

Article

Multi-Index Stage-Based Characterization and Residual-Life Estimation of Splitting Fatigue Damage in Asphalt Mixtures

Youyun Li ¹, Ziyu Wei ^{1,*} and Guangyu Sun ^{2,3}

¹ School of Transportation, Changsha University of Science and Technology, Changsha 410114, China

² State Key Laboratory of Electrical Insulation and Power Equipment, School of Electrical Engineering, Xi'an Jiaotong University, Xi'an 710049, China

³ Power and Wide-band-gap Electronics Research Laboratory (POWERlab), Institute of Electrical and Micro Engineering, École Polytechnique Fédérale de Lausanne (EPFL), CH-1015 Lausanne, Switzerland

* Correspondence: 24101030068@csust.edu.cn

Abstract

To overcome the inability of single life-based indices to represent the nonlinear degradation of asphalt-mixture bearing capacity, this study proposes a multi-index framework for splitting-fatigue damage characterization and residual-life estimation. AC-13C mixtures were tested through indirect tensile strength, constant-amplitude splitting-fatigue, and post-fatigue residual-strength tests. The framework integrates the life-based damage index D_f , residual strength ratio λ , performance damage index D_s , damage lag coefficient L , and peak displacement response, thereby distinguishing life consumption, strength loss, deformation amplification, and their asynchronous evolution. A constrained three-stage residual-strength decay model was further established to identify damage position from measured residual strength and estimate the residual-life ratio, providing greater interpretability and predictive utility for damage-position inversion and residual-life estimation than single-index or single-stage approaches. Under $S = 0.3$, 15°C , and 10 Hz , damage stages were preliminarily bounded by $D_1 = 0.50$ and $D_2 = 0.65$. From $D_f = 0.20$ to 0.80 , λ decreased from 0.928 to 0.380 , while peak displacement increased from 3.076 to 4.788 mm ; the late-stage attenuation and deformation rates were 4.23 and 4.44 times those in the middle stage, respectively. The model achieved $R^2 = 0.9942$, $\text{RMSE} = 0.0158$, and $\text{MAPE} = 2.06\%$, demonstrating reliable damage characterization and residual-life inversion.

Keywords: asphalt mixture; fatigue damage; residual strength; multi-index characterization; residual-life estimation



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1. Introduction

Fatigue cracking is one of the principal deterioration mechanisms in asphalt pavements and develops through the initiation, propagation, and coalescence of microcracks under repeated traffic loading and environmental actions. The evolution of fatigue damage is affected by stress level, temperature, loading frequency, stress state, mixture properties, viscoelastic response, and internal structural heterogeneity [1–4]. As damage accumulates, the effective load-bearing area decreases, and load transfer within the pavement structure becomes progressively less efficient, which may accelerate crack propagation and secondary distresses. Therefore, a reliable fatigue assessment should characterize not only the consumed fatigue life but also the associated loss of bearing capacity and deformation resistance. This issue is particularly important for residual-life estimation and

maintenance decision-making, because the same life fraction may correspond to different levels of mechanical degradation.

Existing fatigue evaluation methods for asphalt mixtures mainly include S–N relationships, stiffness degradation indices, energy-based approaches, fracture mechanics, and viscoelastic continuum damage models [5–7]. These methods provide important tools for describing fatigue resistance and damage evolution. Energy-based methods and continuum damage models are particularly useful for mechanistic interpretation and prediction under different loading conditions [8–12]. However, their parameter identification procedures may be relatively complex, and their engineering application often depends on model assumptions and material-specific calibration. More importantly, a single life ratio, stiffness parameter, or model variable cannot fully represent the simultaneous evolution of fatigue life consumption, bearing-capacity degradation, and deformation response.

To obtain additional mechanical evidence of fatigue damage, recent studies have increasingly employed residual-strength measurements, mesoscopic imaging, acoustic emission monitoring, residual-strain analysis, and damage-based contact models [13–18]. These approaches have improved the understanding of crack initiation, crack propagation, internal structural deterioration, and macroscopic performance loss. Nevertheless, most existing studies focus on a single observation scale or a single response variable. Consequently, it remains difficult to establish a unified quantitative relationship among life progression, residual strength, deformation development, and the remaining fatigue life of asphalt mixtures.

Splitting fatigue testing is useful for investigating asphalt-mixture damage because it produces a combined tensile–compressive stress state under relatively simple loading conditions. However, fatigue damage does not generally accumulate linearly with the number of loading cycles. In the early stage, aggregate interlocking, viscoelastic adjustment of the asphalt mortar, and redistribution of local contacts may allow the specimen to retain a relatively high bearing capacity. During stable damage accumulation, microcracks gradually connect and strength degradation becomes more evident. Once a dominant crack forms, the effective load-bearing area decreases rapidly, peak displacement increases, and accelerated damage may occur. Residual-strength and residual-strain studies have provided useful evidence for describing these nonlinear damage characteristics [19–21]. However, a quantitative framework that simultaneously identifies the damage stage, measures the asynchrony between life progression and performance degradation, and estimates residual life from residual strength is still lacking.

Taken together, previous studies suggest that fatigue-life progression and mechanical performance degradation in asphalt mixtures may not be synchronized. A specimen may retain considerable residual strength during the early and middle stages of fatigue, followed by rapid strength loss and deformation growth when dominant cracking develops. Therefore, the fatigue state cannot be adequately evaluated using the life ratio alone. This study hypothesizes that the coordinated evolution of residual strength and deformation response can better identify fatigue damage stages and that residual strength can be used to estimate the current damage position and remaining life. However, an integrated framework linking life progression, strength degradation, deformation response, damage-stage identification, and residual-life estimation is still lacking. This constitutes the main research gap addressed in this study.

To address the above research gap, this study develops an integrated framework for splitting-fatigue damage characterization and residual-life estimation of AC-13C asphalt mixtures. Indirect tensile strength and constant-amplitude splitting-fatigue tests are first conducted to determine the reference strength and fatigue-life relationship. Post-fatigue residual-strength tests are then performed at selected life fractions. The measured strength

and deformation responses are used to calculate D_f , λ , D_s , L , and peak displacement, based on which the transitions among slow, stable, and accelerated damage stages are identified. Finally, a constrained three-stage residual-strength decay model is established to determine the current damage position and estimate the residual life through model inversion. Thus, the proposed technical route links material characterization, fatigue-life calibration, multi-index damage analysis, stage identification, and residual-life estimation in a unified framework.

