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A Local Scots Pine Volume Equation from Felled Stems in Drymos Forest, Greece

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Abstract

Local stem-volume estimates require both suitable measurements and evidence of predictive accuracy. We developed a local equation and field table for Scots pine (*Pinus sylvestris* L.) in naturally established, monospecific groups in Drymos Forest, Greece. The sample comprised 60 measured stems from 59 trees, including two stems of one forked tree. Diameter at breast height (DBH) ranged from 11 to 48 cm, and standing height ranged from 6 to 25.5 m. Over-bark sectional reference volumes included an approximated stump and tip. Five equations were compared by cross-validation. The selected diameter–height equation had root mean squared error (RMSE) of 0.0905 m^3, 39.29% lower than the diameter-only model. Keeping stems with shared coordinates together during validation gave similar errors. The 2014 regional equation had RMSE of 0.1000 m^3, but the resampling comparison did not establish superior accuracy for the new equation. Pressler estimates with a basal cylinder had RMSE of 0.1083 m^3 in this sample. The field table provides local mean-volume estimates, accompanied by an assessment of prediction uncertainty and defined limits on DBH–height combinations. Its use is conditional on comparable Drymos conditions and matching measurement conventions; performance in other forests remains untested.

Keywords: forest inventory; tree allometry; destructive sampling; cross-validation; prediction uncertainty; local calibration

1. Introduction

Forest inventories commonly use diameter at breast height (DBH) and total height to estimate stem volume. The relationship depends on stem form and on which part of the stem is included. Whole-stem volume, volume above a specified stump, and merchantable volume to a minimum top diameter describe different quantities. A useful volume equation must therefore specify its stem boundaries and measurement conventions, as well as its predictors [1,2].

International databases compile volume and biomass equations for many tree species, including European species, with differences in the tree components represented and their ranges of applicability [3]. Generalized equations offer a starting point where local measurements are scarce [4], but their performance needs to be checked in the forest where they will be used. In the central Rhodope Mountains, Scots pine grows in mixed forests across contrasting site conditions, as documented by local site-quality research [5]. This variation makes comparison with local measurements and existing regional equations particularly relevant.

Differences in growing space, stand structure and site quality can alter height and taper at a given DBH, so a regional equation need not have the same error in every local

Check for updates

Academic Editor: Ion Mihai

Received: 20 September 2026
Revised: 19 October 2026
Accepted: 29 October 2026
Published: 2 November 2026

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group [1,6]. Conversely, a locally fitted equation is not necessarily more accurate than an existing model; that advantage must be demonstrated using comparable reference volumes and predictions assessed outside the fitting sample [7,8].

Kitikidou et al. [9] developed Scots pine volume models for the Central Rhodope Mountains from 158 standing trees, using diameter, height, and an optically determined form factor to estimate reference volume. Fallias and Diamantopoulou [10] subsequently investigated sectional Scots pine volume in the Pieria Mountains through standing-tree measurements and diameter-reduction models. These studies provide relevant comparisons, but neither establishes the prediction error of a simple DBH–height table against the present felled-stem reference in Drymos.

Compatible taper and volume systems for Scots pine have been developed in Northwestern Spain and Northeastern Turkey [11,12]. They can estimate diameter at a chosen height or volume to a specified utilization limit. Depending on the system, their application may require additional parameters, upper-stem measurements, or local calibration. Model choice should reflect the required output and the measurements available [6]. For inventories that need total stem volume, an equation using only DBH and standing height offers a simpler field application.

Methods based on an upper-stem measurement offer another way to estimate volume without felling the tree. Pressler's method performed well in an evaluation using maritime and radiata pine stem measurements, although performance differed between the two species [13]. Our previous studies in the Central Rhodope Mountains examined standing and felled measurements of the same European beech trees and developed stem-carbon models [14,15]. Paired standing and felled measurements allow an analogous assessment for Scots pine, provided that the volume boundaries and height origins are made explicit.

Reliable local calibration requires more than a close fit: the reference-volume definition, sampled tree sizes, and prediction uncertainty must be documented. Sparse size coverage or a few influential observations can otherwise obscure the limits of a small destructive sample [8,16–18].

Our objective was to develop a local stem-volume equation and field table for naturally established Scots pine groups in Drymos. We asked whether standing height improves prediction beyond DBH alone, whether the inside/outside harvesting-area category adds predictive information, and how the selected equation compares with Pressler estimates and the published regional equation. We expected height to improve prediction because stems with similar DBH can differ in stature and form. We also quantified uncertainty and the joint diameter–height domain for field use.

2. Materials and Methods

2.1. Study Area and Sampled Trees

The target population was Scots pine stems in naturally established, monospecific groups in Drymos Forest, within the joint diameter–height domain represented by the sample. Fieldwork took place from 2023–2024 during the wider Central Rhodope campaign, using the standing and felled measurement protocol reported for European beech [14,15]. The wider study area comprises the public forest complexes of Drymos, Kalyva–Margariti, and Oraio within the Xanthi Forestry Directorate in Northeastern Greece (Figure 1). Scots pine occurs within a forest mosaic with other species [5,14]. The regional reference point is 41°16′20.28″ N, 24°42′19.92″ E at approximately 1040 m elevation; individual pine elevations were unavailable.

