



Article

New Objects, Questions, and Methods in the History of Mathematics

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Abstract: This article sums up recent developments in the history of mathematics. The range of mathematics considered has considerably broadened, expanding well beyond the traditional field of original research. As new topics have been brought under consideration, methodologies borrowed from neighboring academic fields have been fruitfully put into use. In the first section, we describe how well-known questions—about the concept of proof and the nature of algebra—have been reconsidered with new questions and analytical concepts. We then sketch up some of the new research topics, among others the history of mathematical education, the inclusion of actors previously neglected, and the prominent role of bureaucracies in the cultural development of mathematics. The last section briefly retraces the development of the Zilsel thesis as a case study illustrating the previous points. Introduced in the mid-20th century, the theory that early modern craftsmen once played a decisive role in the mathematization of nature has recently led to very diverse fruitful studies about the nature and development of mathematical knowledge.

Keywords: historiography of mathematics; algebra; proof; education; numeracy; practitioners; Edgar Zilsel



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1. Introduction

The history of mathematics is nowadays a research field recognized and discussed enough to have its own historiography.¹ In fact, one of its most significant features is its ability to reflect both on its specificity compared to other historical disciplines, and its rapid evolution in the last couple of generations by integrating concepts and methods from the humanities. The institutionalization of the history of mathematics is intertwined with the development of modern mathematical communities, “the writing of the history of mathematics [playing] an important role in the shaping of mathematical identity during the period [1880–1940]” (Remmert et al. 2016b, pp. 2–3).

Ever since, the history of mathematics has been a stable—and largely autonomous—subgroup within the history of science and technology. In the early 20th century, it was mainly practiced by mathematicians, and occasionally by philosophers. Sociologists, philologists, and historians have then brought their own focus and approaches. While this once generated heated methodological debates, a large consensus now acknowledges the necessity of relying on a wide array of questions and problems, and the relevance of using different approaches to tackle the important issues of the history of mathematics (Schneider 2016).

Still, the history of mathematics deals with formidable challenges as it attempts to contextualize the emergence and evolution of concepts, ideas, notations, and their developments in different times and places. While these issues are not specific to mathematics *per se*, the task is extremely broad if we think of the variety of cultures in which mathematics have developed since antiquity, with their own language, symbolism and representations (or

lack thereof), rationality, or mindset. Several important questions, such as the use of modern mathematical notations, or the relationship between philosophical and mathematical concepts, are very general issues. Collaborations between specialists of, say, ancient Greece and South-East Asia, are trying to offer, if not answers, at least meaningful methodological frames. Moreover, as the history of mathematics gained some independence from the mathematical community, interactions have developed in the last decades with the history of science, itself influenced by the history of knowledge. This has led to complex questions about what makes mathematics a specific kind of knowledge—if it is at all—and how to account for its development while taking into consideration all relevant and related fields of human knowledge.

In this article, we will briefly sketch important developments of the field in the last couple of decades. The first part will outline how traditional inquiries have been reoriented thanks to new research methods by using two archetypal examples, the history of modern algebra² and the all-important concept of mathematical proof, which has been the subject of numerous historical accounts since the 20th century. Historical narratives once centered on “great men” and a retrospective view of the development of mathematics have been challenged, and gradually sidelined, by more diverse approaches. The second part of this article will sketch how the last decades have witnessed a broadening of interests. Research programs that once focused mostly on academic mathematics have gradually turned to new places, from museums to schools or factories. Similarly, a vast trove of mathematicians’ and practitioners’ works is now being investigated; besides, the uses of mathematics and the utmost importance of quantification for the history of knowledge and administrations have engaged the researchers’ attention. The last part focuses on a particular example, Edgar Zilsel’s thesis about the role of mathematics in the development of modern sciences, to illustrate recent developments in the discipline.

