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Analysis of Mathematical Methods of Integral Expert Evaluation for Predictive Diagnostics of Technical Systems Based on the Kemeny Median

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Abstract: At present, diagnostics of the current technical condition of high-voltage power equipment in power systems have become more important. This allows the estimation of the real technical condition of power equipment more accurately with its removal into repair based on the results of the diagnostics. This paper presents the comparative analysis of expert evaluations with the use of the arithmetical mean and median values of expert evaluations. In this case, individual expert opinions, influenced by a level of competence, correspond to each other in a different manner, depending on the applied approach. As the comparison of the consistency of expert opinions is the basis for decision-making, it is recommended to make a decision on the technical condition using median estimations because these estimations are less subjected to distortions from single outliers of judgments. This provides more reliable information for making key decisions. Three approaches are considered in this paper: the method of arithmetical mean estimations, the method of median estimations based on the Kemeny median method, and the analytic hierarchy process of Saaty. The considered methods allow decisions on power equipment operation to be made very quickly; namely, if the power equipment is in an operable state and may remain operated, or it has considerable defects and should be removed from operation for routine maintenance, or it has reached the final technical state and needs to be removed from operation.

Keywords: current diagnostics; expert evaluation; consistency; Kemeny median; analytic hierarchy process

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1. Introduction

The development of power systems and the increase in the requirements for the quality of their functioning are significantly determined by the technical condition of the power equipment and the level of its operation. At present, power transmission and distribution enterprises are frequently facing problems regarding industry reforms, stricter regulatory requirements, growing consumer demands, and more stringent requirements for system reliability. At the same time, electricity companies should ensure the economic efficiency of their costs and the full use of invested funds to increase performance and system reliability [1–3].

In this regard, the optimization of power equipment's lifetime is one of the most important tasks of power system management [4].

The planned operation of power grid facilities is regulated by relevant regulations based on a preventive maintenance system. The maintenance and repair of power grid facilities due to their technical condition increases the requirements for planning methods and models [5,6].

Solving the problem of assessing the technical condition of the power equipment of electrical networks is largely associated with the implementation of efficient methods of control and technical monitoring for power equipment that is required to maintain the optimal condition and performance of the power systems and equipment [7]. By implementing advanced diagnostic and monitoring solutions, power plant operators can increase the operating efficiency with the minimization of unplanned downtime and maintenance costs [8–11].

The early detection and diagnostics of failures can improve the reliability and efficiency of power generation and transmission and distribution processes, as well as prevent potentially catastrophic failures and long maintenance shutdowns caused by operating under degraded conditions [12].

The diagnostic system has shown high efficiency in solving the problems of the early detection of defects in high-voltage power equipment, forecasting the development of defects and assessing their danger, determining the amount of repair and restoration works, and the optimization of maintenance and repair for power equipment [13].

Power transmission includes the following stages: voltage transformation from the power plant busbars to the level corresponding to the criteria of economic expediency, power transmission to the load centers, and power transformation into the rated voltage of electricity consumers. Power is transmitted through the electric networks of the power system to the load centers, as a rule, via overhead lines [14].

In the process of the diagnostics of the current technical condition of power equipment based on expert evaluations, an expert usually handles the linguistic variables. Consequently, it is necessary to determine the margin at which power equipment should be removed for repair immediately or should be left in operation with more frequent control. The process of digitalization of the obtained data provides numerical estimations. Based on these numerical estimations, a decision-maker (DM) may decide on the further operation of the power equipment, its removal for repair, or its replacement. At the same time, a detected defect may be a fault symptom, which indirectly indicates a faulty element; therefore, a fault situation should be studied as a whole with the search for possible alternatives [15–17].

Decision-making is a process of accumulating data, analyzing information about an object, and choosing the optimal decision from several possible variants. Decision-making based on expert evaluations can be undertaken after inspecting the power equipment and detecting the failure symptoms [18–20].

The term "evaluation of the current technical condition" can be introduced [21]. It implies the data collection from the transformer equipment under operation and the analysis of this information to make a definite conclusion about the technical condition of the equipment [22]. Such diagnostics are performed after inspecting the power equipment and detecting the failure symptoms [23–25].

The modern power system is characterized by high power, high voltage, intelligence, high reliability, and sustainable development based on renewable energy sources [26]. A significant increase in the operating voltage demands higher requirements for the coordination of the power equipment insulation [18,19]. Therefore, the monitoring and evaluation of the insulation conditions becomes more important. The intelligence of the power system is primarily based on the intelligence of power equipment, which is characterized by high efficiency and reliability. Therefore, in order to achieve a higher level of intelligence for power equipment, it should be accurately controlled and quickly diagnosed.