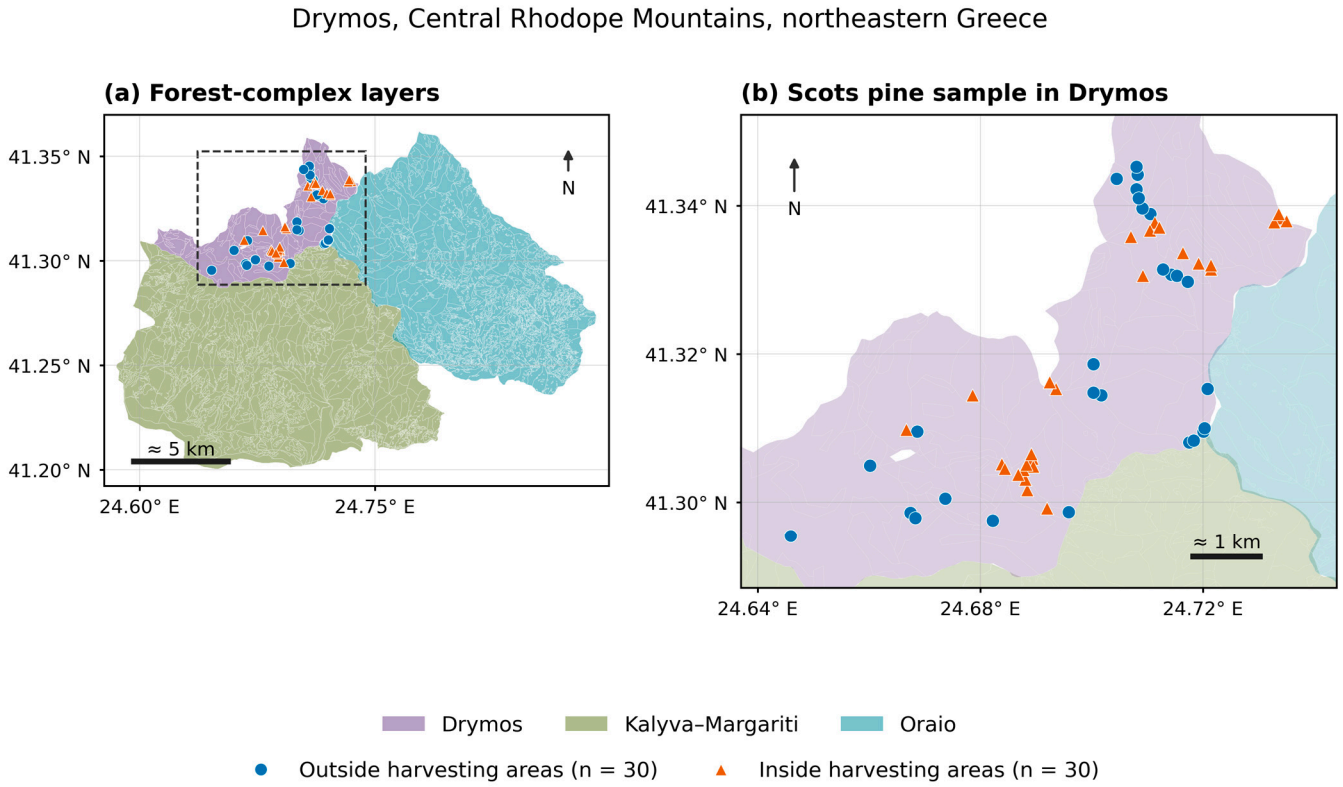


Figure 1. Location of the Scots pine sample in the Central Rhodope Mountains. (**a**) Drymos, Kalyva–Margariti, and Oraio; the dashed rectangle locates panel (**b**). (**b**) All 60 measured stems were in Drymos, with 30 inside and 30 outside areas scheduled for harvesting. Five coincident pairs leave 55 plotted positions: one pair comprises the two stems of a forked tree and four pairs comprise neighbouring trees (Table S6). Symbols have not been displaced. Boundaries and coordinates are from the field map. Axes show WGS84 longitude and latitude; scale bars approximate horizontal distance at 41.3° N. Displayed coordinate precision is not an estimate of positional accuracy.

The climate summary for these complexes reported by Triantafyllidou et al. [15] places the regional reference point in Köppen–Geiger class Csb with warm, dry summers; elevation and terrain cause substantial local variation. Their TerraClimate averages from 1991–2020, extracted using Google Earth Engine (Google LLC, Mountain View, CA, USA), give mean annual temperature of 9.00 °C and annual precipitation of 696.1 mm for Drymos. Corresponding values are 12.22 °C and 617.6 mm for Kalyva–Margariti, and 10.30 °C and 684.8 mm for Oraio. These are long-term averages over the climate-extraction areas, not weather measurements at the sampled stems from 2023–2024.

The field map was matched to the 60 measurement records by identifier and checked against forest-complex boundaries. Clarification from the field investigator established that records 98Π and 99Π represent two stems of the same forked tree; the other four coincident pairs are neighbouring trees. Thus, the analytical unit is the measured stem: 60 stems from 59 trees at 55 recorded positions. The repeated coordinates do not represent duplicated measurement records. Table S6 lists the pairs and their dimensions. Source coordinates are preserved without displacement; their decimal representation does not establish GPS accuracy, for which instrument and recording metadata were unavailable.

The sample contains 30 stems inside and 30 outside areas scheduled for harvesting. This balanced allocation enabled an observational comparison of the two categories, rather than reproducing their population proportions. Diameter classes guided selection to cover a range of sizes, following the shared field approach [14]. Field selection was constrained by the limited, scattered naturally established pine groups and the need to find remaining

diameter classes within the relevant harvesting category; some selected stems were therefore close neighbours. The exact pine class boundaries, within-class selection probabilities, and population diameter frequencies were not available in the analytical records. We consequently fitted an unweighted conditional stem–volume relationship. Neither the fitted coefficients nor the error metrics constitute a design-weighted population calibration; both reflect the sampled size and category distributions.

The sampled stems grew in naturally established, monospecific groups of Scots pine within the wider forest mosaic. These were local pure groups, not necessarily entire pure stands, and originated through natural regeneration rather than planting. Scots pine was scarce in Kalyva–Margariti and Oraio during the wider campaign; all sampled naturally established groups were in Drymos.

The indicator G records whether a stem was inside an area scheduled for harvesting ($G = 1$) or outside such an area ($G = 0$). It describes sampling context, with no randomized treatment allocation or before–after comparison. Nine management-compartment codes were recorded for inside stems, with 1–9 stems per compartment (Supplementary Section S5). Equivalent outside codes and complete natural-group identifiers were unavailable. Stand age, density, basal area, slope, and aspect were not recorded in the analytical dataset, so their effects on form could not be evaluated. Figure 2 and Table 1 show the different size distributions of the two categories.

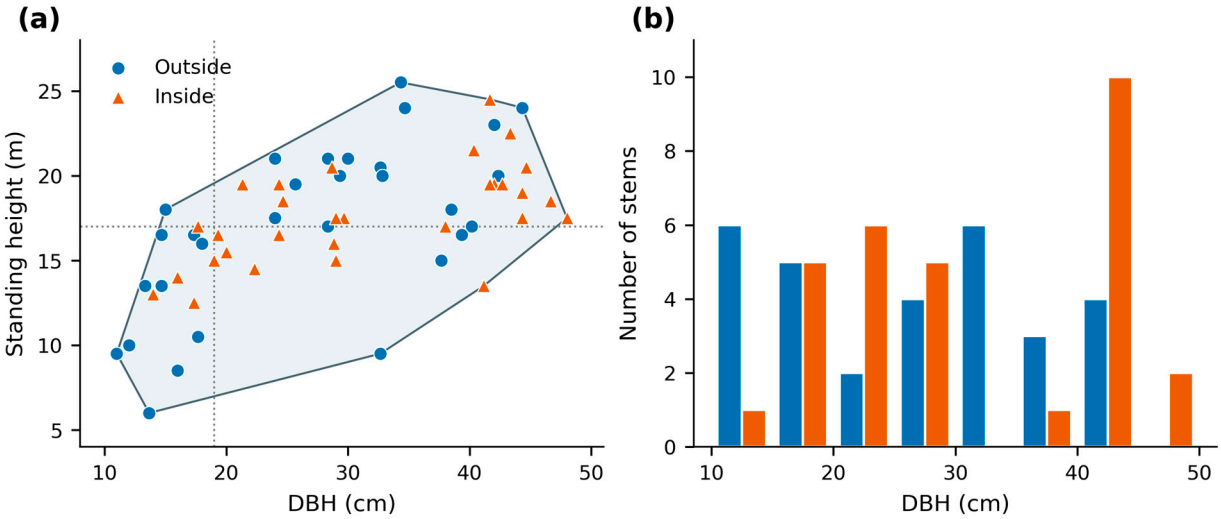


Figure 2. Sizes of the 60 sampled Scots pine stems. (**a**) Standing height against DBH. The shaded polygon encloses all observed diameter–height pairs and defines the range of combinations returned by the new table. Dotted lines show the lower DBH and height limits reported for the 2014 regional equation: 19 cm and 17 m. (**b**) Diameter distributions of stems inside and outside areas scheduled for harvesting.

Table 1. Dimensions of the final Scots pine sample.

Variable	All Stems ($n = 60$)	Outside ($n = 30$)	Inside ($n = 30$)
DBH (cm)	29.15 ± 10.99 (11.00–48.00)	26.82 ± 10.60 (11.00–44.33)	31.48 ± 11.05 (14.00–48.00)
Standing height (m)	17.29 ± 4.06 (6.00–25.50)	16.95 ± 5.01 (6.00–25.50)	17.63 ± 2.85 (12.50–24.50)

Table 1. *Cont.*

Variable	All Stems ($n = 60$)	Outside ($n = 30$)	Inside ($n = 30$)
Felled ground-to-tip length (m)	17.40 ± 4.01 (7.30–25.80)	16.99 ± 4.74 (7.30–25.80)	17.81 ± 3.16 (10.80–24.30)
Stump diameter (cm)	34.08 ± 12.08 (15.00–53.33)	32.18 ± 11.91 (15.00–53.00)	35.99 ± 12.15 (17.00–53.33)
Reference volume (m^3)	0.5805 ± 0.4020 (0.0597–1.4227)	0.5118 ± 0.3698 (0.0597–1.4223)	0.6491 ± 0.4270 (0.0953–1.4227)

Values are mean ± sample standard deviation, with range in parentheses. Inside and outside refer to areas scheduled for harvesting. All sampled stems came from naturally established, monospecific Scots pine groups. Reference volume includes the 0.30 m stump approximation and the section from 0.30 to 1.30 m.

