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Article

On Theoretical Incomprehensibility †

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† Nature is my lady and mistress and blindly obeying her I'm free to do what I want.

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Abstract: This contribution tentatively outlines the presumed conceptual duality between the issues of *incompleteness* and *incomprehensibility*—The first being more formal in nature and able to be declined in various ways until specified in the literature as *theoretical incompleteness*. This is *theoretical* and not temporary, which is admissible and the completion prosecutable. As considered in the literature, theoretical incompleteness refers to *uncertainty principles* in physics, incompleteness in mathematics, oracles for the Turing Machine, *logical openness* as the multiplicity of models focusing on coherence more than the optimum selections, *fuzziness*, *quasiness*, e.g., quasi-crystals, quasi-systems, and quasi-periodicity, which are intended as the space of equivalences that allow for coherent processes of emergence. The issue of incomprehensibility cannot be considered without reference to an agent endowed with cognitive abilities. In this article, we consider incomprehensibility as understood here as not generally scientifically explicable, i.e., with the available knowledge, as such incomprehensibility may be temporary, pending theoretical and technological advances, or deemed to be absolute as coincident with eventual *definitive*, theoretical non-explicability, and incomprehensibility. We considered the *theoretically incomprehensibility* mostly in three main ways: as the inexhaustibility of the multiplicity of constructivist reality as given by the theoretically incomprehensible endless loop of incomprehensible–comprehensible, and by existential questions. Moreover, *theoretical incomprehensibility is intended as evidence of the logical openness of both the world and of understanding itself.* The role of theoretical incomprehensibility is intended as a source of theoretical research issues such as paradoxes and paradigm shifts, where it is a matter of having cognitive strategies and approaches to look for, cohabit, combine, and use comprehensibility and (theoretical) incomprehensibility. The usefulness of imaginary numbers comes to mind. Can we support such research for local, temporary, and theoretical incomprehensibility with suitable approaches such as software tools, for instance, that simulate the logical frameworks of incomprehensibility? Is this a step toward a kind of *artificial creativity leading to paradigm shifts*? The most significant novelty of the article lies in the focus on the concept of theoretical incomprehensibility and distinguishing it from incomprehensibility and considering different forms of understanding. It is a matter of identifying strategies to *act* and *coexist* with the theoretically incomprehensible, to *represent* and *use* it, for example when dealing with imaginary numbers and quantum contexts where classical comprehensibility is theoretically impossible. Can we think of forms of non-classical understanding? In this article, these topics are developed in conceptual and philosophical ways.

Keywords: comprehensibility; constructivism; emergence; incompleteness; incomprehensibility; logical openness

1. Introduction

We first consider the issue introduced in the literature regarding *theoretical incompleteness* [1,2]. This refers to uncertainty principles in physics, incompleteness in mathematics, the necessity of oracles for the Turing Machine, the *logical openness* in the context of the cybernetics of the second order (constructivist invention of models and *cognitive reality*), the concept of *fuzziness* and *quasiness* in physics (when classical without referring to quantum physics), and approaches to model coherence in processes of emergence within a context of equivalences.

The subject of incomprehensibility, the inability to give meaning, cannot be considered without reference to an agent endowed with a cognitive system. A cognitive system is to be understood, in short, as a system of interactions between and at different levels of cognitive activities, such as those related to attention, perception, language, the affective and emotional sphere, memory, inferential system, attribution of meaning, and to logical activity. In artificial systems, limited specific 'cognitive', and mainly logical activities, are possible, including the ability to learn.

We thus refer to agents that are *living* [3,4], suitable to perform cognitive systems, and specifically to *human understanding* [5]. However, for both types of systems, the incomprehensibility has a *temporary* nature that is waiting to have subsequent, increased cognitive resources that are suitable to allow understanding in case *trained* for Artificial Neural Networks (ANN). Additionally, incomprehensibility may have an intrinsic nature whose insuperability as theoretical incompleteness represents the guarantees for *intrinsic and comprehensive logical openness* intended as an unlimited number of degrees of freedom when the system includes the environment (in principle independent), making the system incomplete.

Theoretical incomprehensibility is intended as *incompleteness, non-completability of understanding*, and as the divergence of local understandabilities establishing successions of scientific theories to be considered not necessarily convergent to a final, exhaustive one. Theoretical incomprehensibility is the expression of logical openness and introduced in the literature and in the article as a necessary condition of the *space of constructivist options* for local understandability not *improperly extendable* in the absolute (the *poison* in Eve's apple).

Theoretical incompleteness and theoretical incomprehensibility are intended to correspond to the freedom of becoming only partially regulable, decidable, and controllable within the conceptual framework of the existence of laws (classical physics). It is a poor strategy to consider the world as functioning in a framework of finite options while it emerges and collapses from quantum options.

The theoretical problem of our understanding is that it admits, includes, and is not only compatible, but *contemplates* the completion of understanding. However, this assumption is inherently inconsistent. Understanding would be a function of a final goal that does not exist, while there are many local goals that are not only subsequent, but alternative and possibly increasing wider.

Theoretical incomprehensibility can be understood as a signal, a representation, a guarantee of the intrinsic logical openness of the world, which is represented by the understanding of not being always able to understand and that understanding is a strategy of limited validity. The understanding of the theoretical incomprehensibility would bring *antinomically* to the completion, to the negation of logical openness, and to the impossibility of comprehending understanding as constituted by the cognitive unlimited dynamics of hypotheses, sequencings, and accumulations of possible alternatives.

In conceptual correspondence with the theoretical incompleteness of the emergence (intended as a continuous, unpredictable, but coherent acquisition of multiple non-equivalent properties) of the world and with the principles of uncertainty, theoretical incompleteness can be indicated by the logical openness of understanding itself. An understanding that would admit itself to be closed and complete would admit its own inconsistence with itself. Theoretical incomprehensibility should be intended as an *inexhaustible mechanism* for the emergence and production of local, temporal inconsistencies leading to *paradigm shifts*. This is also related to the power of cases of non-demonstrabilities for issues requiring a paradigm shift, e.g., the Euclidean fifth postulate and the effectiveness of imaginary numbers. This is related to the power of potentially inexhaustible non-demonstrable issues requiring and inducing paradigm shifts. We should concentrate on and search for such issues, as well as invent approaches (logical, experimental, computational, . . .) that are suitable, for instance, for recognizing, representing, inventing, transposing, generalizing, simulating, and comparing facts of incomprehensibility such as paradoxes, unprovable obviousness, and postulated assumptions in logics, mathematics, physics, linguistics (semantics), and philosophy.