A smart grid is a highly intelligent and widely distributed network, which consists of advanced monitoring technologies, high-performance electronic devices, more reliable information technologies, and communication technologies [23,25]. The implementation of various electronic devices (such as a rectifier, variable frequency drive, or thyristor) into a power system can result in the occurrence of more complex harmonics that differ from existing waveforms and, therefore, lead to accelerated insulation aging, power equipment overheating, and a reduction in the power equipment lifetime. In addition, wind power has become the most widely used new energy in power generation [27]. The integration of large-scale wind power generation with widely used electronic devices will seriously threaten the system stability due to the lack of effective inertia and damping [11,14], which consequently results in harmonic problems. All the above characteristics demand new requirements for condition monitoring and the diagnostics of power equipment. The condition monitoring system is already becoming a main component of predictive technical maintenance. A well-implemented condition monitoring system allows the early identification of any problems and reduces the risk of failures and unplanned downtime, which increases the reliability and availability of electrical installations. Due to the rapid progress of modernization, power equipment can be operated continuously and safely, which is related to the production level of an enterprise, production safety, quality control, and other aspects of the smooth failure development of power equipment [28,29]. The diagnostics of power equipment failures or, in other words, risk assessments are real analyses of the exact location and nature of failures.

In this era of global digitalization, multi-criteria decision-making methods (MCDM) are useful tools that help decision-makers choose the most effective options in the case of discrete problems and are widely used in almost all areas of management, including solving multi-criteria tasks in the field of energy. As risk assessment methods are a constantly evolving field of research and practice, different types of fundamental MCDM, such as AHP (analytical hierarchy process), the SAW method (simple additive weighting), PROMETHEE (preference ranking organization method for enrichment evaluation) [30], ELECTRE (elimination and choice expressing the reality) [31], TOPSIS (a technique for order of preference by similarity to ideal solution) [32], and others play a significant role in the development of traditional risk assessments [33–35]. In a broad sense, the above MCDM methods can be divided into two main groups of methodology: compensatory and proactive decision-making. The first group should include AHP (a mathematical tool for a systematic approach to complex decision-making problems. It does not prescribe any "correct" solution to the decision-maker but allows him to find an option (alternative) in an interactive mode that best agrees with his understanding of the essence of the problem and the requirements for its solution.) [36]; SAW (one of the methods used to solve decision-making problems with multiple attributes. The usefulness of the basic concept of the SAW method is to find the number of weighted performance ratings for each alternative for all attributes: the normalized value of the criteria for alternatives is multiplied by the importance of the criteria, and the alternative with the highest score is selected as preferred.) [37]; and TOPSIS (a compensatory aggregation method based on the concept that the best alternative should have the smallest geometric distance to the positive ideal solution (PIS) and the largest geometric distance from the negative ideal solution (NIS).) [32], while PROMETHEE (characterized by many types of preference functions that are used to determine the differences between alternatives in judgments) [38] and ELECTRE (a method of increasing the number of superior approaches to extend the theory of fuzzy sets, namely, a complex spherical fuzzy set) [39] are the most used higher-level MCDM methods, in which the advanced approach is based on a pairwise comparison of alternatives in such a way that the least favorable alternatives are excluded. Sometimes the methods of advanced ranking are used to obtain the main solution to the problem of decision-making instead of ranking alternatives or optimal solutions [40].

However, it should be noted that, as it is currently one of the easiest to use and, as a result, a frequently used MCDM method, AHP allows you to model a complex problem in a hierarchical structure, showing the relationship between the goal and criteria tasks, as well as sub-criteria and alternatives, thus giving decision-makers a general overview of the problems and, as a result, relieving them of the need to go into the mathematical details of calculations [35,41]. So, for example, in [42], AHP finds its application as a decision support method when choosing the most effective solution based on a number of evaluation criteria (the Saaty method). In addition, there are many synergies regarding various developments of the fuzzy MCDM method [38,43–45] with AHP, which, in the future, will be able to help adapt the characteristics of the method to other specific decision-making problems, allowing the analysis of the proposed decision-making problem individually and for a group decision to be made based on individual results [45]. For example, in [45], the authors propose the PRISM-AHP (partial risk map) methodology as a hybrid risk assessment method designed for areas of operation sensitive to safety and reliability, which can create more sensitive ratings than the original AHP method.

The present study attempts to find cause-and-effect relations between the fault symptoms and real reasons using real defects of a power transformer that is influenced by the type of obtained evaluations. In the context of the research focus, the closest to this study are the research papers [46,47] which present statistical estimates of expert opinions based on the arithmetic mean (which is a very approximate averaging of opinions), as well as the work [48], in which the above method is described using a pairwise comparison of expert assessments with their sequential selection.