2.2. Standing and Felled Measurements

Standing DBH, D, was measured over bark at 1.30 m with a diameter caliper to the nearest 0.5 cm for each reading. Three directions were measured: 0°, 45°, and 135° relative to the first direction, including two perpendicular readings. The arithmetic mean was retained without further rounding. Stump diameter was also the mean of three directional readings. Standing total height, H, was measured to the nearest 0.5 m with a Haga hypsometer (Haga GmbH + Co. KG, Nuremberg, Germany) from approximately 25–35 m, where the top and base were visible. These instruments and conventions followed the shared field protocol [14]. Standing heights derive from the recorded top and base readings. A Spiegel Relaskop (Relaskop-Technik Vertriebsges. m.b.H., Salzburg, Austria) was used for Pressler measurements. L_1 is the scale reading at breast height and L_2 is the reading where the stem diameter equals half of DBH. Their difference is converted to the vertical distance above breast height as described in Section 2.4. The same observer took all Haga and Relaskop readings and recorded all field data.

After felling, the distance from the cut to the tip was measured with a tape. Over-bark diameters were recorded at the cut and at one-metre intervals along the felled stem. The cutting height was 0.30 m. Thus, D_{log1} was measured one metre beyond the cut, corresponding to 1.30 m above ground; D_{log2} corresponded to 2.30 m, with later diameters one metre apart. D_{log1} was a single caliper reading in the direction determined by the stem's resting position after felling. Adding 0.30 m to the cut-to-tip length gave the ground-to-tip length in the worksheet. This felled length was used to calculate reference volume. The equation uses standing height because that is the measurement available before a stem is felled. Caliper measurements on standing stems, stumps and felled stems, and tape measurements of stem length were made by a two-person crew whose members alternated tasks.

One-metre sections retained the spacing of the shared field protocol and provided a practical, consistent resolution for sectional integration. They were not selected through an optimization of section length. The recorded section endpoints and the alternative frustum calculation allow sensitivity to geometry at this spacing to be examined.

All recorded diameters were over bark. The reference volumes and field table therefore include bark and exclude branches and roots. We checked stem identifiers, positive dimensions, uninterrupted diameter sequences, and consistency between the final diameter position and total length. No missing diameter was filled in, and no measured profile was smoothed. Standing DBH and D_{log1} were compared descriptively at the same nominal height (1.30 m). This compares a three-direction mean with a single directional reading, so differences may reflect measurement direction and averaging, as well as measurement error.

No discrepancy threshold was used to exclude or alter observations. Crew-member identity was not recorded for each reading, and no repeated full measurements were available to estimate diameter repeatability or observer effects. For upper-stem positions beyond $D_{\log 1}$, the number and directions of readings were not documented.

2.3. Sectional Reference Volume and Quality Checks

We defined the primary response as sectional stem volume from ground level to the recorded tip, including an approximated 0.30 m stump. Let d_0 denote stump diameter and d_j the jth upper-stem diameter, with all diameters in centimetres. Cross-sectional area is $A_j = \pi d_j^2/40{,}000\ \mathrm{m}^2$. If m upper diameters are available and L is the cut-to-tip length in metres, the unmeasured terminal length is $T = L - m$. The reference volume was calculated as

$$V_{\mathrm{ref}} = sA_0 + \sum_{j=0}^{m-1} \frac{A_j + A_{j+1}}{2}\Delta l + \frac{A_m T}{3} \quad (1)$$

where $s = 0.30$ m and $\Delta l = 1$ m. The terms represent a stump cylinder, consecutive one-metre Smalian sections, and a terminal cone. Reference volume thus depends on geometric approximations between measured cross-sections [1,2]; physical volume was not measured by displacement. We retained an alternative response excluding the stump cylinder for sensitivity analysis.

We independently recalculated each stem's sectional volume from its recorded diameters and length and compared it with the finalized worksheet value. The sum included the basal section from 0.30 to 1.30 m and every subsequent one-metre section.

Figure 3 compares stem shape among stems of different sizes. For each stem, measured diameter was divided by standing DBH, and measurement height above ground was divided by felled ground-to-tip length. A relative diameter of 1 therefore equals DBH, and a relative height of 0.5 is halfway along the total stem length. Each line joins that stem's measured positions, beginning at the 0.30 m cut and ending at its last measured diameter. The assumed ground-level cylinder and terminal cone used for volume calculation are not plotted as additional measurements.

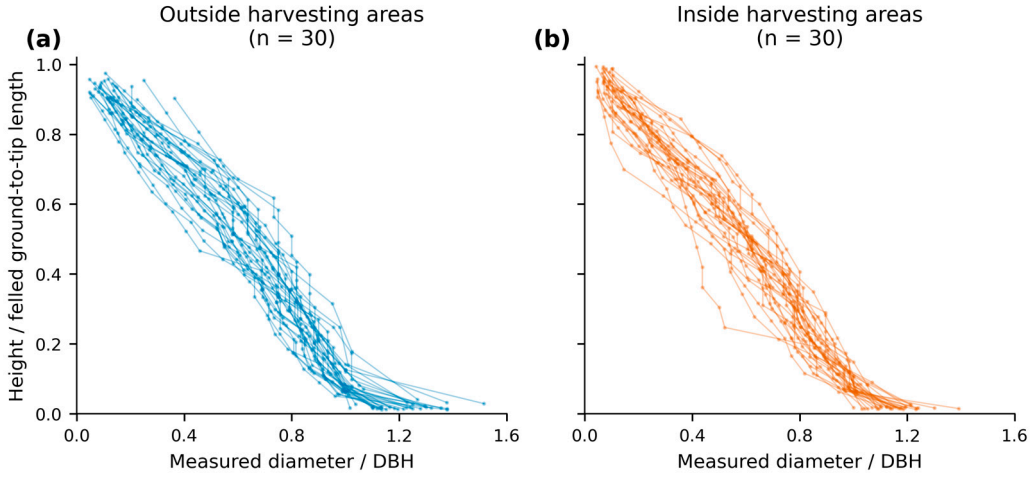


Figure 3. Stem profiles of Scots pine (**a**) outside and (**b**) inside areas scheduled for harvesting. Each line represents one stem and joins its measured diameters along the stem. The horizontal axis is measured diameter divided by standing DBH; the vertical axis is height above ground divided by felled ground-to-tip length. These ratios allow stems of different sizes to be compared. Lines start at the 0.30 m stump measurement and stop at the last measured diameter. No assumed ground-level diameter or zero-diameter tip is shown as a measured point. Both panels use the same scales.

We required the last positive measured diameter to lie below the recorded tip; Table S5 summarizes terminal geometry. Sensitivity analyses examined omission of the stem with the longest terminal section, removal of all tip volumes, and replacement of Smalian sections by conical frusta. The latter compares two geometric approximations of the same measured profiles.