Furthermore, existential questions are incompatible with any answers and then considered as theoretically incomprehensible. The resulting cognitive strategy is to look for such incomprehensibilities and develop frameworks that convert the current comprehensibility into incomprehensibilities in a continuous constructivist *theoretically incomprehensible* endless loop. Thus, incomprehensibility can be a source of theoretical research issues. Can we support such research for local, temporary incomprehensibility with suitable software-based approaches, for instance, by simulating logical frameworks of incomprehensibility? Is this a step toward a kind of *artificial creativity*? We will develop these topics in conceptual and philosophical ways.

2. Incompleteness

The theme of incompleteness is not trivialized simply as a lack of completeness, for example, of scientific theories [1] or as a negative property but is frequently used in science as a positive property characterizing the complexity of phenomena.

For example, *theoretical incompleteness* [2] is closely related to the *uncertainty principles* in physics whereby, in certain phenomena, the search for greater accuracy in knowing the value of a variable implies a reduction in the knowledge of the value assumed by another. This is the measurement of *homologous components* such as position and momentum. This is the well-known uncertainty principle [6], first introduced in 1927 by Werner Heisenberg. Moreover, this principle can be considered with the *principle of complementarity* introduced by Neils Bohr [7] in which the corpuscular and undulatory aspects of a physical phenomenon cannot be observed simultaneously: the experimental observation of one prevents the observation of the other.

Finally, from a generic point of view, the property of incompleteness may be partial, in other words, refer only to aspects, a specific property, or to multiple properties. It may mean not yet complete, be chronically incomplete and, in theory, not completable with a single approach or model, as in the case of complexity for which multiple non-equivalent and variable models are required for the processes of emergence, as considered by the DYnamic uSAge of Models (DYSAM) (see [8], pp. 64–75).

As for the processes of self-organization and emergence, they are assumed, in short, to possibly occur within populations whose components are in interaction between them (a necessary, but not a sufficient condition). We have *interaction* when the behavior of one influences that of the others. These processes of collective interaction are considered to establish *self-organization* when the observer detects regular and repetitive sequences of new properties that are *acquired* and not reducible to those of the components. Examples include the repetitive behavior of a swarm around a light, a flock around a pile of garbage, the formation of traffic queues, whirlpools of liquids, and *spontaneous* synchronizations (applause, objects on vibrating surfaces, fireflies). These processes are considered to establish *emergence* when the observer detects that sequences of new properties are not regular, not repetitive, but *coherent*. Examples include *collective behaviors* such as markets, termite mounds, flocks, herds and swarms that have variable and changeable shapes, density, and directions, but which acquire and maintain coherence over time, scale invariance, and long-range correlations. The emerging properties are dependent on the observer who decides the scaling (for example, the order of the magnitude of the distance between the components of a collective behavior where the behavior can be undetectable when the order of magnitude is very small or very large compared to the observer), on knowing what to detect, and be compared over time, for example, the shapes and macroscopic dynamics of general movements such as contraction, dilation, circularity, or linearity, and varying in density over time [9–12].

Regarding DYSAM, we related this to the multiplicities of the characteristics and problems of an emergent complex system such as a social problem that may be political and economic, an illness that may have biochemical, organic, and psychological natures, and phase transition-like processes in physics that may have classical and quantum natures.

In the case of *structural dynamics* (a simple example of elementary *dynamics of structures* is given by phase transitions changing the structures of matter) of processes of emergence, its hypothetical and non-presumable *completeness* would instead represent a fact of closedness, denial of equivalences, and of unpredictability in reality due, for example, to *fluctuations* (i.e., deviations of the current temporal evolution from its average within a system subject to random forces).

We introduce below the theme of *logical openness* as opposed to *logical closedness*, where the states assumed by the system-environment are in finite number and predetermined. More precisely, we define a model as logically closed when:

1. A complete and formal description of the relationships between the model state variables is available;
2. A complete and explicit description of the interactions between the system and its environment is available; and
3. The knowledge of the preceding two points allows the deduction of all possible states that the system can assume together with its structural characteristics.

For example, a simple temperature control system with a thermostat can be considered as logically closed and complete because the influence of the environment is reduced to its possible temperature variations to which the thermostat reacts.

A system is intended as logically open when there is a violation of at least one of the three points above. Logical openness ([13–16], [17] (pp. 49–50), [18]) can be considered as an unlimited number of the degrees of freedom when the system includes the environment (in principle independent), making the system incomplete with regard to the environment and its influence (see Table 1).

Table 1. A schematic comparison between aspects of logically closed systems and logically open systems.

Logically Closed Systems	Logically Open Systems
Deductive and inductive	Deductive, inductive, and abductive
Avoids contradictions	Use contradictions
Insensitive to context	Context-sensitive
Does not change the rules, at most the parameters	Change the Rules
Non-Flexible	Changes its structure but maintains consistency
Does not learn	Learn
Work on the basis of mono-strategies	Work on the basis of multiple strategies, such as DYSAM *
Object-oriented	Process-oriented
Observer considered external, generator of relativism	The observer is an integral part of the system and generator of cognitive existence
Passive	Active
Conceptual framework of objectivism	Use of objectivism *and* constructivism

* DYnamic uSAge of Models (DYSAM).

In the case where the number of models is limited and stable, one could hypothetically take them into account as a single *complete* approach independent of the circumscribed internal dynamics. However, this hypothetical completeness is unrealistic for both the complex phenomena of emergence with the acquisition of property, and for the constructivist recognition of new problems not posed before, for new representations, and for the loop of cascades solutions—new problems.

These dynamics are addressed with the concept of *abduction*, introduced by Sanders Peirce, where it is a matter of the invention of hypotheses [19]. This is in contrast, or in addition in the DYSAM-like way, to the deduction and induction. The theme is related to creativity, as in the *second order cybernetics*, when logical openness consists of moving from playing the same game with all of the associated parametric variations admitted to playing and inventing another non-equivalent game [18].