2. Experimental Program and Identification Method for Splitting Fatigue Damage Parameters

2.1. Indirect Tensile Strength Test

The test material was an AC-13C asphalt mixture. Standard Marshall specimens with dimensions of $\Phi 101.6 \text{ mm} \times 63.5 \text{ mm}$ were used. All specimens were prepared using a consistent compaction procedure, and the gradation, asphalt content, and air voids were controlled within the same target ranges to reduce the influence of material variability on damage parameter identification. During testing, the temperature was maintained at 15°C and a sinusoidal loading waveform was adopted. A 70# base asphalt binder was used, and the coarse and fine aggregates were limestone aggregates. Limestone mineral powder was used as the mineral filler. The mixture gradation was designed according to the Technical Specifications for Construction of Highway Asphalt Pavements (JTG F40—2004), and the optimum asphalt content was determined to be 5.1% by total mixture mass, corresponding to an asphalt-aggregate ratio of 5.37%. Standard Marshall specimens with a diameter of 101.6 mm and a height of 63.5 mm were prepared using the Marshall compaction method. Each specimen was compacted with 75 blows on each side, and the target air-void content was controlled within the design range required for AC-13C mixtures, as shown in Table 1.

Table 1. Aggregate gradation of the AC-13C asphalt mixture.

Aggregate Gradation	Percentage by Mass Passing the Following Square Sieve Openings (Sieve Opening in mm; Passing Percentage in %)										Reference Asphalt Content (%)
	16	13.2	9.5	4.75	2.36	1.18	0.6	0.3	0.15	0.075	
Upper specification limit	100	100	85	68	50	38	28	20	15	8	4.5~6.5
Lower specification limit	100	90	68	38	24	15	10	7	5	4	
Specification midpoint	100	95	76.5	53	37	26.5	19	13.5	10	6	
Actual value	100	94	72	51	36	28	19	13	9	6	5.1 *

* 5.1: denotes the asphalt content by total mixture mass. The selected asphalt content was 5.1%, corresponding to an asphalt-aggregate ratio of 5.37%.

The indirect tensile strength test was conducted as schematically shown in Figure 1. The UTM hydraulic servo testing system was used to record the peak load P_T during diametral compression. The indirect tensile strength of the unfatigued reference specimen, denoted by R_T , was subsequently calculated from P_T using Equation (1). The mean, standard deviation, and coefficient of variation of the parallel specimens were then calculated.

The peak load and corresponding vertical deformation during the splitting test were recorded by the data acquisition system. Because standard Marshall specimens were used, the indirect tensile strength was calculated as follows $R_T = 0.006287P_T/h$, where P_T is the load applied by the loading strip in N, and h is the specimen height in mm. The results are listed in Table 2.

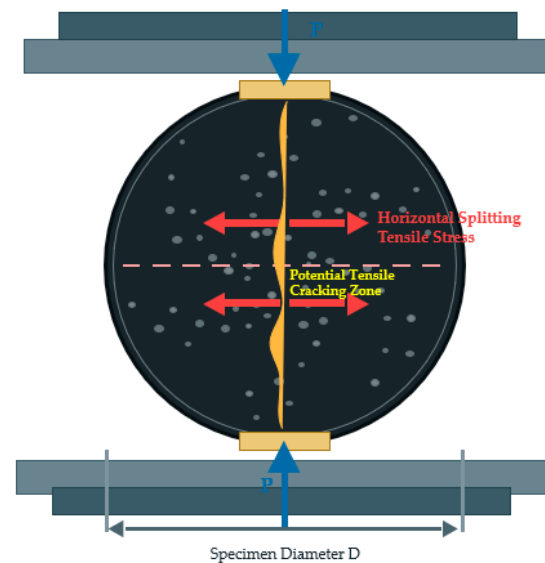


Figure 1. Schematic illustration of the experimental methodology.

Table 2. Indirect tensile strength of AC-13C asphalt mixture.

Asphalt Mixture Type		Sample					
	Specimen	Specimen 1	Specimen 2	Specimen 3	Mean	Standard deviation (S)	Coefficient of variation (CV)
AC-13C	Maximum UTM load ((kN))	11.85	10.99	11.35	11.40	0.432	3.79%
	Indirect tensile strength R_T (MPa)	1.173	1.088	1.124	1.128	0.043	3.78%

2.2. Determination of Splitting Fatigue Life of Asphalt Mixture

Asphalt mixtures undergo significant fatigue damage accumulation under repeated traffic loading. Under specified stress- or strain-controlled conditions, the number of loading cycles experienced by a specimen when it reaches the predefined fatigue failure criterion is generally defined as fatigue life. Fatigue life reflects not only the cracking resistance of the material but also its durability under coupled loading and environmental actions.

A preliminary fatigue life test was conducted on AC-13C asphalt mixture at stress levels of 0.2, 0.3, 0.4, and 0.5. A sinusoidal loading waveform with a frequency of 10 Hz was used. The measured fatigue lives are listed in Table 3.

The splitting fatigue tests were conducted at 15 °C under stress-controlled sinusoidal loading at a frequency of 10 Hz. During testing, the UTM system continuously recorded the applied load and vertical deformation. Fatigue failure was defined as the first loading cycle at which a continuous crack formed along the loading diameter and the specimen could no longer sustain the prescribed peak cyclic load, accompanied by a rapid increase in vertical deformation. The corresponding number of loading cycles was defined as the fatigue life N_f . Three independent specimens were tested at each stress level, and the mean value was used for the subsequent determination of fatigue-life fractions.

A log-log regression method was used to establish the empirical relationship between stress ratio and fatigue life. The S-N fatigue life equation of the asphalt mixture was obtained and used as a unified life benchmark for subsequent calculations and life-fraction settings. The S-N fatigue life equation adopted for the stress-controlled splitting fatigue test is as follows:

$$\lg N_f = A - B \lg S \quad (1)$$

where N_f is the number of loading cycles at specimen failure; S is the ratio of fatigue loading stress to indirect tensile strength; A represents the fatigue life level of the material under the reference stress condition; and B reflects the sensitivity of fatigue life to changes in stress ratio. A larger B indicates greater sensitivity to the loading level.

Table 3. Fatigue life of AC-13C asphalt mixture under different stress levels.

Stress Level	Fatigue Load (kN)	Sample	Fatigue Life (Cycles)	Mean Fatigue Life (Cycles)
0.2	2.28	Specimen 1	32,137	30,938
		Specimen 2	26,845	
		Specimen 3	33,831	
0.3	3.42	Specimen 4	8731	8341
		Specimen 5	7940	
		Specimen 6	8352	
0.4	4.56	Specimen 7	2089	1887
		Specimen 8	1902	
		Specimen 9	1671	
0.5	5.70	Specimen 10	1293	1086
		Specimen 11	1103	
		Specimen 12	862	

Based on the test results in Table 2, the regression results fitted using the S-N fatigue equation are shown in Table 4.

Table 4. Fitted results of fatigue life.