2.4. Operational and Published Benchmarks

We calculated two versions of the Pressler estimate to show how inclusion of the basal stem affects comparison with ground-to-tip reference volume. Let $g = \pi D^2/40{,}000$ be the breast-height cross-sectional area in m^2 and $h\square = (D/100)(L_2 - L_1)$ the measured height difference in metres between breast height and the position where diameter is half of DBH. The expression stored in the worksheet, labelled "Pressler without basal cylinder" in the tables, was

$$V_P = \frac{2}{3} g h_r = \frac{\pi}{6}(D/100)^3(L_2 - L_1) \quad (2)$$

The second version, labelled "Pressler with basal cylinder", was $V_{PG} = V_P + 1.30g$. The added term represents a cylinder from ground level to 1.30 m, with diameter equal to DBH. It extends the above-breast-height estimate to approximate ground-to-tip volume. If the half-DBH position is instead expressed as height above ground, $h_p = h_r + 1.30$, the same calculation is $V_{PG} = (2/3)g(h_p + 0.65)$. The two expressions must therefore be interpreted together with their height origin.

We applied the whole-area Scots pine equation of Kitikidou et al. [9] without changing its published coefficient: $V_{2014} = 0.318(D/100)^2 H$. D is in centimetres, H is in metres, and volume is in cubic metres. We evaluated it on all 60 stems and on the 34 stems whose DBH and height both lay within the earlier reported ranges, 19–72 cm and 17–41 m. These separate ranges do not recover all diameter–height combinations represented in the earlier sample. Its reference volumes were also estimated by a different method. The comparison therefore tests how the published equation performs on the present stems under our reference-volume definition.

2.5. Candidate Equations and Conversion from Logarithmic Volume

Five candidate equations represented common ways of using diameter and height (Table 2). M1 used log DBH alone. M2 used log Q, where $Q = D^2H$, and M3 used separate log D and log H terms. M4 was an original-scale linear equation in Q with an intercept, and M5 was the corresponding equation constrained through the origin. All models used the same 60 verified reference volumes and standing-height predictor. Ordinary least squares was applied on the scale specified in Table 2. We retained full numerical precision during fitting and prediction.

For a model fitted to log volume, simply taking the exponential of the prediction generally gives a value below the arithmetic mean volume. This conversion issue is well established in tree allometry [19,20]. We used the smearing correction of Duan [21]: $S = n^{-1}\Sigma\exp(e_i)$, where e_i is the difference between observed and fitted log volume for a training stem. The mean-volume prediction is $S\ \exp(x\square\hat{\beta})$. Thus, the correction is the average of the exponentiated fitting errors and is recalculated whenever the model is refitted. We examined residual patterns and sensitivity results to assess whether one correction factor was adequate across the observed stem sizes.

Model choice emphasized original-scale predictive error. Root mean squared error (RMSE) was the primary criterion, accompanied by mean absolute error (MAE), mean absolute percentage error (MAPE), mean signed error, and aggregate relative bias. For $e_i = \hat{V}_i - V_{ref,i}$, MAPE $= (100/n)\Sigma|e_i/V_{ref,i}|$, and aggregate relative bias $= 100\Sigma e_i/\Sigma V_{ref,i}$. The latter is volume-weighted and differs from the mean of stem-level percentage errors.

Predictive $R^2 = 1 - \Sigma e_i^2 / \Sigma (V_{\text{ref},i} - \overline{V}_{\text{ref}})^2$ was not the squared correlation between observations and predictions. Small reference volumes can produce large percentage errors for a given absolute error, so MAPE was interpreted alongside RMSE and MAE.

Table 2. Candidate equations and fitted response scales.

Model	Equation	Coefficients p	Fit Scale
M1	$\ln V = a + b \ln D$	2	Log volume
M2	$\ln V = a + b \ln(D^2 H)$	2	Log volume
M3	$\ln V = a + b \ln D + c \ln H$	3	Log volume
M4	$V = a + bD^2 H$	2	Volume
M5	$V = bD^2 H$	1	Volume

D is DBH in cm, H is standing height in m, and V is sectional reference volume in m^3. The coefficient count p includes the intercept when present: M1, M2 and M4 have two coefficients, M3 has three, and M5 has one without an intercept. Log-model mean predictions include a smearing factor calculated from the fitting sample. Table S1 supplies all coefficients.

2.6. Cross-Validation and Comparison of Sampling Categories

In leave-one-out cross-validation (LOOCV), one measured stem was removed, the model was fitted to the other 59 stems, and the omitted stem's volume was predicted. We repeated this for every stem. These are the held-out predictions reported below: the predicted stem did not contribute to the fitted coefficients, smearing factor, or prediction interval. This stem-level calculation does not make the observations biologically or spatially independent; the additional checks below address the identified shared locations. For comparison, the supplement reports apparent fit on the same observations used to fit the model.

Selecting a model and reporting its performance from the same cross-validation results can give an optimistic assessment [7]. We therefore repeated the selection within each training sample, using nested LOOCV. One stem was first reserved for testing. Within the other 59 stems, a second leave-one-out procedure compared the five candidate equations by RMSE. The chosen equation was then fitted to those 59 stems and used to predict the reserved stem. This entire procedure was repeated for all 60 stems. We also checked the effect of identical map coordinates. At each of the 55 recorded locations, we removed all stems recorded there together, fitted M2 to the remaining stems, and predicted the removed stem or pair. This prevents a pair with identical coordinates from being split between fitting and testing but does not separate all neighbouring stems. The coordinate-group check keeps both stems of the forked tree together. Both checks concern the Drymos sample; transfer to different forests requires independent data [22].

To test whether location relative to harvesting areas improved prediction, we added a binary variable G to model M2: $G = 1$ for an inside stem and $G = 0$ for an outside stem. This variable allows the estimated volume to differ between the two categories after accounting for D^2H. A second extension also allowed the slope against log Q to differ, with log Q centred at $Q = 30^2 \times 18$. Partial F tests compared these extensions with the common equation, and LOOCV assessed their prediction errors. Finally, we fitted the common equation to the 30 outside stems and predicted the 30 inside stems, then reversed the exercise. This tests prediction between the two sampling categories; it does not estimate variation among all forest compartments.

2.7. Uncertainty and Robustness

We retained 5000 bootstrap samples drawn with replacement as 30 inside and 30 outside stems, refitting the equation and smearing factor each time. The middle 95% of coefficient or mean-volume estimates defined percentile intervals [23]. This calculation preserves the analytical category balance but assumes exchangeable stems within each category; it is not a reconstruction of the unequal-probability field design. A separate calculation resampled the existing observation–LOOCV prediction pairs within categories to describe variation in prediction-error metrics. Comparisons between methods used the same sampled pairs. These performance intervals hold predictions fixed and do not repeat model fitting or estimate between-forest transfer uncertainty.

To assess the known spatial pairing, we added a coordinate-group bootstrap with 5000 refits: 27 outside and 28 inside positions were sampled with replacement, retaining all stems at each selected position. The number of resampled stems could therefore vary. This preserves the forked-tree pair and neighbouring coincident pairs but cannot account for unrecorded clustering among different positions. We also refitted M2 after omitting 98Π, 99Π, or both. These are sensitivity checks; all 60 measured stems remain in the primary fit (Supplementary Section S6).

A prediction interval for one stem is wider than a confidence interval for mean volume because individual stems with the same DBH and height can differ in stem shape and volume. We calculated approximate 95% individual-stem prediction intervals on the log scale as

$$x^{\mathrm{T}}\hat{\beta} \pm t_{0.975,\, n-p} s_{\mathrm{e}} \sqrt{1 + x^{\mathrm{T}}(X^{\mathrm{T}}X)^{-1}x} \quad (3)$$

In this expression, s_{e} is the residual standard deviation, p is the number of coefficients, X contains the predictors for the fitting sample, and x contains the predictors for the new stem. The last term accounts for uncertainty in the fitted equation at that stem's dimensions. We exponentiated the lower and upper limits to express them in m^3 without multiplying them by the mean smearing factor. The calculation assumes approximately independent, normally distributed log errors with stable variance. It treats the supplied DBH and height as fixed measurements. We assessed how often these intervals contained the omitted stems' reference volumes. They do not separately include new measurement errors, errors in the sectional reference, or differences between forests.