Completeness is also incompatible with the settling of unique events (completeness is compatible with repeatability) at certain levels of description (an approach does not have intrinsically unlimited validity) and even with *decisiveness*. As introduced by Alan Turing, a problem is decidable if there is an *algorithm* that produces the corresponding solution in a finite time for each instance of the input data.

This is the concept of *effective computability* carried out by an abstract machine, i.e., the Turing Machine (TM), a computer that is all computationally equivalent [20].

A problem is "undecidable" if there is no algorithm that produces the corresponding solution in a finite time for each instance of the input data (equivalent to the fact that an algorithm cannot produce the solution in a finite time). This concerns the non-reducibility to a procedure as a general property based on incompleteness, since not only can there be no effective computational algorithms to solve problems, but there can be phenomena that are not algorithmizable.

As for the first case, undecidability, a typical example is the *classic halting problem* for the Turing Machine [20]. With great sensitivity, Turing himself wanted to introduce an issue of 'completion' by inserting an element of theoretical non-completability such as the concept of Oracle [21]. The Oracle represents another logic, possibly incommensurable, that interferes and acts on that in use.

In this regard, in mathematical logic, Kurt Gödel demonstrated two theorems on the incompleteness of mathematics [22].

This result can be extended to any theory that can be recursively represented (using recursive functions and predicates), and for which it is possible to arithmetize the syntax of the theory. A generalization is that it is not possible to use a coherent system to demonstrate its own coherence. Incompleteness concerns indecision as a characteristic of a particular problem, for example, the non-theoretical availability of an algorithm that can answer questions about the problem.

Regarding the second case of the non-reducibility to a procedure, we consider that even in case of availability of effective computational algorithms, the finite precision or finite memory (in case for symbolic manipulation) implies theoretical incompleteness [23–25]. Moreover, another example is given by the non-explicit, non-symbolic computation of Artificial Neural Networks (ANN), see, for example, [26,27]. As is widely known, they are *multi-layered weighted nets* according to which the computation is distributed. This processing is represented in a non-analytical way through weighted connections (weights can change during the process) and levels. For this reason, the processing is considered non-explicit and not analytically represented, which is why it is called *sub-symbolic*, even if the program performing the ANN is an explicit algorithm. If we look instant per instant at the calculation carried out by an ANN, the computation is incomprehensible, so we have to wait for the final result. This also applies to other computational processes such as Cellular Automata. The computation acquires property not formally prescribed like learning.

The objective of such *machine learning* [28] is that a machine is able to *generalize from experience*, which is to *induce*. Furthermore, the architecture of the ANN is used in combination with various approaches of a statistical and probabilistic nature for activities such as classification, clustering, data mining, linguistic, profiling, recognition of patterns, and simulations. Otherwise, for entities provided with cognitive systems at different levels of complexity (*Homo Sapiens Sapiens* are assumed to have a level of greater complexity), learning is a question of dealing with situations in learning modalities using various interrelated capacities such as emotional, inferential, linguistic, memorization, perception, and representative abilities.

Particular classes of ANNs, such as those with non-Turing computable weights, and Recurrent-ANNs [29,30] show a non-Turing behavior for which the principles of *hypercomputation* [31,32]. Furthermore, several approaches for *naturally-inspired computation* [33] have been introduced.

A non-mathematical example of non-reducibility to a procedure concerns *occupational safety* [34], which is not exhaustible by rules and regulations. In fact, rules and regulations cannot exhaust and complete the cases to be treated as anomalous behaviors due to emotionality of any kind and environmental fluctuations (the implementation of unique events such as combinations of the quantity of gases and critical manner in chemical industries and the breakage of dynamic balances with collapses in construction sites). In addition, we can consider the huge philosophical field concerning the non-reducibility of morality to law.

Let us consider at this point another example of *incompleteness.* In geometry, an axiom of order introduced by David Hilbert states that if A and C are two points, then there is always at least one-point B on the AC line, such that C is between A and B.

In scientific literature, multiple and dynamic incompleteness can be understood as expressed by the concept of *quasiness* where examples include quasi-particles, quasi-electric fields, quasi-periodicity, and quasicrystals, which are a particular solid form where atoms are arranged in a deterministic but not repetitive structure that is not periodic as in the case of normal crystals, honeycombs, and hives. Quasicrystals have quasi-periodic patterns for which the local arrangement of the material is fixed and regular, but not periodic throughout the material, i.e., the property is incompletely respected in multiple possible ways [35]. Quasi-systems [17] were also introduced.

We conclude this section by noting how the theoretical incompleteness is delineated as the representation of the freedom of becoming.

3. Incomprehensibility

The theme of incomprehensibility has several aspects. For example, it is possible to refer to the cognitive impossibility or inability to attribute meaning (the impossibility sought in cryptography).

Incomprehensibility may regard the difficulty or impossibility of giving meaning to texts or voicemails in unknown languages, or because they are *incompletely* available, as they are damaged or mixed with interferences. Incomprehensibility may relate to poorly, incompletely, and contradictorily presented issues and texts. In addition, incomprehensibility could relate to pathological situations affecting cognitive processes. In the case of police investigations, incomprehensibility could then relate to behaviors seemingly self-inflicted or meaningless to be deepened. Incomprehensibility may be, moreover, due to mental pathologies.

In this section, we shift the theme in particular reference to the context of the non-understandability of phenomena that can be considered scientifically such as relating to Nature and social systems constituted by the living, and in particular, by *Homo Sapiens Sapiens*.

At this point, we consider that there are phenomena and propositions that are understandable but inexplicable in correspondence to the fact that the domain of the understandable is broader than that of the explainable. In turn, there are phenomena and propositions that are understandable and explainable.

The domain of the inexplicable is therefore either partially contained in the domain of the understandable and overlaps the domain of the incomprehensible.