2. Back to the Classics: Rereading Mathematics and Rewriting Its History

In their introductions to two special issues of *Science in Context* on the history of ancient and modern mathematics, respectively, Netz (2003) and Corry (2004) discussed various recent works aiming at rewriting parts of the history of mathematics. These works propose new, contextualized re-readings, going beyond a purely modern mathematical reading to understand the current state of the discipline—the well-named “royal road to me” jokingly referred to by Grattan-Guinness (1990a) to characterize history as written by some mathematicians.

2.1. Algebra in the 19th Century

The historiography of algebra perfectly encapsulates these evolutions. Depending on the time and place, algebra can mean a method of solving problems, universal and symbolic arithmetic, the analysis of equations, or the study of algebraic structures, among other things. It is thus an example worth discussing, as a discipline emblematic of the emergence of structural mathematics, while being a mathematical domain regularly described as providing a universal language and method.

In his work on the rise of modern algebra and the concept of mathematical structure in the 20th century, Corry showed the feedback effect of the structural representation of mathematics on historical narratives built on selected ‘milestone works’, as well as the selection of the ‘good parts’ and ‘good men’ of mathematics (Corry 1996). Algebraic writing has indeed very often been used to analyze ancient texts as a universal and historical interpretative tool. This has led to many debates such as the controversy initiated by Sabetai Unguru, who vehemently criticized the anachronistic use of modern notations to interpret the *Elements* of Euclid as “geometric algebra” (Schneider 2016).³ Nevertheless, studying the historicity of such representations, and understanding the dynamics of mathematics as well as the diversity of collective and material organizations involved has been the goal of several recent works related to the history of algebra.

The process of disciplinarization, which links cognitive and social issues, began in the 19th century with the progressive professionalization and specialization of scholars (Stichweh 2001). At this time, algebra could certainly not be considered a unified and coherent discipline. Its very definition was then multiple and “linked to the specific practice developed in particular mathematical networks”, whether academic or educational (Brechenmacher and Ehrhardt 2010, p. 611). Historians of mathematics have applied concepts from the history of science to articulate socio-cultural contexts, precise analyses of texts and “modes of production and communication” (Rowe 2002, p. 117) in mathematics.

The notion of “research school” was used by historians to identify and describe how scholars of the so-called “English algebraic school” developed a symbolic algebra in the first half of the 19th century. This algebra was meant to be independent of geometry, and understood by its practitioners as a science of operations. As such, it played a role in the process of institutionalization of science in England in a context of deep political, cultural, and educative reforms (Durand-Richard 1996, following and developing Nový (1968)). The notion of “research school” was also applied analytically in Parshall (2004) to modern algebra at the University of Chicago, in order to highlight the specificity of mathematics compared to experimental sciences. This notably allowed researchers to emphasize the greater independence of mathematics students, and the lesser importance of the geographical proximity of the school’s members, compared to experimental sciences. In the same comparative spirit, Epple (2011) imported and transformed Hans-Jörg Rheinberger’s notion of “epistemic things”, originally aimed at describing experimental systems, to show the necessity to follow “epistemic configurations of research” composed by mathematical objects and techniques embedded in epistemological, intellectual, or cultural contexts.

Introducing concepts from humanities and social sciences in the history of mathematics is also an efficient and promising way to better understand the dynamics of mathematics. This is true of algebraic practices, for instance through the Pierre Bourdieu’s notion of “research field” used to describe a body of research linking algebraic equations, number theory, and analysis (Goldstein and Schappacher 2007). Likewise, the sociological concept of culture has been adapted to make sense of what 19th-century mathematicians referred to as “geometric equations” (Lê 2016).

Another interesting case is Evariste Galois, who since his tragic death has been the subject of numerous hagiographies. To study the “construction of [this] mathematical icon”, Ehrhardt (2011) used the notions of reading (from Roger Chartier), *habitus* (from Bourdieu) and collective memory (from Maurice Halbwachs). This has enabled her to reconstruct the mathematical, institutional, and social *habitus* of the territories crossed by Galois and to analyze the constitution of a collective memory around his character and work.

