

Article

A General Procedure for Basic Kinematic Chain Formation and Topology Selection for Planar Mechanisms

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Abstract

In a complete kinematic synthesis process, a designer must select a planar linkage topology that is well suited to their problem situation. This involves weighing a set of competing priorities. For example, is it better to choose a simple topology like a four-bar mechanism that will be cheaper to produce, or a complex topology like an eight-bar mechanism that can produce intricate motions but will also be more expensive and more difficult to synthesize? The process of selecting the topology is broadly known as type synthesis, or sometimes structure synthesis, and has been studied in the past. However, past works on planar linkage type synthesis have overemphasized isomorphism detection, identifying the complete set of unique topologies up to a certain number of links, while the central problem of choosing the ideal topology has often been overlooked. In this work, a general procedure for forming basic kinematic chains (BKC), a simplified topological representation, is presented. Then, a set of rules and design principles is provided that can help a designer narrow the infinite possible BKC options down to a manageable set. A few practical examples are provided to demonstrate the concepts and show that the procedure is effective. A literature review is also provided that examines past works, as well as introducing alternative approaches, such as simultaneous algorithmic methods and spatial methods.

Keywords: type synthesis; linkage synthesis; topology; planar linkages; topological analysis; basic kinematic chain; BKC; mechanisms

1. Introduction

Kinematic synthesis is the overarching process by which a mechanism is designed. The process can be subdivided into four steps: number synthesis, type synthesis, dimensional synthesis, and kinematic analysis, as shown in Figure 1. Number synthesis, which may also include topological synthesis and analysis, is the step in which a designer determines how many links should be included in the mechanism, and broadly what types of links those should be (i.e., binary, ternary, etc.). Type synthesis is the process by which a specific kinematic topology and inversion are selected, as well as the joint types and locations. Dimensional synthesis assigns dimensions and angles to each link in the chosen topology to solve the prescribed problem. Kinematic analysis is a final quality control step in which the synthesized mechanism's properties (mechanical advantage, velocity, coupler curve, transmission angles, etc.) are analyzed. Depending on the results of this step, the designer may choose to return to the dimensional synthesis step or even type synthesis to try to improve the final output.



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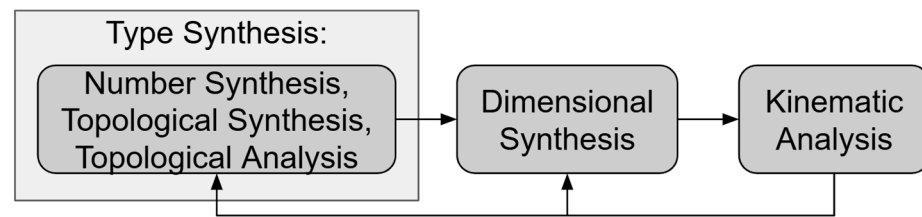


Figure 1. The figure shows one form of the steps required for a complete approach to kinematic synthesis of planar linkage mechanisms. Other authors may combine the type and dimensional synthesis steps into a single process or add more steps for completeness. The kinematic analysis step is a final verification in which the mechanism’s performance is evaluated. Designers may return to the previous steps based on the results of the analysis.

Type synthesis, the emphasis of the present work, is generally not given as much attention as dimensional synthesis, possibly because its importance (and the extent of its impact when done well) is less obvious than other steps in the kinematic synthesis process. With that said, there is a case to be made that type synthesis is the most important step of mechanism design. If the wrong topology is selected, even a perfect dimensional synthesis process cannot produce the best solution. Erdman, while speculating about the future of computer-aided mechanism design, argued, ‘the most critical stage of the entire design process is the choice of the most appropriate type of mechanism [i.e., mechanism topology] [1].’ For example, if a two-degree-of-freedom kinematic chain is selected when a 1-DOF topology would have sufficed, the solution will be unnecessarily complex and costly. On the other hand, if a simple four-bar mechanism is selected when a more complex topology would have performed better, a designer is setting themselves up for a frustrating dimensional synthesis process and is unlikely to produce a high-quality final solution. Type synthesis is also valuable for the classification of mechanisms. James Watt used type synthesis principles to steer clear of existing patents when developing mechanisms for his steam engines during the late 18th century. To this day, patent differentiation remains an important use for type synthesis.

Unfortunately, despite extensive research in kinematic synthesis, there are relatively few resources for a generalized process for optimal topological selection for a given kinematic task. The field of machine design could greatly benefit from developing a general method to predict the best type of mechanism to satisfy a specific design problem. Any rules or principles identified in this area could help in developing a general theory of machine design and ultimately can be incorporated into a machine design expert or automated synthesis system. Professor Robert Willis described this goal in his 1841 text, *Principles of Mechanisms*. He said that mechanism design should be approached in a systematic, mathematical manner to determine ‘all the forms and arrangements that are applicable to the desired purpose’ from which the designer could choose the most suitable [2]. Franz Reuleaux agreed with these sentiments a generation later in the pages of his classic text, *Theoretische Kinematik* of 1875 [3]. Since the time of Willis and Reuleaux, many authors have furthered the investigation into type synthesis, making many advancements, but also leaving opportunities for additional research. These works will be discussed in the following section.

1.1. Literature Review

One of the sub-categories of type synthesis that has been given a great deal of attention in the literature is identifying isomorphism in complex linkages. Two kinematic chains or topologies are said to be isomorphic if they are functionally identical. For linkages with a small number of links, this problem is relatively easy to identify. However, as the

number of links increases, isomorphic subchains become exponentially more challenging to find. Isomorphism matters because if an algorithm is looking for solutions in many different topologies, it is a waste of effort to examine more than one candidate in an identical isomorphic set. On the other hand, if a topology is rejected for isomorphism incorrectly, the search will ignore a possible topology that could have been optimal [4].

As mentioned, many researchers have given this topic their attention. Titus and Erdman used a form of graph theory and rook polynomials in the identification of isomorphic chains [5]. Rizvi et al. use adjacency matrices and what they call the ‘unique chain code’ to identify isomorphic chains. The unique chain code represents a summation of the number of instant centers of a kinematic chain and the sum of degrees of freedom of the attached links [6]. Several others have adopted a similar approach, using a characteristic polynomial or other form of chain code to differentiate topologies, many of which have roots in graph theory [4,7–18]. Hasan demonstrated a variety of similar isomorphism detection techniques, including joint–joint adjacency matrices [15,17,19–21]. Dargar et al. investigated the first and second link adjacency values as a method for isomorphism detection among kinematic inversions and discrete kinematic chains [19,20]. Other unique approaches for isomorphism detection derived from chemistry, information theory, and others [19,22–24]. Kong et al. created a method based on artificial neural networks, though Rao expressed uncertainty with the approach, preferring the Hamming number technique [11,18,25–29]. In other works, Rao applied genetic algorithms to isomorphism detection and kinematic chain selection, intending to optimize for specific design parameters [30,31].

Another common approach to type synthesis is to reference a kinematic atlas. Atlases are resources that usually focus on one specific aspect of kinematics. For example, Hrones and Nelson created an atlas that provides a large collection of coupler curves for various configurations and dimensions of four-bar linkages [32]. Several later works expanded upon this original work by refining or expanding it with new mechanism types [33–39]. These resources provide examples of how different topologies and link dimensions may be used to create specific motions and coupler curves. If a designer finds a suitable solution in an atlas, the type and dimensional synthesis have already been completed on their behalf. Atlases usually do not provide kinematic analysis of key properties to help select the optimal design. Even so, they can be valuable as a starting point for identifying what broad topologies might be most effective for performing a given task.

The proposed method presented in this work divides the type and dimensional synthesis processes into separate steps in the overarching kinematic synthesis process. Several authors have taken a different approach in which type and dimensional synthesis occur simultaneously, typically accomplished using a computational method which operates automatically [40,41]. Kim et al. wrote several papers about simultaneous solvers using a spring-supported rigid block model to represent mechanisms, and other authors have also pursued simultaneous approaches [42–46]. These simultaneous methods can be effective, but they have limitations. The practice of algorithmically uncovering the optimal topology is computationally expensive and time-consuming. Even if this sort of brute force optimization approach becomes the common method, the inclusion of foreknowledge about topological performance could improve the efficiency of these programs by reducing the volume of extraneous calculations on kinematic chains that are unlikely to produce a quality solution.

The present work will emphasize the study of type synthesis for planar, rigid-body mechanisms. All the links in the mechanism are assumed to be completely inflexible, and all motion happens in a set of parallel planes. When these assumptions are not made, the problem situation is very different and justifies entirely unique fields of study, namely compliant and spatial linkage designs. A few works in these areas are briefly mentioned

for completeness, but again, problems dealing with compliant or spatial problem situations are considered out of scope and unrelated to the present study. For example, Murphy researched the idea of building on rigid-link type synthesis methods to perform type synthesis of topologies with flexible members [47–49]. Zhu et al. wrote an effective review on compliant linkage type synthesis, emphasizing the topology optimization approach [50]. Spatial linkages operate in three-dimensional space, rather than moving in a set of parallel planes, and also require a type-selection process. Many of the existing methods use screw theory, a mathematical tool for representing spatial motions and forces, to identify feasible configurations that meet motion requirements [51–53]. Again, the focus in the present work is on rigid planar linkages, so none of these methods will be discussed in detail.

1.2. Limitations and Overview

As this literature review has shown, type synthesis as a broad step in the kinematic synthesis process has been given much attention in kinematics research. With that said, relatively few of the works address the question that is at the core of the topic of planar type synthesis and is the focus of this work, that is, ‘how might a prospective designer select the best possible topology for the unique problem they want to solve?’ Much of the research in type synthesis is dedicated to the complete enumeration of non-isomorphic kinematic chains up to a certain complexity level, or to obviating the type synthesis step through algorithmic automation of topology selection. These works are valuable, but they largely ignore the key challenge of predicting topological performance for a given kinematic task, and correspondingly, selecting the optimal topology. To select the best topology, there needs to be some system for predicting how a topology will perform at a given task.

Rao has completed work in anticipating the performance of topologies according to a kinematic property called compactness that is determined based on the distance (as the least number of joints) between two links. Rao demonstrates the concept on the Watt and Stephenson chains, then applies it to platform-type robots [54,55]. Dargar also researched the ideas of stiffness and compactness as they relate to kinematic chains, and modeled chains as springs in series to obtain a new view of the stiffness of the system [56]. Vijay et al. applied this compactness lens to the selection of robotic hands, along with stiffness and effectiveness at forward kinematic tasks [57]. Rao and Jagadeesh apply a similar concept to dynamic sensitivity, arguing that there is a correlation between dynamic performance and the number of joints separating a link from the ground. They find that the kinematic chains they rate favorably for static performance (i.e., high mechanical advantage) tend to have greater dynamic sensitivity, indicating a tradeoff in the topological selection [58].

Another gap in the literature is the investigation of kinematic chains with higher-pair connections. In this paper, F_2 joints, or higher-pair connections, are defined as any joint between two or more links that removes only a single degree of freedom. This differs from the common revolute and prismatic joints, which remove two degrees of freedom. F_2 joints (examples include pin-in-slot, cams with slipping, and fork joints) are often more complex to model but are still commonly used in modern product designs. The notable exception to this lack of research is gear pairs, a joint that only removes one degree of freedom (typically rotation), which have been extensively researched. However, the previously mentioned point on predicting the quality of kinematic chains also applies to geared linkages. One example of F_2 joints in the type synthesis literature is the paper from Sayeed that investigates isomorphism of higher-pair connections using graph theory [9].

To reiterate the findings of the literature review, type synthesis is an important portion of the planar linkage synthesis process. The complete set of non-isomorphic topologies is well established up to as many as 19 links, but a consistent process for making an optimal topology selection is a clear area for additional work. Additionally, the existing literature does not show clear processes for understanding and synthesizing F_2 -type joints. The remainder of the present manuscript focuses on addressing these two shortcomings. First, a high-level overview of the steps intrinsic in type synthesis is presented. Then, steps for basic kinematic chain (BKC) formation are presented, including certain F_2 joints arising in common machine elements. Finally, a procedure for selecting the optimal BKC for a given application is outlined. This work provides the foundations of a consistent framework that may be applied to any planar kinematic synthesis problem.

2. Materials and Methods—Type Synthesis Methodology

Type synthesis consists of many identifiable steps. For example, the questions listed below may be considered in the following order during the type synthesis process. First, the desired degree of freedom value is known from the requirements of the task to be solved. The steps of the type synthesis process are seen in Table 1.

Table 1. Key design questions in the type synthesis process.

Number	Design Questions
1	How many links and joints are required for the desired degrees of freedom? (See Equation (1)).
2	How many of each link type are needed for this link set? (i.e., binary, ternary, quaternary, see Equation (2)).
3	How many different link sets satisfy the desired degrees of freedom?
4	How many linkage topologies can be formed from these sets of links?
5	How many unique topologies are available from which to choose?
6	How many ways can a ground link be chosen for each topology? (kinematic inversions).
7	How can one predict if any topologic inversions are inherently better than all others for the task at hand?
8	How many ways can the types of joints required to satisfy the task be distributed throughout the linkage?
9	How many different links could serve as the input drivers?

Type synthesis can be subdivided into topological synthesis, topological analysis, and number synthesis. Questions 1 through 3 constitute number synthesis, questions 4 through 6 are topological synthesis, and questions 7 through 9 are typical of topological analysis. Figure 1 lists the steps of the synthesis process. The first step in type synthesis is to determine the number and type of links needed to form linkages with the correct degree of freedom. This can be done by using a modified form of the Gruebler equation, Equation (1).

$$F = 3(N - 1) - 2F_1 - F_2 \quad (1)$$

where

F is the number of degrees of freedom of the mechanism;

N is the number of links in the mechanism;

F_1 is the number of lower-pair joints in the mechanism (pinned joints, sliders, etc.);

F_2 is the number of higher-pair joints in the mechanism (gears, cams, pins-in-slots, etc.).

