

Article

Calculating Combined Hazard Thresholds for Rainfall-Induced Hazards Based on Copula Functions

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Abstract

Against the background of global climate change and rapid urbanization, the frequency of extreme hydrological events has increased markedly, and hydrological and geological hazards induced by extreme heavy rainfall have occurred frequently. Considering the joint occurrence characteristics of heavy rainfall hazards, statistical methods are used in this study to develop a mathematical model for calculating hazard–risk combinations and disaster-triggering thresholds that integrates copula functions and the Kendall return period. First, a bivariate joint-distribution probability model is established using copula functions. Then, the Kendall return period under the same recurrence level is calculated to infer combined hazard thresholds for multiple disaster-causing factors. Taking rainfall-induced flash floods and landslides as examples, combined hazard thresholds are calculated. Rainfall and discharge measured at the Heyuan hydrological station in the period from 2001 to 2020 are taken as a case study. The results show that (1) the optimal bivariate copula function for annual maximum 1-day rainfall and annual maximum discharge is the Clayton Copula; (2) similarly, the optimal bivariate combined probability function for rainfall-induced landslides is the Gumbel Copula; and (3) the combined hazard thresholds of multiple disaster-causing factors for rainfall-induced flash floods and landslides differ substantially from the thresholds based on single disaster-causing factors. Taking the return period of 100 years as an example, the univariate thresholds of discharge and rainfall are 8530.38 m³/s and 217.82 mm, while the combined thresholds decrease to 5793.10 m³/s and 175.22 mm. Meanwhile, the univariate thresholds of rainfall and landslide probability are 195.02 mm and 0.82, but the combined thresholds decrease to 170.81 mm and 0.78, respectively. In other words, conventional single-factor threshold calculation methods can severely underestimate the actual hazard level, and coupling among multiple factors can induce a significant disaster superposition and amplification effect. This study reveals the risk-coupling mechanism of heavy rainfall hazards and provides technical support for the accurate identification, prediction, and early warning of rainfall-induced flash floods and landslides under climate change.



Academic Editor: Osvaldo Luiz Leal De Moraes

Received: 29 June 2026

Revised: 23 August 2026

Accepted: 25 August 2026

Published: 1 September 2026

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This article is an open access article distributed under the terms and conditions of the [Creative Commons Attribution \(CC BY\)](https://creativecommons.org/licenses/by/4.0/) license.**Keywords:** rainfall; flash flood; landslide; threshold; copula

1. Introduction

Extreme rainfall events have become more frequent and intense under climate change. These heavy rainfall events often trigger a range of natural hazards, including landslides, debris flows, and flash floods, which pose serious threats to both human lives and infrastructure. According to global disaster databases, rainfall-induced hazards have caused

thousands of fatalities and billions of dollars in economic losses over the past few decades. It is worth noting that rainfall-induced landslides often trigger a series of secondary hazards, thereby forming typical hydrogeological hazard chains [1]. For example, rainfall-induced landslides may move along sliding surfaces toward river valleys, block rivers, and form barrier lakes with water levels significantly higher than those downstream. Owing to the scouring effect of heavy rainfall and the loose structure of landslide deposits, landslide dams may rapidly become unstable and fail, and the resulting outburst floods often pose severe threats to the lives and property of residents along rivers [2–4]. The domino effect manifested in such landslide-dammed river hazard chains greatly increases their complexity and ultimately leads to a further expansion of disaster risk [5]. The distinction between multiple jointly occurring hazards and cascading hazards in which one event follows shortly after another through explicit cause–effect chains is well discussed; both of them are driven by extreme rainfall events [6,7]. However, threshold estimation often focuses on single hazards rather than multiple.

Given the increasing trend of extreme rainfall, there is an urgent need for reliable early warning systems. A key component of such systems is the identification of reliable hazard thresholds that define the rainfall conditions under which hazardous events are likely to occur. Accurate threshold estimation can provide critical lead time for evacuation and risk mitigation, thereby reducing the potential impact of these devastating events. Understanding and improving the methods for calculating thresholds for heavy rainfall hazards is of both scientific and practical importance. Threshold determining techniques can be cataloged as empirical, probabilistic, or physically based methods [8,9]. Empirical methods, such as the Intensity–Duration threshold for landslides [10] and the antecedent precipitation index (API) threshold for flash floods [11], have seen the most application as they are easy to determine and suitable for quick warnings. A disadvantage of these empirical methods is that they are sensitive to the exact definition of a “rainfall event,” which may differ between specific regions or watersheds. There is an established body of literature on physical methods, which involve uncertainty in their parameters and physical models and are limited to jointly occurring or compounding rainfall hazards [12,13].

Heavy rainfall hazards have complex causes, and their formation and occurrence are affected by multiple environmental conditions [14]. Certain environmental conditions are monitored for landslide hazards and are indispensable for informed decision-making, rapid response, and the prevention of casualties or property loss. These environmental factors include rainfall conditions, soil conditions [15], and groundwater dynamics [16,17]. To improve the accuracy of forecasting and early warnings for rainfall-induced landslides and flash floods, prevent disaster risks, and reduce disaster losses, copula functions have been widely applied in the field of flood hazard prediction. Joint probability distribution models have been established to estimate the probability of concurrence [18–21], including bivariate and trivariate frequency analysis. Favre et al. investigated the application of copula functions in multivariate extreme-value distribution modeling and analyzed the joint probability distribution of flood peak and flood volume [22]. Copula functions have also been widely used in hydrological frequency analysis [23]. However, few studies have employed copula functions to analyze the thresholds of jointly occurring or compound hazards, especially hydrological events and geological disasters.

In order to address these gaps, smoothly apply copula functions to compound rainfall-induced hazard prediction, and build a combined probability model for threshold estimation, a new framework for calculating hazard–risk combinations and hazard thresholds is proposed in this study. Guangdong Province is located in a subtropical monsoon climate zone in China, with mean annual precipitation exceeding 1800 mm. Rainfall is highly concentrated in the rainy season, with April to September accounting for approximately

80% of annual precipitation, and heavy rainfall events occur frequently. In the mountainous areas of northern and western Guangdong, granite weathering crusts and Quaternary loose deposits are widely distributed, providing geological conditions for rainfall infiltration and the accumulation of pore-water pressure. Therefore, rainfall-induced landslides are the main landslide geological hazards in Guangdong Province. Taking Heyuan city in Guangdong Province as a case study, this study contributes a framework for establishing combined hazard thresholds for rainfall-induced flooding and landslide hazards, demonstrating how a copula function can determine the combined threshold and how the combined threshold is an improved triggering threshold for rainfall-induced hazards. Ultimately, this study's findings offer guidance for practitioners and policymakers in the Heyuan area and similarly constrained regions, where severe rainfall episodes can trigger floods and landslides.

2. Materials and Methods

2.1. Study Area

Situated in the northeastern part of Guangdong Province, China, Heyuan City features predominantly mountainous and hilly terrain. The landscape is defined by steep mountains and conical peaks, shaped by a well-developed river network and strong surface cutting. This ongoing natural process has created numerous V-shaped valleys, with slope angles generally ranging between 20° and 75°. With its unique climate, the region experiences multiple heavy rainstorms annually, making it highly prone to geological disasters, particularly landslides. The specific location of the Heyuan hydrological station, which is central to this study, is shown in Figure 1.

