

Editorial

# Many Body Quantum Chaos

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**Abstract:** This editorial remembers Shmuel Fishman, one of the founding fathers of the research field “quantum chaos”, and puts into context his contributions to the scientific community with respect to the twelve papers that form the special issue.

**Keywords:** quantum chaos; quantum kicked rotor; Anderson localisation; dynamical localisation; Gross-Pitaevskii equation

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Shmuel Fishman, a friend, a highly esteemed physicist and a professor emeritus at the Technion, Israel Institute of Technology, passed away on 2 April 2019, 70 years old [1]. Very much moved by his sudden death, I decided to dedicate to him this Special Issue that has found a substantial number of contributors. Shmuel was one of the pioneers of the research field “Quantum Chaos” [2,3]. Over his long career Shmuel made numerous contributions to the field, including understanding of phase transitions, driven dynamical systems, and nonlinear effects in general.

Shmuel provided fundamental insight in the phenomenon of quantum suppression of classical chaotic diffusion, first identified by Giulio Casati et al. [4] in the dynamical behaviors of quantum kicked rotators. Whereas the momentum of the classical kicked rotator diffuses, its quantum counterpart will eventually localize [5]. Fishman, together with his colleagues D. R. Grempel and Richard Prange, showed that this phenomenon has a deep similarity to Anderson localization, in which interference between trajectories leads to localization of wavefunctions. Indeed the nowadays universally accepted name for the phenomenon “dynamical localization” was prompted by their work. Their seminal paper [6] proved this analogy by connecting the seemingly different fields. Connections of this type in a transversal manner were characteristic of Shmuel’s research.

Around the year 2000, in collaboration with Italo Guarneri and Laura Rebuzzini at Como, Shmuel developed a theory [7,8] that explained the resonances observed when free-falling atoms are periodically kicked [9,10]. The method he invented for the analysis of that phenomenon was heavily used later in many other works, e.g., by us for the description of kicked cold atoms and Bose-Einstein condensates [11–15].

Shmuel also studied a phase transition observed in a chain of ions inside a harmonic trap, a problem taken up by our contribution [16]. One of Fishman’s research interests in the last decade again was the study of transport in disordered systems [17], in which Anderson localization is possibly distorted due to other competing effects, such as interactions between the particles or external driving. These systems are relevant to state-of-the-art experiments in nonlinear optics and cold atoms. Interaction is here modelled by a nonlinear potential in the Schrödinger equation, i.e., by the mean-field Gross-Pitaevskii equation [18,19].

Phase transitions and many-body effects in ultracold bosonic systems are also investigated in the contribution by Nitsch et al. [20], whilst ultracold fermionic conductance is the topic of Kolovsky’s and

Maksimov's contribution [21]. One-particle localization effects are delved into by Torres-Herrera and Santos [22], and many-body effects and thermalisation (in contrast to a form of localisation generalised to many-body systems) in an isolated quantum system are the subject of the paper by Frahm et al. [23].

Hyperbolic maps as minimal models of quantum chaos are studied in the contributions by Mantica [24] and Yoshino et al. [25]. Quantum dissipation is the common topic of the papers by Giachetti et al. [26] and by de Bettin et al. [27]. Loho Choudhury and Großmann [28] are using Husimi functions for a semiclassical analysis of correlation functions. Next to all these contributions, I am sure Shmuel would have loved to discuss about exotic physics such as reported in the contribution on a quantum model for cold dark matter [29]. Finally, the quantum kicked rotor has recently found still another application, namely in the realization of quantum walks [30–32]. Our contribution [33] proves the analogy between a continuous-time quantum walk [34] with the kicked-rotor evolution at quantum resonance conditions [12], a research motivated by my joint work with Shmuel!

In the name of Shmuel Fishman I am very grateful to all contributors to this Special Issue. May it find many readers and inspire future research lines along Shmuel's path!

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