After Equation (1) is used to determine the correct number of links and joints, Equation (2) is used to figure out the quantity of each type of link needed to achieve the intended degrees of freedom. Here, ‘type’ refers to the number of joints fixed to a given link. Links with two joints are called binary, those with three joints are called ternary, then quaternary, and so on. The solutions of this equation are the sets of higher-order links (those with more than two connections) that satisfy the desired degree of freedom value.

$$N - (F + 3) = T + 2Q + 3P \quad (2)$$

where

N is the total number of links in the mechanism;

T is the number of ternary links;

Q is the number of quaternary links;

P is the number of pentagonal (quinary) links;

F is the degree of freedom required to perform the desired task.

Note that binary links are excluded from the equation because they are not higher-order links. Instead, the number of binary links is calculated by taking the total number of links and subtracting the number of higher-order links calculated from the equation. For example, the Watt chain has six links and one degree of freedom. Using these values, Equation (2) is rewritten as $6 - (1 + 3) = 2 = T + 2Q + 3P$. There are two values of the variables T, Q, and P that solve the equation, which are when T equals 2, or if Q equals 1. If T equals 2, then there must be 4 binary links, because the total number of links N must add up to 6. This (T = 2, B = 4) is the combination of links used in the Watt and Stephenson topologies. The other combination that satisfies Equation 2 (Q = 1, B = 5) does not produce any useful (non-isomorphic) combinations. Each unique set of N links is known as a kinematic link-set solution (KLSS). The method of using basic kinematic chains (BKC) to analyze these link-set solutions is shown in Section 2.2.

Freudenstein and Maki suggest using the following form rather than the classic Gruebler’s equation, which is more complex but also provides additional flexibility to the designer [59]. Specifically, this version allows a user to consider spatial mechanisms;

$$F = \lambda(l - j - 1) + \sum_{i=1}^j f_i \quad (3)$$

And

$$L_{IND} = j - l + 1 \quad (4)$$

In these equations,

F = number of degrees of freedom of the mechanism.

l = number of links of the mechanism (including the fixed link; all links are considered as rigid bodies having at least two joints).

j = number of joints of the mechanism; each joint is assumed as binary (i.e., connecting two links); if two or more links share a common axis of rotation (i.e., the joints appear to overlap), the number of joints $j = N - 1$, where N = the number of links at the common joint axis. Instances of shared joint axes like this will be referred to as a ‘multiple joint.’

f_i = degrees of freedom of the i th joint; this is the freedom of the relative motion between the connected links. This will be 1 for F_1 joints and 2 for F_2 joints. Spatial joint types may have more degrees of freedom.

λ = degree of freedom of the space within which the mechanism operates; for an unconstrained link moving in a plane, $\lambda = 3$, and for an unconstrained link moving in 3D space, $\lambda = 6$.

L_{IND} = number of independent circuits or closed loops in the mechanism.

Equations (3) and (4) can be combined to form Equation (5):

$$\sum f_i = F + \lambda L_{IND} \quad (5)$$

Take, for example, the case of designing a planar motion with a four-bar linkage that has a single degree of freedom. In that case, $F = 1$, and $\lambda = 3$. A four-bar linkage has one closed loop, so $L_{IND} = 1$. Then, from Equation (3),

$$\sum f_i = 4$$

Inputting these terms into Equation (4) yields the following expression for the four-bar linkage.

$$j = L_{IND} + 1 - 1 = 1 + 4 - 1 = 4$$

Thus, the number of joints is four; a quick inspection of a four-bar linkage confirms that this result aligns with expectations. Equation (3) shows that a five-bar chain with five F_1 joints yields a mechanism with two degrees of freedom:

$$F = 3(5 - 5 - 1) + 5 = +2$$

If five joints allow a single degree of freedom between connecting links, and one joint allows two degrees of freedom of relative motion (e.g., gear connection), a five-bar chain has a single degree of freedom. In the case of a six-bar chain with two loops ($L_{IND} = 2$), from Equation (4)

$$j = 2 + 6 - 1 = 7$$

and from Equation (3) (assuming pin and slider joints),

$$F = 3(6 - 7 - 1) + 7 = 1$$

2.1. Basic Kinematic Chains (BKC)s

The collection of links comprising each potential solution are assembled into figures using pin joints at all link-connection points. These figures define the topological structure of the linkages formed from this set and are called isomers. These isomers are guaranteed to have the desired overall degree of freedom. Each isomer obtained from all kinematic link-set solutions for a desired degree of freedom value and number of links is called a basic kinematic chain (BKC), sometimes called a Basic Kinematic Diagram (BKD). Figure 2 shows the complete set of BKC's for the case of 6 links and one degree of freedom, and outlines the process for finding the set. From the Gruebler equation (Equation (1)), if there are no F_2 joints present, the number of F_1 joints must be seven for a six-bar chain to have one degree of freedom. The BKC's C-H of Figure 2 have zero-degree-of-freedom subchains (triangles formed with all F_1 -type joints are rigid) and thus are isomorphic and not useful. These chains are unnecessarily complex versions of a simpler one. For example, the mobile component of isomer C in Figure 2 reduces to a simple four-bar chain because of the rigid triangular subchain on the left.

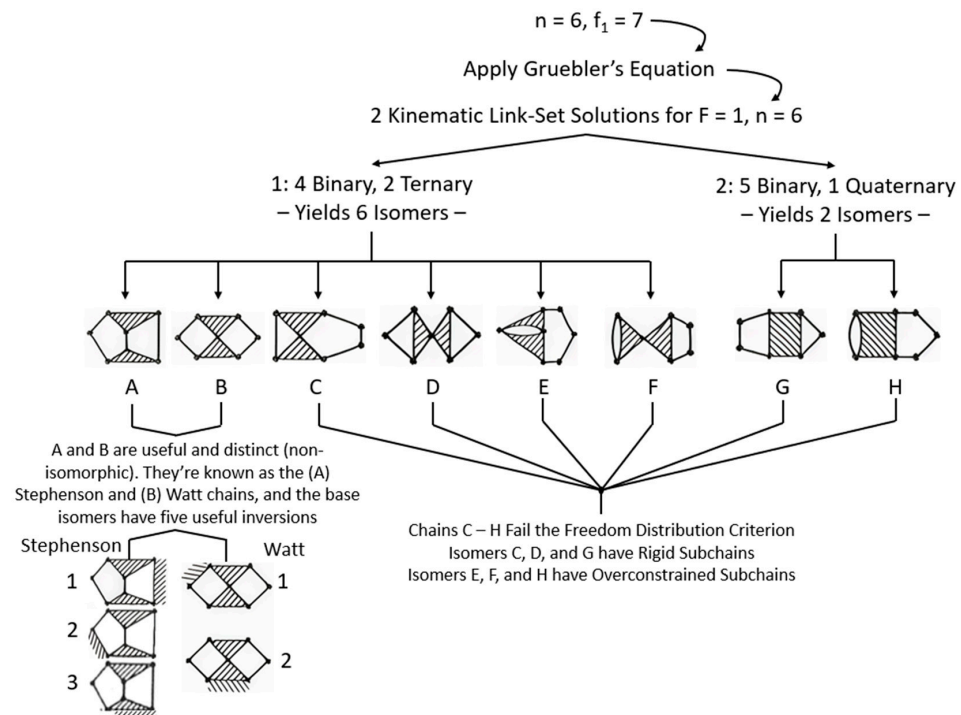


Figure 2. Demonstration of type synthesis for 6-bar, one-DOF chains. Specifically, the topological derivation of the well-known Watt and Stephenson chains is shown. A similar figure was originally published in *Advanced Mechanism Design Vol I* p.529 [60].

The one-degree-of-freedom six-bar chains in the figure have two link-set solutions, found using Equation (2). As was shown earlier, the two solution link sets are first, 5 binary links and 1 quaternary link and second, 4 binary and 2 ternary links. All isomers of the first set (5B 1Q) fail because they contain rigid subchains which, when removed, cause the BKC to degenerate to other simpler linkages. For example, chain G simplifies to a four-bar linkage once the rigid triangle on the right side is removed. Similarly, 2/3 of the isomers (C-F) from the second set fail as they contain rigid subchains. That leaves the two topologies labeled 'A' and 'B', which are commonly known as the Stephenson and Watt topologies, respectively.

The final step is identifying all topological inversions of a given kinematic chain. These are formed by grounding each link in a BKC, one at a time, and determining which resulting mechanisms are topologically unique. For example, a Watt I is topologically different than a Watt II and may be capable of performing quite different tasks. In the Watt I, a binary link is grounded, while in the Watt II, one of the ternary links is grounded. The Stephenson chain has three unique inversions shown in Figure 2, while the Watt chain only has two. Few methods exist to determine which of these topological inversions is best suited for a particular task.

The six-bar 1-DOF case shown in Figure 2 is a relatively simple example to illustrate the concept of topology enumeration, but the complexity of this process increases quickly as the number of links or degrees of freedom increase. As noted in the literature review, many authors have studied topology enumeration, referring to generating the complete set of non-isomorphic kinematic chains up to a certain number of links. Hon-Sen Yan developed an important table on this topic that is replicated here with modifications. Table 2 shows the number of unique kinematic chains with varying degrees of freedom for each number of links from 4 to 12. Note that this is specifically unique chains and does not include kinematic inversions. For example, the useful combinations of six links and one degree of freedom reported in the table is two, representing the Watt and Stephenson chains, rather

than five, including all the unique kinematic inversions. It is also important to note that the table does not consider topologies that use F_2 joints, which is why odd-numbered link combinations do not yield linkages with odd numbers of degrees-of-freedom, and vice versa—even numbered combinations of links do not produce linkages with an even number of degrees of freedom.

Table 2. Number of useful, unique kinematic chains depending on the number of links and the desired degrees of freedom. The table does not include unique kinematic inversions or F_2 joints. This table is modified from a similar table originally reported by Hong-Sen Yan in [61].

Links	Degrees of Freedom						
-	1	2	3	4	5	6	7
4	1	0	0	0	0	0	0
5	0	1	0	0	0	0	0
6	2	0	1	0	0	0	0
7	0	4	0	1	0	0	0
8	16	0	7	0	1	0	0
9	0	40	0	10	0	1	0
10	230	0	98	0	14	0	1
11	0	839	0	189	0	19	0
12	6862	0	2442	0	354	0	24

The 16 useful combinations of 8-link chains with a single degree of freedom is a common result in linkage synthesis. An example of each of these topologies can be seen in [61].

2.2. Special Rules and Techniques for BKC Formation

The next section shows how BKC, such as the ones in Figure 2, can be converted to different mechanisms by the substitution of pin joints for other types of joints or machine elements. A BKC is a closed, planar structure consisting of a lower pair of non-multiple revolute joints and polygonal-shaped links. For example, a binary link is portrayed as a line, and a ternary link is shown as a triangle. Recall that type synthesis provides the designer with topological, not dimensional, information, so it does not matter if the BKC model bears little geometric resemblance to the final mechanism. It is a model of the mechanism topology that is ‘degree-of-freedom equivalent.’ Comparisons of different researchers’ works are made easier if all are using the same mechanism models. The BKC model is preferred because it is the most ‘basic’ visual representation of a topology.

A set of rules called the transformation laws is given in Table 3. They preserve or manipulate the degrees of freedom of a BKC as described in each law and are used in two ways. In a design synthesis mode, they are used to transform the BKC into a less abstract object, either another ‘not as basic’ kinematic chain or a model of a mechanism denoting the type of joints desired between the links (if not revolute).

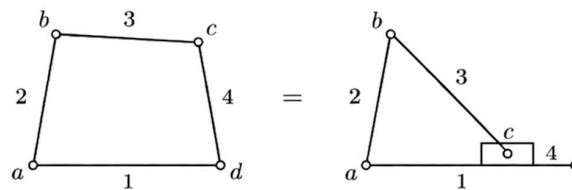
In an analysis mode, the transformation rules are used to simplify all geometric, dimensional, and higher-pair joint information to determine the underlying topology used for a particular design. This is helpful when trying to determine if the same topology is consistently being employed to solve identical or similar tasks. If so, then the question as to why this topology is inherently better suited for the task can be studied and, hopefully, new design rules can be formulated and applied to future design work. A visual representation of each design rule is shown in Figure 3.

Table 3. Transformation laws for forming BKC's.

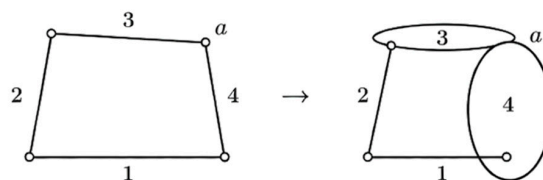
Law #	Transformation Law Description ¹
1	A sliding lower-pair joint can be substituted for a pin lower-pair joint without changing the degrees of freedom.
2	Substitution of a higher-pair joint for a lower-pair joint without removal of a link will increase the degrees of freedom by 1.
3	Removal of any binary link will decrease the degrees of freedom by 1.
4	(Directly from 2 and 3) Removal of a pin-connected binary link and substitution of a higher-pair joint for the binary link and its 2 lower-pair joints will not change the degrees of freedom. (The opposite is also true, i.e., the substitution of a pin-connected binary link for a higher-pair joint will not affect the degrees of freedom).
5	Partial shrinkage of a higher-order link will create a multiple joint but will not change the degrees of freedom.
6	Total shrinkage of a higher-order link will create a multiple joint, and decrease the degrees of freedom by 1. This is similar to law 3. Laws 3 and 6 can be combined to read: the removal of any binary link decreases the degrees of freedom by 1, but the removal of a higher-order link also creates a multiple joint.
7	Allowing a mechanism to be grounded through a single joint instead of a binary link increases the degrees of freedom by 1 and creates a multiple joint at the ground.

¹ Figure 3 shows an example of each of these laws.

