiness. Finally, the paper quantifies the relationship between the draft tube pressure fluctuation amplitude and the auxiliary flow rate needed to mitigate these instabilities.

Keywords: turbines; draft tube vortex rope suppression; radial-axial water jet injection



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1. Introduction

Renewable energy has undergone significant development worldwide over recent decades. Europe and also Romania have experienced the same trend and solar together with wind renewables were the main focus. In the last 8 years, almost 10% of Romanian energy has come from such renewable sources. However, the main challenge regarding this type of energy is associated with its volatility and variable production. For this reason,

rapid compensation of this fluctuation is ensured in Romania by hydroelectric power plants [1].

Hydropower units, especially ones equipped with fixed blade hydraulic turbines (e.g., Francis; propeller), are designed to operate close to the best efficiency point (BEP), typically allowing only a limited load variation of allowable vibrations and pressure pulsations. These machines are designed with a stationary stay vane, adjustable guide vanes (single regulation), and a radial-axial runner with fixed pitch blades. Their design is oriented to minimize the energy losses in the draft tube cone by manipulating swirling flow leaving the runner for peak overall efficiency. At off-design operation, the losses in the draft tube cone sharply increase, and flow instabilities occur and hinder the good operation [1,2]. Hence, the main purpose of the draft tube cone is to convert the excess kinetic energy into static pressure, recovering significant amounts of hydraulic energy, leading to an increase in the turbine's efficiency.

To meet the energy market demands, an extended operation range between -35% to 20% out of nominal discharge at BEP is recommended as guideline for good engineering practice [2]. This operation has as consequences the development of so-called vortex rope in the draft tube cone of such machines. This hydrodynamic instability emerges in the boundary layer between the main flow region (located near the draft tube cone wall) and the central stalled region [3–5]. Instabilities originating in the draft tube cone (e.g., vortex rope) are associated with large-amplitude pressure pulsations that propagate along the hydraulic circuit that leads to increasing hydraulic losses and thus reduce turbine efficiency. Moreover, extended operation under such regimes may lead to fatigue loads and stresses of material [2]. Hence, incidents such as crack formation on runner blades or detachment of the crown-mounted ogive requires shutting down the units and involves costs for repair [6,7].

In industrial practice, pressure pulsation induced by such hydrodynamic instabilities are mitigated by constraining the operating range of the turbine. As a result, the flexible compensating unit's role is not fulfilled. Over the years, many methods have been proposed to address this problem (e.g., suppressing hydrodynamic instabilities from the draft tube cone), but most of the approaches address symptoms rather than the root cause. Some well-known methods include filling the central stalled region of the draft tube cone with air or solid bodies [8,9] or attenuating the swirling flow near the wall of the conical diffuser by installing fins or ribs of other geometries [10]. The Laboratory of Hydraulic Machinery and the Research Centre for Engineering of Systems with Complex Fluids (RCESCF) at Politehnica University Timisoara are focused on investigation of hydraulic machines. The test rig developed under the two research teams explored the decelerated swirling flow in the conical diffuser representative for hydraulic turbine draft tube (Figure 1).

Under partial-load conditions, a vortex rope is generated in the draft tube cone (e.g., swirl apparatus and the test section, represented with blue color in Figure 1). This phenomenon produces significant pressure pulsations. In the last decade, a flow control technique patented by Resiga et al. [11,12] by injecting an axial water jet through the runner's crown along the draft tube cone was proposed. The laboratory-scale testing was recently carried out by Bosioc et al. [13,14]. The method addresses the root cause of instability and acts both on the dynamical response by reducing the pressure fluctuations and on the flow structure by eliminating the central stalled region, suppressing vortex rope formation. Consequently, the water jet injection has been tested on model turbines in additional laboratories and subsequently implemented in a Francis turbine prototype [15,16]. However, the major drawback that blocks large-scale deployment of this technique is the relative high volume of water required for axial injection, which exceeds 15% of the main discharge. The auxiliary circuit is marked with red in Figure 1 and should be tested in

controlled conditions using different control methods by adding water or air into the test section, particularly in the draft tube cone.