Diagnostics included residual plots, the Shapiro–Wilk normality test, a studentized Koenker variance test against log Q, leverage, and Cook's distance [16,24,25]. HC3 covariance estimates provided a further check on coefficient uncertainty [26]. We considered these diagnostics together with predictive performance and influence, following guidance on forest biometric model evaluation [27]. Sensitivity refits omitted, in turn, the largest-diameter stem, the stem with maximum Cook's distance, and the stem with the longest terminal section. All stems remained in the primary analysis.

2.8. Construction and Use of the Field Table

After fitting the selected equation to all 60 stems, we calculated volumes at 2 cm DBH and 2 m standing-height increments, starting at 12 cm and 6 m. These increments provide a readable field grid rather than measurement-resolution or accuracy limits. We restricted predictions to the convex hull of the observed standing DBH–height pairs in Figure 2a: the smallest convex polygon enclosing those pairs. Each grid combination was checked against all hull-edge inequalities, allowing a numerical tolerance of 10^{-9}. The separate diameter and height ranges alone are insufficient. We also counted sampled stems within ±4 cm DBH and ±2 m height. This pragmatic window spans two diameter increments and one height increment on either side; it was not optimized or validated as an accuracy threshold. The count describes local data support and does not determine the uncertainty interval.

Table S4 gives the complete grid of mean volumes. Mean confidence intervals and individual-stem prediction intervals are defined in Section 2.7 and illustrated in Section 3.4.

3. Results

3.1. Sample Dimensions and Sectional Reference Volumes

All 60 stems were in Drymos. The five coincident pairs were retained as separate measured stems, with their field relationships documented in Table S6. Inside stems had larger mean DBH and volume, whereas outside stems covered a wider height range (Table 1). The 60 stump diameters and 962 upper-stem diameters formed uninterrupted, non-increasing sequences. Figure 3 compares the measured profiles after scaling by DBH and felled ground-to-tip length.

The independent sectional calculation agreed with all 60 finalized worksheet volumes within 10^{-10} m^3. Total sample volume was 34.8292 m^3. The stump cylinders contributed 1.8452 m^3 or 5.30% of the total; the remaining volume was assigned to the felled stems above the 0.30 m cut.

Terminal cones contributed 0.0123 m^3 in total; the largest individual contribution was 0.701% of reference volume. All terminal lengths were positive, from 0.10 to 1.90 m, with the longest in stem 304Π. Standing height minus felled ground-to-tip length averaged −0.11 m, but the RMSE was 1.04 m and individual differences ranged from −2.4 to +2.2 m. Thus, the small mean difference concealed appreciable stem-level disagreement between the operational height predictor and the length used in the reference calculation. Standing DBH minus $D_{\log 1}$ averaged +0.047 cm, with an RMSE of 0.681 cm and a range from −2.000 to +1.667 cm. These comparisons describe agreement between measurements, not independent estimates of instrument accuracy.

3.2. Model Selection and Predictive Performance

M2, the log-linear model in $Q = D^2H$, had the lowest LOOCV RMSE, 0.0905 m^3, with MAE of 0.0594 m^3, MAPE of 10.60%, and predictive R^2 of 0.9484 (Table 3). Its aggregate relative bias was −0.021%. The DBH-only model had RMSE of 0.1491 m^3, so adding height through Q reduced this criterion by 39.29%. The paired resampling interval for the RMSE reduction was 0.0280–0.0867 m^3. This interval was calculated by resampling the existing stem–prediction pairs, retaining 30 inside and 30 outside stems; it does not repeat model fitting.

Holding out all stems sharing a location gave M2 RMSE of 0.0907 m^3, MAE of 0.0595 m^3, and MAPE of 10.62% across 55 validation rounds. Each round omitted one stem or one of the five pairs, keeping both stems of the forked tree together. The close agreement with stem-level LOOCV shows little sensitivity to these recorded pairs; it does not exclude dependence among other nearby stems or validate prediction in independent stands.

M3, with separate exponents for D and H, had nearly identical prediction error to M2 (RMSE 0.0917 versus 0.0905 m^3). M2 was retained for its slightly lower RMSE and two fitted coefficients rather than three; the difference does not imply a substantial accuracy advantage. The M3 exponents were 1.7248 for D and 0.8564 for H, and the M2 restriction $\beta_D = 2\beta_H$ was consistent with them ($t = 0.061$, $p = 0.951$). Original-scale M4 and M5 had RMSE values of 0.0935 and 0.1004 m^3. The nested procedure selected M2 in all 60 outer training samples. Its full-sample mean-volume equation was

$$\hat{V} = S \times \exp(a) \times \left(D^2H\right)^b \quad (4)$$

Table 3. Original-scale predictive performance against sectional reference volume.

Equation	RMSE (m^3)	MAE (m^3)	MAPE (%)	Relative Bias (%)	Predictive R^2
M1: DBH	0.1491	0.1045	19.85	1.70	0.8601
M2: DBH^2H	0.0905	0.0594	10.60	−0.02	0.9484
M3: DBH + H	0.0917	0.0603	10.82	−0.04	0.9471
M4: linear DBH^2H	0.0935	0.0643	14.99	0.04	0.9450
M5: origin DBH^2H	0.1004	0.0741	16.30	−3.83	0.9366
Pressler without basal cylinder	0.1647	0.1262	27.69	−21.04	0.8294
Pressler with basal cylinder	0.1083	0.0816	16.23	−4.01	0.9261
Regional 2014	0.1000	0.0717	15.29	−0.52	0.9371

M1–M5 use predictions made after omitting the stem being predicted. Pressler and the 2014 equation were applied without fitting coefficients to these 60 stems. Positive bias indicates overestimation. "Without basal cylinder" is the original worksheet Pressler calculation; "with basal cylinder" adds $1.30g$, as defined in Section 2.4. M2 was selected in all 60 nested validation rounds. The resampling interval for the RMSE difference between M2 and the 2014 equation included zero.

In Equation (4), $S = 1.007619$ is the smearing factor, $a = -8.960123$, and $b = 0.861158$; D is over-bark DBH in centimetres, H is standing height in metres, and $\hat{V}$ is volume in cubic metres. The stratified stem-bootstrap 95% interval for b was 0.8282–0.8994, and the HC3 interval was 0.8251–0.8972. Apparent RMSE was 0.0874 m^3. Figure 4 shows held-out predictions; Tables S1 and S3 report coefficients and descriptive intervals for predictive metrics.

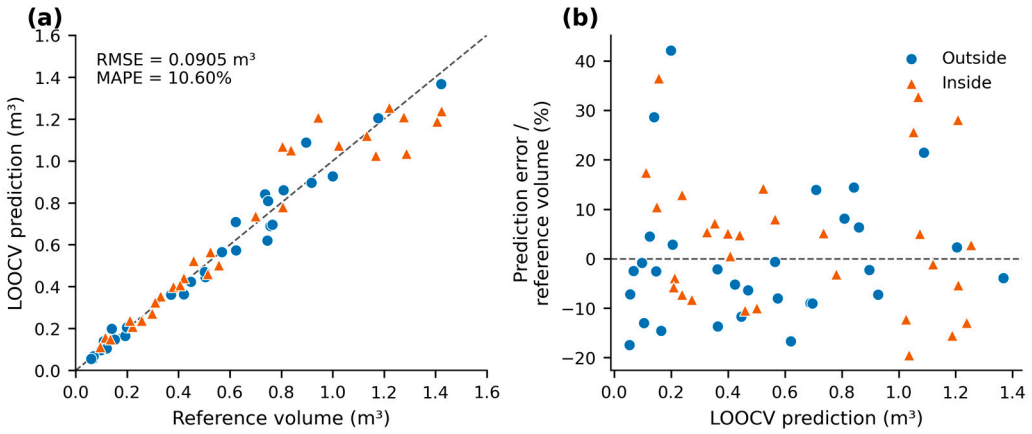


Figure 4. Predictions from model M2 when each stem was omitted from fitting. (**a**) Predicted against sectional reference volume; the dashed line indicates equality. (**b**) Signed percentage error against predicted volume; positive values indicate overestimation. Each prediction and its smearing correction were calculated using the other 59 stems. Blue circles represent stems outside areas scheduled for harvesting; orange triangles represent stems inside those areas.