A	B	Regression Equation	Coefficient of Determinatio R^2 (%)
1.85	3.80	$\lg N_f = 1.85 - 3.80 \lg S$	98.72

2.3. Residual Splitting Strength Test of AC-13C Asphalt Mixture

2.3.1. Classification of Fatigue Damage Levels

According to Miner’s linear cumulative damage theory [22], under single-level constant-amplitude loading, each loading cycle is assumed to cause the same damage increment. The life damage increment corresponding to a single cycle is $1/N_f$. After a specimen has experienced n loading cycles, its cumulative life damage can be expressed as:

$$D_f = \sum (1/N_f) = n/N_f \quad (2)$$

where D_f is the life-based damage index, n is the number of applied loading cycles, and N_f is the number of cycles at fatigue failure. In this study, Miner’s theory is introduced not to assume a strictly linear relationship between fatigue damage and the number of cycles in asphalt mixtures, but to use $D_f = n/N_f$ as a normalized life-progress index for determining fatigue loading termination points and comparing different damage stages on a common basis. Actual performance degradation is further characterized by the residual strength ratio λ , performance damage index D_s , and damage lag coefficient L .

2.3.2. Fatigue Residual Strength Test Results

To determine the damage levels according to Miner’s linear damage theory, fatigue loading was terminated at $0.20N_f$, $0.50N_f$, $0.65N_f$, and $0.80N_f$, and the corresponding residual splitting strength, denoted as R_{Tr} , was measured. Based on the preliminary fatigue-life results in Table 2, a stress level of 0.3 was selected as a representative intermediate loading condition for the residual-strength tests. At a stress level of 0.2, the mean fatigue life

reached 30,938 cycles, resulting in a substantially longer testing duration. In contrast, stress levels of 0.4 and 0.5 produced mean fatigue lives of only 1887 and 1086 cycles, respectively, which could lead to rapid damage development and reduce the resolution of the prescribed fatigue stages. The stress level of 0.3 provided a moderate damage evolution. Because the residual splitting-strength test is destructive, different replicate specimens were used at each fatigue damage level. Five independent specimens were tested at each prescribed life fraction. Therefore, the residual-strength database contained 20 independent residual-strength specimens in total. The residual strength parameters obtained from multiple specimens during the fatigue damage process of the AC-13C splitting test are shown in Table 5.

Table 5. Residual strength parameters in the AC-13C splitting test under fatigue damage.

Specimen	Splitting Test Parameter	Life-Based Damage Index D_f			
		0.20	0.50	0.65	0.80
Sample 1	Peak load P_{Tr}	10.958	9.575	8.627	4.258
	Residual splitting strength R_{Tr}	1.085	0.948	0.854	0.422
	Vertical displacement at peak load	2.98	3.16	3.36	4.64
Sample 2	Peak load P_{Tr}	10.216	9.309	8.304	4.519
	Residual splitting strength R_{Tr}	1.011	0.922	0.822	0.447
	Vertical displacement at peak load	3.06	3.29	3.64	4.87
Sample 3	Peak load P_{Tr}	10.553	9.184	8.292	4.116
	Residual splitting strength R_{Tr}	1.045	0.909	0.821	0.408
	Vertical displacement at peak load	3.22	3.37	3.58	4.81
Sample 4	Peak load P_{Tr}	10.704	9.446	8.515	4.238
	Residual splitting strength R_{Tr}	1.060	0.935	0.843	0.420
	Vertical displacement at peak load	3.12	3.34	3.50	4.70
Sample 5	Peak load P_{Tr}	10.432	9.251	8.218	4.486
	Residual splitting strength R_{Tr}	1.033	0.916	0.814	0.444
	Vertical displacement at peak load	3.00	3.22	3.69	4.92
Mean test parameter	Peak load P_{Tr}	10.573	9.353	8.391	4.323
	Residual splitting strength R_{Tr}	1.047	0.926	0.831	0.428
	Vertical displacement at peak load	3.076	3.276	3.554	4.788

Note: P_{Tr} is expressed in kN, R_{Tr} is expressed in MPa, and the vertical displacement at peak load is expressed in mm.

2.4. Multi-Index Damage Characterization and Theoretical Framework

2.4.1. Definition of Basic Damage Indices

To distinguish fatigue progression from performance degradation, the residual strength ratio is defined as:

$$\lambda = R_{Tr}/R_T \quad (3)$$

The life-progression-based damage index is defined as:

$$D_f = n/N_f \quad (4)$$

The performance damage index based on macroscopic mechanical degradation is defined as:

$$D_s = 1 - \lambda = 1 - R_{Tr}/R_T \quad (5)$$

where D_f denotes the life progression index based on the number of loading cycles. The residual strength ratio λ and the performance damage index D_s are calculated from the residual splitting strength after fatigue loading and mainly reflect the degradation of the material's load-bearing capacity; therefore, they are used as strength degradation indices.

Together, they describe fatigue damage from the perspectives of life consumption and performance loss.

To describe the asynchrony between life progression and performance degradation, the damage lag coefficient is further defined as:

$$L = D_s/D_f \quad (6)$$

This coefficient quantitatively characterizes the synchronization difference between fatigue life progression and mechanical performance degradation. It reveals whether performance damage lags behind, remains synchronized with, or advances ahead of the life process, thereby addressing the limitation that D_f alone cannot accurately represent the actual damage state. When $L < 1$, performance degradation lags behind life consumption; when $L \approx 1$, performance degradation is generally synchronized with fatigue progression; and when $L > 1$, the rate of strength loss exceeds the apparent growth rate of life progression, indicating that the material may have entered an accelerated damage or instability stage. $L \neq 1$ indicates that the linear life-ratio damage index cannot directly reflect the degree of material performance degradation. In this study, L is used to quantify this asynchrony and thereby compensate for the limitations of using Miner's life damage index alone.

To clarify the information structure of the proposed framework, λ , D_f , and X_i are treated as the primary quantities, whereas D_s and L are derived auxiliary indices. Specifically, $D_s = 1 - \lambda$ is the complementary representation of strength degradation, and $L = D_s/D_f$ is a normalized indicator of the relationship between performance degradation and life progression. Therefore, D_s and L are not statistically independent variables. They are retained to provide more interpretable descriptions of strength loss and damage lag.