Neglecting the case of unexplainable understandability, we limited ourselves here to considering the case of *comprehensibility*, in short, as *explicability* with the conceptual tools available to the cognitive device in use, or the brain in the biological reality of its body (more properly *cognitive system* and *mind* as we will see below). Basically, we should be able to answer the question *'why?'*, and represent, predict, and apply using logical inferences such as inferential processes and explanatory theories. Therefore, we assumed here the simplified equivalence between comprehensibility and explicability, understandable and explainable, as between incomprehensibility and inexplicability. Explicability would coincide with the availability of symbolic answers to symbolic questions. This is not the case, for example, for questions about complex phenomena and processes with multiplicities irreducible to each other, as introduced above with DYSAM. It is matter of non *zippability* in formal representations such as processes of emergence [26] and for representations and simulations carried out with sub-symbolic devices, such as neural networks for which the behavior cannot be explained by reference to symbols but is attributable to the connection weights and levels in a context of equivalences [36].

In this regard, we mention how in a traditional view, a model is better when it can explain more with less, according to Chaitin's *compression is comprehension* [37].

Compression as comprehension should be intended as the fact that the models, however, compress more observations from natural phenomena in a reduced number of models [a]. Besides, empirical data sets should be intended as algorithmically incompressible since individual data are determined partly by perturbations and causal factors irreducible to any pattern. Consequently, they exhibit maximal algorithmic complexity, maximal entropy and zero redundancy and a string is algorithmically random when it is incompressible.

Due to their perturbed nature, empirical data sets should be intended as algorithmically random strings of digits [38,39].

Moreover, we can consider understandability as *achieved* when someone understands and we can transfer the understood, we know that there is someone who has understood, even if not everyone is able to do so.

However, there may be various, equivalent and non-equivalent ways of understanding what is believed to be the explicative *why.* The explication can be given by an explanation, by a falsifiable scientific theory, by the fact that we did not perform a ritual correctly, that we have not prayed well, because of astral influences, because of default destiny, etc.

We specify that incomprehensibility is understood here as not general scientific explicability, i.e., with the available knowledge. Incomprehensibility may be temporary, pending theoretical and technological advances, or deemed to be absolute as coincident with eventual *definitive* non-explicability. Can we prove or demonstrate non-explicability such as non-completeness? Or can it be taken as admissible anyway? In the case of definitive non-explicability, the process of local comprehensibility does not converge to complete comprehensibility, but diverges in theoretical incomprehensibility (see Section 3), *absolute incomprehensiveness*. The *comprehensibility of the incomprehensibility would* be, in its 'complete' representation, at least as a limit, as denial, incommensurability, different cardinality, other than the understandable, as *imaginariness* (*imaginary numbers*, which are widely used and effective, come to mind), while the *non-understandability of the incomprehensible* would be in its intractability and theoretical non-representability and irreducibility to the variation of the understandable, as its negation.

As mentioned below, the ability to understand can be *reductively* understood as a property of the available knowledge *and* of the cognitive device in use (for example, the property of neural and reticular complexity of the brain understood as necessary for the cognitive system and the mind). For example, mechanics was first thought to explain the world, then thermodynamics, then physics, followed by electromagnetism and then quantum field theory. Are we done?

We can understand that we have not understood until we understand that we cannot understand, that namely *the strategy of understanding does not apply in absolute* (for animals, we would say that by trying to eat, they can understand that not everything is edible). After all, it would mainly involve—In a simplistic view—The activity of an organ like the brain.

Other organs have other activities. Should the brain be extended (even artificially)? Have we missed other organs? Do we need an evolutionary leap?

Or perhaps the current understanding would have no limits and it is only a question of increasing *more and more* (is it a matter of increments converging to complete understanding?) the understood?

Many species have a brain that is used for their role in nature (surviving by seeking food, reproducing, defending, seeking the best environment—Migrations and lethargy—etc.).

The theme of comprehensibility (and incomprehensibility) is not only conjugated with the theoretical resources used and abductively increasable, but also with the *device* biologically used for now—The brain and the mind capable of developing and using cognitive resources adapted to it.

In fact, as considered by the *cognitive sciences* [40–42], the emerging dynamic system of cognitive processes, the *mind*, does not coincide and cannot be reduced and simplified to the brain (mentioned in the immense literature [43]) even if as an inferential device is generally considered crucially necessary and capable of studying itself.

Computationalism and its derivatives are outdated approaches replaced by considering the *cognitive system* and its *hosting* body from which there is an emergence of the mind ([8], pp. 387–405).

Related issues that are not discussed here include artificial, collective, networked *intelligence* and the artificial *mind*.

The discourse then becomes more complicated when evaluating the availability of *self-consciousness* without considering that the brain has its own evolutionary history and local properties such as aging, the context, the learned, and the health of *its* body when considering the number of neurons and their network, see, for example, [44].

The extent of the species' understanding appears to be an improper use, at our peril (the fact of Eve). However, comprehensibility is neither necessary nor sufficient to act in the world.

Comprehensibility—Explicability is however *intrinsically insufficient* in that it has a provisional and local nature of being incapable of explaining *new, subsequent* problems (generated, for example, by side effects of uses, understood or not, such as pollution, medicines, and exposure to previously unknown radiation), or that had not been previously considered (for example, gravitation, electromagnetism, and radiation).

Explicability is local as it relates to questions that we can consider that change over time without presumption of converging to another ideally *definitive*, assumed as existing and attainable.

Local incomprehensibility, i.e., temporary, would be due to knowledge not yet available. However, understand that something is incomprehensible seems to be an oxymoron. We understand that a door is closed because we know that the doors can be opened. This does not imply that all doors can be opened.

Hypothetically we might aspire to a completely understood world. A world characterized by the theoretical completeness of the explained.

The fact is that the world and its emergent becoming are incompatible with the completeness that would *explain it as* reduced within the limits of the comprehensibility of our thinkability.

Facts of incomprehensibility represent guarantees that this *collapse* on the completeness cannot happen. The world remains theoretically more than we, even cognitively, are. It is logically open and logically non-closable (see the concept of *logical openness* introduced above).

The theoretical problem of our understanding is that it admits, includes, and is not only compatible, but also *contemplates* its completion. This is, however, inherently inconsistent. This would imply that the world runs according to a final goal that does not exist, whereas there are many local goals that are not necessarily succeeding in a unique way, as well as convergent goals. This somehow seems to correspond to the deterministic illusion of believing that the more we go into detail, the more we approach the effective final knowledge (the last particles of matter. . .).