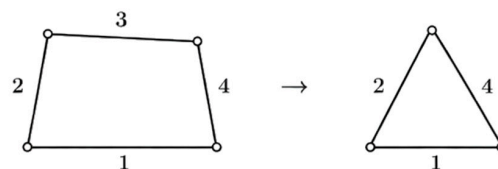
Law 1



Law 2 (Not degree of freedom equivalent)



Law 3 (Not degree of freedom equivalent)



Law 4

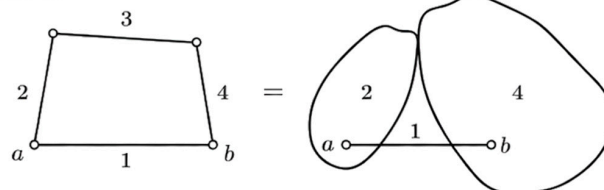
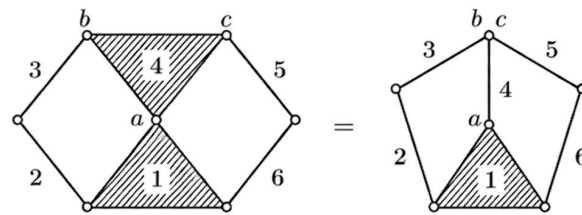
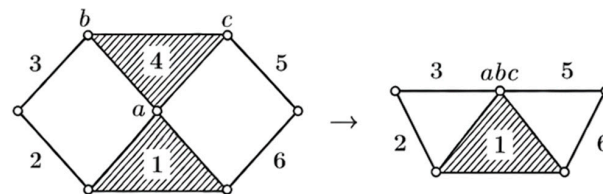


Figure 3. Cont.

Law 5



Law 6 (Not degree of freedom equivalent)



Law 7 (Not degree of freedom equivalent)

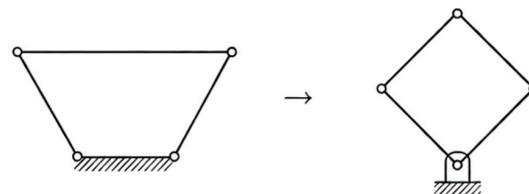


Figure 3. Sketches showing the effect of the transformation laws in simple kinematic chains. The numbers in the BKC's on the left correspond to the same elements in the BKC's on the right after the transformation laws are applied.

Law one shows that a slider (prismatic) joint is degree-of-freedom interchangeable with a pinned joint. This property is useful for forcing relative straight-line motion between two links. Laws two, three, and four are useful for simplifying topologies with F_2 joints or converting an existing linkage to include an F_2 joint. Law five shows that collapsing a ternary link into a binary link does preserve degrees of freedom, but collapsing the whole link into a single joint is not degree-of-freedom equivalent, as shown in law six. This property is useful for compressing links, or for using fewer joints. Designs may also be simplified by combining joints that are too close together on a single link. Finally, law seven shows that grounding a link and grounding a single joint on that link are not degree-of-freedom equivalent. This is important to note, as it shows a linkage may not be mounted on a shaft while maintaining its motion properties.

It is much easier to examine BKC's than to investigate all the possible mechanisms that can be derived from them. The reason for gathering all unique BKC's is that if one is missing, a potentially useful mechanism topology will be lost to the designer. Also, BKC's can model all mechanisms. Examples of common elements with their kinematic equivalents are discussed next.

2.3. Modeling Common Machine Elements as BKC's

Belts, chains, and cables are flexible elements most used in friction or sprocket drives. Their primary purpose is to act as a power-transmission machine element. As such, their kinematic task is a simple function generation case, maintaining a constant angular velocity ratio or timing between two shafts. Ribbed timing belts are an important member of this machine element family. Curiously, one of the first uses of a member of this family, a chain, was as a coupler link in the Newcomen steam engines of the early 18th century. Naturally,

these links only transmitted motion and performed work when in tension. These elements are used to transmit power between shafts, which may be quite a distance away from each other or may not be parallel to each other. They are cheap and easily replaced, they can be adjusted to account for manufacturing errors, they allow adjustments between the shafts, they can power more than one shaft simultaneously, and they decrease the transmission of shock loads through a machine. They are not modeled in the most obvious manner, which would be to use a link for each side of the belt. Instead, a binary link is used to span the two pulleys on the tension side only. The slack side is kinematically redundant and modeling this side with a binary would yield an incorrect-degree-of-freedom BKC. In general, the slack side of the belt in a multi-pulley system, which lies between the drive and last driven pulleys, is always omitted. Therefore, a BKC which can be transformed to include a belt or chain drive must have one degree of freedom and a binary coupler link (or a series of binary coupler links if the system includes more than two shafts). The links that model the pulleys almost always need to be pinned to the ground through a revolute joint. Figure 4 shows two BKC that can model these mechanisms.

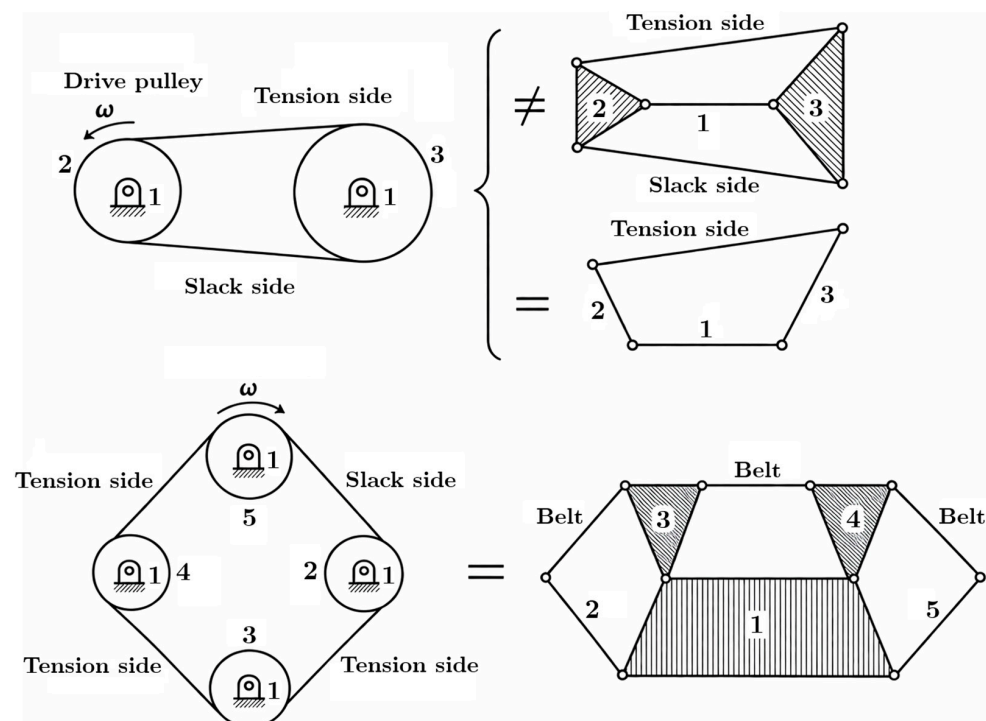


Figure 4. Belt and pulley systems and their BKC. The link numbers correspond between the left- and right-hand sides of the figure. For example, link 1 in the pulley system on the left is represented by link 1 in the BKC on the right.

Springs have many uses in machines, including maintaining closure of the higher-pair cam joint in a cam-and-follower system, providing a preload in a mechanism assembly, being a flexible support member as in an automobile chassis, storing potential energy, or acting as a counter-balance. Both linear and torsional springs are modeled as a binary dyad (two binary links pin-jointed in series). Automotive valve springs and axel springs are joined to the ground link while preloading and counter-balancing springs are not limited to a special site within the BKC. In all cases, the only requirement is that the spring ends must be attached to two different links in the machine, usually two adjacent links. To incorporate a spring into a machine requires the BKC to have a dyad, which is to be replaced by the spring. The two six-bar BKC are the least complex topologies capable of making use of a spring in a non-trivial capacity. Figure 5 depicts a compression spring guaranteeing closure

of the higher-pair cam joint in an automotive valve train system and the BKC from which it was derived.

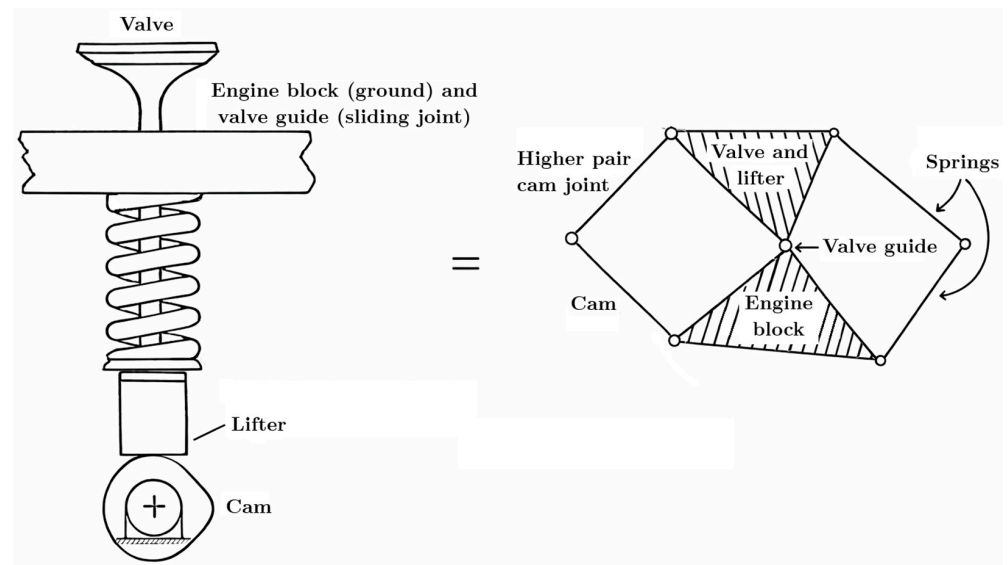


Figure 5. Spring maintains the closure of the cam–follower joint in an automotive valve assembly.

Hydraulic and pneumatic cylinders are almost always used as linear input drivers, especially when large forces are needed. When used as an input driver, this mechanism element is often pinned to the machine ground link. Although the common usage of a cylinder/shaft assembly dictates it to be modeled by a binary dyad (the lower-pair pin joint between the links ultimately becomes a lower-pair slider), the cylinder can become the base for attachment points for other links and in this guise become a higher-order link in the BKC. The normal characteristic required of a BKC to model a hydraulic/pneumatic cylinder is a binary dyad pinned to the ground. Figure 6 illustrates the structure of the BKC required to model this machine element and a mechanism incorporating this BKC.

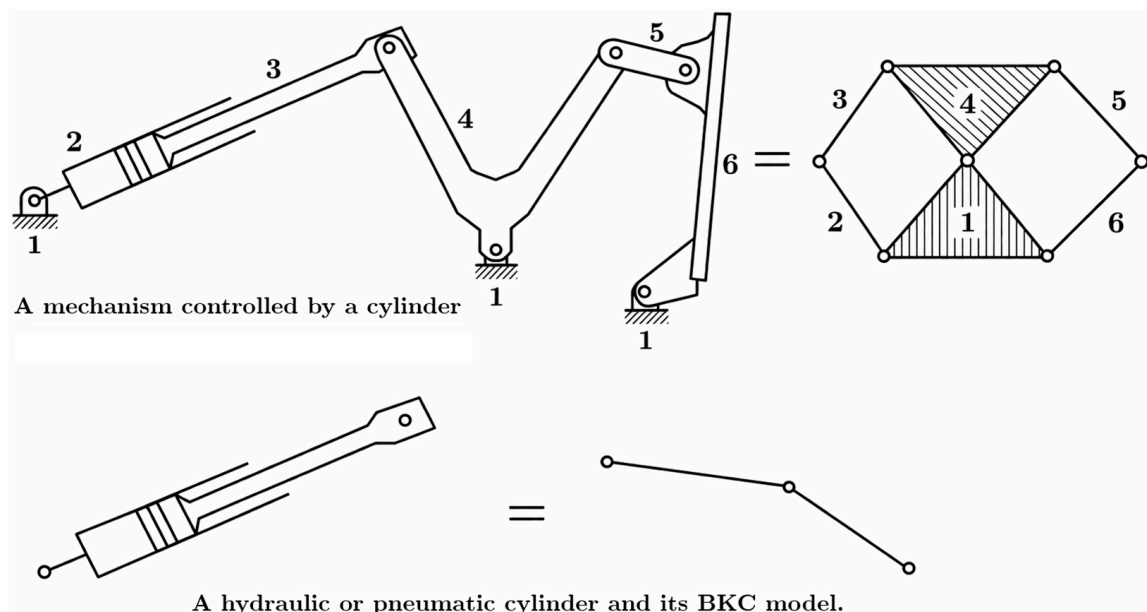
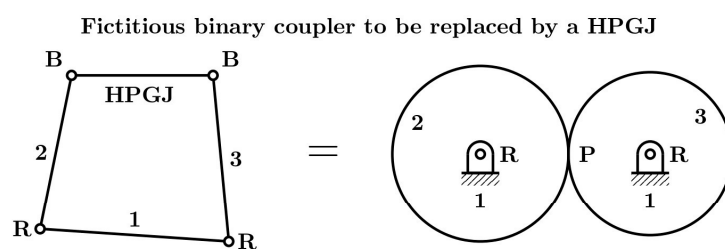


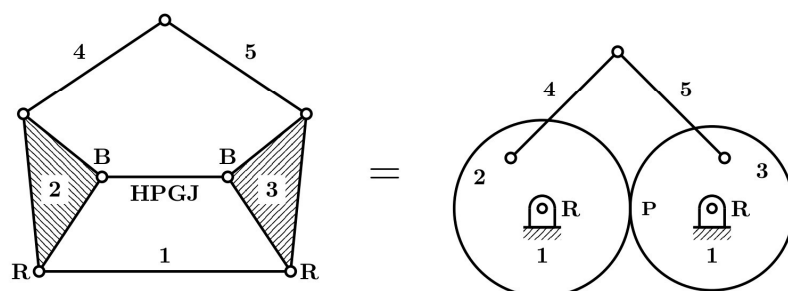
Figure 6. Modeling hydraulic and pneumatic cylinders. Each numbered link corresponds to the same numbered element in the BKC.