2.4.2. Three-Stage Damage Identification Method

Splitting fatigue damage in asphalt mixtures is not a linear development process [23,24]. Based on the slope of the λ - D_f curve, the growth rate of the D_s - D_f curve, and the evolution characteristics of L , the damage process was preliminarily divided into three stages for the present dataset. This interpretation applies to the AC-13C asphalt mixture tested at $S = 0.3$, 15°C , and 10 Hz . In the early stage, λ remains at a high level while D_s and L are low. In the middle stage, λ decreases steadily, D_s continues to increase, and L gradually approaches 1; In the late stage, λ decreases rapidly, D_s and L increase significantly, and strength degradation accelerates. The preliminary reference boundaries are denoted as D_1 and D_2 , where D_1 represents the transition reference point from initial slow damage to stable accumulation, and D_2 represents the transition reference point from stable accumulation to rapid instability. To provide a consistent operational basis for stage identification, the rates of change of multiple indices within adjacent life-fraction intervals were compared. The attenuation rate of the residual strength ratio, the growth rate of the performance damage index, the rate of change of the damage lag coefficient, and the growth rate of peak displacement were calculated. When the rates of change of multiple indices increase simultaneously within an adjacent interval, and both strength degradation and displacement growth show clear acceleration, that interval is considered to correspond to a damage-stage transition, and the endpoint of the adjacent interval is taken as the reference boundary. Because only four damage levels ($D_f = 0.20, 0.50, 0.65$, and 0.80) were available, this comparison included only three adjacent intervals. Therefore, the identified transitions should be regarded as preliminary descriptive results rather than formal mathematical inflection-point detection.

2.4.3. Residual Strength Decay Model and Life-Damage Inversion Method

To further transform damage characterization into engineering prediction, a degradation model can be established between the residual strength ratio λ and the life-based damage index D_f . Because splitting fatigue damage has stage-dependent characteristics,

a three-stage piecewise model is adopted in this study to describe the evolution of the residual strength ratio with D_f .

$$\text{When } 0 \leq D_f \leq D_1 : \lambda(D_f) = 1 - aD_f;$$

$$\text{When } D_1 < D_f \leq D_2 : \lambda(D_f) = \lambda_1 - b(D_f - D_1)^n; \quad (7)$$

$$\text{When } D_2 < D_f \leq 1 : \lambda(D_f) = \lambda_2 e^{[-c(D_f - D_2)]};$$

where $\lambda_1 = 1 - aD_1$, $\lambda_2 = \lambda_1 - b(D_2 - D_1)^n$; a , b , and c are parameters to be identified. Because only one measured damage level, $D_f = 0.80$, was available beyond the second-stage boundary $D_2 = 0.65$, the functional form of the late accelerated-damage stage cannot be independently verified using the present dataset. The exponent n was therefore fixed at 1.0, and the parameter c was estimated subject to the continuity and monotonicity constraints. $D_1 = 0.50$ and $D_2 = 0.65$ were adopted as preliminary reference boundaries based on the multi-index damage analysis. The fitting constraints were $a > 0$, $b > 0$, $c > 0$ and $\lambda(D_f)$ was required to decrease monotonically over the entire fatigue-life interval.

After the residual splitting strength R_{Tr} is obtained, the residual strength ratio is first calculated $\lambda = R_{Tr}/R_T$, and the fitted $\lambda(D_f)$ relationship is then used to back-calculate the current life damage position D_f^* . To characterize the proportion of residual life relative to the total fatigue life of the specimen, the residual life ratio is defined as:

$$R_{LF} = 1 - D_f^* \quad (8)$$

If the fatigue life N_f at the corresponding stress level is known, the number of remaining cycles can be estimated as:

$$N_{rem} = (1 - D_f^*)N_f \quad (9)$$

This procedure enables quantitative inversion from the residual strength state to the residual life ratio. Unlike simple curve fitting, it improves the practical value of the model for residual-life estimation.

The parameters of the three-stage residual strength decay model were identified using constrained nonlinear least squares, with the following objective function:

$$J = \sum W_k [\lambda_{exp,k} - \lambda_{pre,k}]^2 \quad (10)$$

where $\lambda_{exp,k}$ is the measured residual strength ratio, $\lambda_{pre,k}$ is the model-predicted value, and w_k is the weighting coefficient. Model fitting performance was comprehensively evaluated using the coefficient of determination R^2 , root mean square error (RMSE), mean absolute percentage error (MAPE) and residual distribution.

2.4.4. Definition of the Peak Displacement Response Index

In the residual splitting strength test, the vertical displacement U_{tr} corresponding to the peak load characterizes the deformation development before the specimen reaches its ultimate bearing state, reflecting crack opening, skeleton rearrangement, and local damage propagation. Peak displacement can reveal the fatigue damage accumulation process from the perspective of deformation response and can therefore serve as an important supplementary index for strength degradation analysis.

To compare deformation responses at different fatigue damage stages and reduce the influence of displacement dimension and specimen variability, a normalized displacement index was introduced. Because no unfatigued reference specimen was included in this study, the mean peak displacement U_{20} at $D_f = 0.20$ under the same stress level was used as a relative reference, and the relative displacement ratio was defined as:

$$X_i = U_{tr}/U_{20} \quad (11)$$

A larger X_i indicates more pronounced deformation accumulation and crack opening before the specimen reaches the ultimate load. Joint analysis of the evolution relationships among λ , D_s , L , and X_i can be used to evaluate the coordination between bearing capacity degradation and deformation development, and to provide deformation-response evidence for identifying the transition from stable accumulation to late-stage accelerated damage.

2.5. Construction of the Test Database

The values of λ , D_f , D_s and L were calculated consistently, and a test database was constructed for stage classification and model regression, as shown in Table 6:

Table 6. Test database.

S_i	Sample	Fatigue Damage Level	N_f	n^*	R_T	R_{Tr}	λ	D_f	D_s	L	Vertical Displacement at Peak Load
0.3	Sample 1	0.20	8341	1668	1.128	1.085	0.962	0.200	0.038	0.191	2.980
0.3	Sample 1	0.50	8341	4171	1.128	0.948	0.840	0.500	0.160	0.319	3.160
0.3	Sample 1	0.65	8341	5422	1.128	0.854	0.757	0.650	0.243	0.374	3.360
0.3	Sample 1	0.80	8341	6673	1.128	0.422	0.374	0.800	0.626	0.782	4.640
0.3	Sample 2	0.20	8341	1668	1.128	1.011	0.896	0.200	0.104	0.519	3.060
0.3	Sample 2	0.50	8341	4171	1.128	0.922	0.817	0.500	0.183	0.365	3.290
0.3	Sample 2	0.65	8341	5422	1.128	0.822	0.729	0.650	0.271	0.417	3.640
0.3	Sample 2	0.80	8341	6673	1.128	0.447	0.396	0.800	0.604	0.755	4.870
0.3	Sample 3	0.20	8341	1668	1.128	1.045	0.926	0.200	0.074	0.368	3.220
0.3	Sample 3	0.50	8341	4171	1.128	0.909	0.806	0.500	0.194	0.388	3.370
0.3	Sample 3	0.65	8341	5422	1.128	0.821	0.728	0.650	0.272	0.419	3.580
0.3	Sample 3	0.80	8341	6673	1.128	0.408	0.362	0.800	0.638	0.798	4.810
0.3	Sample 4	0.20	8341	1668	1.128	1.060	0.940	0.200	0.060	0.301	3.120
0.3	Sample 4	0.50	8341	4171	1.128	0.935	0.829	0.500	0.171	0.342	3.340
0.3	Sample 4	0.65	8341	5422	1.128	0.843	0.747	0.650	0.253	0.389	3.500
0.3	Sample 4	0.80	8341	6673	1.128	0.420	0.372	0.800	0.628	0.785	4.700
0.3	Sample 5	0.20	8341	1668	1.128	1.033	0.916	0.200	0.084	0.421	3.000
0.3	Sample 5	0.50	8341	4171	1.128	0.916	0.812	0.500	0.188	0.376	3.220
0.3	Sample 5	0.65	8341	5422	1.128	0.814	0.722	0.650	0.278	0.428	3.690
0.3	Sample 5	0.80	8341	6673	1.128	0.444	0.394	0.800	0.606	0.758	4.920
0.3	Mean	0.20	8341	1668	1.128	1.047	0.928	0.200	0.072	0.360	3.076
0.3	Mean	0.50	8341	4171	1.128	0.926	0.821	0.500	0.179	0.358	3.276
0.3	Mean	0.65	8341	5422	1.128	0.831	0.737	0.650	0.263	0.405	3.554
0.3	Mean	0.80	8341	6673	1.128	0.428	0.380	0.800	0.620	0.775	4.788