However, the development of science and technology provides this sensation. What reaction would Bach have to hearing his own music coming from a box?

Understanding and explicability would be inherently local and not global and would not exhaust the *size* of the world. Understanding is effective for our species, for the interest of species, and allows us to anticipate and design.

One is amazed at the comprehensibility of the world. It is like being surprised that a mother and child speak the same language. *We are part of the world and understanding it is our way of belonging to it*.

Indeed, the species-mission of our subspecies, *Homo Sapiens Sapiens* [45,46], might just be to understand to act. Understanding would have the same nature as the ability to *know* how to fly for birds and pollinate for bees without presumption.

Understanding has been discussed in a thoughtful, founding manner, for example as "*understanding understanding*" in [47]. However, it may also be sufficient for understanding not only for there to be anything else, but also for there to be incomprehensibility, or something incommensurable, of a different *cardinality* (in mathematics, and in particular in set theory, Cantor's theorem states that given a collection of any cardinality—Number of elements—There is always a greater set of cardinalities).

A *comprehensible* aspect of incomprehensibility concerns the limits of the explanatory power of theories and models when they are inadequate and ineffective in the face of certain problems. The possibility of increasing understanding begins with the understanding that something is, at present, incomprehensible.

Paraphrasing Von Foerster [47], there are no *anomalies* in the environment. If a given phenomenon seems strange and incomprehensible, it is intended for the theoretical framework to be used to understand this phenomenon as inadequate. This requires inventing new conceptual frameworks, explanatory theories, and approaches within the cognitive process of the reformulation of the interpretative model named *abduction* (see above) as its purpose is to "normalize" anomalies [48,49].

For example, phenomena considered in the domain of electromagnetism were incomprehensible for thermodynamics. Recognizing and considering them as incomprehensible has made the theory of electromagnetism possible. We will not address the aspect of *incompleteness of scientific theories* (see, for example, [50]), which refer to the incompleteness of mathematics, as introduced above.

This process of reformulation, abduction, could be understood to have a final limit of convergence or could be intended as being *always*, *repetitively* applicable, mathematically *ad infinitum*.

The correspondence between non-computability as the unavailability of an effective computational algorithm (a Turing Machine as introduced above) and not comprehensibility as the non-availability of theories of opportune explanatory power comes to mind. Is it possible to think of correspondence between hypercomputation (see above) and a hypothetical *hypercomprehensibility*, all to be ideated, for example, by postulating the cumulative applicability at the *infinity* of abductive approaches?

Examples of conceptual correspondence of *jumps of cardinality* with regard to comprehensibility, that is to say hypercomprehensibility, are given by current theories which presume significant distances from a classical understanding of the universe such as the string theory [51]. We also mention quantum theories such as quantum field theory [52,53] where in place of the objects, there are fields [54], everything is connected through the *entanglement* [55], and no classical iterations are necessary as they are properties of the *quantum vacuum*. It is the quantum void that gives properties to matter as being always connected—*Entangled*—not being the void lack of matter [56]. Wave-particle duality was introduced by Louis De Broglie, redefined and developed, for example, in [57–59] as *quasi-particles* (called *magnon*) that share traditional particle properties with the exception of localization [60], and the time as *imaginary* in the statistical theory of fields [61], just to mention a few *deviations* from the classical understanding (see also [17], pp. 221–248).

In the constructivist vision (see above), experiments are intended as questions to nature responding by making them happen. There are no answers without questions. However, events can turn into answers if we find the appropriate questions. Nature is not a huge mine to be dug and potentially exhausted with cognitive facts (theories, experiments, and technological resources), rather, it is available to be explored through the abductive invention of questions in multiple and non-equivalent ways (in finite or infinite numbers, as for the theoretical incomprehensibility introduced below).

Local incomprehensibility, or limited understandability, could thus be an abductive engine for the development of more powerful successive comprehensibility.

We will limit ourselves to considering issues that can then be elaborated from various points of view in future research.

3.1. Theoretical Incomprehensibility

The theme of the *intrinsic, absolute, or theoretical incomprehensibility* above-mentioned as considered in correspondence with the theoretical incompleteness, is not very popular in the literature if not implicitly and sporadically regarding, for example, psychiatric or theological issues (with divisions on the *admissibility* of theology itself. This is also because it seems to represent intrinsically arid territories for research and without the prospect of applicability of some interest. As we have said, the research on theoretical incomprehensibility seems to be an oxymoron.

For instance, madness was considered as a fact of strangeness, otherness, incomprehensibility and irrationality. See, for instance, Michael Foucault [62].

We have seen how incompleteness relates to understood properties of the becoming and its representations. It is also a property of the thinkable and thought, as we have seen for the incompleteness of mathematics. This relates to the relationship between the incompleteness and the *consistency of* *the thinkable* (its *autonomous* property?) which, for example, rejects theoretical approaches aimed at demonstrating errors such as attempts to prove the absurdity of the fifth postulated in Euclidian geometry [63,64]. How can we interpret, for instance, the non-comprehension of the distribution of prime numbers, which is combined with their use for cryptography [65]? What *is* this impossibility supposed to mean? Another interesting research issue in Systemics relates to conceptually replacing the interaction-based mechanism among the constituent elements when dealing with collective systems whose number of elements tends to potentially be infinite (analytically intractable).

This incomprehensibility concerns mainly the becoming and not, as expected, the thinkable, or the thought being the incomprehensibility of the thought hardly permissible if not for pathological cases, as in the fact of madness.

The issue of incomprehensibility has, however, a profound interdisciplinary nature involving scientific issues such as logics, mathematics, physics, neuroscience and mind, philosophy of science, and anthropology. We can think of a *science of understanding* irreducible to neurology and given by *cognitive sciences* (see, for example, the fundamental [42]). *However, with regard to irreducibility, everything has to happen in some way, even if the way by which it happens is other than what happens (event)*. The 'modality' *recognizable* at the level of representation used by the observer can concern the becoming of phenomena and processes possibly equivalent for the establishment, settling, and emergence of the event. Here, we consider the presumption of the existence (not necessarily re-constructible backwards) of the processes, in some cases possibly equivalent, which led to an event.