The primary use of gears is as a coupling device to transmit power and rotary motion. Like belts, this use is a simple function generation task. The gear pair joint is a higher-pair joint capable of both rolling and sliding contact, indicating the properties a candidate BKC must have. A pair of meshing gears is modeled by a four-bar loop (see Figure 7A). The two gears are on opposite sides of the loop, and the gear pair joint and ground link connecting the gear centers are on opposite sides of the loop. The four pin joints are labeled RRBB, where the Rs correspond to the gear centers of rotation while the Bs correspond to the tangent line common to both gear base circles. The two pin joints labeled B are to be eliminated. Another interpretation for a set of gears was presented in Table 1.2 of Vol 1 [60]. A lower-pair equivalent to a four-bar instantaneously matches the angular velocity ratio of the set of gears where two pivots are at the center bearing of the gears and the other two pivots are at the center of curvature of each gear face.

HPGJ = Higher-pair gear joint



(A) Gear pair machine element is modeled by a 4-bar BKC.



(B) Stephenson I one-degree-of-freedom BKC is model for a classic geared 5-bar mechanism.

Figure 7. Modeling a gear pair and incorporating this element into a BKC. The numbered links of the mechanism correspond to the numbered elements in the BKC on the left. (A) The BKC for the most basic case of a gear pair, two grounded interlocking gears. The ‘fictitious’ binary link B allows the BKC to move with one degree of freedom. (B) A five-bar linkage is modeled using the six-bar Stephenson topology. This linkage may be used to recreate the path of a four-bar mechanism while driving both grounded links [62].

When attempting the inverse problem of substituting a gear set into a BKC, satisfactory 4-bar loops must have at least one binary link. This binary link becomes the fictitious coupler link between the two base circle pin joints. This link is to be replaced by the higher-pair gear pair joint as discussed in transformation law 4 above. Figure 7 shows how gears can be introduced into a variety of BKC. One sketch shows how the Stephenson I linkage can model a geared 5-bar linkage. This supports the theory that geared 5-bar linkages are derived from 6-bar one-degree-of-freedom BKC.

Single-arm planetary gear trains are basically two-degree-of-freedom devices. These are different from the one-degree-of-freedom gear trains and have different BKC. These mechanisms have the structure of a one-degree-of-freedom BKC with the second degree of

freedom provided by rotation of the entire BKC about a single ground pivot instead of a ground link. This pivot corresponds to the common main shaft centerline. A new set of BKC's especially suited for synthesizing planetary gear trains is possible by expanding the RRBB loop to a RARBB two-degree-of-freedom loop. The extra A denotes that the planet gear arm link joint to the ground is considered separate from the input sun gear ground pivot. This makes it appear that the gears are rotating about two different centerlines, which is impossible if the mesh between the gears is to be maintained. This is a low price to pay to keep one canonical set of BKC's.

If a BKC is chosen for a planetary gear train at this stage of the design, then the extra rule of starting with a two-degree-of-freedom chain with 5-sided loops must be kept in mind. More complex mechanisms are formed by adding a binary dyad, one binary link for each additional gear, and one binary link for the fictitious coupler, which will be eliminated by the 4th transformation law later in the design process to model the higher-pair gear joint. In general, a train with N meshing sun gears has N binary dyads, one binary link for the arm, and two links of order $N + 1$, one of which is the planet gear (which is a compound gear in the more complex mechanisms) and the other is the ground link. From these original two-DOF BKC's, the multiple joint BKC form can be derived by performing transformation laws 6 and 7. Figure 8 demonstrates these points. Figure 7.19 of *Vol I* shows several 5-link planetary gear trains enumerated from BKC's [60].

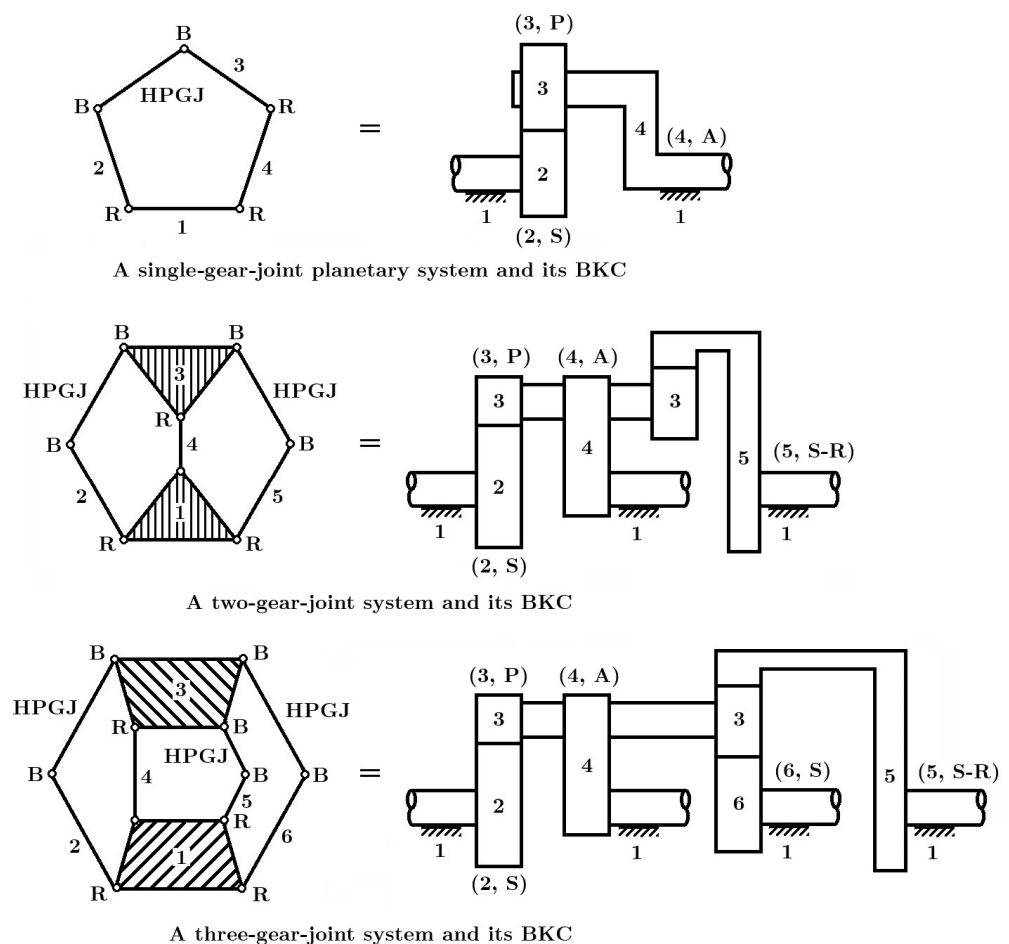


Figure 8. The BKC's of two-degree-of-freedom planetary gear trains. In the BKC's, HPGJ indicates the higher-pair gear joint. 'S' indicates a sun gear, 'P' indicates a planet gear, and 'A' denotes the arm of the gear train. 'S-R' is a ring gear, a special case of the sun gear. The element numbers in the BKC's on the left correspond to the gear numbers in the planetary systems on the right.

Cam-and-follower elements are modeled in a similar manner as gears. The style of the higher-pair joint is identical for both; the gear pair joint is a special case of a cam joint. Therefore, it should not be surprising that the characteristics required in a candidate BKC are like those needed for a gear pair. However, there is one important difference. While a meshing gear set requires a four-bar loop in the BKC, a cam–follower mechanism does not.

Figure 9 shows two examples of cam mechanisms and their BKC. Again, one can conceive of a link between the centers of curvature of the faces of the cam and cam follower with a pin joint on each end as a replacement for the higher-pair cam joint. Section 6.10 of *Vol I* shows how BKC can be applied to cam-modulated linkages—combinations of cams and multi-loop linkages [60].

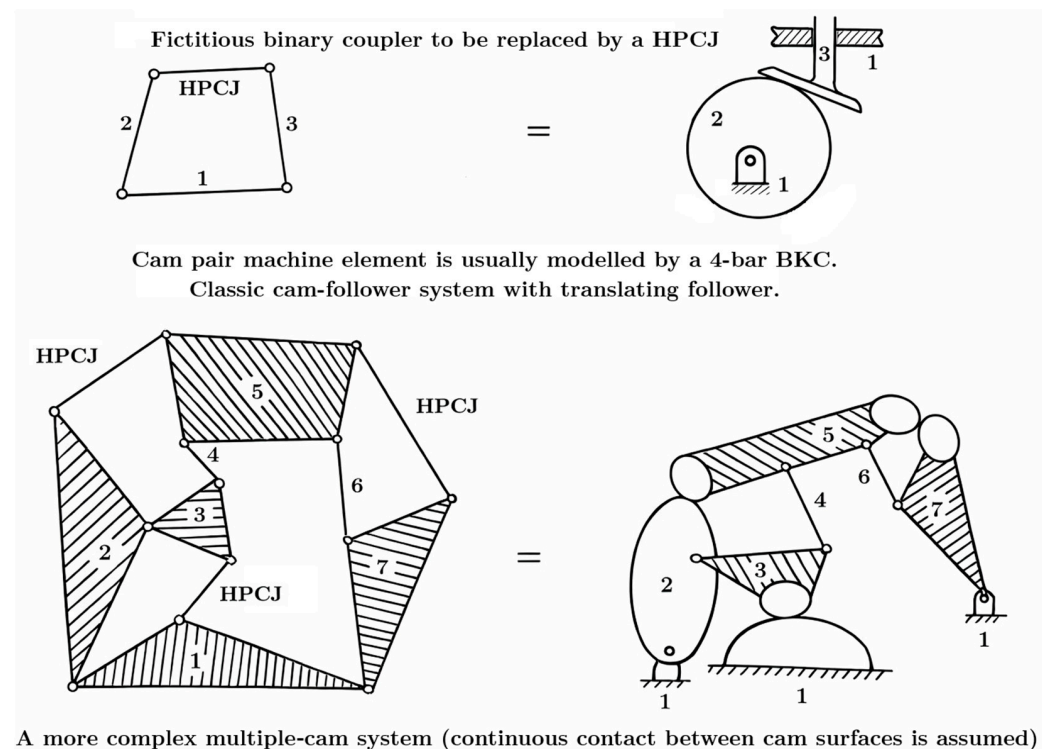


Figure 9. Two different types of cam systems and their BKC. The numbers shown in the BKC on the left correspond to the link numbers shown in the mechanisms on the right.

2.4. Topological Design Selection Principles

Once the set of candidate BKC has been identified, the final step of type synthesis is to select one (or sometimes a few) to synthesize in the dimensional analysis step. What follows are some examples of general design rules that illustrate BKC and kinematic inversion selection methods. Rules of this nature are meant to systematize the experience that machine designers have drawn upon in their work. In addition to hard and fast design rules that restrict the design space, there are several design principles of type synthesis which can inform topological selection. These are not ‘rules’ in a strict sense. Rather, they are a set of tools a designer might use to predict the effect of the choice of topology on certain performance metrics, and in particular, they serve as a method of comparing any two BKC to each other. The first of these principles, compactness, has already been briefly introduced during the literature review.

2.4.1. Compactness

Rao introduced compactness as metric that predicts the minimum space a mechanism can occupy, but it also has significance for the static performance of mechanisms [54]. The compactness is defined as the sum of all elements in the distance matrix. The distance

matrix is a square matrix where each element d_{ij} is the shortest distance (as the least number of joints) between link i and link j . As a result, all the elements along the diagonal $i = j$ are zero, and all other elements are necessarily non-zero—there must be at least one joint between any two links. Consider the Stephenson III topology shown in Figure 2. The distance matrix D_S for this topology may be written as shown in Equation (6). The formulation is different due to a different link numbering scheme, but the result agrees with that reported in [54].

$$D_S = \begin{bmatrix} 0 & 1 & 2 & 1 & 2 & 1 \\ 1 & 0 & 1 & 2 & 2 & 2 \\ 2 & 1 & 0 & 1 & 1 & 2 \\ 1 & 2 & 1 & 0 & 2 & 2 \\ 2 & 2 & 1 & 2 & 0 & 1 \\ 1 & 2 & 2 & 2 & 1 & 0 \end{bmatrix}, \sum D_S = 46 \quad (6)$$

The compactness is calculated as the sum of all the elements in the distance matrix. In this case, for the Stephenson BKC, the compactness score is 46. All inversions of a given topology will have the same overall compactness sum, but not all topologies with equal numbers of links have the same compactness score. For example, the other main 1-DOF six-bar topology, the Watt chain (see Figure 6), has a compactness score of 50, as shown in Equation (7). Again, this result agrees with that reported in [54].

$$D_W = \begin{bmatrix} 0 & 1 & 2 & 1 & 2 & 3 \\ 1 & 0 & 1 & 2 & 3 & 2 \\ 2 & 1 & 0 & 1 & 2 & 1 \\ 1 & 2 & 1 & 0 & 1 & 2 \\ 2 & 3 & 2 & 1 & 0 & 1 \\ 3 & 2 & 1 & 2 & 1 & 0 \end{bmatrix}, \sum D_W = 50 \quad (7)$$

As compactness is defined here, a lower score indicates a more compact mechanism, so the Stephenson chain is more compact than the Watt chain. To be clear, this does not mean that all Stephenson linkages are smaller or more compact than all Watt chains; compactness is not tied to physical dimensions because BKC/topologies are dimensionless. A more helpful way to think about compactness is that if two different topologies are created using the same set of links, the more compact mechanism will usually have a smaller overall footprint. Compactness is applicable over the complete solution set, or the average solution, but does not necessarily hold for a side-by-side comparison of any two dimensioned solutions.