The identified sources indicate that today, there is a significant gap in this area of research, with the exception of a primitive approach based on arithmetic mean, as a result of which the purpose of this work is the mathematical justification of a new direction, which is characterized as prognostic diagnostics. The authors conducted a study of real recommendations on the use of statistical methods for the analysis of expert assessments, depending on the current technical condition of power equipment. The main motivation for the work is due to the desire and need to reduce financial and material costs, as well as the cost of measuring automated diagnostic systems in general, which currently can be up to 30% of the cost of the electrical equipment itself. To achieve this goal, a comparative analysis of three mathematical methods of the complex assessment of the opinions of a group of experts was carried out: arithmetic mean, Kemeny median, and pairwise comparison according to Saaty.

The structure of the paper includes an introduction and the methodology of an integral assessment of expert opinions based on three approaches, namely:

- Arithmetic mean;
- The Kemeny median (as the median from the point of view of probability theory and mathematical statistics more correctly reflects the property of the statistical population);
- The pairwise comparison of expert assessments based on the Saaty method, including the results of expert judgments from several experts to diagnose the current state of electrical equipment, as well as a discussion of the results, conclusion, and a list of references.

2. Materials and Methods

At present, predictive diagnostics and control are being developed with an appropriate justification. They are based on a prediction that is determinative, in some cases, for decision-making. The processing of expert opinions is called the method of expert evaluations, in which expert opinions are expressed in qualitative and quantitative forms. When analyzing expert opinions, various statistical methods can be used, while the main widely used methods of mathematical processing of expert assessments can be distinguished as follows: checking the consistency of expert opinions and averaging expert opinions within a consistent group.

The expert evaluation process was carried out by forming an expert group. Then, an expert survey was conducted, and quantitative estimates were obtained. Next, statistical processing of the results and determination of the consistency of opinions were performed.

In accordance with the general scientific conception of stability, which recommends the use of different methods for processing the same data in order to find the conclusions obtained simultaneously by different methods, it is reasonable to use two methods simultaneously, such as the standard deviation method and the Kemeny median method.

2.1. Evaluation of the Consistency of Expert Opinions Based on the Standard Deviation Method

If the number of experts in the group is equal to ($i = 1, 2, \ldots, n$) and the number of evaluated factors is ($j = 1, 2, \ldots, m$), then the consistency evaluation can be performed.

The expected value of the evaluation for the j-th factor is

$$M(x_j) = \frac{\sum_{i=1}^{n} x_{ij}}{n} \tag{1}$$

where x_{ij}—evaluation of the i-th expert for the j-th factor and n—number of experts.

The variance of the evaluation for the j-th factor is calculated as

$$D(x_j) = \frac{\sum_{i=1}^{n} (x_{ij} - M(x_j))^2}{n-1} \tag{2}$$

The standard deviation of the evaluation for the j-th factor is

$$\sigma(x_j) = \sqrt{D(x_j)} \tag{3}$$

The consistency factor for consistency evaluation of all experts ($i = 1, 2, \ldots, n$) for each factor of ($j = 1, 2, \ldots, m$) is

$$\mu_j = 1 - \frac{\sigma(x_j)}{M(x_j)} \tag{4}$$

If the obtained result is in the range of $\mu_j = 0.7–0.9$, it demonstrates the high consistency of experts with no conspiracy between them [46].

A flowchart of the process for describing the stages of the proposed method is shown in Figure 1.

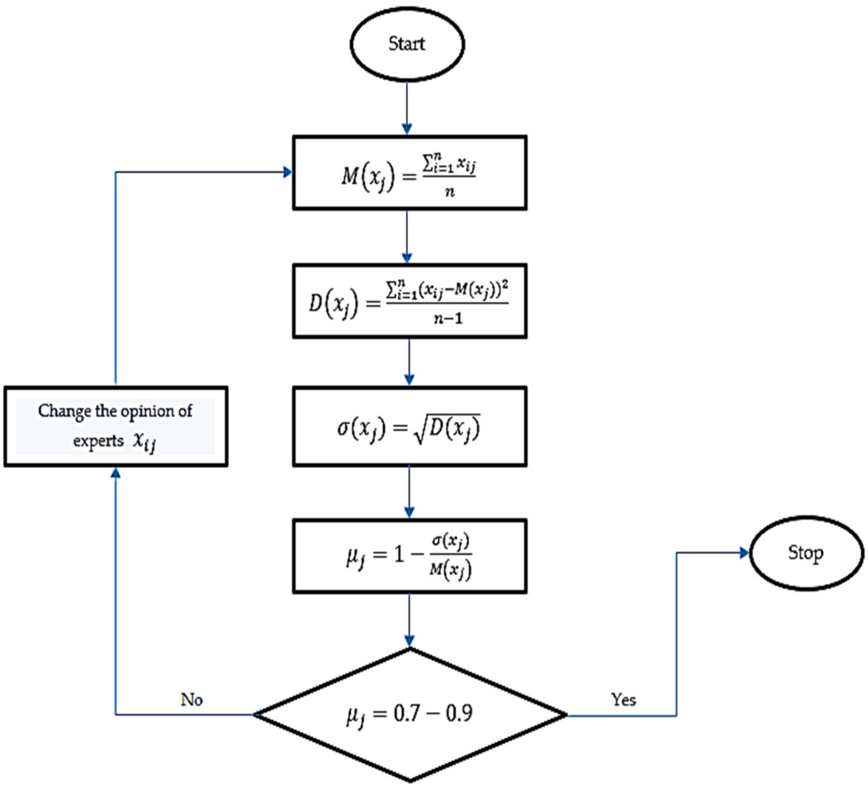

Figure 1. Flowchart of the process of the mean-square deviation method.