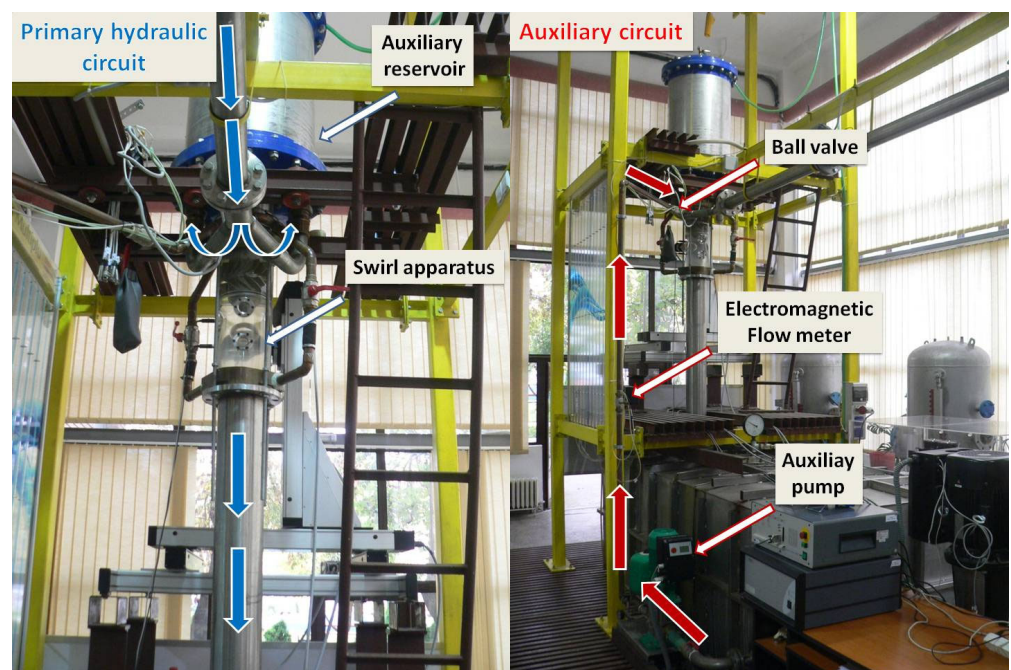
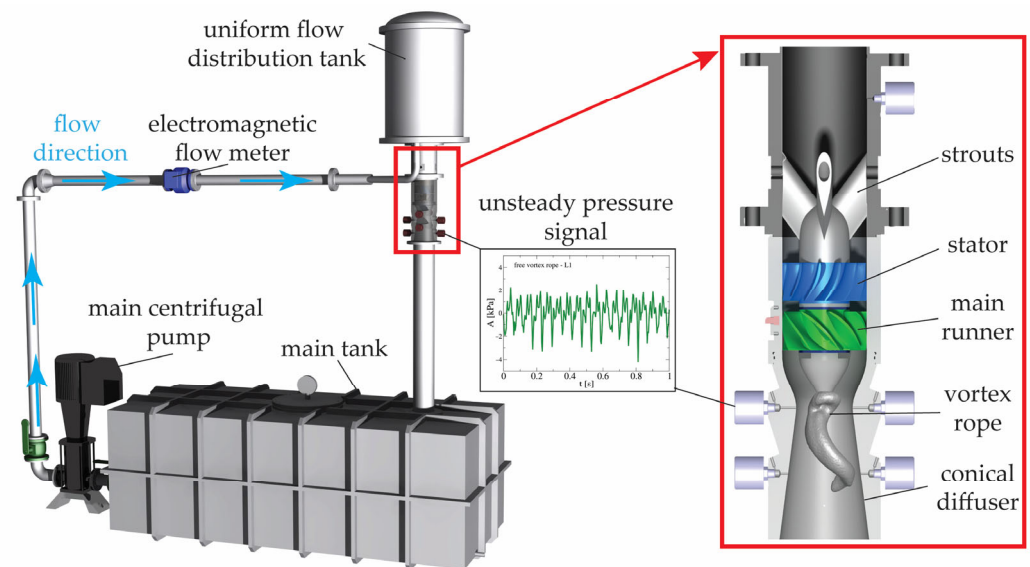


Figure 1. The test rig from the laboratory similar to the numerical model employed in this paper with the primary hydraulic circuit where the test section is mounted (blue) and auxiliary hydraulic circuit (red), which is used for testing different control methods.

2. Materials and Methods

Recently, a series of numerical studies has revived interest in water injection through the runner's crown, with [16,17] reporting comparable reduction in flow instabilities under part-load operating conditions. Two separate research groups have focused on modifying the ogive or nozzle geometry in a manner that the volume of water required for injection can be reduced. Their findings indicate that a radial-axial water jet of 10% from the total discharge can mitigate the vortex rope and associated hydrodynamic instabilities. Hence, this paper aims to numerically explore a new concept that employs an actuator mounted at the tip of the ogive. The actuator, together with a control valve, guides a radial-axial

water jet inside the draft tube cone. As illustrated in Figure 2, the swirling flow generator is composed of the fixed stay vanes, the runner, and the downstream ogive equipped with an actuator (shown in red) that can be moved and positioned accordingly. The main flow rate is indicated with blue arrow while the auxiliary circuit feeds the radial-axial jet. The ogive is fixed (non-rotating), while the actuator is constrained to translate axially along the axis of the conical diffuser. Compared to the purely axial water jet control method already validated experimentally, the proposed radial-axial jet concept offers the following advantages: (i) improved jet efficiency by lowering the flow rate from around 15% to roughly 10% and (ii) enhanced flexibility since the displacement system can deliver quasi-axial, radial-axial, and purely radial jet and address the different hydrodynamic instabilities that arise in the draft tube cone.