3.3. Inside–Outside Comparison and Existing Volume Estimates

Adding the binary inside/outside variable G increased LOOCV RMSE to 0.0932 m^3. Its log-scale coefficient was −0.0371, corresponding to an estimated volume difference of −3.64% for inside stems at the same D^2H, with a conventional 95% interval from −9.90% to +3.06%. The partial test gave $F_{1,57} = 1.222$ and $p = 0.274$. Allowing both the intercept and

slope to differ between categories also failed to improve RMSE (0.0936 m^3; joint $F_{2,56} = 1.392$, $p = 0.257$). We therefore retained one equation for the sample. The interval still allows a meaningful difference between categories, so the results do not demonstrate that their volume relationships are identical.

Fitting the equation to outside stems and predicting inside stems gave RMSE of 0.1086 m^3 and aggregate relative bias of +0.97%. Fitting it to inside stems and predicting outside stems gave RMSE of 0.0622 m^3 and bias of −1.34%. The absolute errors differed alongside the different stem-size distributions; MAPE was similar at 11.33% and 11.17%. Table S2 separately reports errors for each category from the equation fitted across both categories and from the fixed Pressler and 2014 calculations.

Pressler without the basal cylinder gave RMSE of 0.1647 m^3, MAPE of 27.69%, and aggregate relative bias of −21.04%. Including the 1.30g cylinder reduced these to 0.1083 m^3, 16.23%, and −4.01%, respectively (Table 3; Figure 5). The regional equation of Kitikidou et al. [9] gave RMSE of 0.1000 m^3, MAPE of 15.29%, and aggregate relative bias of −0.52%. M2 had approximately 9.5% lower RMSE than the 2014 equation. However, the paired resampling interval for RMSE2014–RMSEM2 was −0.0049 to +0.0212 m^3 and included zero. Among the 34 stems within both earlier size ranges, RMSE was 0.1265 m^3 for the regional equation and 0.1170 m^3 for M2 cross-validation predictions; MAPE was 12.10% and 9.97%, respectively.

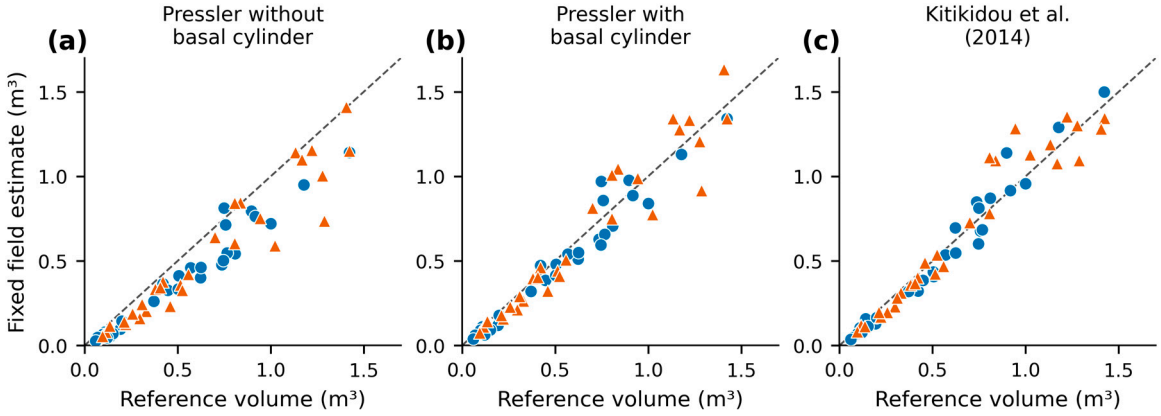


Figure 5. Existing volume calculations compared with the sectional reference. (**a**) Pressler without the basal cylinder, as originally calculated in the worksheet. (**b**) Pressler with the added cylinder, 1.30 g, representing the stem below breast height. (**c**) The whole-area equation of Kitikidou et al. [9]. Dashed lines show equality. Blue circles represent outside stems, and orange triangles represent inside stems. No coefficients were fitted for these calculations. For comparison, Table 3 reports M2 predictions made after omitting each stem from fitting.

3.4. Diagnostics and Uncertainty

The log residuals showed a modest departure from normality (Shapiro–Wilk $p = 0.0413$), whereas the Koenker test did not indicate variance changing with log Q ($p = 0.719$). Six stems exceeded the descriptive Cook's-distance threshold of $4/n$. Stem 97Π had both maximum leverage (0.108) and maximum Cook's distance (0.133); diagnostic plots are supplied in Figure S1. No observation was removed from the primary fit. Omitting this stem changed fitted mean predictions over the retained observations by, at most, 2.13%; omitting the largest stem changed them by at most 1.00% (Table 4).

Table 4. Sensitivity of model M2 to influential records, volume geometry, and height.

Variation	*n*	Exponent *b*	LOOCV RMSE (m^3)	Max. Fitted Change (%)	Mean Target Change (%)
Omit largest DBH	59	0.8658	0.0843	1.00	—
Omit maximum Cook's distance	59	0.8688	0.0915	2.13	—
Omit longest terminal section	59	0.8619	0.0912	0.34	—
Conical frustum sections	60	0.8621	0.0906	—	−0.291
Remove terminal cones	60	0.8620	0.0905	—	−0.076
Exclude stump cylinder	60	0.8711	0.0898	—	−5.836
Use felled length	60	0.8819	0.0745	—	0

Maximum fitted change is the largest absolute percentage difference between refitted and primary-model mean predictions over the retained observations. Mean target change is the signed arithmetic mean of $100(V_{\text{alternative}}/V_{\text{ref}} - 1)$ across stems; negative values indicate a smaller reference target. RMSE values for different targets or subsets are descriptive and are not interchangeable validation tests. The felled-length model is diagnostic, not the deployed table.

Excluding stem 304ΓΙ, which had the longest terminal section (1.90 m), changed fitted predictions over retained observations by, at most, 0.34%. Replacing Smalian sections with conical frusta reduced individual reference volumes by 0.291% on average, with little change in predictive error. Omitting tip cones had an even smaller effect. By contrast, removing the stump changed the response boundary and reduced stem volume by 5.84% on average. Using measured felled length in place of standing height reduced LOOCV RMSE to 0.0745 m^3; this diagnostic describes the potential role of height measurement and does not define an operational standing-stem model.

Omitting both stems of the forked tree changed fitted mean predictions over the remaining 58 stems by at most 0.97%; omitting either stem alone changed them by at most 0.74% (Table S8). The coordinate-group bootstrap gave an exponent interval of 0.8291–0.8971 and mean-interval widths within approximately −2% to +5% of the original stem-bootstrap widths across supported grid points (Table S7). These checks indicate limited sensitivity to the identified pairs, not independence of the full sample.