* $n = \text{round}(N_f D_f)$, where $N_f = 8341$ cycles is the mean fatigue life at $S = 0.3$.

3. Results and Discussion

3.1. Multi-Index Damage Evolution of Life and Performance

Based on the regression database, the λ - D_f , D_s - D_f and L - D_f curves were plotted, as shown in Figure 2:

In the figure, the measured values of parallel specimens are shown as scatter points, and the mean values at each life fraction are shown as line plots. From the variation amplitude of the λ - D_f curve, λ decreases relatively slowly from $D_f = 0.20$ to $D_f = 0.65$. When D_f increases from 0.65 to 0.80, λ rapidly decreases from 0.737 to 0.380, indicating a markedly larger reduction and an accelerated decline in bearing capacity during the late fatigue stage.

Corresponding to the attenuation of λ , D_s continuously increases with D_f , showing a trend consistent with the λ - D_f curve. In the early and middle stages, D_s increases slowly, indicating that although the specimen has experienced a certain proportion of fatigue cycles, the macroscopic strength loss remains relatively limited. When D_f increases to 0.80, D_s increases significantly, implying that fatigue damage gradually shifts from stable accumulation to rapid development. This difference indicates that D_f alone cannot fully

reflect the actual degradation of material bearing capacity, whereas D_s supplements the damage characterization from the perspective of performance loss.

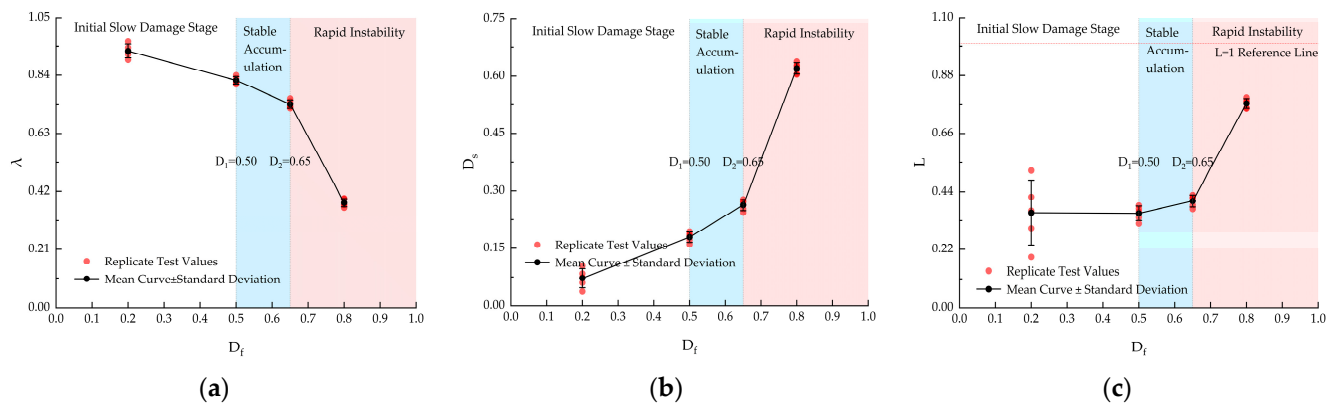


Figure 2. Evolution relationships between multi-index damage parameters and D_f . (a) λ - D_f curve. (b) D_s - D_f curve. (c) L - D_f curve.

The L - D_f curve shows that the damage lag coefficient L generally increases with D_f . Overall, all L values in this test dataset are less than 1, indicating that performance degradation lags behind life progression within the selected fatigue termination points. In other words, although the specimen has consumed a certain proportion of fatigue life, the residual strength does not decrease by the same proportion. However, when D_f increases from 0.65 to 0.80, L rapidly increases from 0.405 to 0.775, indicating that the lag of performance degradation relative to life progression is substantially reduced, strength degradation begins to accelerate, and the internal damage state changes significantly.

Based on the variation characteristics of λ , D_s , L , and peak displacement response at the four life fractions, the splitting fatigue damage process was preliminarily divided into stages, as shown in Table 7.

Table 7. Boundary identification results for the three stages of splitting fatigue damage.

Stage	D_f Range	Main Characteristics
Initial slow-damage stage	$D_f \leq 0.50$	λ remains high; D_s and L are low; macroscopic bearing capacity degradation is limited
Middle stable-accumulation stage	$0.50 < D_f \leq 0.65$	λ decreases steadily; D_s continues to increase; L increases slowly
Late rapid-instability stage	$D_f > 0.65$	λ decreases rapidly; D_s and L increase significantly; strength degradation accelerates

From the rates of change in adjacent intervals, the λ attenuation rate is small and peak displacement increases slowly in the $D_f = 0.20 \sim 0.50$ interval, indicating that this stage is dominated by slow damage. In the $D_f = 0.50 \sim 0.65$ interval, the rates of change in λ and D_s increase and L begins to rise, indicating that damage enters the stable accumulation stage. When $D_f = 0.65 \sim 0.80$, the λ attenuation rate and peak displacement growth rate increase markedly, reaching approximately 4.23 and 4.44 times those in the middle stage, respectively, indicating that the material enters a late accelerated damage stage.