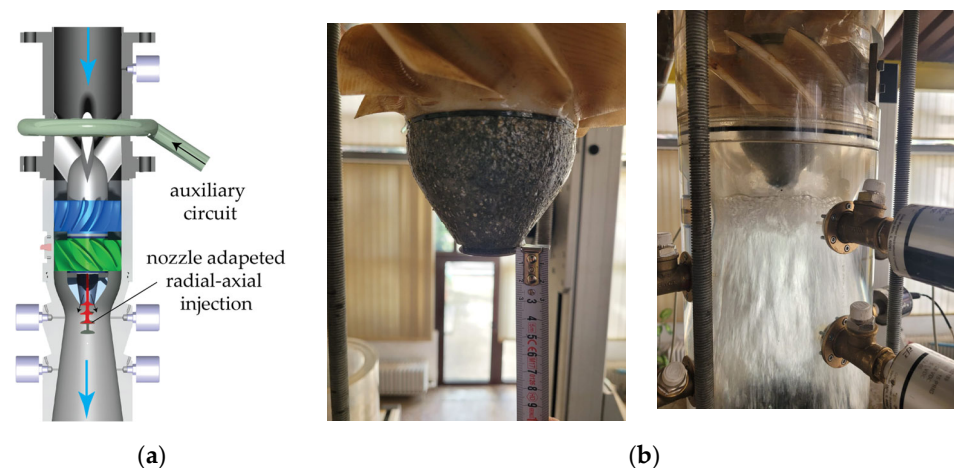


Figure 2. Swirl generator apparatus with the nozzle adapted for radial-axial injection installed on the swirl generator apparatus in design phase (a) and tested in the laboratory for validation and deep characterization (b). The main flow rate is indicated with blue arrow while the auxiliary circuit feeds the radial-axial jet.

The objective of the new-concept efficient jet is to increase operational flexibility for fixed-blade hydraulic turbines, thereby preventing efficiency alteration due to the mismatch between a decelerated swirling flow and the draft tube cone under off-design operating conditions. Moreover, the control method does not affect the good operation of the runner and may be used only when the hydraulic machine operates under off-design conditions. The strategy of modulating the decelerated swirling flow in the conical diffuser has already been numerically explored [18]. Building on the existing results, the present work seeks to determine an efficient configuration to control the flow at the inlet of the draft tube cone. Hence, three criteria were defined to evaluate the control method: (i) minimizing hydraulic losses, (ii) maximizing pressure recovery, and (iii) reducing the pressure pulsation at the wall of the draft tube cone. The implementation of a radial-axial water jet facilitates the fulfilment of these conditions with the least energy required for radial-axial water injection.

The laboratory test rig that is analogous to the numerical model employed in this study is depicted in Figure 1. The computational model used to analyze the radial-axial water jet in the draft tube cone is directly associated with the swirl apparatus installed on the test rig in terms of geometry and boundary conditions. This simple construction apparatus allows testing of the various flow control strategies to mitigate the hydrodynamic instabilities induced by the vortex rope phenomenon. Integrated into the main hydraulic circuit of the test rig, the apparatus consists of primary two components: the swirl generator and the test section. The swirl generator itself has three main components: (i) the strouts, (ii) the stator, and (iii) the main runner, all installed together on a hub inside the plexiglass test

section. The test section has a constant diameter of 0.15 m at the level of the swirl apparatus. Downstream to this section, the test section has a conical diffuser (8.5°). At the inlet of the conical diffuser, the stator and the runner produce a swirling flow analogous to that observed downstream of a Francis FLINDT operating at 70% of the nominal discharge [13]. The principal dimensions of the swirling flow generator together with the geometric parameters of the radial-axial water jet concept are summarized in Table 1.

Table 1. Hydraulic parameters of the swirling flow generator and investigated operating points.

Parameters	Value	Unit
Nominal discharge Q_n	0.03	[m ³ /s]
Main runner–rotational speed n	920	[rpm]
Main runner–tip diameter D_t	0.150	[m]
Main runner–hub diameter D_h	0.09	[m]
Main runner–blade number z	10	[-]
Investigated operating points		
Nozzle opening	5	[mm]
Control 1—Flow rate injection	1.5	[L/s]
Control 1—Percent from main flow	5	[%]
Control 2—Flow rate injection	3	[L/s]
Control 2—Percent from main flow	10	[%]

3. Numerical Setup and Validation

The computational domain corresponding to the convergent-divergent part of the swirling flow apparatus was developed at Politehnica University Timisoara [19]. The convergent section is bounded upstream by the annular inlet and downstream by the throat, as illustrated in Figure 3. In the same figure, the entire computational domain considered is presented together with the jet injection section. The discretization employs a hybrid mesh consisting of about 2.8 million elements. The mesh study previously performed by our team confirmed that above 2 million elements mesh could describe accurately enough the flow behavior in the test section of the swirling flow apparatus [20]. The relatively high Reynolds number ($\approx 3.8 \times 10^5$) is associated with a flow dominated by inertial forces; hence, the wall treatment can be neglected.