2.2. Data Sources

In this study, rainfall and discharge data from Heyuan hydrological station and landslide probabilities were analyzed. Maximum 1-day rainfall was adopted as the rainfall indicator, and maximum annual discharge was adopted as the discharge indicator, so that the most severe flood events could be identified and listed in the database. Maximum 1-day rainfall and maximum annual discharge were measured by Heyuan hydrological station during the period from 2001 to 2020. Landslide probability refers to the likelihood of a landslide occurring in a given area; the results from a landslide susceptibility model were adopted [14].

When we calculate thresholds for rainfall-induced flash floods, the annual maximum discharge (m^3/s) is used as the indicator of flood hazard intensity, and the corresponding annual maximum 1-day rainfall forms the rainfall–flood data series. When we calculate thresholds for rainfall-induced landslides, the annual maximum 1-day rainfall is used as the rainfall indicator, and the corresponding landslide susceptibility in a given year serves as the rainfall–landslide data series.

2.3. Copula Functions

2.3.1. Definition of Copula Functions

A copula function is a multivariate joint probability distribution function defined on the uniformly distributed domain $[0, 1]$. It connects the marginal distribution functions of multiple random variables to construct a joint distribution. Suppose that the marginal distribution functions of the random variables are given, N is the number of random variables, and x_i is the value of the random variable X_i . Then, the joint distribution function F of the random variables is

$$F_{X_1, X_2, \dots, X_N}(x_1, x_2, \dots, x_N) = P\{X_1 \leq x_1, X_2 \leq x_2, \dots, X_N \leq x_N\} \quad (1)$$

The multivariate distribution function F can be written as

$$C(F_{X_1}(x_1), F_X(x_2), \dots, F_{X_N}(x_N)) = F_{X_1, X_2, \dots, X_N}(x_1, x_2, \dots, x_N) \quad (2)$$

where C is the copula function. In essence, it is the multivariate joint distribution function of random variables $X_1, X_2, \dots, X_N$ with marginal distributions $F_{X_1}(x_1), F_{X_2}(x_2), \dots, F_{X_N}(x_N)$, and can fully describe the dependence characteristics among these variables.

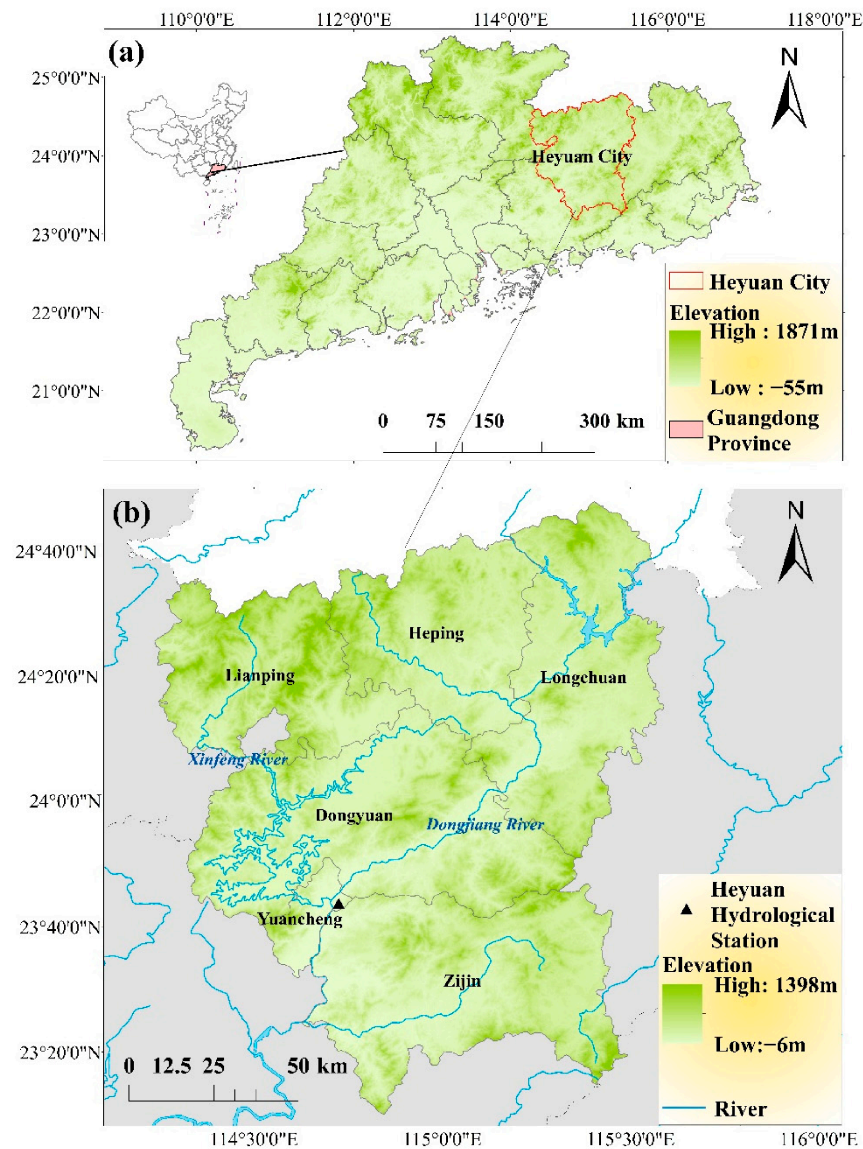


Figure 1. Study area: (a) Guangdong Province, China; (b) Heyuan City, Guangdong Province.

Common copula functions used in hydrology and meteorology include empirical copula functions, elliptical copula functions, and Archimedean copula functions. Among them, Archimedean copula functions are the most widely applied because they are computationally simple, can be used to construct diverse and highly adaptable multivariate joint distribution functions, and meet most application requirements. Therefore, this study selected three common forms of Archimedean copulas—namely, the Clayton Copula, Frank Copula, and Gumbel–Hougaard (GH) Copula—for analysis. Their bivariate and trivariate functional expressions are shown in Table 1.

Table 1. Expressions of common Archimedean copula functions.

Type	Function
Clayton	$(u_1^{-\theta} + u_2^{-\theta} - 1)^{-1/\theta}$
Frank	$-1/\theta * \ln\{1 + [\exp(-\theta u_1) - 1][\exp(-\theta u_2) - 1]/[\exp(-\theta) - 1]^2\}$
Gumbel-Hougaard (GH)	$\exp\{-[(-\ln u_1)^\theta + (-\ln u_2)^\theta + (-\ln u_3)^\theta]^{1/\theta}\}$

Parameter estimation is the core of copula functions. Common methods include the moment-like method of inversion of association of measures (MOM), maximum likelihood (ML), and optimal curve fitting. The association of measures method is applicable to bivariate copula functions, where the parameter is estimated according to the mathematical relationship between the parameter and the Kendall rank correlation coefficient. The maximum-likelihood method solves for the parameters of the copula joint distribution function by constructing a likelihood function. The principle of optimal curve fitting is to use a certain criterion, such as the minimum sum of squared deviations, to identify the parameters that provide the best fit between the theoretical frequency curve and the empirical frequency curve. In this study, the maximum-likelihood method is used to calculate bivariate copula parameters, with the calculation steps given in Equations (3)–(5):

$$L(\theta) = \prod_{i=1}^n F(x_i; \theta) = \prod_{i=1}^n [\partial F(x_i; \theta) / \partial x_i] \quad (3)$$

$$\ln[L(\theta)] = \sum_{i=1}^n \ln[F(x_i; \theta)] \quad (4)$$

$$\partial \ln[L(\theta)] / \partial \theta = 0 \quad (5)$$

where $L(\theta)$ is the likelihood function; $F(x_i; \theta)$ is the marginal density distribution function; and θ is the parameter to be estimated.