2.2. Status and Forms of Proofs in Mathematics

The notion of proof also appears to be central to the construction of a historiography of mathematics in the 19th century in the framework of European imperialism and legitimization of the scientific method. It was used as a social distinction to show the superiority of ancient Greek mathematics with their geometric demonstrations—and hence western mathematics—over oriental mathematics, then characterized by particular computations, intuitive rules, and absence of proofs (Chemla 2012; Charette 2012). In this context, Euclid’s *Elements* and Archimedes’ works are paradigmatic examples of texts based on demonstration, understood as required in mathematics, in an axiomatic-deductive form as long as the main value of proof is supposed to be its irrefutability.

These assumptions about what the sole purpose and form of proofs should be have led to a biased and incomplete view of the history of mathematics. This holds for history of ancient mathematics, which lacked some functions and types of proofs, and wholly disregarded the proofs of the correctness of algorithms, e.g., the fact that algorithms give the expected and correct results (Chemla 2012). Beyond irrefutability, a demonstration can namely consist of explanations to bring understanding, especially in a context of teaching. Recent reappraisings of Mesopotamian mathematical tablets carefully considered

the terminology used, the mathematical reasoning applied, and the organization of the texts. They have shown that the geometrical and material terms used in the texts provide a diagrammatic explanation of the procedures of resolution, whereas these methods had been so far seen as numerical steps without any justification (Høyrup 2012).

These recent analyses show the value of reading ancient texts anew, of considering multiple ways of ‘proving’, in order “to study systematically the devices [...] that various human collectives constructed for ‘understanding’ and interpreting the ‘meaning’ of operations” (Chemla 2012, p. 42). Loosening the requirement that irrefutable proofs are necessary in the practice of “valuable” mathematics also reverberates on the historiography of more recent mathematics. Many historical accounts of Pierre de Fermat, one of the most famous mathematicians of the 17th century, once pointed out apparent paradoxes: his available texts are surprisingly short on proofs, especially in number theory. A micro-social analysis of Fermat’s correspondence network has shed light on the concrete modalities of mathematical exchanges in the early modern period. Challenges and answers in the form of solutions in large numbers were prioritized over axiomatico-deductive proofs (Goldstein 2009).

These different examples show the value of constructing a social history of mathematics, which does not neglect the detailed analysis of technical contents and which is not confined to studying institutions or mathematics explicitly applied to social issues. It thus transcends the traditional dichotomy between an internalist history that would only deal with mathematical content and an externalist history that would only deal with institutional aspects. This ultimately shows the necessity for the historian of a “double play of scales, between the local and the global, but also between the short and the long term” (Ehrhardt 2010, p. 493).

3. New Questions about New Topics

Recent efforts have been made not only to study mathematical sources with new methods, but to considerably broaden the very idea of what constituted past mathematics worthy of historical study. Mathematics was once synonymous with the knowledge patiently accumulated by a small number of people, scholars in monasteries, universities, and more recently seminars. In the past two decades, numerous publications have dealt with the development of mathematics in museums, technical schools, public departments, as well as private companies. Quantitative and prosopographic studies on institutions have shown the diversity of actors involved, with very different professional profiles. A collective project dedicated to the study of one single city was for instance able to describe all actors, from university professors to school teachers, their literary production, as well as their professional interactions (Rollet and Bruneau 2017). Another collective project on the concrete fabrication of mathematical knowledge aims at analyzing “how heritage is made through selection, appropriation, adaptation, or normalization of knowledge and practices” in their material, conceptual, and symbolic dimensions (Ehrhardt et al. 2022, p. 12). This historiographical evolution has had profound implications for what is considered as science. Mathematics is not only about research, be it abstract or applied, but also about teaching and popularization, as well as appropriation by technicians, engineers, and all kind of practitioners.