2.4.2. Reach

Recall that kinematic inversions of an isomer are unique mechanism layouts created by grounding different links in a basic kinematic chain. Reach is a topological property that is specific to each kinematic inversion. It is defined as the least number of joints between the furthest link and the fixed frame. The reach of a given kinematic inversion is the maximum value of the corresponding row (the row is selected according to the reference number of the grounded link) in the distance matrix. Consider distance matrix D_W given in Equation (7), the distance matrix for the Watt mechanism using the numbering scheme shown in A. To choose the Watt II inversion, the designer grounds either of the ternary links, which correspond to rows three and four in this case. The minimum number of joints between either of the ternary links and any other link in the topology never exceeds 2. If,

however, the designer grounds any of the binary links (the Watt I inversion), there will always be at least one link that is at least 3 joints away from the fixed frame.

As the number of joints between a given link and the fixed frame increases, the complexity of the motion it could produce also increases. A link pinned to the ground with a revolute joint can only produce a circular path about the fixed frame. The next step up, a link spaced two joints from the ground (like the coupler of a four-bar linkage), is constrained on at least one joint to a circular path about the fixed frame. Even so, referring to the four-bar atlas prepared by Hrones and Nelson, this combination can already produce complex motions [32,33]. In the case of a link that is at least three joints removed from the fixed frame, none of the joints constraining that link is necessarily restricted to a circular path, greatly amplifying the theoretical complexity of its motion. This effect continues as reach continues to increase.

The reach can be viewed as the ability of a mechanism to unfurl from some compressed position. A kinematic inversion with a long reach will be able to use more of the length of each link to stretch away from the ground pivots. A scissor-lift mechanism is a good practical example of this principle, as seen in Figure 10. The linkage looks like a series of letter Xs stacked on top of each other, allowing the system to convert a relatively small lateral motion at the base of the mechanism into a large vertical movement. It has a remarkable ability to compress into a small space and then unfold into a large structure. Despite only having 10 links (taking each slider as a separate link), the linkage has an impressive reach of 5 joints. The ability of a linkage to compress is a design property and is not guaranteed by a large reach; defects like change points or branch defects can prevent a final dimensioned design from collapsing into itself. With that said, choosing a high-reach topology is a good place to start when designing collapsible linkages.

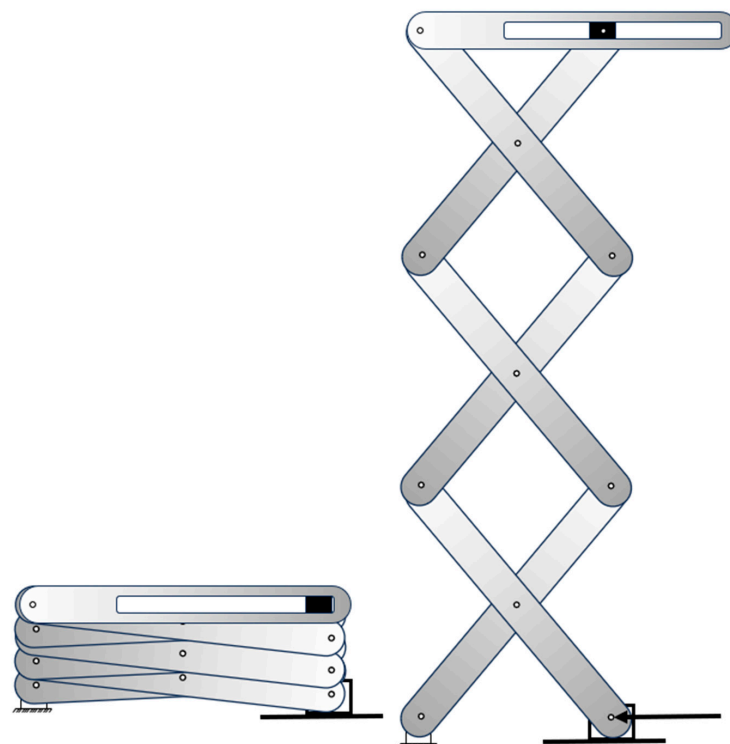


Figure 10. A 10-bar scissor-lift mechanism is shown in a collapsed and partially extended form. For the purposes of the BKC, this topology could also be considered an 8-bar topology with two pin-in-slot F_2 joints. The linkage has a reach of five and could be further extended by adding an additional 'X' on top.

It is also worth noting that a large reach can come at the cost of stability and static performance. In its applications, the scissor lift is generally used to raise people and equipment up to a target, like an overhead light that has gone out. As a result, the forces it experiences are mostly directed straight down the structure into the ground pivots, making it reasonably safe for workers. If any lateral forces were applied at the top while the lift is extended, though, they would create tremendous torques about the base of the mechanism. Stability is discussed in more detail in the following section.

2.4.3. Stability

Stability is an important kinematic property for any mechanism, but especially for those tasked with lifting heavy loads. Typically, this property is applied to low-speed and high-force applications where static forces dominate. That puts it somewhat at odds with dynamic performance, where inertial forces dominate.

Stability, like any of the other properties mentioned here, is influenced by the dimensional synthesis step and the manufacturing quality of the physical mechanism, but the topological selection still has various impacts on the performance of the constructed mechanism. The first is the number of connections to the fixed frame (i.e., ground pivots). Adding more ground connections allows for the forces to be more widely distributed across the fixed frame.

The second is the connectivity of the links. In most cases, binary links may be approximated as two-force members where loads are only carried axially between their two joints. As soon as a third connection or force is added, though, the link can exert forces in other, non-axial directions. Using a ternary link for the fixed frame is a great way to increase stability, as more of the forces are directed into the frame.

2.4.4. Dynamic/Static Performance

Dynamic performance is a key property to consider for any mechanisms that will operate at high speeds. For many mechanisms, this property can be wholly ignored with few consequences, as at low operation speeds static force considerations will dominate to such a degree that dynamic forces may be treated as negligible. At high speeds, though, forces associated with the mass of the links (inertia, angular and linear acceleration, gravitational forces on the center of mass) will begin to dominate, or at least become significant enough that they may no longer be neglected. While mechanisms are inherently not static, the term ‘static performance’ is used to describe low-speed operation of a mechanism, or performance in a stationary configuration when motion has paused. Under these conditions, a free-body diagram based on an instantaneous snapshot of a moving mechanism will be dominated by static forces.

Dynamic performance will be heavily influenced by the construction of the mechanism, as choices like using lightweight materials or making higher-order links as frames rather than solid links can greatly reduce dynamic forces. The dimensional synthesis step will also be significant, as long links add greater mass and increase torques, and issues like branching or mangling links due to poor transmission angles will become more severe at high speeds. Even so, the type synthesis step influences dynamic performance.

Rao and Jagadeesh, in comparing the results of two of their studies, find that linkage topologies comprising more higher-order links (greater connectivity, ternary, quaternary links) will tend to have better dynamic performance, but poorer static performance [54,58].

2.4.5. Complexity and Manufacturing

This category is perhaps the most straightforward. As the number of links increases, so too does the complexity of synthesizing and eventually constructing the mechanism. Similarly, using specialized joint types like F_2 joints, or requiring that a certain revolute

joint be replaced by a slider, will also increase the synthesis complexity and challenges associated with constructing the mechanism. Pin joints are always to be preferred over sliding and higher-pair joints. Pin joints enjoy lower initial and maintenance costs, frictional losses, wear, noise, and heat buildup than other types of joints. This should not restrict a designer from using sliders or higher-pair cam joints, because sometimes these are the only joints that will satisfy the needs of specific problems, but it is generally worth looking for a pin-jointed solution first.

One area of analytical complexity to emphasize is the number of transmission angles present in the mechanism. A four-bar mechanism contains a single transmission point where the coupler meets the follower link. The angle of this joint should be kept as close to 90 degrees as possible throughout the motion to maximize the efficiency of force transmission. As the angle dips closer to 0 degrees, more of the force being transmitted through the two-force binary links is directed into the ground pivots or into deforming the links (stretching/compressing). As the number of links in a mechanism increases, so too does the number of instances of relevant transmission points.

Consider the Watt II, which resembles two four-bar mechanisms attached together via a shared ternary link in the center. Each of these four-bars has its own transmission angle that varies independently of the others except in special case geometries like the Chebyshev parallel-motion linkage that is created using cognates. The transmission points also have the added cost of introducing new places where branch or change-point defects can form. These defects are far from guaranteed to appear in a synthesized mechanism, but the probability that they will appear in a synthesized mechanism increases with each new transmission point. The primary consequence is that the analytical difficulty of finding a solution free from defects and containing favorable transmission angles throughout its motion is greatly increased for high-link-count topologies. The benefits of the extra links will frequently outweigh the cost associated with the extra transmission angles, but it is a significant oversight to imagine that effective dimensional synthesis of a complex topology is as simple as evaluating a couple of extra loops.

Another cost with increasing the number of links is that the manufacturing tolerances stack up as the number of parts increases. With more links, the precision of the output will decrease. Precise construction can mitigate these losses, but decreasing tolerances will also make the project more expensive.

2.4.6. Kinematic Task, Six-Bar Examples

Not all inversions of a given BKC are equally effective at solving a given kinematic task. Perhaps the most demonstrative example of this is a comparison between the Watt I and Watt II mechanisms. Though they share the same BKC, the two inversions are strikingly different in their optimal applications. The Watt II is equivalent to a four-bar mechanism with an attached driving dyad. This makes it no more effective for motion or path generation tasks than a four-bar mechanism, except for the specific situation of enabling a crank input of a rocker-rocker four-bar mechanism. For the function generation task, however, the Watt II is quite effective. Its dual four-bar topology allows it to model complex function relationships between input and output. The Watt I, on the other hand, is perhaps the most effective of the six-bar topologies for motion or path generation tasks. As discussed above, it has the longest reach of any 1-DOF six-bar mechanism, enabling complex motions and substantial travel distances relative to the length of the links. The base loop of the Watt I between the two ground pivots is the same as a generic four-bar mechanism. This means that for the function generation task, the Watt I has the complexity and transmission points of a six-bar mechanism, but offers no benefits over a four-bar mechanism for the actual kinematic task.

In addition to its long reach, the Watt I BKC is also capable of creating parallel motion through the Roberts–Chebyshev Cognate Theorem, which is unique among the 4- and 6-bar BKC (and which is preferred over the 8-bar BKC shown in Figure 11, composed of two identical 4-bar mechanisms connected by a tie bar).

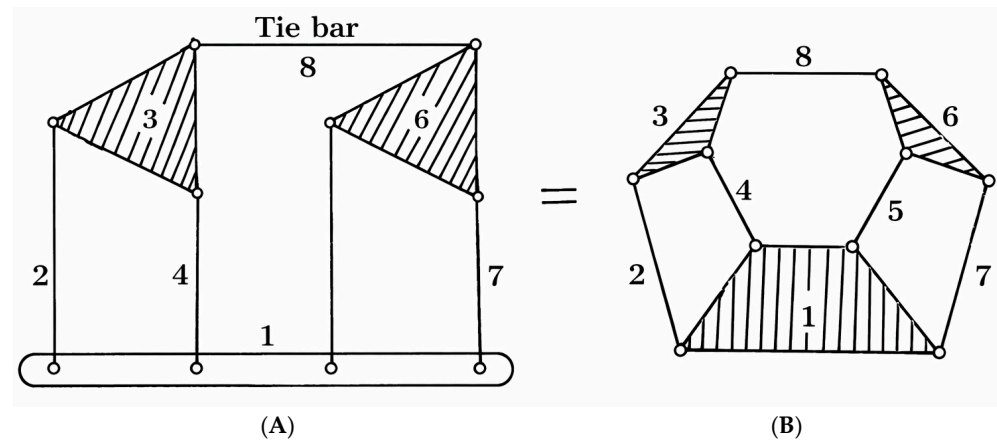


Figure 11. (A)—Two identical 4-bar mechanisms tied together produce parallel motion. (B)—It is possible to achieve change-point motion from this 8-bar BKC.

Parallel-motion generation is one of multiple specific problem types in which the required kinematic task dictates the ultimate choice of topology. Another example is a dwell linkage, where one classic solution is created with the Stephenson III BKC (see Figure 12). A path generator 4-bar linkage is synthesized to obtain a pseudo-straight-line or circular-arc coupler curve. An output (dwell dyad) is attached to this coupler point using either a slider or a straight binary link. This is an instance where a path generator drives a function generator, which is the opposite of the typical case. Figure 12 depicts the strategy for choosing the BKC used in dwell linkages, in which link 6 has a dwell as slider 5 moves along the straight-line portion of the coupler curve of the embedded 4-bar mechanism.