For copula functions of three or more dimensions, the association of measures method is no longer applicable. In this case, the inference function for margins (IFM) method can be used to estimate parameters. Specifically, the log-likelihood function of the marginal distributions is first established to solve for the marginal distribution parameters α_i , and then the copula log-likelihood function is established to solve for the copula parameter θ .

In order to reasonably construct the joint distribution model, correlation analysis among the variables is first conducted. Nonlinear dependence among variables can be described using the Kendall rank correlation coefficient τ , which is calculated from the sample as follows:

$$T = [2/n(n-1)] \cdot \sum_{i=1}^{n-1} \sum_{j=i+1}^n \text{sign}((x_i - x_j)(y_i - y_j)) \quad (6)$$

where (x_i, y_i) and (x_j, y_j) are observed sample points; n is the sample length; and $\text{sign}(\cdot)$ is the sign function; namely,

$$\text{sign}(x) = 1(x > 0), 0(x = 0), -1(x < 0) \quad (7)$$

2.3.2. Basic Steps for Applying Copula Functions

Applying copula functions to solve concurrence problems among hydrological events mainly includes the following steps.

1. Determination of marginal distribution functions

The marginal distribution functions are the foundation for constructing a copula function. Before conducting frequency analysis using the multivariate copula joint distribution, it is necessary to determine the marginal distribution function for each variable. Since the skewness of the probability distributions differs among the variables, this study adopts five

marginal distributions commonly used in hydrology, namely the P-III distribution, generalized extreme value (GEV) distribution, Lognormal distribution, Weibull distribution, and Gamma distribution. Goodness-of-fit testing is conducted using the Kolmogorov–Smirnov test, the ordinary least squares (OLS) criterion, the Akaike information criterion (AIC), and the coefficient of determination R^2 to determine the optimal marginal distributions of each variable.

2. Copula function selection and goodness-of-fit testing

The parameter θ in the copula function is calculated using the association of measures method—that is, by establishing the relationship between the Kendall rank correlation coefficient τ and θ to solve for the value of θ . The formula for calculating the Kendall rank correlation coefficient τ is

$$\tau = \sum_{i=1}^{n-1} \sum_{j=i+1}^n \text{sign}[(x_i - x_j)(y_i - y_j)] \quad (8)$$

After parameter calculation, the fit with the empirical copula is evaluated using the OLS criterion, AIC, D^2 , R^2 , and the K-S test, thereby determining the optimal joint distribution model. The calculation methods for these indicators are as follows:

$$OLS = \sqrt{\frac{1}{N} \sum_{i=1}^N (\hat{y}_i - y_i)^2} \quad (9)$$

$$AIC = N \cdot \ln(MSE) + 2m \quad (10)$$

$$MSE = (1/N) \cdot \sum_{i=1}^N (\hat{y}_i - y_i)^2 \quad (11)$$

$$D^2 = \sum_{i=1}^N (\hat{y}_i - y_i)^2 \quad (12)$$

$$R^2 = \{\sum_{i=1}^N [(y_i - \text{mean}(y)) (\hat{y}_i - \text{mean}(\hat{y}))]\}^2 / \{\sum_{i=1}^N (y_i - \text{mean}(y))^2 [\sum_{i=1}^N (\hat{y}_i - \text{mean}(\hat{y}))^2]\} \quad (13)$$

where $\hat{y}_i$ and y_i denote the empirical frequency values and theoretical frequency values, respectively; $\text{mean}(\hat{y})$ and $\text{mean}(y)$ denote their mean values; N is the sample size; and m is the number of model parameters. Smaller D^2 , OLS, and AIC values indicate better fitting performance, whereas a larger R^2 indicates better fitting performance.

The K-S test (D):

$$D = \max_{1 \leq i \leq n} \{ |F(x_i) - (i-1)/n|, |F(x_i) - i/n| \} \quad (14)$$

where $F(x_i)$ is the theoretical distribution of variable x . A smaller D value indicates a better fit; the return value $P \in (0, 1)$ indicates the probability of passing the test, with a larger value indicating a higher fitting probability; $H = 0$ indicates that the test is passed, whereas $H = 1$ indicates that the test is not passed. The significance level is set to 0.05.

2.4. Kendall Return Period

The return period is a design-standard concept in hydraulic engineering used to measure the magnitude of hydrological events [24]. When an event exceeds the design threshold for a given return period, it is defined as a hazardous event [25]. The univariate return period can be calculated directly as the reciprocal of the exceedance probability. However, in multi-factor disaster-causing scenarios, considering only a single factor cannot satisfy the needs of hazard identification. The joint return period and co-occurrence return period are commonly used for calculating multivariate return periods, and their definitions are as follows:

The “OR” return period (joint return period) refers to the return period in which at least one variable exceeds a specified value. Its calculation expression is

$$T_{X,Y}^{\vee} = 1/P \cdot (X > x \cup Y > y) = 1/[1 - C(u, v)] \quad (15)$$

where $C(u, v)$ is the joint distribution of two variables, and u and v are the univariate frequency distributions of X and Y , respectively.

The “AND” return period (co-occurrence return period) refers to the return period in which the variables simultaneously exceed specified values. Its calculation expression is

$$T_{X,Y}^{\wedge} = 1/P \cdot (X > x \cup Y > y) = 1/[1 - u - v + C(u, v)] \quad (16)$$

where $C(u, v)$ is the joint distribution of two variables, and u and v are the univariate frequency distributions of variables X and Y , respectively.

However, analyses based on the “OR” and “AND” return periods have certain limitations. When distinguishing safe events from hazardous events, the “OR” return period may enlarge the hazardous domain, whereas the “AND” return period may shrink the hazardous domain [26]. To address this issue, Salvadori and De Michele proposed the Kendall return period, which is the average time interval for an event to occur in the supercritical region [24]. The bivariate Kendall return period is defined as

$$T_k = 1/P \cdot [C(u_1, u_2) > t] = 1/[1 - K_C(t)] \quad (17)$$

$$K_C(t) = t - \varphi(t)/\varphi'(t) \quad (18)$$

where K_C is the Kendall distribution function, and its value is the quantile of the distribution function at probability t ; $\varphi(t)$ is the generator of the Archimedean copula function; and $\varphi'(t)$ is its first right derivative.

The Kendall return period divides the hazard threshold according to joint probability values, avoiding over- or underestimation of the hazardous domain in multivariate settings. It can reasonably describe the occurrence patterns of multivariate hydrological events under general conditions. Therefore, this study uses the Kendall return period to calculate risk-combination thresholds for different variables. Under the constraint of the curve defined by Equations (17) and (18), the optimal combination point can be identified with the maximum probability density using the argmax function. This study uses the maximum-likelihood method for threshold calculation. For the optimal threshold combination of two variables (X_{m1}, Y_{m1}) , the joint probability density $f(x, y)$ should reach its maximum, namely,

$$(X_{m1}, Y_{m1}) = \operatorname{argmax}[f(x, y)] \quad (19)$$

$$f(x, y) = c(u_x, u_y) \cdot f_X(x) \cdot f_Y(y) \quad (20)$$

where c is the probability density function of the copula, and $f_X(x)$ and $f_Y(y)$ are the probability density functions of the two variables. Subsequently, the thresholds of the variables are inferred using the inverse functions of the univariate marginal distributions, and the risk-combination thresholds of the variables under different return periods are thereby obtained.