Therefore, $D_1 = 0.50$ and $D_2 = 0.65$ are proposed as preliminary reference boundaries for the AC-13C mixture tested at $S = 0.3$, 15°C , and 10 Hz . These values should not be directly extrapolated to other stress levels. D_1 represents the transition from initial slow damage to stable accumulation, whereas D_2 represents the transition from stable accumulation to late-stage accelerated damage. Because the number of life fractions is

limited, D_1 and D_2 should not be interpreted as strict mathematical inflection points; their boundary positions still require further verification using denser damage fractions and additional test conditions.

3.2. Damage Position Inversion and Residual Life Estimation Based on Residual Strength

To evaluate the characterization and inversion capability of the model for the existing test data, the measured residual strength ratio $\lambda_{exp,k}$ was compared with the model-predicted value $\lambda_{pre,k}$. The results are shown in Tables 8 and 9.

Table 8. Model parameters for the three-stage residual strength decay model.

D_1	D_2	λ_1	λ_2	a	b	c	n
0.500	0.650	0.821	0.737	0.358	0.563	4.419	1.000

Table 9. Fitting accuracy evaluation of the three-stage degradation model.

R^2	RMSE	MAPE	Residual Range
0.9942	0.0158	2.06%	−0.032~0.034

The model accuracy evaluation indicates that the established λ - D_f three-stage degradation model effectively characterizes the evolution of the residual strength ratio. The coefficient of determination R^2 indicates that the model explains approximately 99.42% of the variation in the residual strength ratio, demonstrating a high goodness of fit. The RMSE indicates a small average deviation between the predicted and measured values, and the MAPE indicates a low overall relative error. The consistency evaluation of the three-stage decay model is shown in Figure 3.

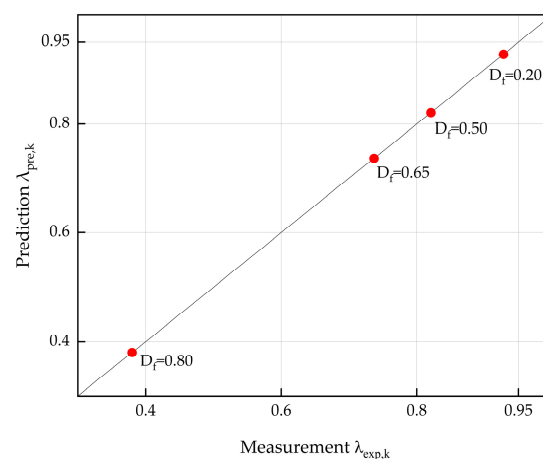


Figure 3. Consistency evaluation of the three-stage residual strength decay model.

Overall, the measured $\lambda_{exp,k}$ values agree well with the model-calculated $\lambda_{pre,k}$ values; the residual range is small, and no obvious systematic bias is observed. This indicates that the three-stage degradation model can accurately describe the attenuation of the residual strength ratio with the life damage position D_f^* , and can be used for subsequent damage position inversion and residual life estimation based on residual splitting strength.

Based on the three-stage residual strength decay model, the measured residual strength ratio $\lambda_{exp,k}$ can be inverted to obtain the current life damage position D_f^* , and the residual life ratio R_{LF} and remaining number of cycles N_{rem} can then be calculated. The residual life results obtained through model inversion are listed in Table 10.

Table 10. Results of residual life prediction.

N_f	D_f	R_{LF}	N_{rem}
8341	0.200	0.800	6673
8341	0.500	0.500	4170
8341	0.650	0.350	2919
8341	0.800	0.200	1668

Under the tested condition of the AC-13C asphalt mixture at $S = 0.3$, $15\text{ }^{\circ}\text{C}$, and 10 Hz , these results indicate that the residual strength ratio can be used to characterize the damage position on the life scale and estimate the residual-life ratio when the fatigue-life benchmark N_f is given. The proposed method realizes a quantitative conversion from strength degradation state to life damage position, providing a reference for residual serviceability evaluation. The residual life ratio also offers a more intuitive basis for evaluating material fatigue resistance and supporting pavement maintenance decisions.

3.3. Coupling Between Strength Degradation and Peak Displacement Response

The vertical displacement at peak load is an important response variable in the residual splitting strength test, reflecting crack opening, skeleton rearrangement, and deformation compatibility. Compared with the strength degradation indices λ and D_s , peak displacement is more sensitive to crack coalescence and local instability in the late fatigue stage. Introducing U_{tr} into damage analysis enables cross-validation of splitting fatigue damage from the perspectives of bearing capacity degradation and deformation development.

The test results show that as D_f increases from 0.200 to 0.800, λ decreases from 0.928 to 0.380, D_s increases from 0.072 to 0.620, and U_{tr} increases from 3.076 to 4.788. Using the mean peak displacement U_{20} at $D_f = 0.200$ as the reference, $X_i = U_{tr}/U_{20}$ the relative displacement ratios at $D_f = 0.500$, 0.650, and 0.800 are 1.065, 1.155, and 1.557, respectively. Thus, displacement increases slowly in the early and middle stages, whereas the deformation response is significantly amplified in the late stage, indicating that the controlling mechanism gradually shifts from dispersed microcracking to dominant crack propagation.

The staged rates further reveal this transition. In the three intervals $D_f = 0.200\sim 0.500$, $0.500\sim 0.650$, and $0.650\sim 0.800$, the λ attenuation rates are 0.357, 0.563, and 2.379, respectively, and the peak displacement growth rates are 0.667, 1.853, and 8.227, respectively. The late-stage strength attenuation rate is approximately 4.23 times that of the middle stage, and the displacement growth rate is approximately 4.44 times that of the middle stage. This indicates that after D_f exceeds D_2 , bearing capacity loss and deformation development accelerate synchronously.

The correlation analysis supports the above interpretation. Using the 20 specimen-level observations in Table 6, Pearson correlation analysis showed a strong negative correlation between λ and U_{tr} ($r = -0.985$, $p < 0.001$), a strong positive correlation between D_s and U_{tr} ($r = 0.985$, $p < 0.001$), and a strong positive correlation between L and U_{tr} ($r = 0.933$, $p < 0.001$). The corresponding linear-fit R^2 values were 0.971, 0.971, and 0.871, respectively. Because the same specimens were measured at multiple damage levels, these results are interpreted as strong associations within the present dataset under the tested conditions of $S = 0.3$, $15\text{ }^{\circ}\text{C}$, and 10 Hz . The coupled relationship between strength degradation and peak displacement response is shown in Figure 4. These results indicate that peak displacement not only characterizes deformation accumulation but also serves as an auxiliary index for identifying fatigue damage stage transitions. In particular, near D_2 , the synchronous variations of λ , D_s , L , and X_i are relatively clear, providing mechanical evidence for determining the late accelerated damage stage.