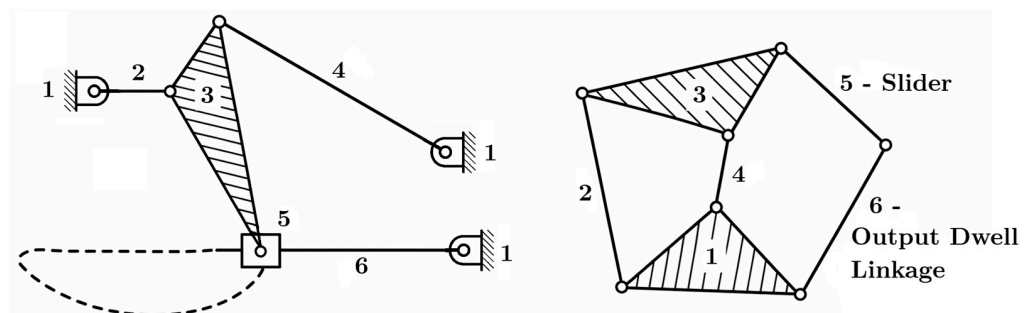


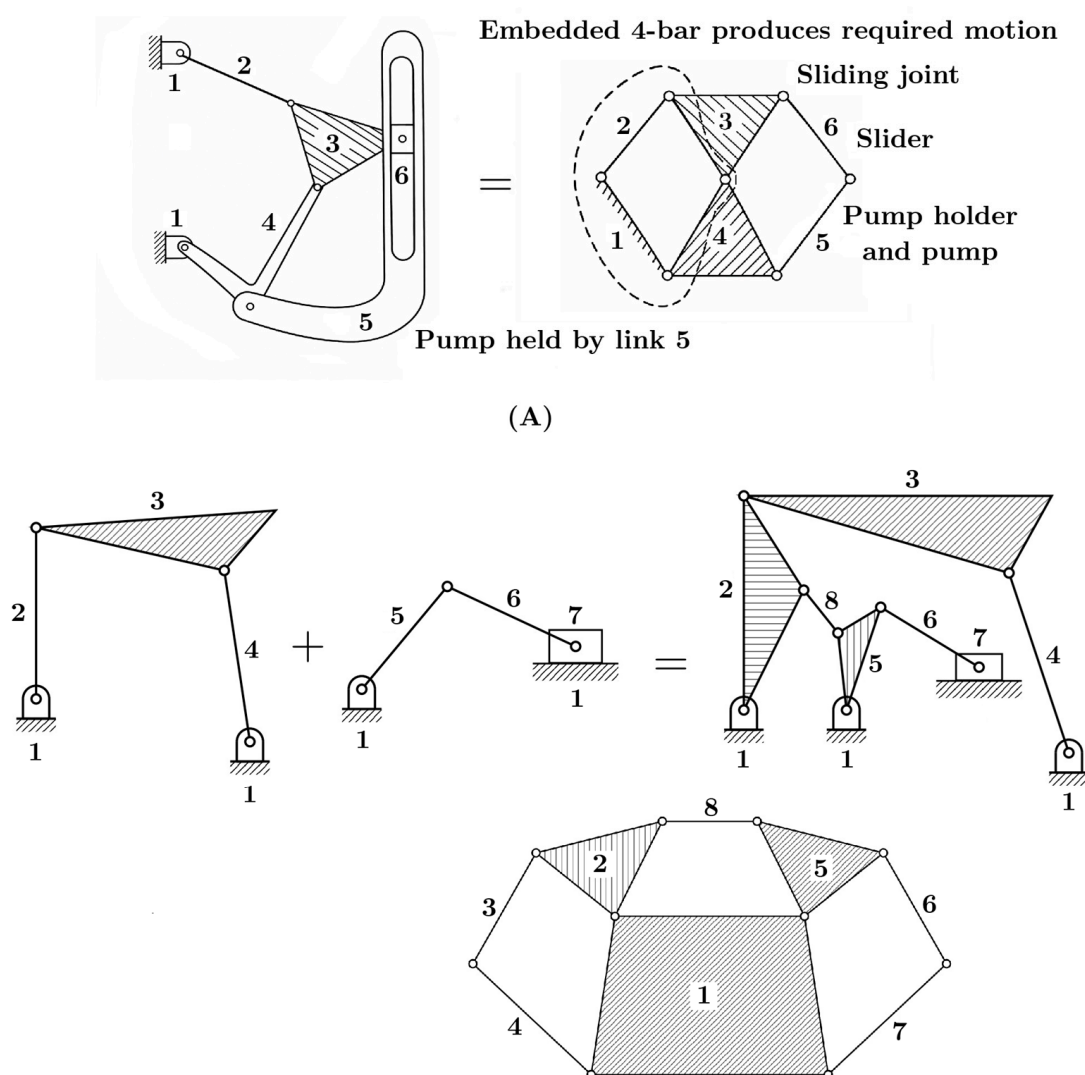
Figure 12. The Stephenson III BKC is the classic dwell linkage topology. Here, link 6 dwells during the straight-line segment of link 5's movement. The link numbers of the mechanism correspond to the element numbers in the BKC on the right side.

Multiple-task machines perform several tasks with the same linkage. The most common method used to design such systems is to use a separate one-degree-of-freedom machine for each task and then tie all these small machines together with a belt drive or other function-generator mechanism to form a large one-degree-of-freedom machine. While simple in concept, this is frequently a wasteful and unnecessarily complex means to fulfill the system requirements.

Machines with many elements are more expensive to build and maintain and require more energy to drive. The BKC of such systems are composed of many simple sub-BKCs concatenated into one large, complex BKC. A more elegant and simpler solution is to create

a mechanism in which different parts of the mechanism perform the various required motions. For example, using two separate one-degree-of-freedom mechanisms for a dual-task problem requires at least an 8-bar BKC. This same task might be solved by a 6-bar BKC. Undeniably, this type of mechanism is more difficult to synthesize, but the potential benefits are worth the effort in considering this approach.

One clever dual-task mechanism was based on the Watt I BKC (see Figure 13). The problem was to deposit material from a pump along a particular path. The design team synthesized a 4-bar linkage to obtain the required path during which the angular velocities of the coupler and follower links rotated in opposite directions. Then, an extra binary dyad was superimposed onto the original coupler and follower links to activate the pump at the appropriate time in the cycle. The pump was attached to one of the two new binary links, and a slider with sliding joint replaced the other binary link and directly activated the pump. Figure 13A shows a labeled BKC and a sketch of this solution, along with a typical strategy of combining two 4-bar BKC into an 8-bar BKC (B).



(B) Two 4-bar linkages responsible for one task each joined together to form a dual-task machine.

Figure 13. The link numbers on the left side correspond to the element numbers in the BKC on the right side. (A) A pump mechanism topology accompanied by its simplified BKC. (B) Two four-bar chains joined together into a single larger mechanism and an accompanying simplified BKC.

The Watt II BKC is frequently used as a function generator or includes a function-generating component. This use is strongly suggested by the topology, which is two 4-bar chains joined together through a bell crank. In other applications, the second half of the BKC is used to obtain the motion or path characteristics, and the first half is a function generator used to either limit the output motion or ensure that a crank can drive the machine. This function-generator portion is frequently referred to as a driving dyad, but an entire 4-bar solution is synthesized. One limitation of this topology is that it cannot supply motion any more complex than a 4-bar topology. However, this topology works very well as a force or motion multiplier. The function-generator side of this topology can drive the remaining links through a motion that would not be possible for the 4-bar alone (often due to poor transmission angles). Figure 14 shows such a situation where the output agitator link in a washing machine transmission rotates through an angle of 210 degrees for each half-revolution of the input crank.

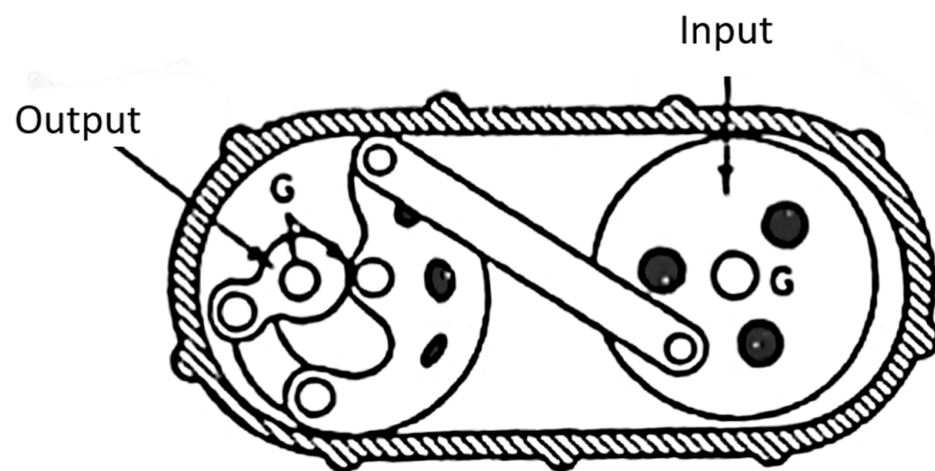


Figure 14. A Watt II BKC washing machine transmission is used to multiply motion. The points labeled 'G' indicate pinned ground connections.

The Stephenson III BKC finds favorable usage in motion and path generation tasks that require a small ground link and good transmission angle capability. The ternary ground link is also more stable than a binary ground. The small ground link characteristic allows the linkage to achieve a longer reach relative to its base. Poor transmission angles in the embedded 4-bar BKC can be overcome by attaching a driving dyad directly to the coupler. The ternary coupler link also makes the coupler more stable and able to support/provide torques.

Figure 15 shows a mechanism derived from an 8-bar BKC (counting the spring as a driving dyad). The overall linkage has an embedded Stephenson III BKC that guides an automobile hood. The hood motion is created by the 4-bar portion of the Stephenson BKC. The extra dyad was included for force stability reasons and to introduce a base for the counter-balancing spring.

2.4.7. Methodology Summary

From the fundamentals of type synthesis, a strong understanding of their problem definition, and a quantitative understanding of topological properties, a designer may arrive at a consistent procedure for type synthesis. That procedure is recorded at a high level in Figure 16. The following sections will provide a few examples showing the type synthesis procedure at work in real design situations.

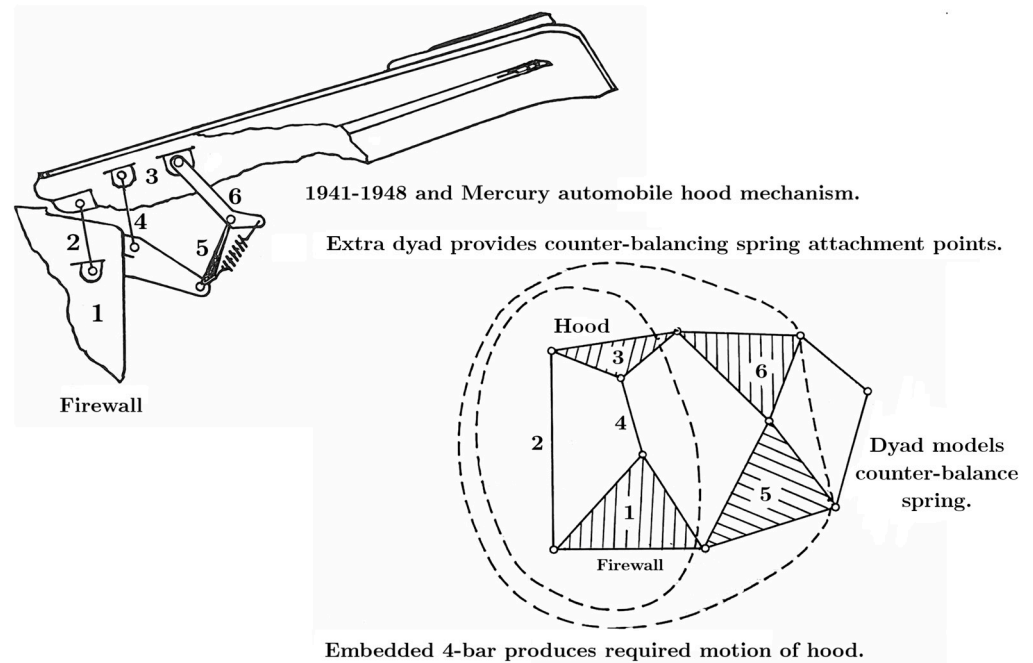


Figure 15. A Stephenson III BKC-derived mechanism to open a car hood. With the added spring, the BKC may be modeled as an equivalent 8-bar mechanism. The link numbers shown in the mechanism correspond to the element numbers in the BKC.

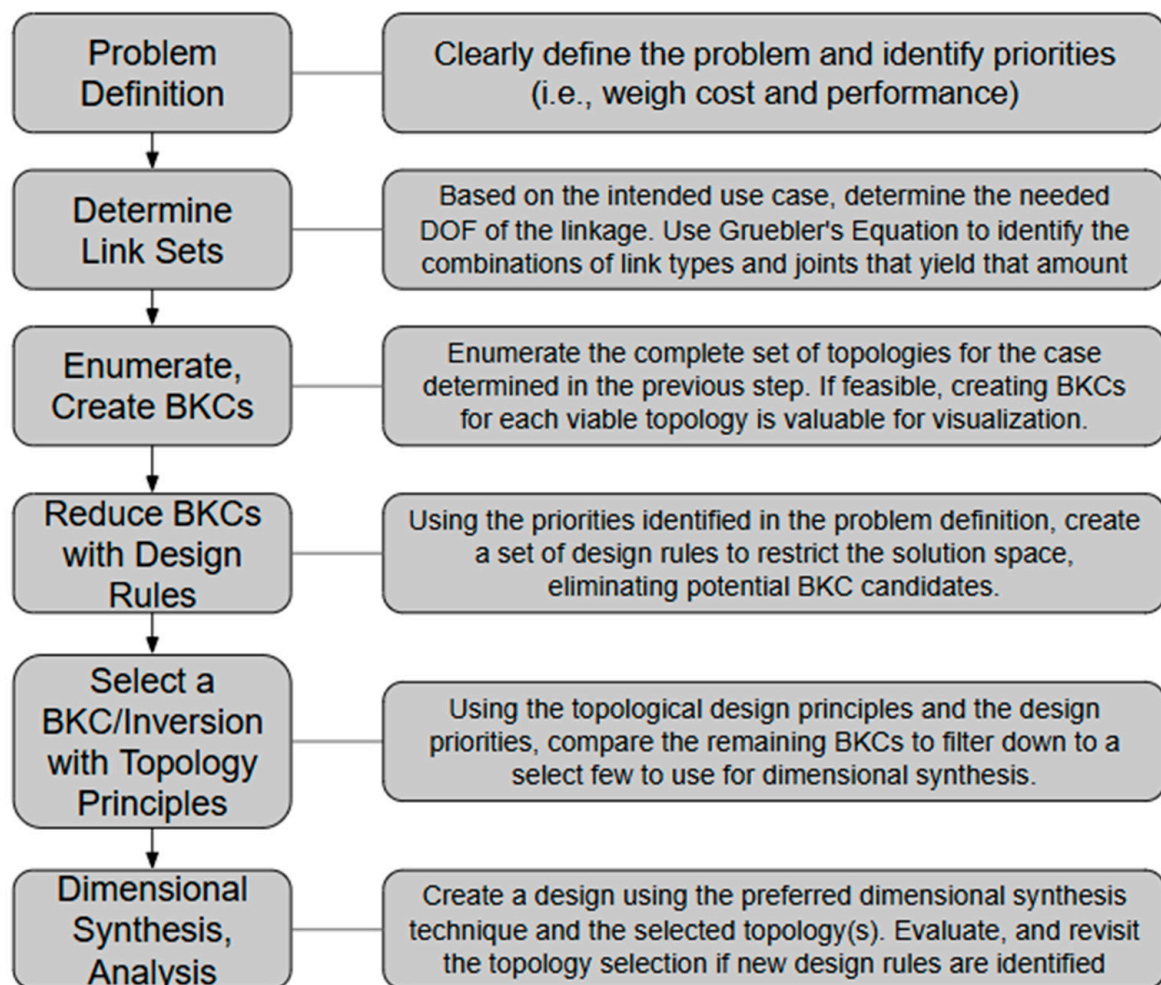


Figure 16. A flow chart reiterating the main steps in the type synthesis process.