The research framework is illustrated in Figure 2.

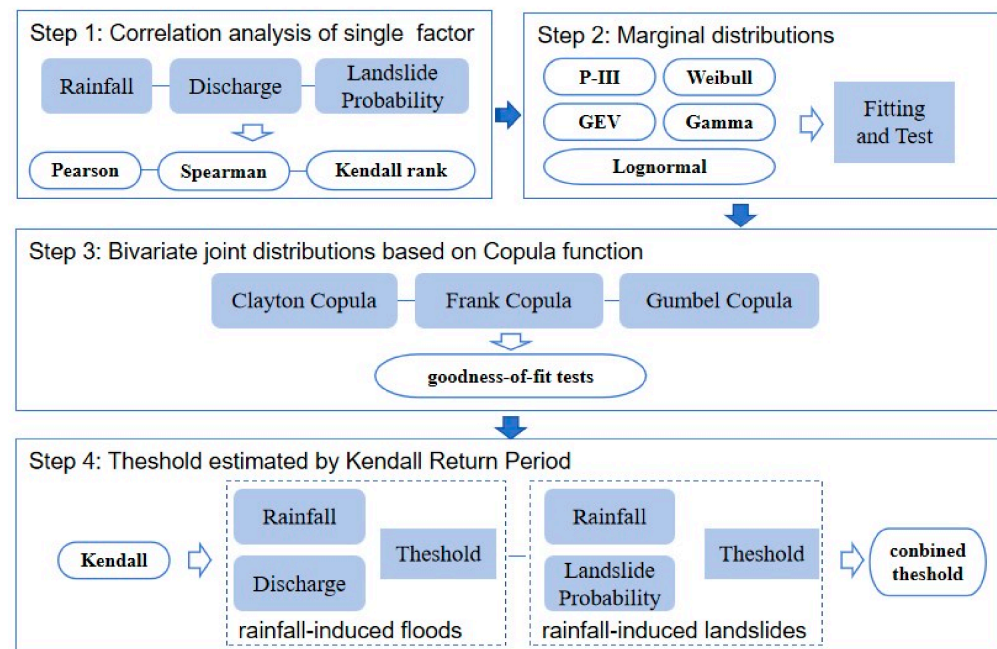


Figure 2. Research framework.

3. Results and Discussion

3.1. Analysis of Multiple Disaster-Causing Factor Combinations

To explore the combination relationships among multiple disaster-causing factors for heavy rainfall hazards, this study selected indicators such as the maximum 1-day rainfall and annual maximum discharge as representative variables and determined the joint distribution based on the results of univariate marginal frequency analysis. On the basis of correlation analysis, copula functions were used to construct bivariate and joint distribution models. The Clayton Copula, Frank Copula, and Gumbel Copula functions from the Archimedean copula family were selected for fitting. The goodness of fit of the copulas with the empirical copula was evaluated using five methods, namely the *OLS* criterion, *AIC*, D^2 , R^2 , and the K-S test, to determine the optimal joint distribution model.

3.1.1. Selection of Marginal Distribution Functions for Single Disaster-Causing Factors (Rainfall–Flash Flood)

In the practical application of copula functions, the correlation coefficients of each single disaster-causing factor need to be calculated first. Therefore, the Pearson correlation coefficient ρ , Spearman correlation coefficient ρ_s , and Kendall rank correlation coefficient τ were used to describe nonlinear dependences between variables. The calculation results are shown in Table 2. The results indicate a weak positive correlation between rainfall and discharge. Several representative marginal distributions were then selected to optimize the univariate marginal distribution functions. The probability distribution functions of different distributions are shown in Table 3.

Table 2. Kendall rank correlation coefficient test.

Correlation Coefficient	Value
Pearson correlation coefficient ρ	0.07
Kendall correlation coefficient τ	0.069
Spearman correlation coefficient ρ_s	0

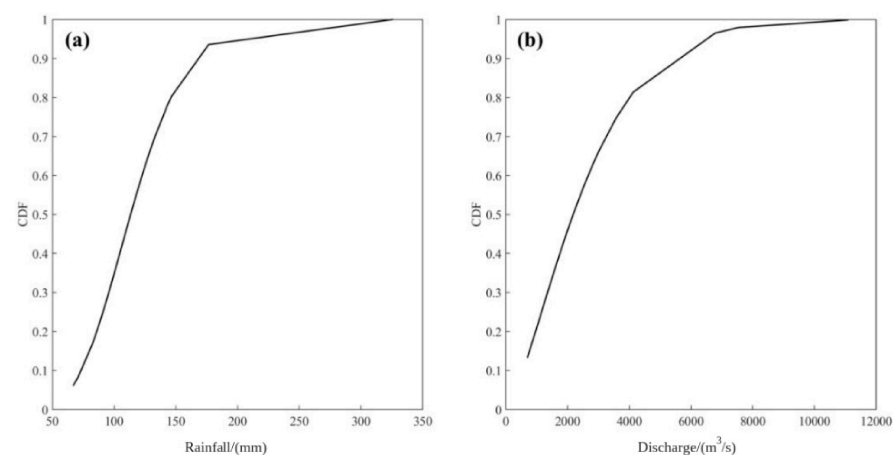
Table 3. Probability distribution functions of marginal distributions.

Distribution	$F(x)$	Parameters		
		Shape	Scale	Location
P-III	$F(x) = \int_{a_0}^x [\beta^\alpha / \Gamma(\alpha)] (x - a_0)^{\alpha-1} \exp[-\beta(x - a_0)] dx, \alpha > 0, x > a_0$	α	β	a_0
GEV	$F(x) = \exp\{-[1 - k((x - \mu)/\alpha)]^{(1/k)}\}, 1 - k((x - \mu)/\alpha) > 0$	k	α	μ
Lognormal	$F(x) = 1/2 + (1/2) \cdot \text{erf}[(\ln(x) - \mu)/(\sqrt{2}\sigma)]$	/	σ	μ
Weibull	$F(x) = 1 - \exp[-(x/a)^b]$	b	a	/
Gamma	$F(x) = \int_0^x [x^{\beta-1} / (\alpha^\beta \cdot \Gamma(\beta))] \exp(-x/\alpha) dx$	β	α	/

Goodness-of-fit testing was conducted using the Kolmogorov–Smirnov test, the *OLS* criterion, *AIC*, and R^2 to determine the optimal marginal distributions of discharge and rainfall. The test results are shown in Table 4. All five distributions passed the K-S test. For annual maximum 1-day rainfall, the *OLS* and *AIC* values were similar, whereas the Gamma distribution had a larger R^2 and a smaller K-S test *D* value; therefore, the Gamma distribution was selected as the optimal marginal distribution for rainfall. For annual maximum discharge, the *OLS* and *AIC* values differed only slightly, but the Weibull distribution had the largest R^2 ; therefore, the Weibull distribution was selected as the optimal marginal distribution for discharge. The fitted marginal distributions are shown in Figure 3.