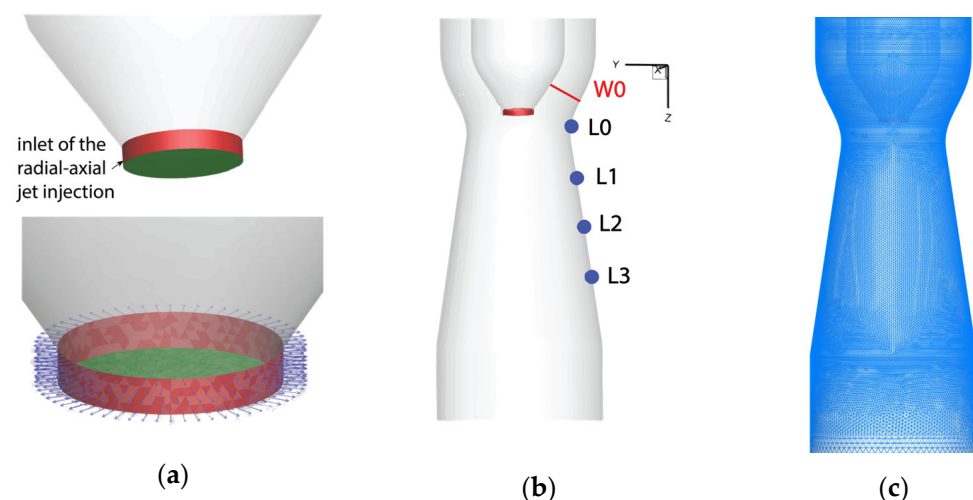


Figure 3. Inlet of the radial-axial jet injection (a), measuring levels L0–L3 placed along the conical diffuser of the test section and W0 LDV measuring window (b), and (c) the 3D discretized domain of the entire test section.

The computational fluid dynamics (CFD) analysis was limited to the test section of the swirl generator apparatus. For each simulated decelerated swirling flow configuration, the boundary conditions are specified by prescribing a velocity profile at the inlet of the computational domain and an outflow condition at the outlet. The inflow boundary conditions were obtained from prior work, in which the upstream flow field was computed. Accordingly, the inlet velocity profiles comprising axial, radial, and circumferential components together with the turbulent quantities (turbulent kinetic energy—TKE; turbulence dissipation rate— ϵ) corresponding to a runner rotational speed of 920 rpm are imposed at the annular inlet section. Figure 4a presents the velocity profiles at the inlet test section. Two volumetric flow rates were investigated for the jet injection device: 1.5 L/s and 3 L/s corresponding to 5 and 10% of the main discharge, respectively.

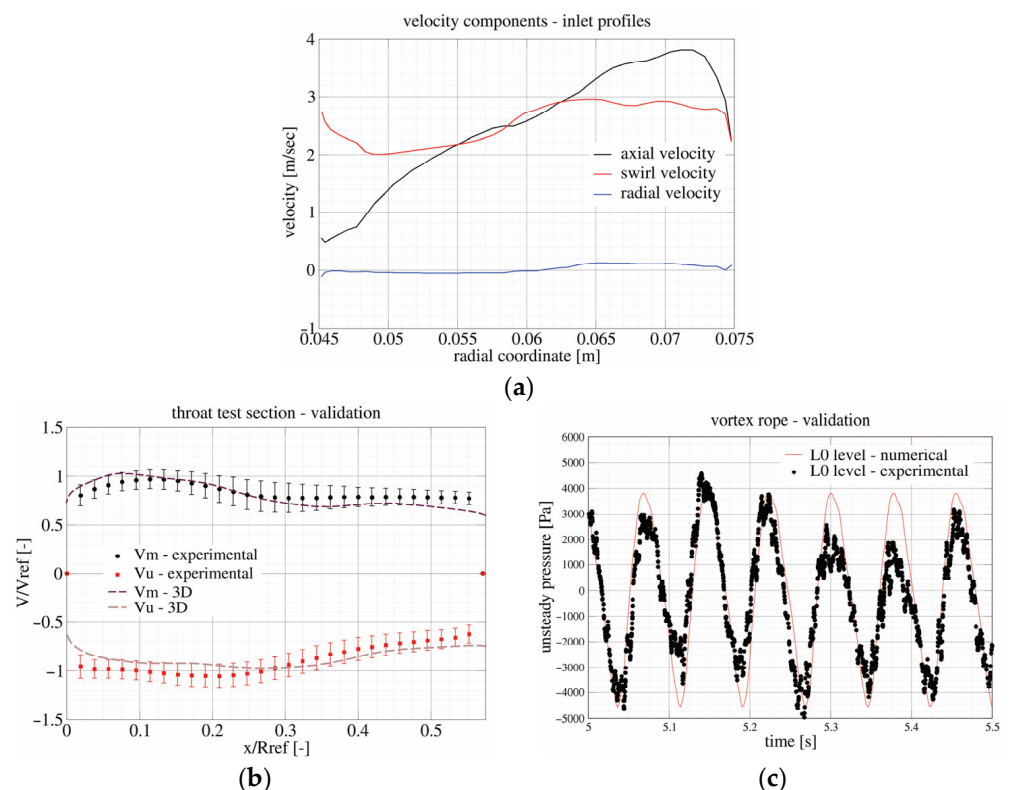


Figure 4. (a) The velocity profiles imposed as boundary conditions at the inlet of the test section, (b) the dimensionless time-averaged meridional (v_m) and tangential (v_u) velocity component validation at the W0 measuring window, and (c) the unsteady pressure distribution validation on L0 measuring level.