3. Results—Design Examples

3.1. Design Example #1—Bookcase Cover

This example should serve as an illustration of the entire type synthesis process. It is desired that a bookcase should include a motor-driven front glass cover to keep out dust. The cover should rotate 90 degrees and translate backward into the upper section of the bookcase over the books, as depicted in Figure 17. The overall design of the linkage should be as small as possible to be unobtrusive, but also must be stable and relatively quiet.

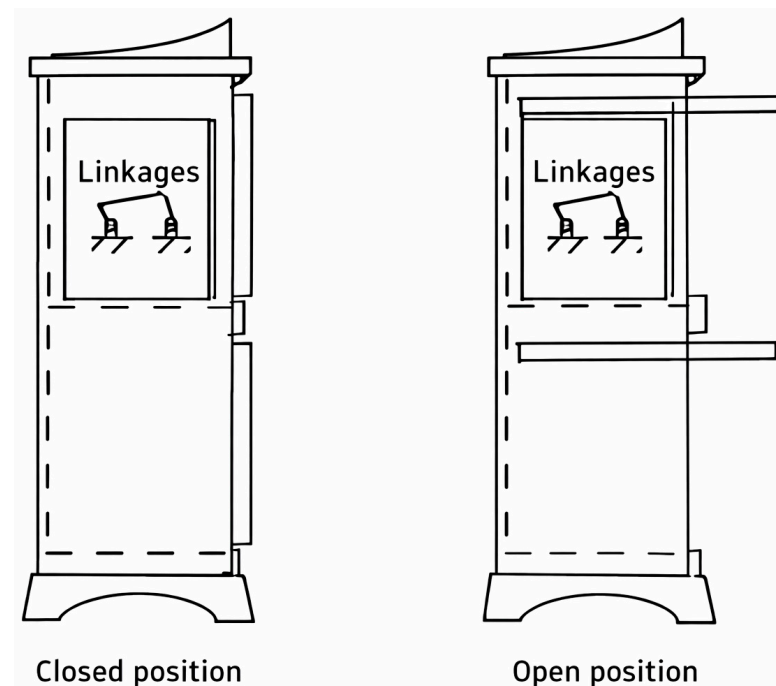


Figure 17. Required motion of the bookcase covers.

The statement above is representative of the information one can expect when beginning the design process. A few important characteristics and constraints are implied by the need statement above. First, the mechanism has a single degree of freedom. This is known because a single controlled input is to result in a single controlled output. This will limit our type synthesis effort to BKC's with an even-number-of-links and a single-degree-of-freedom. Second, the kinematic task is motion generation, given the need to control a floating link along a desired path and orientation. This sets specific requirements as to which link in a candidate BKC can act as the cover with respect to the ground link. Third, the input link is to be a crank to allow a motor to drive the linkage. The transmission angles and input torque required to drive the cover is a concern. Finally, link lengths and ground pivot locations cannot intrude into the bookshelf space, nor should they require extensive modifications to the bookshelf structure.

The comments listed above that are necessary for the solution of this task suggest the following design rules:

- (R1) The cover link must be capable of complex (non-circular) motion.
- (R2) Short members are needed to support the weight of the cover.

A corollary of rule 2: A slider link between one end of the cover and the bookcase may be advantageous.

Rule #2 and its corollary are lumped together because they address the same design point. First, long strings of links spanning between the ground link (bookcase) and the cover should be avoided. Such a construction would require large cross-sectional dimensions of the links to provide bending resistance along with very accurately machined joints. Large

link depth dimensions would be difficult to fit into the small space available. Second, a long string of links is more expensive to construct and would require a considerable amount of space to fold into the available space. Third, a slider can more easily support the cover weight because it can slide directly on the bookcase frame, and the required translational motion of the cover strongly suggests a sliding component. We could relax this constraint at a future time and study the resulting mechanisms if it is deemed worthwhile.

(R3) The mechanism must create a force couple (applied torque) on the cover.

This rule arises because the cover will be controlled by a single crank input. However, a simple single-loop crank mechanism chain can supply only a simple torque to the cover via one side of the mechanism, resulting in poor motion transfer at some point during the duty cycle. Requiring a second, concurrently actuated input would introduce an unnecessary complication upon the user and therefore is undesirable. The strategy, then, is to create the force couple by splitting the single user-supplied input motion within the mechanism. The entire mechanism will still have only one degree of freedom, and the user supplies only one input, but internally the mechanism creates a force couple upon the cover.

Based on these design rules, topological rules are created that define specific structural requirements that a successful BKC and mechanism must possess. Note that a different set of design rules will create a different set of topological rules, resulting in different sets of candidate BKC. Topological rules are designated TR#. Some rules are specific to simple gear train inputs. These rules will be designated as TR#-SG.

(TR1) The cover link cannot be adjacent to the ground.

TR1 directly follows from rule R1, as a grounded link is only capable of creating circular motion with respect to the ground, and the cover must follow a complex path. A dyad separating the cover and ground is initially considered adequate.

(TR2) The ground link should not be binary.

TR2 follows from R3, which requires a force couple be applied to the cover. This feature will be easier to achieve with more ground pivots to distribute forces to.

(TR3) The cover link should not be binary.

TR3 stems from R2 and R3. Increasing the number of links supporting the cover can decrease its deflection and better support its weight. These support links form chains spanning from ground to the cover. Although it is feasible that a force couple can be generated on a binary link cover, it is easier to create this force system on a higher-order link.

(TR4) A multi-loop BKC is required. This follows from R3. Creating a force-couple system with a single input requires a topology that splits the input into two chains, each driving the cover. The links that drive the cover should be part of separate loops or redundant sections of the chain that serve no purpose will appear. Multi-loop single one-DOF BKC always have higher-order links.

At this point, we can start to describe the form that successful BKC will have. They are multi-loop mechanisms with at least three higher-order links (ground, input crank, and cover). Also, the cover link is a floating link not adjacent to the ground, but a slider can be adjacent to the ground and could also be joined to the cover.

A gear driver system can split the input torque within a mechanism. The special rules required for a gear input driver are listed below. These are in addition to the above list.

(TR-SG1) A four-bar loop must be adjacent to the ground. The coupler of this four-bar must be a binary link.

This is to form an RRBB loop. Recall that the Rs correspond to the gear centers of rotation, while the Bs correspond to the tangent line common to both gear base circles. The two pin joints labeled B are to be eliminated. Also, note that both gears are pinned to the ground, so neither is a sector gear on the ground. This could be relaxed if a sector gear on the ground is deemed to be feasible.

(TR-SG2) The two geared links must be higher-order links.

Four higher-order links result. This is necessary to ensure that the gear train is useful. Both gears are needed to drive different loops of the BKC, which ultimately drive opposite ends of the cover. The higher-order links are the ground, two gear input cranks, and the output cover.

Finally, a few dimensional rules can be listed here. These influence BKC selection to a far lesser degree than the rules above, but these points are deemed to be of such importance that they are collected at this point in the discussion.

(DR1) The use of short links and sliders is preferable due to deflection considerations.

(DR2) Neither the ground nor the cover should be sector gears unless they are of small size.

(DR3) Achieving a stationary position when the cover is fully open would be ideal.

We will apply these design rules to the BKC's, modeling all as one-degree-of-freedom mechanisms up to eight links. We are endeavoring to supersede the performance of the slider crank chain. However, this mechanism does have many of the desired parts, a crank input, a slider shoe on the bookcase, and a cover link. However, the BKC is not complex enough because it is only a single-loop chain capable of producing only a simple torque upon the cover. The six bars are the first multi-loop single-degree-of-freedom chains. The five six-bar topologic inversions all fail for a variety of reasons. These specifics are listed to document the thought process and to be able to return to these options if any of the violated design rules are relaxed later.

Stephenson I fails because both the ground and cover links are binaries.

Stephenson II fails because the ground is a binary chain, and the loop adjacent to the ground is a five-bar.

Stephenson III fails because the ground link would be a sector gear.

Watt I fails because the ground would be a binary sector gear and the cover a binary.

Watt II fails because the cover would be a binary, and one gear is not utilized.

As noted in Table 2, there are 16 eight-bar BKC's and 71 unique kinematic inversions of these base topologies [61]. After eliminating topologies that violate the rules above, there are only three mechanisms out of the possible 71 that warrant further attention. The three inversions that satisfy the design rules are labeled in Figure 18. The 'HS' designates the fictitious binary link, which models the higher-pair rotational/sliding joint between the gears. A similar set of rules for planetary gear train drivers yielded only six eight-bar mechanisms. Other types of drivers might yield additional candidate mechanisms. At this point, the type synthesis process is completed, having uncovered all the forms and arrangements that apply to the desired purpose. Dimensional synthesis efforts may now be employed to determine optimal link lengths to complete the full-dimensional synthesis of this mechanism.

3.2. Design Example #2

One application of type synthesis is to confirm that a design is adequately differentiated from a competing product or design. Using a unique topology certainly wouldn't cover the copying of any proprietary technology, but from a kinematic perspective, using a different topology ensures the underlying linkage solution is unique from any competing design. This case is certainly more compelling than slightly changing some dimensions in an existing design and claiming it is a new product.

This example is inspired by a similar study in [60]. Another similar type analysis example on vehicle suspension systems is shown in [63]. In a use of type synthesis methodology like that just presented, a new rear suspension for a motorcycle was desired. In general, the rear wheel suspension of a motorcycle is a four-bar linkage. However, such a design cannot

provide a large wheel travel with a variable leverage ratio. Therefore, the concept of six-bar linkages has already been adopted for the rear suspension of off-road motorcycles, such as the Honda CR250R Pro-link. (Figure 19A), the Suzuki RM250X full-floater (Figure 19B), and the Kawasaki KX250 Uni-track (Figure 19C). It is desired to find any other six-bar solutions that are different from these three.

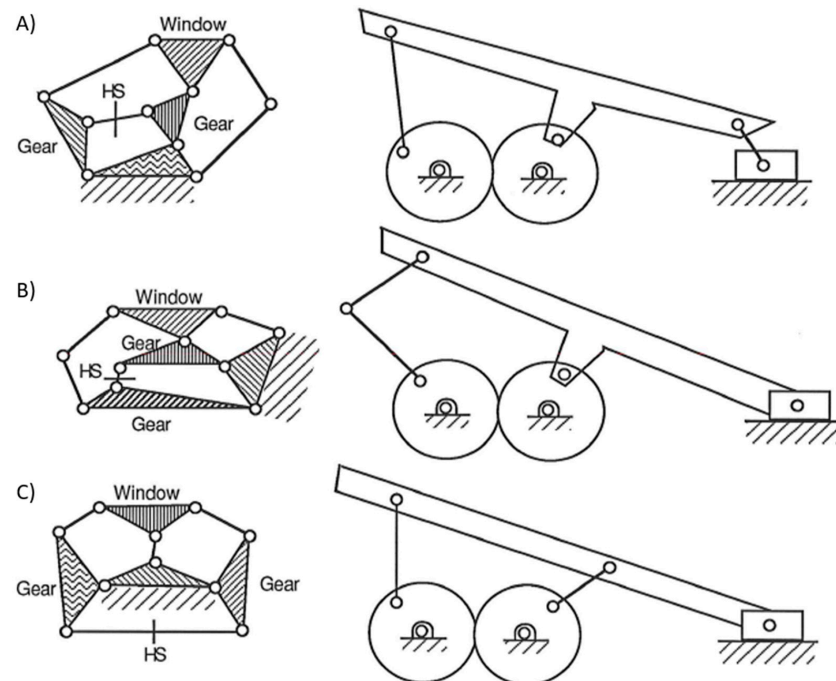


Figure 18. The 3 candidate topologies for the cover guiding mechanism. On the right is one sketch of how a mechanism with that topology might look; note that dimensional synthesis has not been completed for these example sketches.

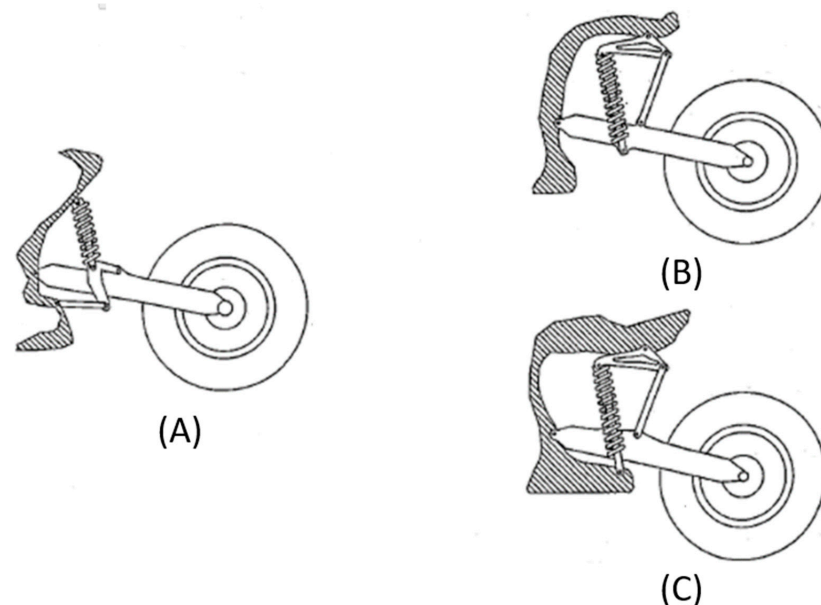


Figure 19. Various candidate suspension designs for a motorcycle rear wheel. The three suspensions have many similarities, but each uses a slightly different topology to accomplish the suspension task. (A) A model of the Honda CR250R Pro-Link motorcycle suspension. (B) A model of the Suzuki RM250X Full-Floater Suspension. (C) A model of the Kawasaki KX250 Uni-Track Suspension.