As initial data, expert evaluations in the form of fuzzy cause-and-effect relations were used. These evaluations represent the subjective probability for the development of a defect in power equipment in the case of deviation of controlled parameters from normative values.

Symptoms and parameters of the defects are given below, where X_i is a symptom and Y_i is a controlled parameter.

X_1—continuous uninterruption of a through short circuit current at the low voltage side of a transformer;

X_2—insufficient electrodynamic strength of windings to short circuit currents;

X_3—cooling system failure;

X_4—reduction in mechanical strength of insulation;

Y_1—winding overheating;

Y_2—winding deformation;

Y_3—moistening and contamination of winding insulation;

Y_4—winding insulation deterioration.

Initial information is summarized in Table 1.

Table 1. Reasons and failures of transformer windings.

		Reasons			
		X_1	X_2	X_3	X_4
Failures	Y_1	**0.9**	**0.5**	**0.8**	**0.3**
	Y_2	**0.6**	**0.5**	**0.3**	**0.7**
	Y_3	**0.4**	**0.3**	**0.8**	**0.7**
	Y_4	**0.7**	**0.4**	**0.6**	**0.6**

For experimental results, a group of competent expert practitioners related to the operation of electric power systems and power supply systems aged from 29 to 53 years with work experience from 7 to 28 years was selected.

As a comparison, six and nine experts were considered. Table 2 shows expert evaluations for four reasons X_1, X_2, X_3, and X_4 and their consistency for the second parameter Y_2.

Consistency factors for the parameter Y_2 correspond to the recommended level of consistency, being in the range of 0.7–0.9 [47–54]. The consistency of expert opinions for other parameters Y_1, Y_3, and Y_4 was determined in a similar way.

In practical psychology, it is considered that no more than 20% of decisions can be erroneous; in other words, at least 80% of decisions must be correct. The relative error almost always obeys the normal distribution law; that is, all reliable solutions fall into the range $\pm$ 3σ (where σ is the standard error) with a probability of 0.997. Thus, taking the consistency ratio (CR) 0.8 $\pm$ 0.1 = 0.7–0.9, we obtain, with the above probability, an area of reliable solutions, provided that $\pm$0.1 is equivalent to a deviation of $\pm$3σ.

Table 2. Expert evaluations for the second parameter.

	Reasons			
Y_2	X_1	X_2	X_3	X_4
Experts				
1	1.0	0.8	0.3	0.7
2	0.9	0.7	0.4	0.5
3	0.6	0.5	0.3	0.7
4	1.0	0.6	0.5	0.8
5	0.9	0.5	0.4	0.7
6	0.7	0.8	0.5	0.5
7	0.9	0.8	0.3	0.7
8	0.7	0.6	0.5	0.5
9	0.8	0.7	0.4	0.8
Consistency for Y_2				
6 experts				
μ_j	0.807	0.788	0.776	0.811
9 experts				
μ_j	0.830	0.839	0.783	0.812

2.2. Evaluation of the Consistency of Expert Opinions Based on the Kemeny Median Method

The use of the Kemeny median is based on the input of the metric into the set of expert opinions with the axiomatic input of the distance between elements in the expert opinions set. It is important, in this case, how the set opinions are represented because it determines the problem complexity.

Each matrix of pairwise comparisons is represented by an element from the set of expert opinions *P* [54]. In another way, if the metric is introduced and P-set elements are put into the space, elements will be represented by points in this space, which is illustrated schematically in Figure 2.