Table 4. Marginal distribution fitting and testing (Rainfall–Flash Flood).

Hydrological Variable	Marginal Distribution	K-S Test			<i>OLS</i>	R^2	<i>AIC</i>
		<i>H</i>	<i>P</i>	<i>D</i>			
Annual maximum 1-day rainfall	P-III	0.00	0.55	0.14	124.98	0.57	295.69
	GEV	0.00	0.88	0.10	125.00	0.58	295.70
	Lognormal	0.00	0.53	0.14	125.02	0.63	293.71
	Gamma	0.00	0.09	0.22	125.02	0.77	293.71
	Weibull	0.00	0.32	0.17	125.03	0.67	293.71
Annual maximum discharge	P-III	0.00	0.30	0.17	3370.44	0.54	493.37
	GEV	0.00	0.98	0.08	3370.45	0.61	493.37
	Lognormal	0.00	0.48	0.15	3370.47	0.70	491.37
	Weibull	0.00	0.18	0.19	3370.47	0.80	491.37
	Gamma	0.00	0.16	0.20	3370.48	0.76	491.37

**Figure 3.** Marginal distributions of single disaster-causing factors: (a) annual maximum 1-day rainfall; (b) annual maximum discharge.

3.1.2. Selection of Marginal Distribution Functions for Single Disaster-Causing Factors (Rainfall–Landslide)

Goodness-of-fit testing was conducted using the Kolmogorov–Smirnov test, the *OLS* criterion, the *AIC*, and R^2 to determine the optimal marginal distributions of landslide probability and rainfall. The test results are shown in Table 5. All five distributions passed the K-S test. For annual maximum 1-day rainfall, the *OLS* and *AIC* values were similar, whereas the Lognormal distribution had a relatively large R^2 and a small K-S test *D* value; therefore, the Lognormal distribution was selected as the optimal marginal distribution for rainfall. For landslide susceptibility, the *OLS* and *AIC* values differed only slightly, but the Gamma distribution had the largest R^2 ; therefore, the Gamma distribution was selected as the optimal marginal distribution for discharge. The fitted marginal distributions are shown in Figure 4.

Table 5. Marginal distribution fitting and testing (Rainfall–Landslide).

Hydrological Variable	Marginal Distribution	K-S Test			<i>OLS</i>	R^2	<i>AIC</i>
		<i>H</i>	<i>P</i>	<i>D</i>			
Annual maximum 1-day rainfall	P-III	0.00	1.00	0.07	2010.01	0.99	310.24
	GEV	0.00	0.99	0.09	2010.01	0.99	310.24
	Lognormal	0.00	1.00	0.08	2010.01	0.99	308.24
	Weibull	0.00	0.98	0.10	2010.02	0.98	308.24
	Gamma	0.00	1.00	0.08	2010.01	0.99	308.24
Landslide susceptibility	P-III	0.00	0.80	0.14	1165.50	0.97	288.44
	GEV	0.00	0.76	0.14	1165.50	0.97	288.44
	Lognormal	0.00	0.64	0.16	1165.50	0.96	286.44
	Weibull	0.00	0.89	0.12	1165.52	0.98	286.44
	Gamma	0.00	0.74	0.14	1165.50	0.97	286.44

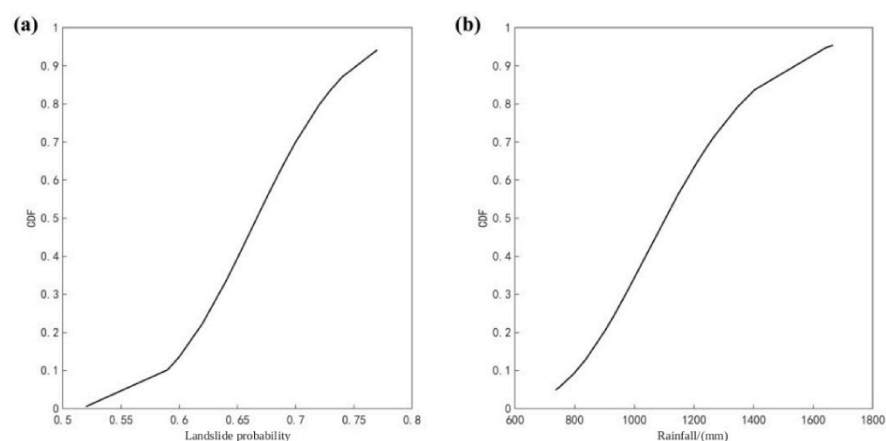


Figure 4. Marginal distributions of single disaster-causing factors: (a) landslide probability; (b) annual maximum 1-day rainfall.

After the marginal distributions of the single disaster-causing factors of rainfall and landslide susceptibility were determined, bivariate joint distributions were constructed using copula functions for annual maximum 1-day rainfall and landslide susceptibility. The elliptical copula family includes the Gaussian copula, the *t*-copula, and the Archimedean copula family. As Archimedean copulas are more flexible in form and can characterize different tail dependence patterns through the selection of different generator functions, the Clayton Copula, Frank Copula, and Gumbel Copula functions were selected for fitting. The parameters and the results of goodness-of-fit tests for each function are shown in Table 6. Table 6 indicates that all three copula functions passed the K-S test. Among them, the

Clayton Copula was determined as the optimal function because it had smaller *OLS* and *AIC* values and the largest R^2 . To further verify the model's reliability, a fitting plot of the theoretical joint distribution probability and empirical joint distribution probability for the Clayton Copula function was produced (Figure 5). The results showed that the data points were uniformly distributed near the 45° line, indicating that the theoretical joint distribution probability fit the empirical joint distribution probability well. Therefore, the Clayton Copula was selected as the optimal joint distribution function, and the joint distribution probability plot of the Clayton Copula was drawn, as shown in Figure 6.

Table 6. Bivariate joint distribution fitting and testing (Rainfall–Flash Flood).

Function Type	D^2	H	K-S Test P	Value	<i>OLS</i>	R^2	<i>AIC</i>
Clayton Copula	0.14	0.00	0.34	0.23	0.00	0.95	−160.01
Frank Copula	0.12	0.00	0.54	0.20	0.13	0.95	−162.65
Gumbel Copula	0.22	0.00	0.54	0.20	0.33	0.94	−145.85

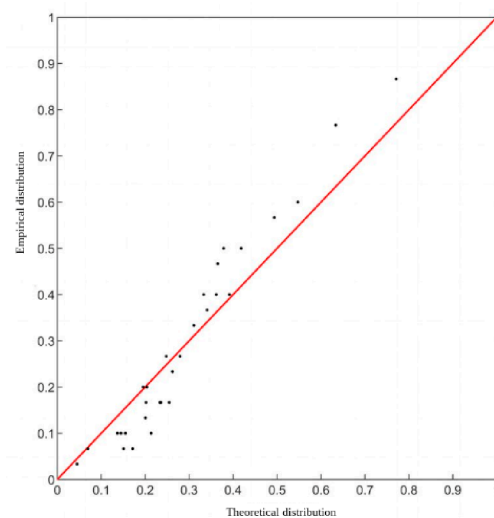


Figure 5. Fitting plot of the bivariate theoretical joint distribution probability and empirical joint distribution probability of the Clayton Copula function.