Three-dimensional (3D), unsteady numerical simulations with and without jet injection were conducted using commercial software Ansys Fluent 2023 R2 to assess the effectiveness of the proposed method in mitigating the vortex rope that develops in single-phase flow. For the computation of turbulent flow within the computational domain, the $k-\omega$ GEKO turbulence model was employed. This turbulence model was recently implemented in Ansys Fluent, which is particularly well-suited to capturing the characteristic flow feature of hydraulic machinery. The main advantage of using this turbulence model consists of its flexibility and broad applicability. It freely incorporates adjustable parameters that can be tuned for specific classes of flows without compromising the base calibration. This capability provides a powerful framework for model optimization. However, it also requires a comprehensive understanding of the influence of the individual coefficients to avoid inappropriate tuning. Moreover, the model is equipped with robust default

settings that allow its use without any modifications. Any settings deviations from these defaults should be justified and supported by high-quality experimental data. For all cases considered, a time step of $\Delta t = 0.002$ s has been considered. Convergence of the numerical solution was achieved with residuals threshold of 10^{-4} . To capture the unsteady evolution of the pressure pulsations associated with vortex rope evolution, pressure probes L0–L3 were positioned on the cone wall of the test section at four axial locations separated by 50 mm, according to the experimental setup.

First, the reference configuration, where the vortex rope develops naturally, was validated against the available experimental data. Once the numerical setup reliance was achieved, the same boundary conditions were applied to the two operating points investigated. The validation results are presented in Figure 4b for the dimensionless time-averaged velocity profiles (meridional and tangential component) together with the unsteady pressure evolution at the wall of the test section (at L0 measuring level) (Figure 4c), respectively.

The aim of this work is restricted to (i) the validation of numerical results against experimental data and (ii) the analysis of the newly proposed radial-axial jet concept. The dimensionless time-averaged velocity field was validated for two components: the meridional component v_m (comprising the axial and radial component) and the tangential or swirl component v_u . As illustrated in Figure 4b, the velocity profiles computed within the convergent part of the test section using the $k-\omega$ GEKO turbulence model reproduce the experimental velocity measurements with good accuracy, with the numerical predictions lying within the experimental U_{RMS} band with small deviations close to the extremities of the dimensionless measuring axis (e.g., 0 and 0.55 of x/R_{ref}). Close to the tip, at 0.55 x/R_{ref} , these discrepancies may be induced by not considering the gap between the runner and the test section in the numerical work. The U_{RMS} band quantifies the fluctuation of each velocity point measured (e.g., v_m , v_u) during the experimental work, and it was calculated by the following formula: $U_{RMS} = \sqrt{\sum_{i=0}^{N-1} \frac{1}{N} \cdot (u_i - \bar{u})^2}$, where N is the number of measured points, u_i the instantaneous value, and $\bar{u}$ the average value of the velocity.

4. Results

The unsteady evolution of the pressure pulsation validation was performed after 5 s of iteration once the case had stabilized, and the numerical results exhibited a signal comparable to the experimental measurements.

The subsequent step consisted of analyzing the flow field and pressure distribution obtained from the numerical simulations for the investigated configurations: a basic, well-developed vortex rope and two cases with 5% and 10% radial-axial injection. Figure 5 presents the flow characterization in a qualitative way (underlining the vortex rope structure at constant iso-pressure of 2000 Pa—shown in gray) and presents the axial velocity component for the three investigated cases. For each operating regime, three snapshots are depicted presenting the vortex rope structure and its evolution for three time-steps. The results indicate that the vortex rope is present in the conical diffuser for all cases considered. It originates in the vicinity of the ogive and extends over approximately two-thirds of the diffuser length. When the radial-axial jet is injected downstream of the ogive, the vortex rope increases in both length and diameter for the 5% flow-rate case, whereas for 10% injection case, the vortex rope length and thickness decrease, and its structure is more fragmented.