The approximate 95% individual-stem intervals included the reference volume of 57 of 60 omitted stems, with mean width of 0.3012 m^3. Holding out shared-coordinate groups also included 57 of 60 reference volumes. Both are internal checks with substantial uncertainty at this sample size. At $D = 30$ cm and $H = 18$ m, estimated mean volume was 0.5458 m^3. The stratified stem-bootstrap 95% mean interval was 0.5284–0.5633 m^3, compared with 0.5275–0.5638 m^3 when resampling whole coordinate groups. The approximate individual interval was wider: 0.4190–0.7002 m^3 (Figure 6). Coordinate-group sensitivity and omission of the forked-tree stems are reported in Tables S7 and S8.

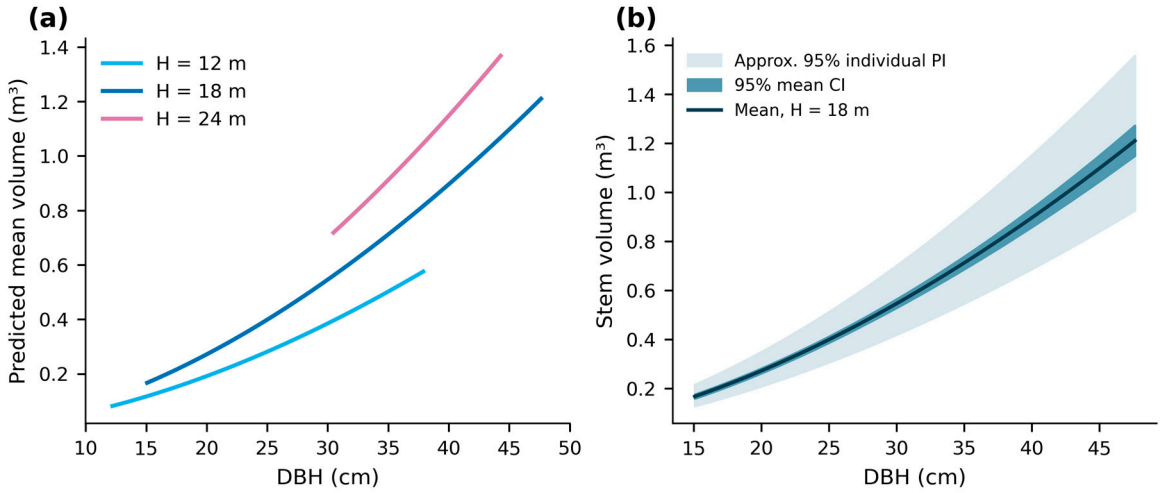


Figure 6. Volume predictions and their uncertainty. (**a**) Estimated mean volume at standing heights of 12, 18, and 24 m. Each curve ends where its DBH–height combination reaches the boundary of the sampled size polygon in Figure 2a. (**b**) At 18 m, the dark band shows the 95% confidence interval for mean volume, calculated from 5000 bootstrap samples with 30 inside and 30 outside stems. The pale band is the approximate 95% prediction interval for one stem, including variation among stems of the same dimensions. Both calculations treat DBH and height as fixed; they do not separately include new measurement error or uncertainty from applying the model in another forest.

3.5. Field Volume Table

Of 190 combinations on the 2 cm DBH × 2 m height grid, 105 lay within the sampled size polygon. Table 5 is a compact extract starting at 10 m; Table S4 retains the full 6–24 m grid, including the six supported combinations at 8 m. No 2 cm grid diameter at 6 m lies inside the hull so that column contains dashes. Tabulated values are estimated conditional means, not exact volumes of individual stems. At 30 cm DBH and 18 m height, six sampled stems fall within the stated proximity window.

Table 5. Compact field-table extract of predicted mean stem volume (m^3).

DBH (cm)	H = 10 m	H = 12 m	H = 14 m	H = 16 m	H = 18 m	H = 20 m	H = 22 m	H = 24 m
12	0.068	—	—	—	—	—	—	—
16	0.111	0.130	0.149	0.167	0.185	—	—	—
20	0.164	0.191	0.219	0.245	0.271	—	—	—
24	0.224	0.262	0.299	0.336	0.372	0.407	—	—
28	0.292	0.342	0.390	0.438	0.485	0.531	0.576	—
32	0.368	0.430	0.491	0.551	0.610	0.668	0.725	0.781
36	—	0.527	0.602	0.675	0.747	0.818	0.888	0.957
40	—	—	0.721	0.809	0.896	0.981	1.065	1.148
44	—	—	—	0.954	1.056	1.156	1.255	1.352
48	—	—	—	—	—	—	—	—

A dash indicates a standing DBH–height pair outside the observed convex hull (Figure 2a). Table S4 contains the complete grid of mean volumes. Mean confidence intervals and approximate individual prediction intervals are illustrated in Figure 6 and Section 3.4. Means are not exact individual-stem volumes, and membership of the hull does not ensure dense local support. Volumes include bark and the 0.30 m stump approximation. Use over-bark DBH at 1.30 m and standing total height.

4. Discussion

The principal finding is that standing DBH and height support a useful local stem-volume relationship when checked against felled-stem sections. Height improved prediction over diameter alone, while the small numerical advantage over the 2014 equation remained inconclusive. The main addition to existing work is therefore a documented local reference, a complete field table, and separate information on size support and prediction uncertainty.

4.1. Stem Boundaries and Reference Volume

The sectional reference and new equation cover the stem from ground level to the tip, including bark and an approximated 0.30 m stump. Stump cylinders accounted for 5.30% of the summed reference volume; the mean of individual-stem stump percentages was 5.84%. The first calculation weights stems by volume, whereas the second weights them equally. Removing the stump changes the response boundary and does not validate the cylinder approximation. This issue concerns the sectional reference; the separate Pressler cylinder in Section 2.4 represents the entire 0–1.30 m interval.

Recording the cross-sections and their height origins allows the reference calculation to be reproduced, as recommended for tree allometry [2,8]. The measured profiles show variation in stem form that a constant form factor cannot capture. The DBH–height equation summarizes this variation for total-volume prediction. Estimating upper-stem diameters or volume to a utilization limit would require a taper model [6].

Both Smalian and frustum calculations use the same measured cross-sections, so their close agreement establishes limited sensitivity to sectional geometry, not accuracy against physical volume. The stump cylinder can underestimate basal volume if diameter increases towards the ground. Ground-level diameters were not measured, so the magnitude and size dependence of any basal-volume error cannot be quantified from these data. Terminal cones contributed little volume. Recording the actual last-section height and total length would avoid reconstructing that position from the section count [2]. For the forked tree, the fork height and geometry of any common basal section were not separately recorded; its two stem estimates should therefore not be interpreted as a validated whole-tree total.

4.2. The Contribution of Height to Volume Prediction

Adding height distinguishes stems of similar DBH but different stature, while the fitted D^2H exponent accommodates variation from a constant form factor. This explains the improvement over diameter alone without requiring separate diameter and height exponents. M2 and M3 performed similarly; the simpler M2 was selected throughout nested validation. This supports a practical two-predictor equation within the local size domain, rather than a universal stem-form relationship.

The form factor expresses stem volume as a fraction of the volume of a cylinder with the same total height and a diameter equal to DBH. When either DBH or height increases while the other is held constant, the fitted equation predicts a larger stem volume but a smaller fraction of the corresponding cylinder volume. This is a mathematical property of the equation within the sampled size range and does not demonstrate how stem form changes as an individual stem grows.

Standing height and felled ground-to-tip length differed little on average, but their RMSE was approximately one metre. Replacing standing height with felled length reduced volume RMSE to 0.0745 m^3. Standing-height error can therefore contribute to residual variation and distort predictor effects, although this comparison does not isolate its magnitude: felled length also enters the sectional reference and is not an independent error-free standard. Haga readings can be affected by top identification, viewing geometry and stem inclination. The data cannot separate these effects from tape or diameter measurement error.