We refer to the way of becoming as an emergence that is other from what emerges from [35], and also for thinking and understanding when incomprehensibilities and non-demonstrabilities open the way to paradigm shifts.

Considering the *ability to understand of not understanding* (recognition of the ineffectiveness of models and cognitive approaches in use) and the *ability to understand of being not able to understand* (theoretical incomprehensibility) as the understanding manifests itself as an inadequate strategy. In the first case, it is a matter of formulating new questions whereby the space of answers currently empty is conceptually refillable, while in the second case, the space of answers is not conceptually refillable with admissible answers.

If the conceptual category of incomprehensibility is permissible, it can therefore only be declined as local, provisional, or it could also be theoretical like, for example, understandability of the inadequacy of understanding. What is the point? What usefulness does it have?

Once more, this can be applied to imaginary numbers, which are very useful and extremely usable. We might think of strategies other than understanding when dealing with incomprehensible facts. Of course, we have to give them the statute of being theoretically incomprehensible and not temporarily by using surrogates, understandable simplifications, and other reductions.

An example of facts of incomprehensibility that are inadequate for the application of scientific explanatory theories is artistic productions, for which to understand is a limited *strategy, ineffective* and *inadequate*, if not a form of reductionism, and the *practices* of the religions established to determine and regulate the alleged reality beyond earthly life. Human beings have always conducted practices, ceremonies, and built places to do something in the face of the enormous incomprehensibility of death. We *celebrate* in front of this incomprehensibility.

The deception of comprehensibility consists in its alleged lack of limits, in its alleged absolute generalizability. The *poison* in Eve's apple lies in this and not in the local use, of the resource of our species, the brain (a more complex organ than in other species) and the emergent mind.

We previously introduced the concept of *logical openness*, which can be understandable, for example, as the ability to change a game, invent new games, i.e., new models according to which to act and understand, through abduction and learning. This is the approach of not to iterating and not insisting on using the same approach (always playing chess, always explaining something with the same model in use, e.g., everything as thermodynamic phenomena, as phenomena to be optimized, . . .), which in turn leads to at least two alternative approaches.

The first approach is that the world is understood cognitively as multidimensional and indeterminate and becomes determined in the act of knowing, that is, the knowledge in use must be dynamically interdisciplinary whereby the same problem, a *same reality* (recognized as such) has representations, properties, and multiple non-equivalent characteristics irreducible to each other.

This reference is to Constructivism and *Invention of Reality* [66]. The configurations considered by research are unavoidably constructivist [67–70], created by the observer, the generator of *cognitive reality*, rather than of relativism, as introduced by De Finetti (see Galavotti [71]). The cognitive strategy expressed by cognitivism is well expressed through the distinction between: (1) trying to understand how something *really* is, and (2) how it is more effective to think that something is (which model to adopt).

Note that the first case (the existence of objective reality independent from the observer) is a particular case of the latter. There is not one single excellent explanation. An infant uses the five senses, irreducible to each other, to learn how to use them coherently, not to choose the best one.

Another example is when the problems and the solutions of a representation are also usable for another. This is the case with equations and models used in a discipline and then used in another by changing the meaning of the variables, of the conversion of a military problem into an economical one, an algebraic problem into a geometrical one, or teaching a discipline by referring to another (explaining history through geography, urban planning), the strategy of exchanging roles, for example, when dealing with couple problems, one is asked to represent the motivations of the other.

The central point is that it is possible *to increase* the number of representations and disciplines used, but the resulting multiple and dynamic representation is not to be theoretically considered as exhaustive or complete. This is comparable to the misunderstanding that the increase in the data and information available involves an increment of knowledge and bearing to the paradox of Big Data [72]. The data are not convergent to the explanation and the knowledge (the poor idea was to make knowledge in production by quantitative increases of data). The underlying idea was that the availability of a large amount of data that could be increased at will would have led to the possibility of selecting, characterizing, and clustering data by data-driven approaches through concordances, correlations, correspondences, interrelationships, and statistics towards *theory-less knowledge*. The reference is to cases of knowledge produced without searching and availability of theories but, instead, by using concordances and correspondences [73,74].

A *second* possible approach is to assume that we should consider the *meta-properties* of the representations, of the explanations, namely, that we consider representations of representations and properties of properties.

This relates to the thematic of *meta-structures* (when the structures of interaction between elements are variable, as in collective behaviors of swarms and flocks the interactions (see above), take place considering, for example, the distance, altitude, speed, direction, and variable temporal duration) in which the representation of the collective behaviors is undertaken by using appropriate properties of the clusters of elements and not by considering single separated elements as structured by fixed, invariable interactions and representation that this later is treatable at the most statistically, but not analytically [10,75].

We depart from exhaustiveness, but we accept and use *fuzziness* [76], *incompleteness* of the explanations [2] and *sloppiness* of the models [77], i.e., models with many parameters of fitting, generally more than five, are intended as sloppy. These models are "poorly constrained" and "poorly conditioned" because it is difficult to use the experimental data to understand what their parameters are. We are interested in properties, recurring or not, possible equivalences, possible combinations, topological forms [78], and distributions of their multiplicity. This reasoning is outlined through analogies and metaphors.

We see aspects of incompleteness in comprehensibility. Sequences of incomplete comprehensibility are not necessarily convergent to comprehensibility, or hypothetically complete. A state of *completed* understanding is not plausible even for the dynamics of a world where solutions generate new problems and where the strategy of *resolving* concerns areas with reduced complexity, with complexity to be *managed* rather than to be *solved* [35]. Complex phenomena as collective behaviors such as traffic signals and vehicles, properties of financial and biological phenomena, e.g., health, should be assumed to be *oriented* rather than *decided or regulated*.

3.2. Theoretical Incomprehensibility of the Loop 'Comprehensible—Incomprehensible' Approaches

As incompleteness is at least a space of equivalences within which there is freedom for the becoming of the processes of emergence, the theoretical incompleteness of the space of comprehensibility can be intended as the property of the space of freedom for multiple understandings. The space of explanations would be incomplete as logically open.

Theoretical incomprehensibility as an expression of the logical openness, can be understood as a necessary condition for local understandabilities not inappropriately extensible or generalizable.