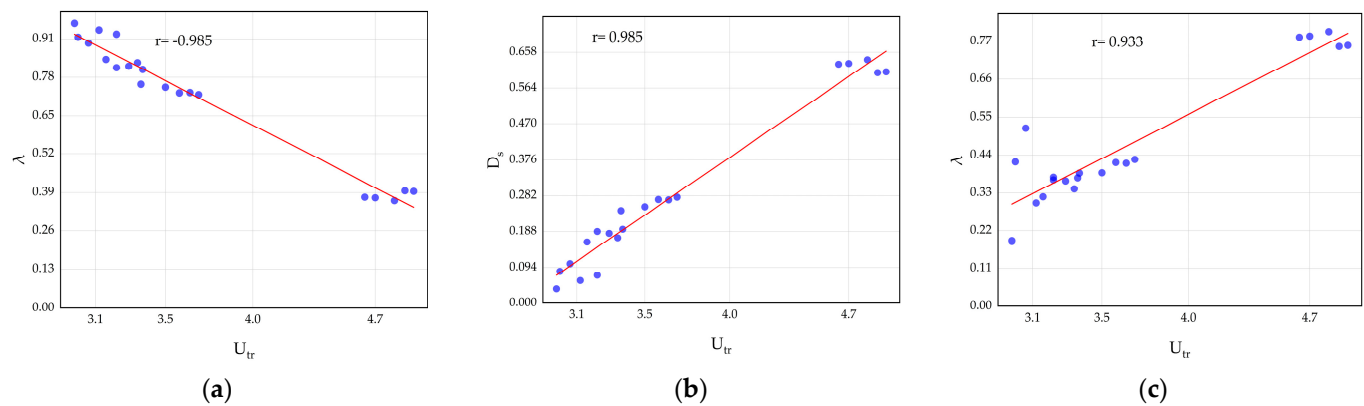


Figure 4. Coupled relationship between strength degradation and peak displacement response. (a) λ - U_{tr} . (b) D_s - U_{tr} . (c) L - U_{tr} .

Overall, the residual strength ratio λ , performance damage index D_s , damage lag coefficient L , and peak displacement response constitute a complementary multi-index characterization system. The indices λ and D_s reflect bearing capacity degradation, L reflects the relative relationship between life progression and performance degradation, and X_i reflects crack opening and deformation amplification. The coordinated results of multiple indices show that splitting fatigue damage in AC-13C asphalt mixture has clear stage-dependent characteristics: the early stage is dominated by defect adjustment and local microcrack initiation; the middle stage is dominated by stable crack propagation and damage accumulation; and the late stage is governed by rapid propagation of a dominant crack, weakening of the load-bearing skeleton, and deformation localization. Therefore, evaluating damage state solely according to the life ratio may overlook the risk of abrupt performance change in the late accelerated damage stage. The high correlations between peak displacement response and λ , D_s , and L further indicate that deformation indices can serve as supplementary evidence for residual strength evaluation and for identifying the critical transition from stable accumulation to rapid instability.

3.4. Limitations and Future Work

Despite these findings, several limitations of the present study should be acknowledged. Although the relatively low coefficients of variation indicate acceptable repeatability under the present test conditions, the limited number of independent specimens restricts the statistical power and external generalizability of the proposed model. The identified stage boundaries and model parameters should consequently be regarded as preliminary and condition-specific. Future studies should employ larger independent datasets, denser fatigue-life fractions, and different mixtures, stress levels, temperatures, and loading conditions to validate and improve the proposed method.

Regarding aggregate gradation, the present study was limited to the AC-13C gradation. The proposed framework is conceptually applicable to other asphalt-mixture gradations because the definitions of D_f , λ , D_s , L , and peak displacement are based on normalized life, residual strength, and deformation responses. However, different gradations may exhibit different aggregate interlocking, air-void structures, stiffness characteristics, and crack-propagation patterns. Therefore, the stage boundaries, residual-strength decay parameters, and model coefficients identified in this study should not be directly transferred to other gradations. For other gradations, the reference strength and fatigue-life relationship should first be established, followed by recalibration and independent validation of the multi-index framework and the three-stage residual-strength model.

Another limitation concerns the sparse sampling of fatigue damage levels. Only four damage levels ($D_f = 0.20, 0.50, 0.65$, and 0.80) were investigated; therefore, the identified stage boundaries should not be regarded as statistically validated mathematical inflection points. Denser sampling, preferably with at least six to eight damage levels and additional measurements near the suspected transition regions, is required to evaluate the uncertainty and reproducibility of the proposed stage boundaries.

Only four unique damage levels, $D_f = 0.20, 0.50, 0.65$, and 0.80 , were available for calibrating the three-stage decay model. Although five independent specimens were destructively tested at each prescribed damage level, these specimens represent repeated observations at the same damage positions rather than additional independent damage levels or independent validation points. Therefore, the high R^2 value indicates goodness of fit within the calibration dataset and should not be interpreted as independent predictive validity. In addition, the residual-life estimation results in Table 10 were obtained by inverting the model at the same damage levels used for calibration. Further validation using additional damage levels and independent test datasets is required.

Because the residual-strength tests were conducted only at a stress level of 0.3 , the stress-level dependence of the identified stage boundaries and calibrated model parameters could not be assessed. Consequently, the present findings should not be directly extrapolated to other stress levels. Future studies should conduct independent residual-strength tests at additional stress levels, such as 0.2 and 0.4 , to evaluate the robustness, generalizability, and parameter transferability of the proposed framework.

4. Conclusions

In this study, indirect tensile strength, constant-amplitude splitting fatigue, and post-fatigue residual strength tests were conducted on AC-13C asphalt mixtures. A multi-index framework integrating the life-based damage index D_f , residual strength ratio λ , performance damage index D_s , damage lag coefficient L , and peak displacement response was established to characterize splitting fatigue damage. In addition, a constrained three-stage residual strength decay model was developed for damage-position inversion and residual-life estimation.

- (1) The proposed multi-index system characterizes splitting fatigue damage from the perspectives of life progression, strength degradation, and deformation development, thereby overcoming the limitation of using the life-based damage index D_f alone.
- (2) Under the tested conditions of $S = 0.3$, 15°C , and 10 Hz , splitting fatigue damage was preliminarily divided into an initial slow-damage stage, a stable-accumulation stage, and a late accelerated-damage stage. The corresponding reference boundaries were $D_1 = 0.50$ and $D_2 = 0.65$. These boundaries are condition-specific and require further verification using denser damage levels and additional test conditions.
- (3) The constrained three-stage residual strength decay model described the nonlinear attenuation of the residual strength ratio. The model achieved $R^2 = 0.9942$, $\text{RMSE} = 0.0158$, and $\text{MAPE} = 2.06\%$. It enabled the residual strength ratio to be converted into the life damage position and residual-life ratio within the tested data range. Further validation is required under different stress levels, loading frequencies, and variable-amplitude loading conditions.
- (4) As fatigue damage increased, the residual strength ratio decreased, whereas the performance damage index and peak displacement increased. After D_f exceeded D_2 , both strength degradation and displacement growth accelerated significantly. Peak displacement can therefore serve as an auxiliary indicator for evaluating residual strength degradation and identifying the late accelerated-damage stage.