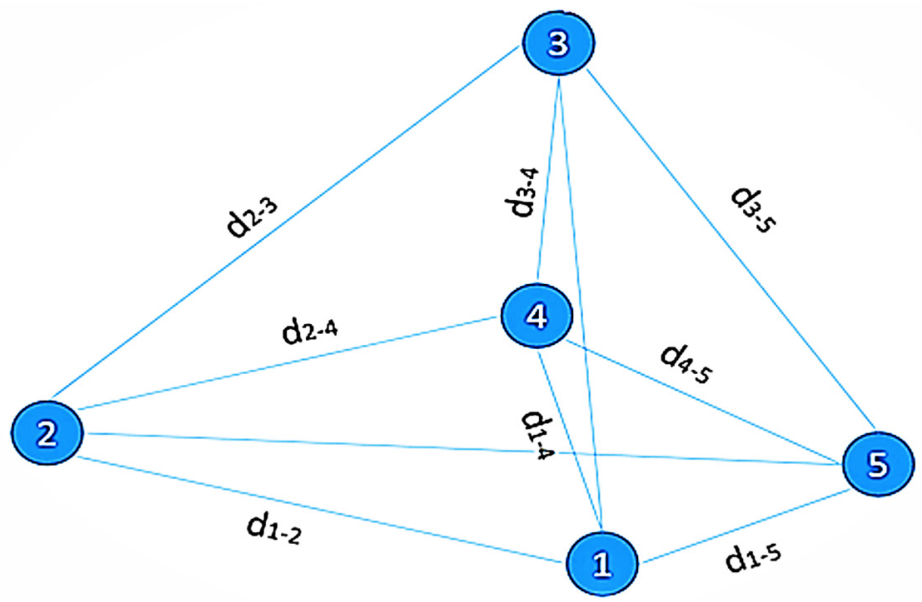

Figure 2. Space of expert opinions.

Therefore, the Kemeny median can be determined as an element of the set P that has the least distance to the other elements. Mathematically, it is the least sum of distances from a fixed element of the set P to the other elements of this set:

$$M^*(P_1,\ \ldots, P_m) = \mathrm{arg}min \sum_{i=1}^{m} d(P, P_i) \tag{5}$$

The consistency of expert opinions using the Kemeny median is proposed to be evaluated using the following formula proposed by Manusov, where the inconsistency index (II) should be first determined:

$$II = \frac{1}{n-1} \sum_{k=1}^{m} \frac{\sum_{i=1}^{n-1} |C_{ik} - C_{kk}|}{m} \tag{6}$$

where n—the number of experts; m—the number of symptoms; i—the current number of an expert; k—the current number of a symptom; c_{ik}—evaluation of the i-th expert for the k-th symptom; and c_{kk}—evaluation of an expert using the Kemeny median method for the k-th symptom.

Then, the consistency ratio is defined as follows:

$$CR = 1 - II. \tag{7}$$

Evaluations in the range of $0.9 \leq CR < 1$ are considered to be consistent, while the consistency over the whole expert examination is determined using the Kemeny median.

The calculation results for the Kemeny median and the consistency evaluation for the second parameter Y_2 are presented in Tables 3 and 4.

Table 3. Kemeny median between the opinion of one expert in relation to the others for the second parameter for 6 experts.

Experts	**1**	**2**	**3**	**4**	**5**	**6**
Sum of distances	2.9	2.7	3.5	3.1	3.0	3.5

Table 4. Kemeny median between the opinion of one expert in relation to the others for the second parameter for 9 experts.

Experts	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Sum of distances	2.1	0.6	4.2	3.0	0.3	0.6	1.2	2.4	1.2

For the second parameter, the median between 6 experts is the opinion of expert No. 2, while the median between 9 experts is the opinion of expert No. 5. The consistency ratio is given in Table 5.

Table 5. Consistency ratio (CR) for the second parameter Y_2.

	Symptoms			
Number of Experts	X_1	X_2	X_3	X_4
6 experts	0.945	0.955	0.970	0.955
9 experts	0.966	0.975	0.981	0.959

Consistency factors for the parameter Y_2 correspond to the recommended level of consistency, being in the range of $0.9 \leq \mathrm{CR} < 1$.

The comparative analysis of expert evaluations is given for 9 experts, as well as the consistency for the second parameter Y_2 with considerable deviations of expert No.1's opinions (Tables 6 and 7).

Table 6. Kemeny median between one expert's opinions in relation to the others in the case of the deviations of one expert's opinions.

Experts	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Sum of distances	2.4	0.5	6.6	3.3	0.6	0.3	1.5	2.1	1.5

Table 7. Consistency of expert opinions with deviations of one expert's opinions.

	Reasons			
Y_2	X_1	X_2	X_3	X_4
		Standard deviation method		
μ_j	0.766	0.724	0.635	0.677
		Kemeny median method		
CR	0.963	0.966	0.972	0.963

In the case of deviations of one expert's opinions, the median value is the opinion of expert No. 6.

The study shows that median estimates are less susceptible to distortion from individual outliers of estimated opinions, so, for example, with consistency through the mean square, the consistency ratio when rejecting the opinions of one of the experts (dissident) went beyond μ_j = 0.6, which does not fall within the generally accepted interval [55].

Along with the above, expert evaluations are widely used in comparative analysis based on the preference of certain solutions and evaluated in scores (intensities of importance) according to the Saaty method [56].