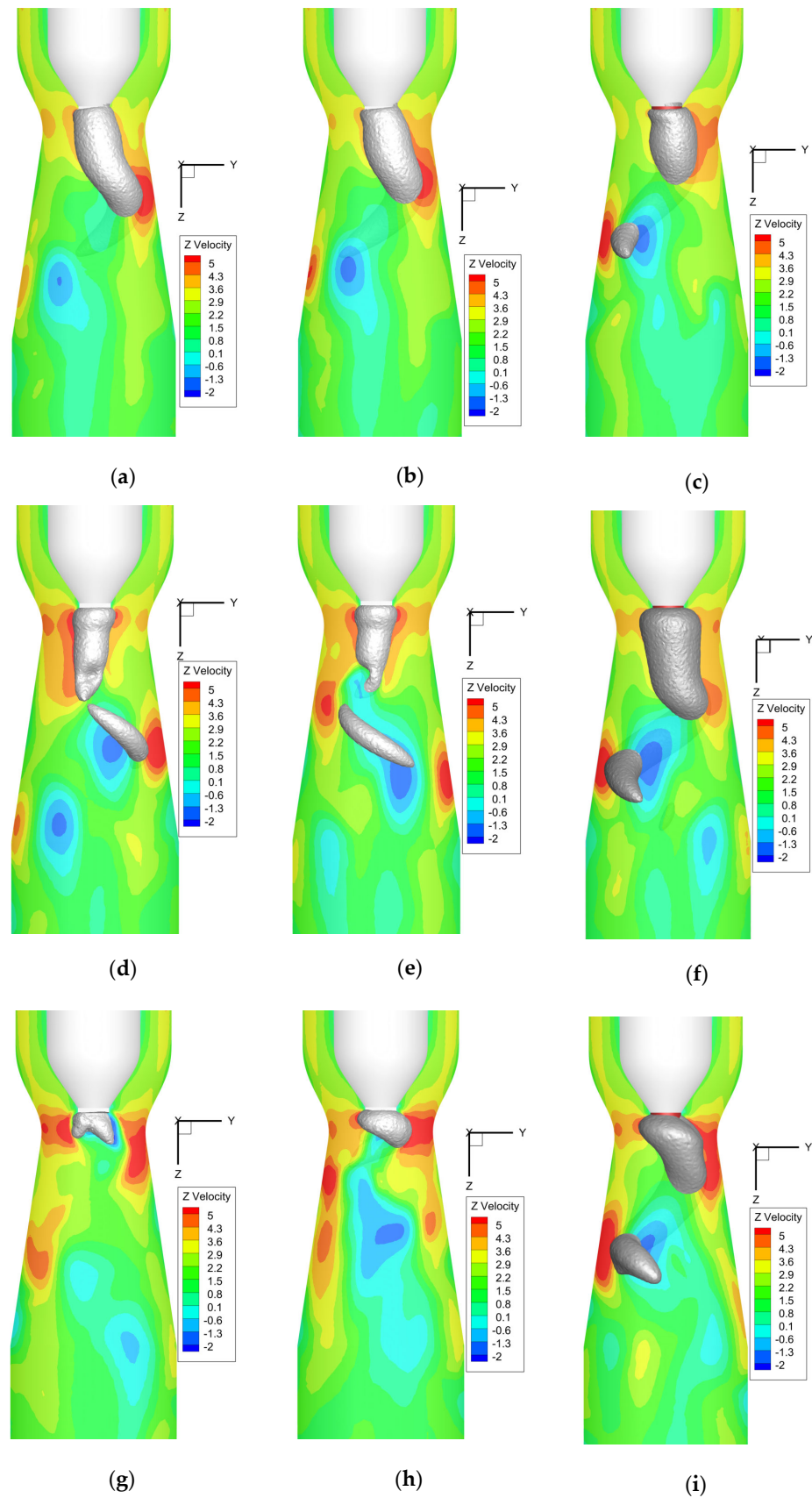


Figure 5. Vortex rope snapshot visualizations obtained from the performed 3D unsteady numerical simulations in the conical diffuser at different time steps: (a–c) the basic vortex rope structure without flow control method, (d–f) vortex rope structure at 5% radial-axial jet injection, and (g–i) vortex rope structure at 10% radial-axial jet injection.

An additional analysis concerns the CFD obtained evolution of the unsteady pressure signals measured at levels L0–L3 located in the divergent part of the test section, as illustrated in Figure 6. The unsteady pressure data are obtained from numerical simulation and subsequently processed using Fast Fourier Transform (FFT). The spectra analysis reveal a dominant vortex rope frequency of around 12.6 Hz. In the absence of radial-axial water jet, the maximum pressure pulsation reaches around 4000 Pa. When the radial-axial flow rate is injected at 5% there is no significant effect on both frequency and amplitudes of pressure pulsations. However, when 10% radial-axial water jet is injected, both the frequency and associated amplitude of the vortex rope were significantly diminished. The fragmented structure of the vortex rope and its dynamics leads to a broad range of noises in the power spectra when 10% radial-axial water jet is injected.