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References

- Huang, T.; Yu, X.; Lin, B.H.; Zhou, L.B.; Qian, G.P.; Wang, C.Y. Study on the fatigue residual strength and damage evolution of asphalt mixtures under different stress states. *China J. Highw. Transp.* **2024**, *37*, 284–293.
- Sudarsanan, N.; Kim, Y.R. A critical review of the fatigue life prediction of asphalt mixtures and pavements. *J. Traffic Transp. Eng.* **2022**, *9*, 808–835. [\[CrossRef\]](#)
- Zhang, Q.P.; Gu, X.Y.; Ding, J.T.; Hu, D.L. Creep damage model and damage evolution of asphalt mixtures. *J. Traffic Transp. Eng.* **2021**, *21*, 104–113.
- Yang, R.; Xue, C.; Cheng, H.; Hu, Y.; Sun, L.; Cui, C. Differentiated fatigue damage characteristics and life prediction of asphalt mixtures under actual service conditions. *Constr. Build. Mater.* **2026**, *509*, 145170. [\[CrossRef\]](#)
- Cheng, H.; Sun, L.; Wang, Y.; Liu, L.; Chen, X. Fatigue test setups and analysis methods for asphalt mixture: A state-of-the-art review. *J. Road Eng.* **2022**, *2*, 279–308. [\[CrossRef\]](#)
- Imoh, U.U.; Akinmade, D.; Movahedi Rad, M. Rutting resistance and fatigue performance of crumb rubber-modified asphalt concrete: Experimental investigation and mechanistic-empirical modeling. *Infrastructures* **2026**, *11*, 133. [\[CrossRef\]](#)
- Ling, M.; Zhang, J.; Fuentes, L.; Walubita, L.F.; Lytton, R.L. A mechanistic framework for tensile fatigue resistance of asphalt mixtures. *Int. J. Fatigue* **2021**, *15*, 106345.
- Wang, C.; Sun, Y.G.; Ren, Z.Y. Identifying fatigue crack initiation and propagation behavior of asphalt binder under dynamic shear. *J. Beijing Univ. Technol.* **2026**, *52*, 282–293.
- He, M.; Yuan, J.; Zhu, X. Fatigue damage characteristics and reliability analysis of asphalt mixtures using damage mechanics. *Constr. Build. Mater.* **2025**, *458*, 139748. [\[CrossRef\]](#)
- Zeng, Z.; Kim, Y.R.; Underwood, B.S.; Guddati, M. Asphalt mixture fatigue damage and failure predictions using the simplified viscoelastic continuum damage (S-VECD) model. *Int. J. Fatigue* **2023**, *174*, 107736. [\[CrossRef\]](#)
- Klug, A.; Ng, A.; Faxina, A. Application of the viscoelastic continuum damage theory to study the fatigue performance of asphalt mixtures—A literature review. *Sustainability* **2022**, *14*, 4973. [\[CrossRef\]](#)
- Su, Y.; Nikraz, H. New analytical-modelling method to interpret flexural bending fatigue response of asphalt mixture using the S-VECD theory. *Int. J. Pavement Eng.* **2022**, *23*, 2829–2845. [\[CrossRef\]](#)
- Chen, A.; Airey, G.D.; Thom, N.; Li, Y. Characterisation of fatigue damage in asphalt mixtures using X-ray computed tomography. *Road Mater. Pavement Des.* **2023**, *24*, 653–671. [\[CrossRef\]](#)
- Cheng, H.; Sun, L.; Wang, Y.; Chen, X. Analysis of fatigue behaviors of asphalt mixture under actual loading waveforms using pseudo-strain-based approaches. *Int. J. Pavement Eng.* **2023**, *24*, 2020269. [\[CrossRef\]](#)
- Wei, H.; Liu, Y.; Li, J.; Wang, F.; Zheng, J.; Yuan, Z. Characterizing fatigue damage evolution in asphalt mixtures using acoustic emission and Gaussian mixture model analysis. *Constr. Build. Mater.* **2023**, *409*, 133973. [\[CrossRef\]](#)
- Câmara, G.; Micaelo, R.; Azevedo, N.M.; Silva, H. Incremental viscoelastic damage contact models for asphalt mixture fracture assessment. *Infrastructures* **2024**, *9*, 118. [\[CrossRef\]](#)
- Gkyrtis, K.; Plati, C.; Loizos, A. Mechanistic analysis of asphalt pavements in support of pavement preservation decision-making. *Infrastructures* **2022**, *7*, 61. [\[CrossRef\]](#)
- Zhang, Y.; Zhang, J.; Ma, T.; Qi, H.; Chen, C. Predicting asphalt mixture fatigue life via four-point bending tests based on viscoelastic continuum damage mechanics. *Case Stud. Constr. Mater.* **2023**, *19*, e02671. [\[CrossRef\]](#)
- Zhang, Z.; Oeser, M. Residual strength model and cumulative damage characterization of asphalt mixture subjected to repeated loading. *Int. J. Fatigue* **2020**, *135*, 105534. [\[CrossRef\]](#)
- Fang, C.; Guo, N.; Leng, Z.; Jiang, J.W.; Li, H.; Lu, G.Y.; Wang, H.P. Kinetics-based fatigue damage investigation of asphalt mixture through residual strain analysis using indirect tensile fatigue test. *Constr. Build. Mater.* **2022**, *352*, 128962. [\[CrossRef\]](#)
- Cheng, H.; Liu, J.; Sun, L.; Liu, L.; Zhang, Y. Fatigue behaviours of asphalt mixture at different temperatures in four-point bending and indirect tensile fatigue tests. *Constr. Build. Mater.* **2021**, *273*, 121675. [\[CrossRef\]](#)

22. Hectors, K.; De Waele, W. Cumulative damage and life prediction models for high-cycle fatigue of metals: A review. *Metals* **2021**, *11*, 204. [[CrossRef](#)]
23. Wei, H.; Wang, Y.; Li, J.; Lu, R.; Liu, Y. Fatigue damage characterization in asphalt mixtures using acoustic emission with information entropy and the maximum Lyapunov exponent. *Low.-Carbon Mater. Green Constr.* **2025**, *3*, 21. [[CrossRef](#)]
24. Zhang, D.; Zhang, Z.; Huang, T.; Huang, Z.; Zhou, Z.; Huang, J. Study on fatigue characteristics of anti-stripping sandstone asphalt mixture after water immersion. *Front. Mater.* **2025**, *12*, 1634717. [[CrossRef](#)]

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