2.3. Evaluation of the Current Technical Condition Based on the Analytic Hierarchy Process

At the first stage, pairwise comparisons of symptoms with each other are performed for every possible reason. Comparison of these symptoms is realized using nine intensities of importance from the fundamental scale of Saaty, where 1—equal importance (or equally preferred) and 9—extreme importance (or absolutely preferred). When comparing an element with itself, we obtain equal importance, so we set "1" at the intersection of line A with column A in position (A, A). Therefore, the main diagonal of the matrix should consist of values equal to 1. Elements of pairwise comparisons are located above the main diagonal, while their corresponding reciprocal values are located under the main diagonal. Then, we set corresponding reciprocals: 1, 1/3, ..., or 1/9. After that, using the matrix of pairwise comparisons, it is necessary to determine a column vector of priorities (CVP) of failure symptoms for every possible reason. In mathematical terms, it means calculating the principal eigenvector, which becomes the vector of priorities after normalizing [28].

After multiplying the matrix of pairwise comparisons using the column vector of priorities, we get a new vector of eigenvalues. Then, we divide the first component of this vector by the first component of the vector of decision evaluation, the second component of this component by the second component of the vector of decision evaluation, and so on. Therefore, we determine a vector of eigenvalues for the matrix. Dividing the sum of components of this vector by the number of components, we obtain an approximation to the value λ_{max} (called the maximum or principal eigenvalue) used for evaluation of consistency representing proportionality of importance. The closer λ_{max} is to n (the number of objects or actions in the matrix), the more consistent the result is. Departure from the consistency can be calculated in the following way:

$$(\lambda_{max} - n)/(n - 1) \quad (8)$$

This value is called a *consistency index* (CI). Then, using the AHP of Saaty, it is necessary to proceed to a *consistency ratio* (CR). This CR is determined by dividing CI by a constant depending on the matrix order. A *CR*, lower or equal to 0.100, is considered to be acceptable.

As parameters, cause-and-effect relations considered above were chosen. Then, the hierarchy was constructed in stages, from the lower level to the upper level, as shown in Figure 3, as follows:

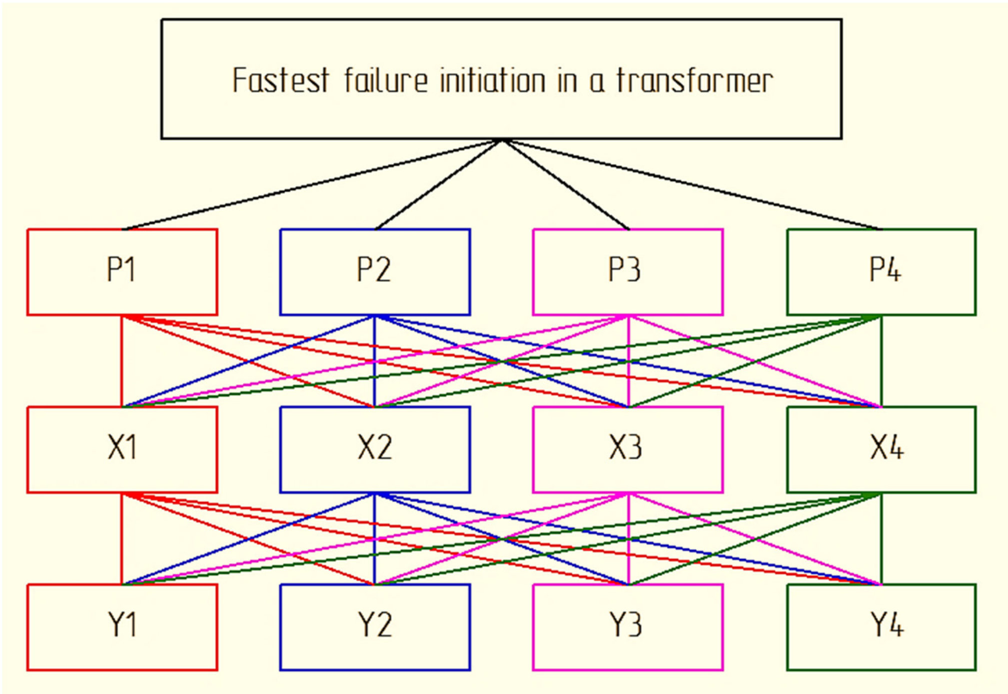

Figure 3. Hierarchical model of fault reasons and symptoms for an oil-filled transformer.

The fourth (lower) level of the hierarchy involves the key symptoms of failures for an oil-filled power transformer: Y_1—winding overheating; Y_2—winding deformation; Y_3—moistening and contamination of winding insulation; and Y_4—winding insulation deterioration.

The third level includes the main reasons for development of the given failures: X_1—continuous uninterruption of a through short circuit current at the low voltage side of a transformer; X_2—insufficient electrodynamic strength of windings to short circuit currents; X_3—cooling system failure; and X_4—reduction in mechanical strength of windings.