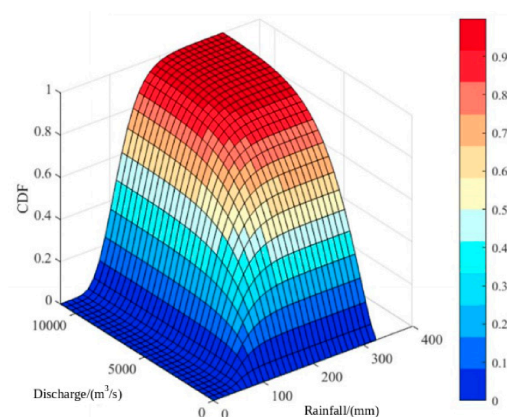


Figure 6. Bivariate joint distribution probability plot based on the Clayton Copula function.

3.1.3. Bivariate Joint Distributions Based on Copula Functions

After the marginal distributions of the single disaster-causing factors of rainfall and landslide probability were determined, bivariate joint distributions were constructed using copula functions for annual maximum 1-day rainfall and landslide probability. The Clayton

Copula, Frank Copula, and Gumbel Copula functions were selected for fitting, and the parameters and results of goodness-of-fit tests for each function are shown in Table 7. Table 7 indicates that all three copula functions passed the K-S test. Among them, the Gumbel Copula was determined as the optimal function because it had smaller OLS and AIC values and the largest R^2 . To further verify the model's reliability, a fitting plot of the theoretical joint distribution probability and empirical joint distribution probability for the Gumbel Copula function was produced (Figure 7). The results showed that the data points were uniformly distributed near the 45° line, indicating that the theoretical joint distribution probability fit the empirical joint distribution probability well. Therefore, the Gumbel Copula was selected as the optimal joint distribution function, and the joint distribution probability plot of the Gumbel Copula was drawn, as shown in Figure 8.

Table 7. Bivariate joint distribution fitting and testing (Rainfall–Landslide).

Function Type	D^2	H	K-S Test P	Value	OLS	R^2	AIC
Clayton Copula	0.11	0.00	0.50	0.25	0.22	0.959	−102.83
Frank Copula	0.08	0.00	0.77	0.20	0.16	0.960	−108.25
Gumbel Copula	0.08	0.00	0.77	0.20	0.14	0.959	−109.26

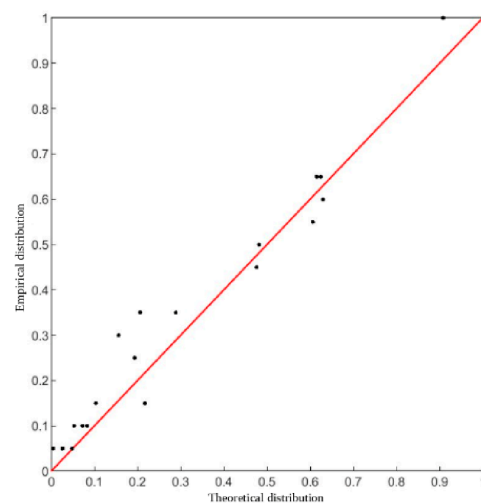


Figure 7. Fitting plot of the bivariate theoretical joint distribution probability and empirical joint distribution probability of the Gumbel Copula function.

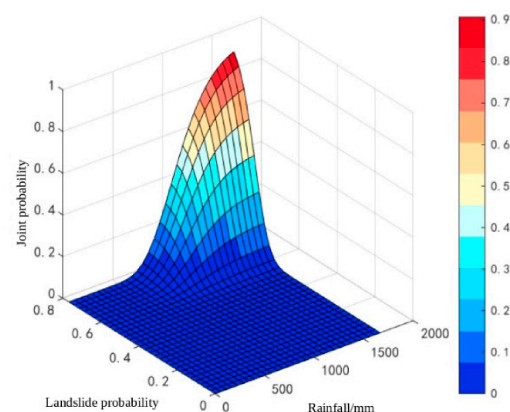


Figure 8. Bivariate joint distribution probability plot based on the Gumbel Copula function.

3.2. Calculation of Disaster-Triggering Thresholds for Heavy Rainfall Hazards

3.2.1. Calculation of Disaster-Triggering Thresholds for Rainfall–Flash Floods

Based on the Clayton Copula function, the “AND” return period, “OR” return period, and Kendall return period were calculated for the joint distribution of rainfall and discharge, and the associated return-period distribution maps and contour maps are shown in Figures 9–11. Then, the disaster-triggering thresholds corresponding to Kendall return periods of 5, 10, 20, 50, 100, and 200 years were calculated using the maximum-likelihood method, as shown in Table 8. The analysis showed that the combined disaster-triggering thresholds of multiple disaster-causing factors were lower than the thresholds of single disaster-causing factors. For example, for a return period of 100 years, the univariate disaster-triggering thresholds of discharge and rainfall were 8530.38 m³/s and 217.82 mm, respectively. The combined disaster-triggering thresholds of discharge and rainfall were 5793.10 m³/s and 175.22 mm, respectively. This analysis indicates that conventional single-hazard threshold calculation methods can severely underestimate the actual hazard level, and coupling among multiple hazards can induce a significant disaster superposition and amplification effect. The hazard thresholds derived from multiple combined hazard factors are often lower than those based on a single factor. For example, in the analysis of rainfall–flood coincidence, the design rainfall or discharge under the joint return period exhibits a “magnifying effect,” indicating that the combined risk is higher than the risk posed by a single factor. This directly demonstrates the necessity of considering combined thresholds, as single-factor thresholds tend to underestimate the actual compound risk level presented by heavy rainfall-induced hazards.

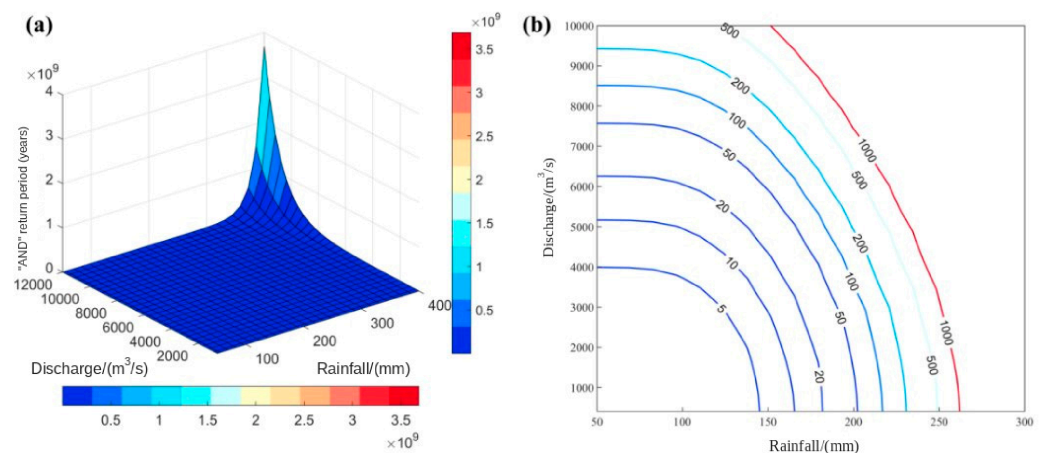


Figure 9. Bivariate AND return period for rainfall and discharge: (a) return-period distribution map; (b) return-period contour map.

Table 8. Hazard-combination disaster-triggering thresholds under different return periods (Rainfall–Flash Flood).