*Similar versions of Figure 19A–C were previously published in the *Advanced Mechanism Design Vol I* textbook [60]. For their original production, they were adapted by former student R. Rizq from the study shown in [64], H.S. Yan’s ‘A Methodology for Creative Mechanism Design,’ 1985. In that paper, Yan uses number synthesis methods to derive 98 new wheel-damping mechanism designs.

The following process is followed:

(A) Determine the type of six-bar linkage that it represents (the wheel is not considered a link).

(B) Using single-degree-of-freedom six-bar BKC’s only, create as many other suspension designs as possible that are different from the three that are given.

(C) The following design requirements apply:

(R1) There must be a fixed link as the frame.

(R2) There must be a shock-absorbing spring or hydraulic cylinder for the suspension to absorb impacts. It will be modeled as two binary links, so there must be a binary dyad chain in the BKC.

(R3) There must be a swing arm supporting the rear wheel, and the swing arm must be adjacent to the fixed frame to adequately support the wheel.

(R4) The fixed link, the shock absorber, and the swing arm must be distinct members.

Solution:

(A) Referring to the three designs in Figure 19, Design A has three ground pivots and the ternary links are separated, so it is a Stephenson III. The ternary links are also separated in Design B, with both connected to a binary ground, so this is the Stephenson I design. The ternaries are adjacent in Design C with a ternary ground, yielding a Watt II six-bar mechanism.

(B—R1) Any of the Stephenson and Watt BKC’s satisfy these criteria.

(B—R2) Figure 20 shows all the remaining possible link and element arrangements after Rule 2 is applied. These must adjacent binary links (i.e., a dyad chain) that may be replaced by a suspension spring or cylinder. This element is denoted as Ss in Figure 20.

(B—R3, R4) These rules require that the swing arm is adjacent to the ground and that each element is unique. After applying the final design requirements, only six designs remain (Figure 20). The element layouts that failed Rule 3 are crossed out.

Figure 21 shows unscaled kinematic diagrams of the six surviving suspension topologies. Note that Figure 21b is the Kawasaki KX250 Uni-trak suspension, Figure 21e is used in the Honda CR250 Pro-link suspension, and Figure 21f is the Suzuki RM250 full-float suspension. Therefore, there are three new types of six-bar rear wheel suspensions for off-road motorcycles as shown in Figure 21a,c,d.

3.3. Design Example Number #3

The principles of type synthesis presented here have a strong qualitative backing, and in the case of special topological properties like compactness and reach, have some numerical backing as well. Even so, it is valuable to provide a quantitative demonstration of the principles to further cement the key points. The intent of this final design example is to provide some preliminary quantitative evidence supporting the design principles and topology selection guidelines identified in Section 2.4.

Type synthesis is only one portion of a complete kinematic synthesis process. This makes it difficult to isolate the impacts of type synthesis on the final mechanism design. Even while holding topology selection constant, the dimensional synthesis step can produce outputs with vastly different kinematic properties depending on the input variables. Examining a single candidate solution, it would be nearly impossible to accurately attribute some percentage of a given property to the dimensional or type synthesis steps. Given

this limitation, the principles will be demonstrated through a quantitative analysis of the two topologies relative to each other. The Watt I and Watt II topologies will be compared side-by-side using an identical grid search for the same kinematic task in order to minimize the impact of the dimensional synthesis step. The problem parameters are provided in Table 4. The Watt I and Watt II, shown in Table 5, are two kinematic inversions of the same topology, and have the same number of links and joints. Despite their similarities, as noted in the earlier analysis, they exhibit significantly different behavior for properties like reach, compactness and stability.

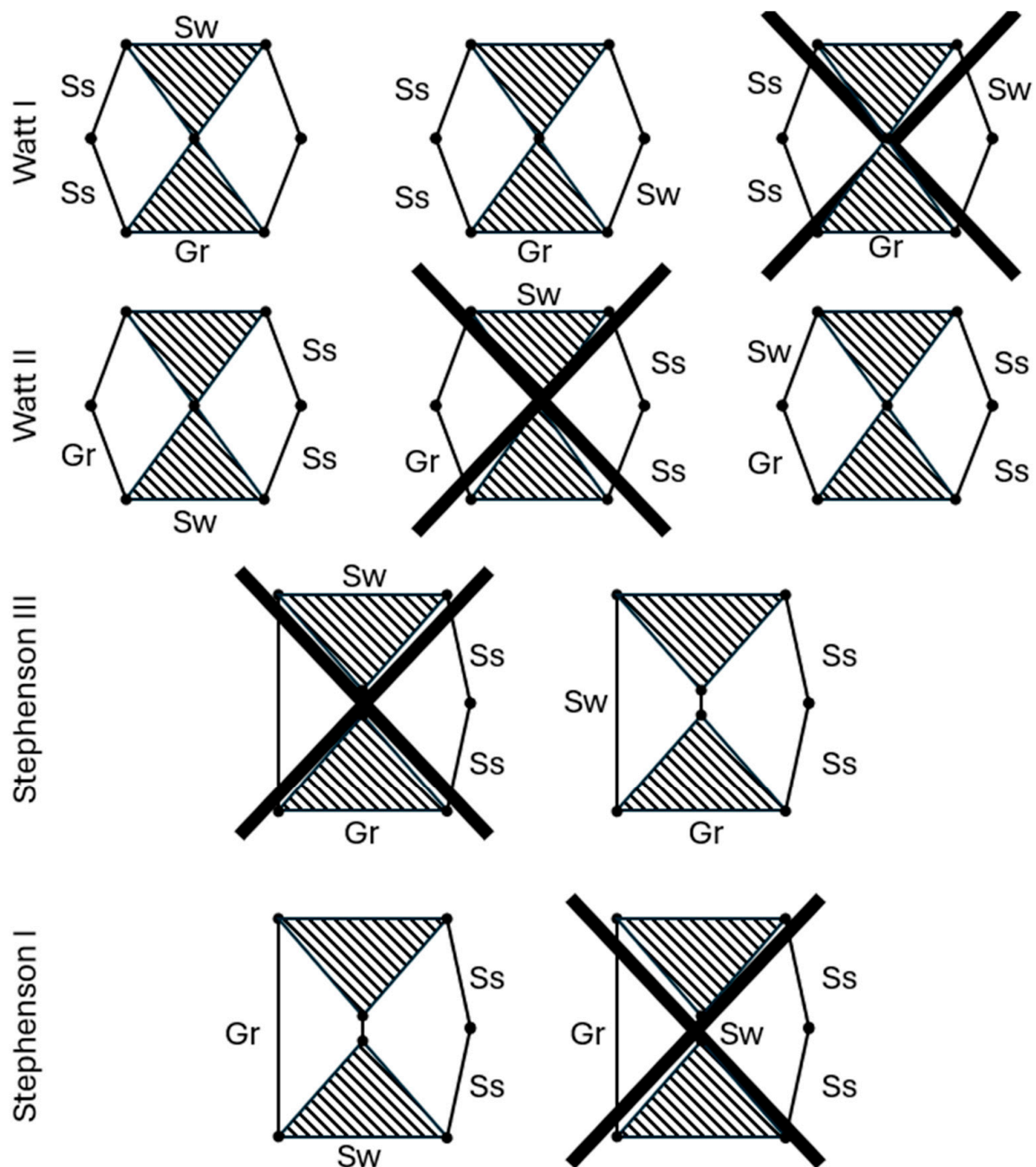


Figure 20. Kinematic diagrams—the topologies that do not meet the requirements are eliminated from consideration. Here Sw indicates the swing arm which holds the wheel, Gr indicates the frame (ground) of the motorcycle, and Ss refers to the two-element suspension spring. The elements that fail design rule 3 (swing arm adjacent to ground) are crossed out in the figure.

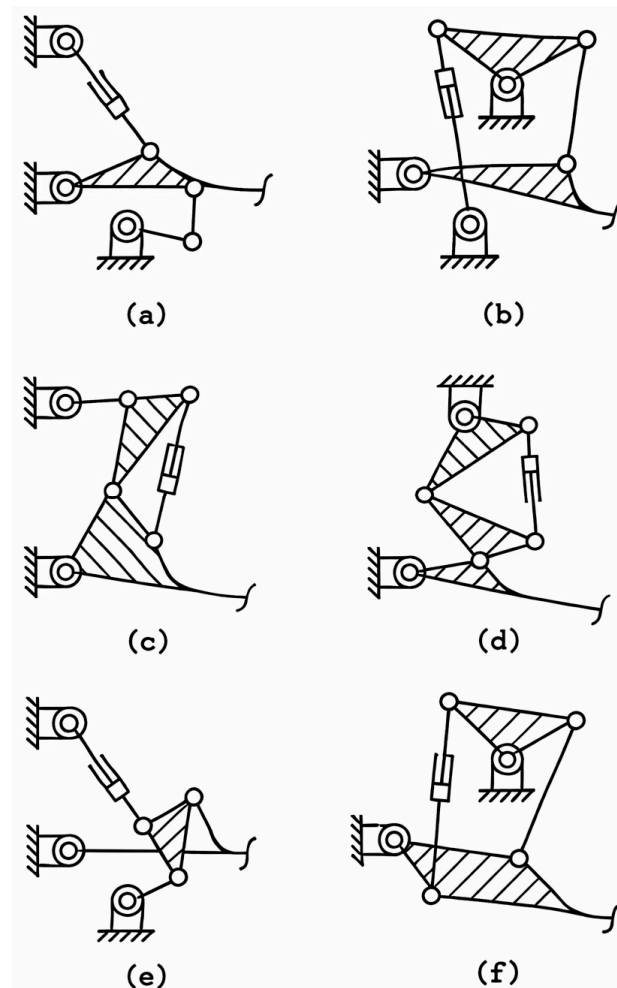


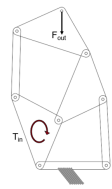
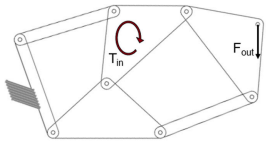
Figure 21. Six surviving suspension topologies. (a) A Watt II topology with a suspension spring between the frame and swing arm. (b) A Watt II topology with a suspension spring between the frame and a dyad that supports the swing arm. (c) A Watt I topology with a suspension spring between the swing arm and an adjacent link. (d) A Watt I topology with a suspension spring stretched between two connected ternary links, both of which are attached to the frame. (e) A Stephenson III topology with a suspension spring between the frame and second ternary link. (f) A Stephenson I topology with a suspension spring between the two ternary links, directly supporting the swing arm.

For the example, a problem situation is designed which requires a high horizontal reach. The Watt I topology has better reach than the Watt II topology, so the expected result is that a more compact solution will be found with the Watt I, and that averaged over the whole search, the Watt I will tend to have a lower total length. Conversely, the Watt II topology is expected to be more stable than the Watt I linkage. While stability has many factors, including the construction of the physical mechanism, one indicator for stability is be the ability of a linkage to resist an external force on the coupler link. To estimate stability, the average mechanical advantage between the input torque and output force is recorded, with a higher value indicating a greater ability to resist an external force on the coupler. The required forces are estimated using the method of instant centers. Table 4 summarizes the problem. For both topologies, 1.68 million different dimensional combinations are considered, created by incrementing the free-choice variables from -180 to 180 degrees.

Table 4. Design example three problem definition.

	Position 1	Position 2	Position 3
Coordinates	$0 + 0i$	$10 + 11i$	$30 + 14i$
Prescribed Angle ($^{\circ}$)	0	5	50

Table 5. Design example three search results.

	Watt I	Watt II
Topologies:		
Average Total Length	239.874	610.696
Minimum Total Length	105.802	136.067
Average Mechanical Advantage	0.1009	1.1363

The results of this comparison provide preliminary evidence that reach criteria are an effective predictor for the overall size of a mechanism performing an extended movement. Similarly, the Watt II topology, with an additional ground pivot and shorter reach, had an average torque advantage nearly $10\times$ higher than the Watt I topology, indicating a much greater resistance to external torques applied to the coupler link.

4. Conclusions

Many researchers have addressed means to discover, categorize, and enumerate all BKC (e.g., isomorphism detection). These works provide a fundamental base for a general machine design theory, but the crucial step of the process within type synthesis—determining a means to predict which topologic inversion is best suited for a specific task—remains elusive. One approach, presented here, is to formulate a set of design rules and principles to narrow down the solution space. The problem statement and the constraints of the specific need define the task that must be performed. When the task is formulated, design rules are defined that form topological requirements for candidate BKC. These requirements whittle the infinite possible kinematic chains to a more manageable number. The selection can be further narrowed down to a handful, or even a single option by applying generalized design principles like compactness, reach, and stability (among others) to choose the best BKC and topologic inversion for this task. The combination of these rules and principles offer a feasible, generalized methodology for type synthesis which may be implemented by human designers or incorporated into an automated synthesis program.

There are several areas for future work on type synthesis. A few categories that were identified in the literature review as being out of scope for the present work, like spatial or compliant linkages, could benefit from crossover with the current work. The modified form of the Gruebler equation is effective for finding the degrees of freedom of spatial linkages, and spatial linkages equally require topology selection. It makes intuitive sense that a corresponding set of design principles like reach and stability would exist for spatial or compliant linkages as well. Identifying quantitative metrics for predicting the performance of topologies according to these and other metrics could be a valuable contribution. For planar linkages, identifying additional criteria or methods for evaluating the characteristics of a topology could also further progress the ideas presented here.

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Abbreviations

The following abbreviations are used in this manuscript:

BKC	Basic kinematic chain
BKD	Basic kinematic diagram
F ₁	A joint that removes two degrees of freedom from a planar mechanism
F ₂	A joint that removes one degree of freedom from a planar mechanism

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