The second level of the hierarchy represents the main reasons influencing the fast transition of a transformer into a non-operable state: P_1—malfunction of relay protection; P_2—seal failure of a casing; P_3—lack of a surge arrester or its malfunction; and P_4—violation of operating regulations.

The first level of the hierarchy is the main objective. It implies determination of the shortest time for a transformer to pass into a non-operable state.

Calculations were performed for each of four possible reasons, as well as for other hierarchy levels described above, based on judgments of experts No. 5 and No. 6.

Using the analytic hierarchy process (AHP), a table of pairwise comparisons was composed for possible fault symptoms by the number of possible reasons, where the column vectors of priorities (*CVP*), principal eigenvalue (λ_{max}), consistency index (CI), and consistency ratio (*CR*) were calculated (Table 8).

Table 8. Pairwise comparisons of symptoms under continuous interruption of a through short circuit current.

			Expert No. 5		
	Y_1	Y_2	Y_3	Y_4	**CVP**
Y_1	1	5	2	3	0.464
Y_2	1/5	1	1/3	1	0.106
Y_3	1/2	3	1	3	0.316
Y_4	1/3	1	1/3	1	0.112
CR			0.021		
CI			0.019		
λ_{max}			4.057		
			Expert No. 6		
Y_1	1	2	5	3	0.522
Y_2	1/2	1	1	1	0.166
Y_3	1/5	1	1	1	0.152
Y_4	1/3	1	1	1	0.158
CR			0.034		
CI			0.031		
λ_{max}			4.093		

In expert No. 5's opinion, the most probable symptom is winding overheating with a priority of 0.464. In second place is moistening and contamination of winding insulation with a priority of 0.316. In third place is winding insulation deterioration with a priority of 0.112. The consistency ratio CR = 0.021, which is lower than 0.100 and corresponds to the condition of consistency by the analytic hierarchy process of Saaty. CI = 0.019. The principal eigenvalue is λ_{max} = 4.05.

In expert No. 6's opinion, the most probable symptom is also winding overheating with a priority of 0.523. In second place is winding deformation with a priority of 0.166. In third place is winding insulation deterioration with a priority of 0.158. The consistency ratio CR = 0.021, which is lower than 0.100. CI = 0.031. The principal eigenvalue is λ_{max} = 4.093.

Thus, the order of failure priority is identical for both experts, only with different values.

To achieve the main objective, it is necessary to make the final integral evaluation of interrelation between reasons of the first level and resulting prioritized evaluations on the possibility of transition of a transformer or other equipment into a non-operable state. This is caused by the fact that a transformer, or any other equipment, may have defects but be in an operable state. To solve the considered problem, it is proposed to use multiplication of column vectors of priorities with all hierarchical levels. Column vectors of priorities for each level of the hierarchy by experts No. 5 and No. 6 are given below in Table 9.

The resulting column vector of the influence of each symptom on the fastest transition of a transformer into a non-operable state can be determined by multiplying matrices of column vectors of priorities from the lower level to the upper level. The resulting integral column vector characterizing the influence of each symptom on the final objective is obtained and shown in Table 10.

Table 9. *CVP* for the fourth level of the hierarchy.

	X_1	X_2	X_3	X_4
Expert No. 5				
Y_1	0.464	0.437	0.457	0.457
Y_2	0.107	0.328	0.122	0.274
Y_3	0.316	0.171	0.146	0.152
Y_4	0.113	0.063	0.274	0.116
Expert No. 6				
Y_1	0.523	0.393	0.349	0.459
Y_2	0.166	0.196	0.218	0.161
Y_3	0.152	0.224	0.349	0.307
Y_4	0.158	0.187	0.083	0.073

Table 10. Integral column vector.

Expert No. 5		**Expert No. 6**	
Y_1	0.451	Y_1	0.435
Y_2	0.219	Y_2	0.186
Y_3	0.105	Y_3	0.164
Y_4	0.019	Y_4	0.105

Thus, if one follows Saaty's expert preferences, then in expert No. 5's opinion, the failure symptom (defect) which forces an oil-filled transformer to transition into a non-operable state in the shortest time with the highest probability is winding overheating with a priority of 0.451. In second place is winding deformation with a priority of 0.219. In third place is moistening and contamination of winding insulation with a priority of 0.105. In expert No. 6's opinion, the most probable symptom is also winding overheating with a priority of 0.435. In second place is winding deformation with a priority of 0.186. In third place is moistening and contamination of winding insulation with a priority of 0.164.

3. Discussion of Results

It is shown that decision-making, as an activity, includes the process of collecting and analyzing information about an object, including expert judgments and preferences [55]. Decision-making based on expert evaluations can be undertaken after inspecting the power equipment and detecting failure symptoms.

This paper proposes a method for determining the inconsistency index and the consistency coefficient in relation to the median of Kemeny, which has never been determined and has not appear before. Without an assessment of the consistency coefficient, the use of the Kemeny median is possible only in sociological surveys and that reduces the quality of these surveys. The expediency of publishing these new results is similar to how Saaty published his formula for the consistency coefficient in an additional article in a scientific journal [56].