Return Period (Years)	Single-Variable Disaster-Triggering Thresholds		Combination Disaster-Triggering Thresholds Under the Kendall Return Period	
	Rainfall/mm	Discharge/(m ³ /s)	Rainfall/mm	Discharge/(m ³ /s)
5	146.16	4002.87	133.63	2896.55
10	165.49	5179.83	143.81	3724.14
20	182.64	6259.92	152.11	4551.72
50	203.27	7585.45	165.52	5308.35
100	217.82	8530.38	175.22	5793.10
200	231.69	9436.10	185.84	6206.90

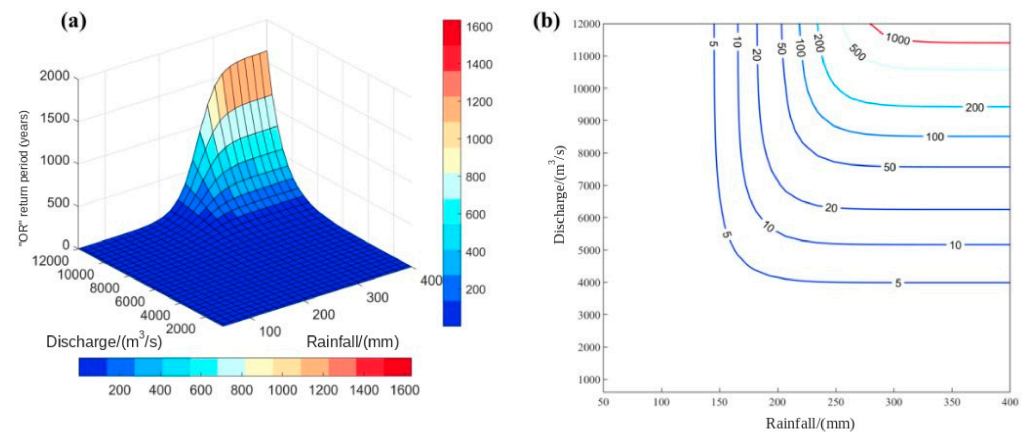


Figure 10. Bivariate OR return period for rainfall and discharge: (a) return-period distribution map; (b) return-period contour map.

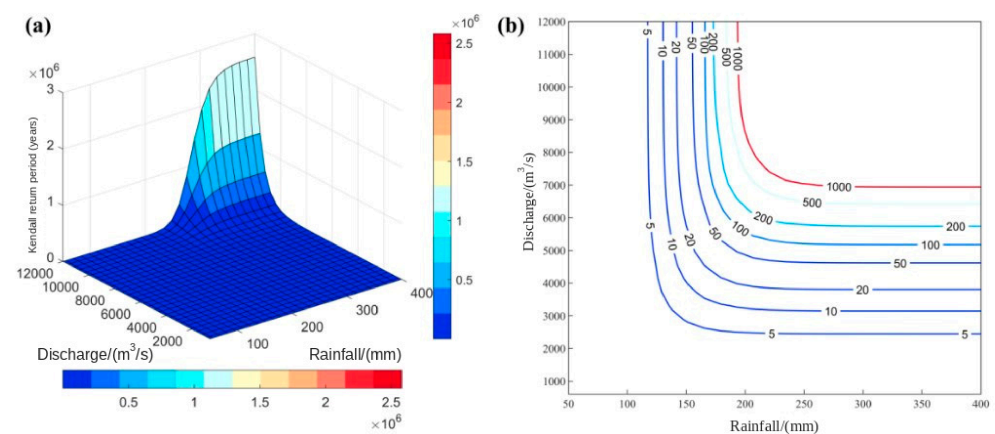


Figure 11. Bivariate Kendall return period for rainfall and discharge: (a) return-period distribution map; (b) return-period contour map.

As shown in Figures 9b, 10b and 11b, the Kendall return period was employed in this study as it delineates the hazard zone according to joint probability values; this effectively prevents over- or underestimation of the risk zone.

3.2.2. Calculation of Disaster-Triggering Thresholds for Rainfall–Landslides

Based on the Gumbel Copula function, the “AND” return period, “OR” return period, and Kendall return period were calculated for the joint distribution of rainfall and landslide probability, and the associated return-period distribution maps and contour maps are shown in Figures 12–14. Then, the disaster-triggering thresholds corresponding to Kendall return periods of 5, 10, 20, 50, 100, and 200 years were calculated using the maximum-likelihood method, as shown in Table 9. The analysis showed that the combined disaster-triggering thresholds of multiple disaster-causing factors were higher than the thresholds of single disaster-causing factors. For example, for a return period of 100 years, the univariate disaster-triggering thresholds of rainfall and landslide probability were 195.02 mm and 0.82, respectively. The combined disaster-triggering thresholds of rainfall and landslide probability were 170.81 mm and 0.78, respectively. This analysis indicates that conventional single-hazard threshold calculation methods are applicable to certain hazard types and that the disaster superposition and amplification effect induced by coupling between rainfall and landslides is obvious.

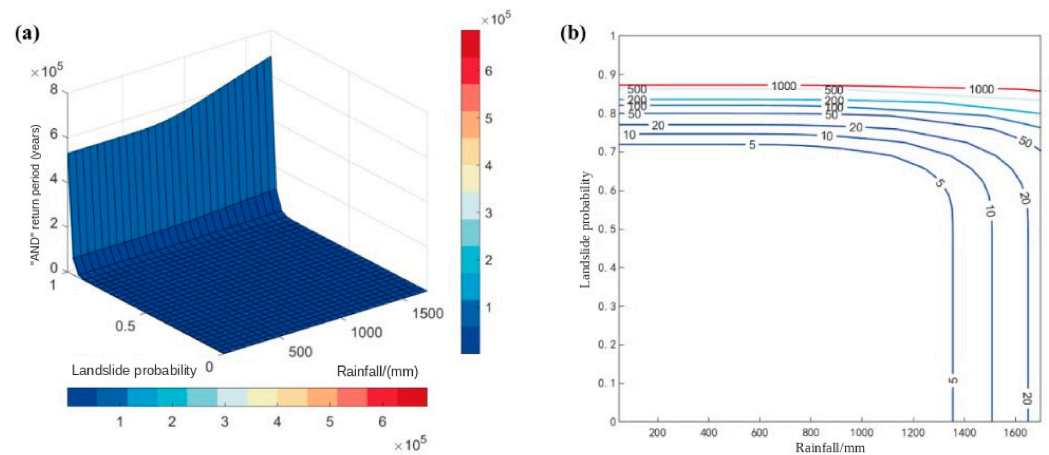


Figure 12. Bivariate AND return period for rainfall and landslide probability: (a) return-period distribution map; (b) return-period contour map.