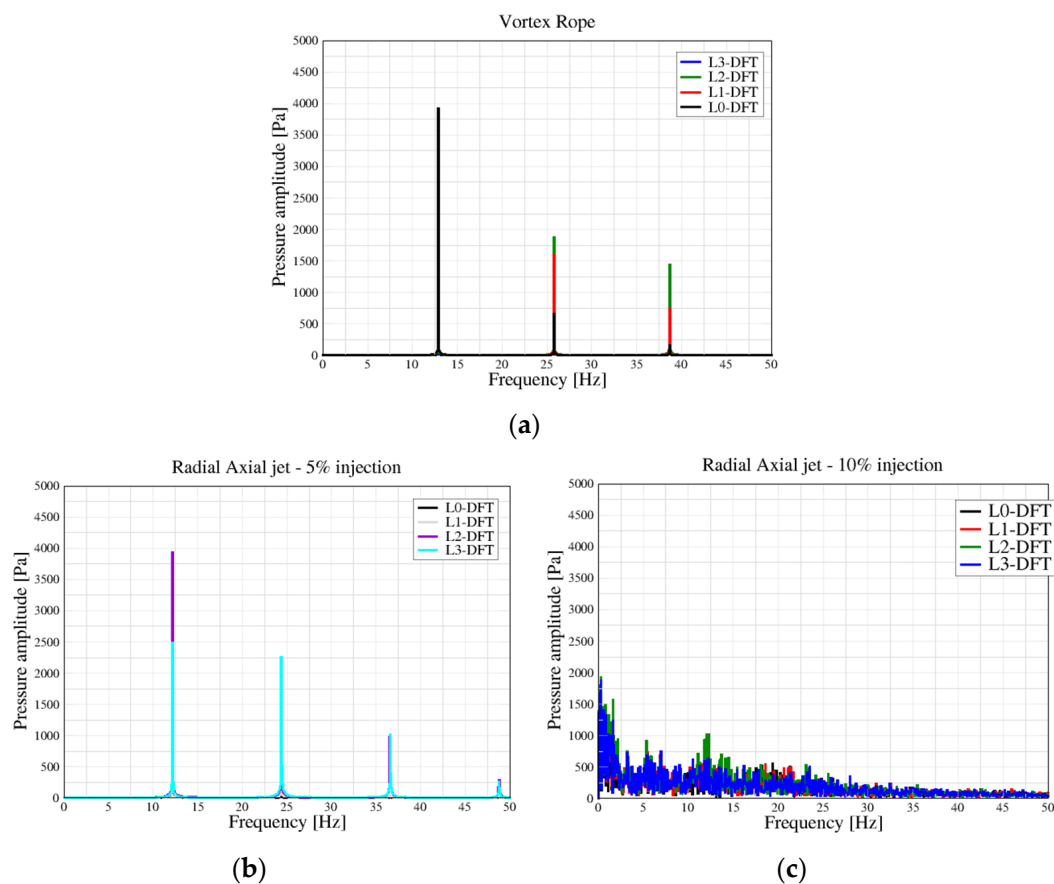


Figure 6. Power spectra of the unsteady pressure signals associated with: (a) the basic vortex rope structure without flow control method, (b) vortex rope structure at 5% radial-axial jet injection, and (c) vortex rope structure at 10% radial-axial jet injection.

Next, to improve the characterization of the flow dynamics within the conical diffuser, Figure 7 shows a contour map of axial and swirl velocity fields, overlaid with streamlines. In the configuration without radial-axial jet injection, the vortex rope structure is clearly defined and extends over more than 50% of the conical diffuser length. Following the introduction of the radial-axial water jet through the ogive, the vortex rope elongates and its inclination angle increases. This increase promotes the progressive breakup of the vortex rope with the fragmentation initiated in the vicinity of the ogive.