3.1. Sharing and Transmitting Mathematical Knowledge

Some 25 years ago, Bruno Belhoste deplored that many historians of mathematics considered “communication, transmission, and popularization of mathematical knowledge as secondary and peripheral activities” (Belhoste 1998, p. 289). Fortunately, numerous works have shown that the once hegemonic distinctions between the high endeavors of scholars and a lower activity of teaching, adapting, or transmitting this knowledge was largely inoperative.

In the Antiquity, for instance, the utmost importance of the transmission of knowledge was recognized. Besides the obvious reason that a much greater number of Babylonian

clay tablets deal with multiplication or inverse tables than with higher mathematics, these sources are often the best available evidence if we are to understand how quantification and computation were seen in these societies (Robson 2008). To quote Christine Proust, “in order to understand Babylonian sexagesimal calculus, the best method is to follow the curriculum and teaching methods of mathematics in the scribal schools of Mesopotamia” (Proust 2005).

In a similar fashion, the question of numeracy (which in many cases turns out to be much more complicated to evaluate than literacy) has recently gathered lots of interest (Danna 2021). A case in point is the introduction of Hindu-Arabic numerals in Europe. The usual narrative correctly states that a latinized version of Al-Khwarizmi’s *Book of Indian computation* circulated in 12th-century scholarly Europe under the generic name *Algorismus*. Leonardo Fibonacci’s *Liber Abaci*, composed in 1202, introduced Western merchants to the new numerals shortly after, or at least, reflected a contemporary “abbaco culture”, partially built on decimal arithmetic (Høyrup 2005). Since the 13th century at least, treatises now called “commercial arithmetic”, written in the vernacular, had been circulating in several Italian cities, but also in Iberia and Provence, and then further north in Europe. They contained arithmetic useful for trade, including the base-ten numeral system and positional algorithms,⁴ but were also the almost exclusive medium for the dissemination of algebra until the Renaissance (Spiesser 2003; Høyrup 2014). The first printed book of mathematics, *The Arte dell’Abbaco* (or *Treviso Arithmetic*, 1478) belonged to the genre of merchant treatises. The interplay between teaching material and advanced mathematics is not limited to ancient periods: the famous *Cours d’Analyse* published by Augustin-Louis Cauchy in 1821 was a textbook, which also brought groundbreaking concepts and a new rigor to the continuity of functions (Belhoste 1991). Although the cumulative aspect of mathematics has sometimes obscured this fact, it has now become clear that our modern, clear-cut distinction between teaching and research makes little sense for previous time periods.

3.2. Who Matters? Considering a Broader Range of Actors

The history of computing, computations, and computers—both human and mechanical—highlights a diversity of actors and mathematical activities. It is at the crossroads of the history of mathematics, history of technology (De Mol and Bullynck 2018), history of gender, and history of labour (Gardey 2008; Grier 2005), to name but a few. The very activity of calculation has very often been devalued by academic mathematicians, in favor of a more conceptual approach to their discipline, and therefore long neglected by historians. Nevertheless, with the rise of computer science, instrument, and popular history, the computational aspect of mathematics, its associated actors, and material aids have been the subject of several book-length studies in the last two decades (Campbell-Kelly et al. 2007; Grier 2005, among others).

The production of trigonometric and logarithmic tables in the late 18th century in Paris, by the engineer Gaspard de Prony, is for instance relatively well known (Grattan-Guinness 1990b). Its organization was based on the concept of the division of labor proposed by Adam Smith and relied on hairdressers to carry out the elementary calculations, as they were economic victims of the Revolution. Large-scale number table projects were also developed for mathematical domains that are *a priori* more conceptual and abstract, such as number theory. The mainly individual initiatives to construct number theory tables in the early modern era were followed by more collective projects from the 1770s onward, generally led by scholars or teachers promoting the heuristic interest of such tables for theory. These endeavors also involved military personnel, engineers, or office workers, who were also number amateurs, some of whom proposed new paper or material devices to optimize calculations. The historical study of the production, use, and circulation of tables also reveals the importance of actual calculations in number theory’s supposedly abstract works of the 19th and 20th centuries (Corry 2008; Bullynck 2010).