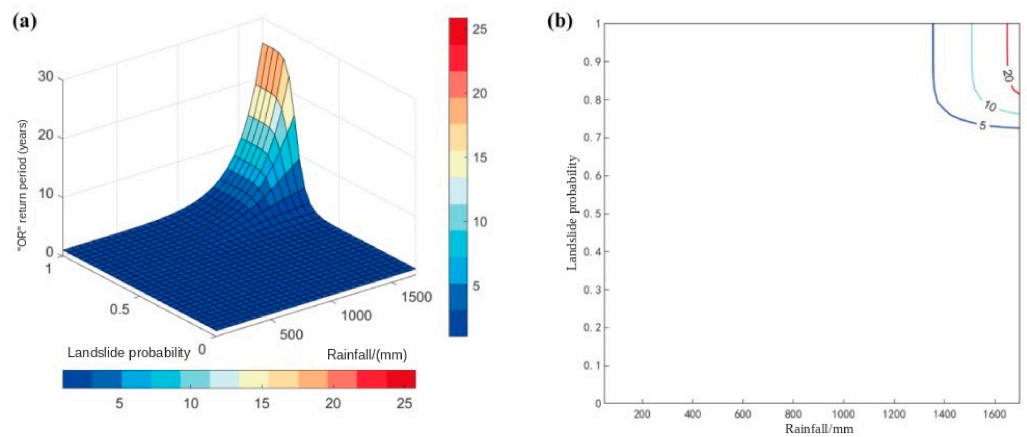


Figure 13. Bivariate OR return period for rainfall and landslide probability: (a) return-period distribution map; (b) return-period contour map.

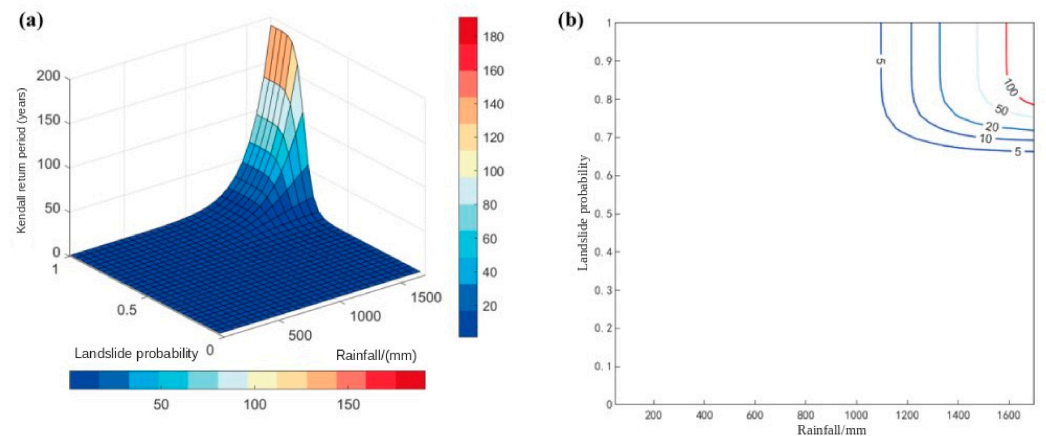


Figure 14. Bivariate Kendall return period for rainfall and landslide probability: (a) return-period distribution map; (b) return-period contour map.

Table 9. Hazard-combination disaster-triggering thresholds under different return periods (Rainfall–Landslide).

Return Period (Years)	Single-Variable Disaster-Triggering Thresholds		Combination Disaster-Triggering Thresholds Under the Kendall Return Period	
	Rainfall/mm	Landslide Probability	Rainfall/mm	Landslide Probability
5	135.60	0.72	122.51	0.70
10	151.02	0.75	133.89	0.72
20	165.06	0.78	145.10	0.74
50	182.43	0.80	159.06	0.77
100	195.02	0.82	170.81	0.78
200	207.30	0.84	180.87	0.81

3.3. Limitations

It should be noted that landslide susceptibility values were used in this study as a measure of landslide probability, but they actually reflect the relative likelihood of landslides in space, not how often they happen over time. The susceptibility value is usually given as a relative number between 0 and 1; better data on triggering factors or landslide events are needed in future studies to improve the probability estimates.

We validated the probability model by means of goodness-of-fit testing using R², the K-S test, the OLS criterion, the AIC, and D₂; uncertainty analysis should be conducted for the high-return period in future work. Moreover, the interaction effects among variables depend on the type of hazard; as such, the results may differ for other rainfall-induced hazards.

4. Conclusions

Considering the joint occurrence characteristics of rainfall-induced hazards, a mathematical model integrating copula functions and the Kendall return period was developed for the calculation of hazard combinations and triggering thresholds. The main conclusions of this study are as follows:

1. A method was introduced for constructing multivariate combination relationships for heavy rainfall hazards using copula functions; copula parameter estimation, model testing, and threshold calculation based on the Kendall return period were described.
2. Bivariate copula functions were used to analyze the joint distribution probabilities of rainfall-induced flash floods and rainfall-induced landslides. The results showed that the optimal univariate marginal distribution for annual maximum 1-day rainfall was the Gamma distribution, and the optimal univariate marginal distribution for annual maximum discharge was the Weibull distribution. The optimal bivariate copula function for annual maximum 1-day rainfall and annual maximum discharge was the Clayton Copula; similarly, the optimal bivariate combined probability function for rainfall-induced landslides was the Gumbel Copula.
3. The combined thresholds of multiple disaster-causing factors for rainfall-induced flash floods and rainfall-induced landslides differed substantially from the thresholds based on single disaster-causing factors. Taking the return period of 100 years as an example, the univariate disaster-triggering thresholds of discharge and rainfall were 8530.38 m³/s and 217.82 mm, while the combined thresholds decreased to 5793.10 m³/s and 175.22 mm. Meanwhile, the univariate disaster-triggering thresholds of rainfall and spatial relative susceptibility were 195.02 mm and 0.82, but the combined thresholds decreased to 170.81 mm and 0.84, respectively. In other words, conventional single-factor threshold calculation methods can severely underestimate

the actual hazard level, and coupling among multiple factors can induce a significant disaster superposition and amplification effect.

The threshold values estimated using the copula-based framework can serve as further validation using observed flood and landslide events, triggering responses when the joint risk exceeds a set limit; therefore, the findings of this study can provide practical support for dealing with rainfall-induced hazards.

Author Contributions: Conceptualization, J.Y. and M.Z.; methodology, J.Y. and M.Z.; validation, M.Z.; formal analysis, J.Y., S.Z., D.Z., T.Z., J.S., S.Y. and M.Z.; investigation, J.Y., S.Z., D.Z., T.Z., J.S., S.Y. and M.Z.; data curation, J.Y., S.Z., D.Z., T.Z., J.S., S.Y. and M.Z.; writing—original draft preparation, J.Y., S.Z., D.Z., T.Z., J.S., S.Y. and M.Z.; writing—review and editing, J.Y., S.Z., D.Z., T.Z., J.S., S.Y. and M.Z.; visualization, J.Y., S.Z., D.Z., T.Z., J.S. and S.Y. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by the National Natural Science Foundation of China (Grant No. 42571088), and China Southern Power Grid Company Science and Technology Project (Grant No. 031600KC23120010).

Data Availability Statement: The raw data supporting the conclusions of this article will be made available by the authors on request.

Acknowledgments: We acknowledge reviewers' and editors' comments.

Conflicts of Interest: The authors declare that this study received funding from China Southern Power Grid Company Science and Technology Project. The funder was not involved in the study design, collection, analysis, interpretation of data, the writing of this article or the decision to submit it for publication.

Abbreviations

The following abbreviations are used in this manuscript:

AIC	Akaike information criterion
CDF	Cumulative distribution function
GEV	Generalized extreme value
GH	Gumbel-Hougaard
IFM	Inference function for marginal method
K-S	Kolmogorov-Smirnov
ML	Maximum likelihood
MOM	Moment-like method of inversion of association of measures
MSE	Mean squared error
OLS	Ordinary least squares
P-III	Pearson type III

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