Along with this, the third method chosen for comparison, namely, statistical methods for assessing consistency, have been repeatedly published in scientific journals and have no authorship as they are based on probability theory [46].

Thus, the proposed method for assessing the consistency of expert opinions in relation to the Kemeny median has a significant novelty and complements the general methodology for assessing the consistency of expert opinions, especially for cases based on the linguistic variables of fuzzy set theories.

At the same time, a comparison of the consistency of expert evaluations is important. The comparison of the consistency of expert opinions is performed in this paper using the following criteria: standard deviation of arithmetical mean estimations and median estimations based on the Kemeny median method. Our investigations showed that median estimations are less subjected to distortions from single outliers of judgments. For example, when evaluating the consistency using arithmetical means, the consistency ratio under the deviation of expert (dissident) opinions exceeds the limits, being equal to μ_j = 0.6, which is out of the conventional consistency interval of μ_j = 0.7–0.9. In addition, the same deviation of evaluation for the second symptom using the Kemeny median method leads to the substitution of expert No. 5 by expert No. 6, who represents the expert group opinion in the best way. However, the consistency of opinions, in general, was decreased, but it did not exceed the conventional consistency interval of μ_j = 0.7–0.9. In turn, the expert evaluations of the possible reasons and consequences of failures organized into four levels of the hierarchy, in spite of several deviations at the medium level, have shown the same result; the winding overheating is the main reason for the failures of an oil-filled power transformer.

In general, the analysis of the above results shows that along with the automation of diagnostic systems, the opinions and judgments of qualified experts represent a very important function for the diagnostic assessment of the current technical condition of high-voltage equipment and other electrical equipment.

However, until now, these opinions, to a necessary extent, could not be presented, on a strictly mathematical basis, as a reliable integral collective assessment. The representation of opinions as the arithmetic mean is a rather rough approximation, not unlike the average body temperature of patients in hospital. Along with this, the use of median estimates for the Kemeny median, similar to any median estimate, gives a more reliable result. As follows from the literature review, the Kemeny median has not previously been used in engineering disciplines, including technical diagnostics. Its application was limited only to sociological and economic disciplines. Thus, the novelty and practical significance of this work, taking into account the proposed new formulas of the inconsistency index and consistency ratio, is innovative.

The new methodology can be applied, in practice, in combination with some, albeit rather simple, automated measurement systems. At the same time, the integral estimates of expert opinions are a prediction of possible defects. It should be considered that this is a new section of diagnostics, namely, predictive diagnostics, that is, diagnostics by prediction.

Some limitations of the proposed method of predictive diagnostics are due to the selection of sufficiently qualified experts with work experience and expressing plausible judgments, which is compensated by a reduction in the financial costs for an automated system, as it does not require additional costs to express expert opinions and due to this, in most cases, coincides with their official duties.

Further research is expected to be devoted to the software implementation and inclusion of this approach in the AHP-TOPSIS system with a view to its wide dissemination for use in other technical fields and not only in the diagnosis of the current state of electrical equipment.

4. Conclusions

In this paper, methods for diagnosing the current technical condition of electrical equipment according to the prediction of an expert group (which should be referred to as predictive diagnostics, for which no stable mathematical apparatus has been chosen at present) are proposed and considered. The main contribution of the authors to predictive diagnostics consists of a reliable choice of the consistency of expert opinions based on the consistency ratio.

Our investigations show that a comparative analysis of decision-making, in particular predictive diagnostics based on the standard deviation estimation, median estimation based on the Kemeny median, and expert preferences based on the Saaty method, gave different results. The solutions based on the Kemeny median seem to be the most weighted, as it basically chooses the expert's opinion, which most closely reflects the collective opinion of the expert group, in this case, the opinion of expert No. 5. As a metaphor, it can be considered that this opinion corresponds to the "conditional center of gravity" of the expert group, as their qualifications are different, which is the reason for giving different expert opinions.

The practicability of the proposed mathematical methodology for predicting the pre-fault and emergency condition of power equipment based on the evaluation of the impact of certain defects can significantly reduce emergency repair and restoration operations due to the transition from scheduled preventive maintenance, which regulates mandatory repairs for a certain period of power equipment operation, to predictive diagnostics based on the current technical condition of each object.

Along with this, the proposed mathematical model for assessing the consistency of experts' opinions on the Kemeny median, namely, the inconsistency index and the consistency ratio, can be recommended to increase the reliability of the results of evaluating public and sociological surveys. The proposed mathematical model for assessing the consistency of expert opinions, namely the consistency index and the consistency coefficient, can be recommended to increase the reliability of the results of evaluating public and sociological surveys, where it has never been used before.

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