The deployed equation retains standing height, the measurement available to inventory users. Holding the other predictor fixed, a 1% change in height changes predicted volume by approximately 0.86%, and a 1% change in DBH by approximately 1.72%.

4.3. Comparison with Regional and Pressler Estimates

M2 produced a lower RMSE than the equation of Kitikidou et al. [9] for the sampled stems. However, the resampling interval for the difference in RMSE included zero. Thus, although M2 performed better numerically, the analysis did not provide clear evidence of superior predictive accuracy. Its contribution is a local table based on measurements along felled stems, with errors assessed on omitted stems and intervals for predicted volume. It also includes stems smaller than those in the earlier calibration: the current minima are 11 cm DBH and 6 m height, compared with 19 cm and 17 m in 2014. Application of either equation should respect the stem sizes, forest conditions, and volume definition used to develop it.

Where the new table returns no estimate, the 2014 equation can be considered as an existing regional alternative if the stem is Scots pine under comparable Central Rhodope conditions and both $D = 19$–72 cm and $H = 17$–41 m are satisfied. Being outside the size range of the new table is not, by itself, a sufficient reason to use the equation of Kitikidou et al. [9]. Our additional comparison used the 34 of our 60 sampled stems whose DBH and standing height both fell within the ranges reported in that study: 19–72 cm and 17–41 m, respectively. This comparison does not establish predictive accuracy for larger stems that were absent from our sample. Before using the 2014 equation for such stems, we recommend comparing its predictions with local reference volumes from stems of similar size.

The related studies also differ in their intended outputs. Fallias and Diamantopoulou [10] modelled sectional Scots pine volume through diameter-reduction relationships, whereas Saygili and Kahriman [12] developed a compatible taper–volume system from felled stems and evaluated it on separate stems. Their predictors and validation designs differ from ours, so their reported goodness of fit cannot be directly ranked against our LOOCV results. The present table estimates total stem volume from measured DBH and standing-stem height. It does not estimate volumes for individual logs or their distribution among product categories. Such estimates can be obtained using stem taper models combined with specifications for log lengths and diameters [6].

Pressler with the specified basal cylinder gave useful sample-specific information about a non-destructive alternative. Its observed errors may suit preliminary assessments in comparable Drymos groups, but suitability depends on the inventory purpose and required precision [28]. No acceptance threshold was specified or tested here, so practical reliability cannot be asserted for other stands, species, or measurement teams. The basal-height convention must be stated when reproducing this implementation; the comparison does not show that Pressler's method itself is wrong. Results for other pine species [13] support further evaluation, with local reference checks before operational adoption elsewhere.

4.4. Sampling Categories and Prediction Uncertainty

Adding the inside/outside indicator did not improve prediction in this sample. Its coefficient interval still allowed approximately 10% lower volume for inside stems at the same D^2H, so the result does not establish equivalence. The categories were observational and differed in size distribution; they should not be interpreted as treatments or complete stand classifications. The shared-location checks changed errors and coefficient uncertainty little, but incomplete group and compartment identifiers prevent a complete assessment of spatial dependence. Equal weighting characterizes the available sampled stems and may yield different coefficients or aggregate error metrics under another size or category distribution.

Mean-volume confidence intervals describe uncertainty in the fitted relationship; individual-stem intervals additionally include residual variation among stems with the same dimensions. Their internal coverage of 57/60 reference volumes does not establish nominal coverage in a new stand. The stem bootstrap and coordinate-group comparison are conditional checks on the present sample, not design-based intervals or estimates of variation between forests. Measurement uncertainty in new DBH and height readings is not separately propagated.

For a complete plot census, users may sum stem-volume predictions when the stems meet the calibration conditions, but uncertainty in that total requires a separate calculation. Shared fitted coefficients can move many predictions in the same direction [29]; adding individual interval widths or assuming error cancellation is therefore inappropriate. An equation with the best individual-tree performance need not be best for plot totals [30]. The small aggregate bias in this sample is encouraging for local inventory development but does not establish unbiased plot or regional totals. No independently measured plot totals were available, and any shared basal section of a forked tree requires explicit accounting.

4.5. Application and Further Sampling

The table is intended for naturally established Scots pine groups in Drymos, using over-bark DBH at 1.30 m and standing total height. Even inside the observed DBH–height polygon, sparse nearby observations can limit support, especially near its boundary; the nearby-stem count describes this support and does not certify accuracy. Users should consider both mean and individual uncertainty and match the ground-to-tip volume definition. Different bark or stump conventions and merchantable-volume applications require a corresponding conversion or model.

The study adds a locally checked, felled-stem reference and accessible uncertainty information for forest inventory in Drymos. It does not demonstrate transfer to Kalyva–Margariti, Oraio or other forests, where no eligible stems entered this sample. Broader destructive sampling across independent groups and size classes would strengthen such assessment [18]. The present contribution is therefore a local equation and field table, with documented size limits and uncertainty, rather than a region-wide volume calibration.

5. Conclusions

Standing height materially improved local Scots pine stem-volume prediction over DBH alone. The parsimonious diameter–height equation and the 2014 regional equation both performed well for these measured stems, but the comparison did not establish superior predictive accuracy for the new equation. Adding the harvesting-area category did not improve prediction; this does not establish equivalence between categories.

The contribution is a Drymos calibration based on sectional measurements of felled stems, accompanied by a field table and explicit uncertainty and size limits. The Pressler comparison is specific to the measured sample and stated basal convention.

The equation can support local inventories of naturally established Scots pine groups in Drymos when measurement conventions and the joint DBH–height domain match. Its limited sample, incomplete stand information, and lack of independent forest or plot-total validation constrain wider use.

Supplementary Materials: The following supporting information can be downloaded at: https://www.mdpi.com/article/10.3390/sci8110333/s1: Table S1. Fitted coefficients and 95% coefficient intervals; Table S2. Predictive errors within each sampling category; Table S3. Apparent fit and descriptive resampling intervals for predictive metrics; Table S4. Mean volume (m^3) on the full 2 cm × 2 m field grid; Table S5. Terminal geometry of the corrected reference profiles; Table S6. The five pairs sharing recorded coordinates; Table S7. Comparison of stratified stem and coordinate-group

resampling; Table S8. Influence checks for the two stems of the forked tree; Figure S1. Diagnostics for M2. (a) Normal quantile plot of full-fit log residuals. (b) Cook's distance against DBH; the dashed line is $4/n$ and 97Π is the maximum-influence record. (c) Standing height minus felled ground-to-tip length. Blue and orange distinguish outside and inside sampling categories in panels (b,c). References [9,21,23,26] are also cited in the Supplementary Materials.

Author Contributions: Conceptualization, M.T., E.M. and K.K.; methodology, M.T., E.M. and K.K.; formal analysis, M.T. and K.K.; investigation, M.T. and K.K.; writing—original draft preparation, M.T. and K.K.; writing—review and editing, M.T., E.M. and K.K.; supervision, K.K. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Institutional Review Board Statement: Not applicable. This study does not involve human participants or experimental animals.

Informed Consent Statement: Not applicable.

Data Availability Statement: Summary data and model results are reported in this article and the supplementary document. The latter contains model coefficients, the complete mean-volume grid, endpoint summaries, coincident-stem records, and additional diagnostics and sensitivity results.

Conflicts of Interest: The authors declare no conflicts of interest.

Abbreviations

CI	confidence interval
DBH	diameter at breast height
HC3	heteroskedasticity-consistent covariance estimator
LOOCV	leave-one-out cross-validation
MAE	mean absolute error
MAPE	mean absolute percentage error
PI	prediction interval
RMSE	root mean squared error

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