Furthermore, understandability is usually combined with *eligibility* with respect to the *rules of the world* (assumed, however, as objectivist and not constructivist), in essence, with respect to the so-called *laws* of classical physics, which are considered as objectively immutable, existing as a characteristic of the world to be *discovered* (as if they were *covered*, as opposed to constructivism considered as generated). Only what is explicable would be admissible. *Tautologically*, in the vision of classical physics, what is permissible would be understandable and explicable, *discoverable*. The concept of eligibility has acquired a tremendous increase in degrees of freedom with quantum physics.

Where does this reasoning based on theoretical incomprehensibility take us? To skepticism? To opening the door to *unlawful explanations*, authorized neither by science nor by efficacy? Should we resign ourselves to our corner of local validity?

The fact is that by now, we understand that understanding is incompletable, and that the understood is incomplete.

The incompleteness allows scientific declinations that refine and specify our knowledge by extending usability.

Theoretical incomprehensibility can mostly be considered in the three following ways.

In the first case, theoretical incomprehensibility is combined with the inexhaustibility and incompleteness of the multiplicity of constructivist reality when we invent a way to think of something in a continuous loop of usages of objectivism–constructivism. A central point is that it is possible to increase the number of representations and disciplines that will be used, but the resulting multiple and dynamic representation is not to be theoretically considered as non-exhaustive. We can consider postulating the cumulative applicability at the *infinity* of the abductive approaches. This is well expressed by logical openness considering coherent usages of resources rather than the selection of the best or optimum one.

In the second case, theoretical incomprehensibility is considered related to the problems of incompleteness in science, such as phenomena for which the *principles of uncertainty* and *of complementarity* apply. In a sort of conceptual correspondence with the principles of uncertainty, *the theoretical incompleteness would be indicated by the logical openness of the understanding itself.* An understanding that would admit itself as closed and complete would admit its own inconsistence with itself. Theoretical incomprehensibility should be intended as an *inexhaustible mechanism* for the production of local and temporal inconsistencies leading to paradigm shifts. This is related to the power of potentially inexhaustible non-demonstrable issues requiring and inducing paradigm shifts. We should concentrate on approaches (logical, experimental, computational, . . .) suitable, for instance, to recognize, *invent*, represent, transpose, generalize, simulate, and compare facts of incomprehensibility, such as paradoxes, inconsistencies, unprovable obviousness taken for granted, postulated assumptions, and violations. Incomprehensibility is also a source of theoretical research issues. The purpose is to normalize them within completely different conceptual frameworks. The cognitive strategy is to look for such incomprehensibilities and make frameworks that convert the current comprehensibility into incomprehensibilities in a continuous *theoretically incomprehensible* endless loop.

For the previous two cases, we provide epistemological examples for which algorithms can be written. However, we may also consider the second-order or recursive theoretical incomprehensibility as a powerful hermeneutic tool in support of a "science of understanding". With reference to the concepts mentioned above, we are supposed to deal with *incompressibility* [79].

In the third case, theoretical incomprehensibility seems, moreover, to also relate to *existential* questions, as outlined below. Many such questions remain unanswered because they are incompatible with an answer and with the *answering itself*. *There are no admissible answers in principle*. Human beings have always conducted practices, ceremonies, and built places to do something in the face of the enormous incomprehensibility of death. We *celebrate* in front of this incomprehensibility.

In Table 2, we present the three key-points of theoretical incomprehensibility, summarizing the discussion introduced and proposing approaches.

Table 2. Three fundamental types of theoretical incomprehensibility.

The theoretically incomprehensibility of inexhaustibility of the multiplicity of constructivist reality
The theoretically incomprehensible endless loop Incomprehensible-comprehensible and *incompressibility*
The theoretically incomprehensibility of existential questions

We extensively discussed the first two points considered in the Table 2. With reference to the last point, existential questions are of the following type.

What is the purpose of life? Is there anyone who cares about us? Why is Nature an attentive mother to the species, but a stepmother insensitive to individuals (in fact, on different time scales, it is also the stepmother of the species in the sense that it makes them extinguish)?

What is the answer to the question of *what happens after death?* What is hidden behind the reference to *destiny* (being at the wrong place at the wrong time has in fact non-zero probability)?

Do such questions have *meaning*? Do they have a level of understandability and meaningfulness adequate to be answered?

These are *existential* questions, incommensurable with rational answers, theoretically incomprehensible, as the wondering the weight of a thought and the color of a taste. The space of possible answers is cognitively void, with only the availability of hypotheses. In answering these questions, the abduction is not subject to any kind of empirical verification, so they remain hypotheses.

However, in the face of such incommensurability, we mention the cognitive process of making *illations*, as introduced by John Henry Newman [80].

The certainty reached by a convergence of probabilities is also called *natural inference*. This certainty is recognized as linked to the action of an "illative sense", a subject to which the entire chapter XI of the book [80] is dedicated. The author writes that it is the mind that reasons, and that controls its own reasonings, not any technical apparatus of words and propositions. This power of judging and concluding, in its perfection, is called the *Illative Sense* [81].

The *Illative Sense* operates not by virtue of education or ability for formal reasoning, but by virtue of natural experience and maturity, based on the practice and context of previously acquired knowledge.

The personal dimension and personal value of the illative sense are clear.

We all are supposed to have an illative sense, a faculty of theoretical reasoning that judges the validity of inferences.

This specifies other aspects of the theoretically incomprehensible, which can be terrifying for those seeking only answers or the syntax of becoming.

Additionally, where does the door overlook? On the brink? In the dark? These questions have always been handled as sources of inspiration and remain unresolved.

This brings to mind the era when intelligence was intended as the ability to solve problems and avoid errors—The era of Good Old-Fashioned Artificial Intelligence (GOFAI) dedicated to manipulating symbols [18]—While in the era of the paradigm of complexity, this is a question of learning, managing, and orienting the emergence of becoming rather than deciding or designing it and using it rather than just avoiding 'mistakes' [82]. It is matter of allowing software programs and algorithms to become autonomous and acquire properties such as learning, as introduced above for ANN.

Even in the face of possible artificial extensions of understanding (*cognitive prostheses*, for example, so-called *augmented reality*, are intended to allow us to act in the incomprehensible), we could presumably have extensions and not categorical jumps.