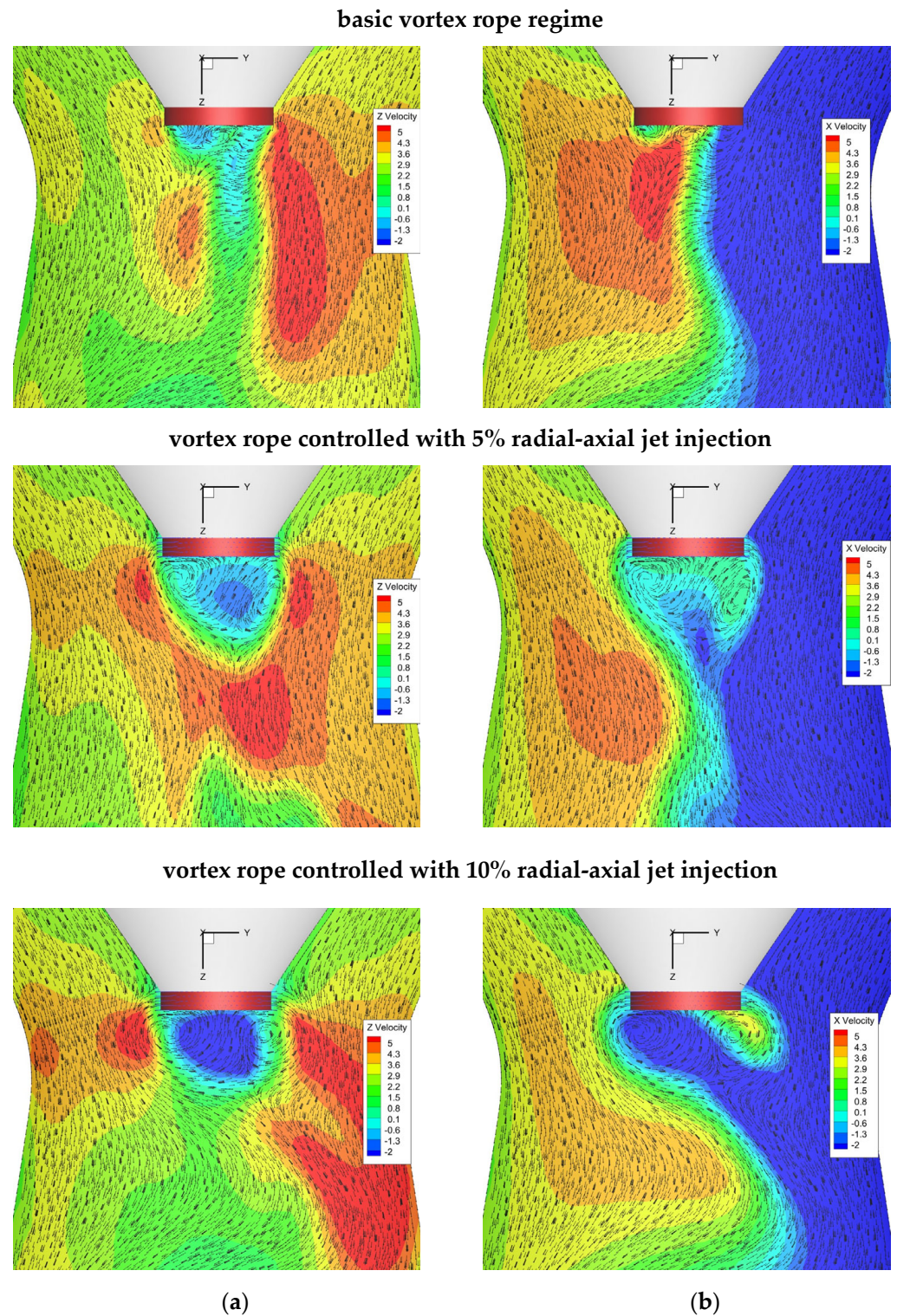


Figure 7. (a) Axial velocity distribution and (b) tangential velocity distribution overlapped with the velocity vectors.

The analysis of the pressure pulsations corroborates the velocity contour results indicating that, once the radial-axial water jet is introduced into the draft tube cone through the ogive, the vortex rope begins to disintegrate because of the modified flow angle. This process leads to a reduction in the dominant frequency and an alteration of the pressure-amplitude distribution. Correspondingly, the Fourier spectra of the pressure pulsation signals evolved from a narrow band signal (specific for vortex rope dominated flow) in a

broadband spectrum associated with the flow controlled by the radial-axial water jet. From an operational standpoint, the breakup of the vortex rope into smaller structures facilitates more flexible operating conditions, characterized by reduced pressure amplitudes and the absence of a dominant pressure pulsation frequency in the draft tube cone.

5. Conclusions

This paper investigates a control strategy to mitigate hydraulic instabilities in the draft tube cones through the application of a radial-axial water jet. Recent numerical studies have demonstrated that water injection, especially when the jet geometry (associated with nozzle geometry configuration) is modulated, can hinder flow instabilities under part-load operating conditions, thereby reducing the required injection flow rate from around 15% to about 10% of the main discharge. In this context, the performed research proposes a novel concept based on an actuator installed on ogive, designed to generate a modulated radial-axial water jet capable of adapting to varying hydrodynamic conditions. The objective is to suppress performance degradation in hydraulic turbines at off-design operating points without interfering with the operation of the main runner. Numerical simulations were performed using a swirl generator apparatus coupled with the radial-axial jet injection device. The computations were performed using Ansys Fluent 2023 R2, employing the $k-\omega$ GEKO turbulence model, which provides enhanced flexibility and accuracy for the simulation of hydraulic machinery flows. Two jet injection flow rates (1.5 L/s and 3 L/s) corresponding to 5% and 10% of the main flow rate, respectively, were investigated. The results show that the introduction of the radial-axial jet modifies the characteristics of the vortex rope forming within the draft tube cone. At an injection flow rate corresponding to 5% of the main discharge, the vortex rope exhibits increased length and thickness, whereas at 10%, the structure is significantly weakened, and its presence is notably reduced. Unsteady pressure analyses further reveal that the maximum amplitudes of the vortex-induced pressure pulsations are significantly diminished at the higher injection flow rate, indicating improvement of the hydraulic stability of the system. Future work will be conducted on evaluating the prototype implementation for the radial-axial water jet control method.

6. Patents

A national patent application (A 2025 00144) was submitted at State Office for Inventions and Trademarks in Romania before publishing the results.

Author Contributions: Conceptualization, A.-I.B., R.-A.S., C.T., A.S. and R.S.-R.; methodology, A.-I.B.; software, A.S. and C.-E.T.; validation, R.-A.S. and A.-I.B.; numerical analysis, A.S., C.-E.T. and R.S.-R.; experimental investigation, C.T.; writing—original draft preparation, R.-A.S.; writing—review and editing, R.-A.S. and A.-I.B.; supervision, R.S.-R.; project administration, A.-I.B.; funding acquisition, A.-I.B. All authors have read and agreed to the published version of the manuscript.

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Abbreviations

The following abbreviations are used in this manuscript:

BEP	Best Efficiency Point
GEKO	Generalize $k-\omega$
FFT	Fast Fourier Transform

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