The possibility is always open to using strategies of the theoretically incomprehensible (e.g., avoid, combine, induce, reduce, replicate, and represent the theoretically incomprehensible).

One should look for this incomprehensibility and not only stumble into it, but consider it when unavoidable.

Hypothetical researches on theoretical incomprehensibility should not only look for cases, but also invent cases in logics, mathematics, physics, linguistics (semantics), and philosophy.

A great cognitive window is provided by the ability to learn, adapt, and act by cohabiting with the theoretically incomprehensible, represent, use, and make it happen, act by analogies and use the *body-brain-mind* as a 'synthesizer and decision maker' for sensations and intuitions.

Presumably, there is a need for continuous coexistence between the understandable and the incomprehensible, being the second combination of local incomprehensibility (temporary, for example, use without understanding) and becoming of events with properties that have a nature of absolute incomprehensibility, like in the points considered in Table 2. We are responsible for the use of the understandable and the not understandable.

Regarding life, even for understanding, can we presume the existence of other forms of comprehension? Would we recognize a *life* different from the present biological one? Would we recognize an extra-terrestrial life? Are there other forms of understanding? The presumption that everything is exhausted from our properties recalls geocentric theory. However, science has been able to overcome it. Is it permissible (in this case, no permission is required from anyone) for science to overcome itself?

4. Conclusions

Theoretical incomprehensibility is not the funeral of knowledge, but its context, which gives meaning. Theoretical incomprehensibility is not reduced to non-understandability, but outlines *another* from the understanding. It can be considered as a logically open space.

Theoretical incomprehensibility is intended as a signal, a representation, and a guarantee of the intrinsic logical openness of the world. The world emerges more than it functions or works.

Theoretical incomprehensibility corresponds to the incompleteness of the emergence of the world and is expressed by its non-exhaustibility, the non-finiteness of local understandabilities, non-improperly extendable in the absolute. Global comprehensibility for the great Laplace machine was the resulting, definitive sum of local understandabilities that are reflexively valid. This is related to the power of potentially inexhaustible non-demonstrable issues requiring and inducing paradigm shifts

In a manner of *antinomic* nature, the understanding of theoretical incomprehensibility would be a negation of the logical openness, which is the impossibility of understanding intended as the dynamics of hypotheses, sequencings, and accumulations.

An understanding that admits itself as closed and complete would be inconsistent with itself, as it would admit its exhaustibility. Openness and non-exhaustibility are instead assumed in the face of the unnecessary conceptual limitation of abduction, of the possibility of acquiring new levels of description, of the establishment of new problems as a consequence of the dynamics of becoming (for example, due in our age to technological growth and environmental changes), of the existence of unresolved problems for disciplines like mathematics and physics, and of the infiniteness in the issues, opening the scenario to *paradigms shifts*.

The inherent deception of comprehensibility consists in its alleged lack of limits and in its alleged absolute generalizability. The *poison* in Eve's apple lies in this and not in the local use and resource of our species.

In conceptual correspondence with the principles of uncertainty, *theoretical incomprehensibility would be indicated by the logical openness of the understanding itself*, an understanding that would admit itself as closed and complete would admit its own inconsistence with itself. The theoretical incomprehensibility should be intended as an *inexhaustible mechanism* for the production of local and temporal inconsistencies leading to paradigm shifts. This is related to the power of cases of non-demonstrabilities for issues requiring a paradigm shift. We should concentrate on approaches (logical, experimental, computational, . . .) suitable, for instance, to recognize, represent, transpose, generalize, simulate, and compare facts of incomprehensibility such as paradoxes, unprovable obviousness, and postulated assumptions.

Hypothetical researches on theoretical incomprehensibility should not only look for cases, but even invent cases in logics, mathematics, physics, linguistics (semantics), and philosophy.

The cognitive strategy is to look for such incomprehensibilities and make frameworks suitable to convert the current comprehensibility into incomprehensibilities in a continuous *theoretically incomprehensible* endless loop. Theoretical incomprehensibility would then be a source of theoretical research issues.

The interest focuses on the possibility of generating new non-equivalent approaches, paradigm shifts, and paradoxes.

Can we support such research with suitable software, for instance by simulating logical frameworks of incomprehensibility? Is this a step toward a kind of *artificial creativity*?

Dealing with theoretical incomprehensibility is a matter of the *inventive* use of strategies and cognitive strategies indirectly, in nonlinear correspondence or *interfaced* with classical understanding. Theoretical incomprehensibility is a subject of possible interest for philosophers, systems scientists modelling processes of emergent acquisition of unexpected, non-equivalent, properties, model builders and social systems designers dealing with the creativity and unexpectedness of complexity, and educators introducing students to the ineffectiveness of using pre-complexity approaches [83] to deal with the post-industrial society [84–88].

The purpose was to facilitate and induce possible conceptual breakthroughs in philosophy and social sciences in hypothetical correspondence to science with quantum physics, complexity, computational sciences, and artificial intelligence.

Theoretical incomprehensibility is intended as a sign, an index of dominant logical openness.

The most significant novelty of the article lies in the focus of the concept of theoretical incomprehensibility, in distinguishing it from the incomprehensibility and the consideration of different forms of understanding. Moreover, theoretical incomprehensibility is detected as a manifestation and a guarantee of incompleteness of the crucial logical openness, for example, for aspects such as non-decidability, principles of uncertainty, non-computable uncertainty, non-complete, non-explicit and non-univocal modelling for the use of non-equivalent models when dealing with complexity by the Dynamic Usage of Models (DYSAM).

Theoretical incomprehensibility is a property of the inexhaustibility of the multiplicity of a constructivist reality, and of the inexhaustibility of the incomprehensible-comprehensible sequences. It presents itself as a subject for philosophical investigation to detect the limits of understanding and to consider them as a property of understanding itself.

Theoretical incomprehensibility is a matter of identifying strategies to *act* and *coexist* with the theoretically incomprehensible, to *represent* and *use* it, for example, when dealing with imaginary numbers and quantum contexts where classical comprehensibility is theoretically impossible.

Such cases, uses and strategies can in turn be represented and simulated in suitable contexts of AI. Can we think of forms of non-classical understanding?

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