The figure of the professional calculator evolved considerably during the modern era. In the 18th century, for instance, astronomers such as Jérôme Lalande housed and paid assistants, men and women trained in mathematics, to carry out their astronomical

calculations, and sometimes allowed them to unofficially join their scholarly networks. These “computing households” thus blurred the boundaries between public and private on the one hand, and amateur and professional on the other hand (Lémonon 2016). From the 19th century onward, the professionalization of science, the transformation of states, and industrialization led to an “expansion of the domain of numeracy” (Gardey 2008, p. 283) and, with the rise of statistics, an “avalanche of printed numbers” (Hacking 1982). This led to an increase in the need for calculations, their instrumentation and mechanization, first in the United States and then in Europe. Professional calculators, generally female, poorly paid and devalued, became more numerous, leading simultaneously to the integration and segregation of women in scientific (Rossiter 1980) and office jobs (Gardey 2008). The automation of computing thus goes hand in hand with the de-skilling of professions, even when (women) computers had deep mathematical skills and developed important technical expertise to operate on electronic computers during World War II, for example (Light 1999).

Recent historical research has also studied scholars’ attempts at mathematizing artisanal practices, thus revealing effective or fantasized circulations of knowledge between mathematicians and artisans, and informing the ways in which mathematics were thought to be applicable to technical or social fields. This can be illustrated by the geometrical and arithmetical reflections developed in the field of the textile industry and weaving by several mathematicians, even though it seems that they were never actually applied to textile production. This is the case for Joachim Jungius’ works in the 17th century, when the access to scientific and technical knowledge through, by observation, experimentation, and induction, was being promoted (Friedman 2022), or Alexandre-Théophile Vandermonde’s at the end of the 18th century, in a context of state-driven rationalization of arts and manufactures (Meyer 2021). In the 1870s and 1880s, Édouard Lucas also developed a mathematical approach to weaving in order to legitimize and popularize the usefulness of abstract number theory by applying it to industry, at a time when the relationship between science, industry, and society was valued (Décaillot 2002). In the same period, weaving and its associated motifs were also areas of encounter between mathematics and other fields such as ornamental art (Boucard and Eckes 2015) or anthropology (Vandendriessche 2014). The analysis of weaving practices has also provided a rich ground for ethnomathematics, a field related to the history of mathematics whose objective is “to identify and analyze mathematical practices implemented, nowadays or in the past, in different human societies [...], outside of academic or school institutions only” (Vandendriessche and Petit 2017, p. 190). The study of ancient textiles and current weaving practices allows researchers to make assumptions about the articulation between technical gestures, “algorithmic and computational” reasoning (Carraro 2021), and “mathematical concepts at work” (Brezine 2009, p. 474), while remaining very careful not to project contemporary and Western mathematical conceptions onto the studied cases (Vandendriessche and Petit 2017, pp. 203–4).

3.3. Quantification and the Modern State

By their very nature, administrations have always been prone to use quantification in all kinds of settings, and necessarily deal with large numbers. Their skilled workforce and abundant textual production have provided historians of mathematics with precious sources to understand some past uses of mathematics. Although this may come as a surprise, the administrative category turns out to be relevant across all cultures and even in antique societies, from the Babylonian kingdoms to early imperial China (Michel and Chemla 2020). Taking into account both the history of teaching and the concrete use of mathematics—the why and the how, so to speak—allow modern historians to get a better sense of what numeracy was like in the past and even, in many cases, to better understand the mathematical content of specific problems and their method of resolution (Middeke-Conlin 2020).

In the early modern period, the rise of modern bureaucracies was largely powered by an intensive use of quantification, for taxation and finance, as well as geometry involved in map-making and architecture, to give but a few examples. Yet the versatility of early

modern mathematical practices has long been overshadowed by the exceptional achievements of contemporary scholars, whose mathematics participated in the rise of the new sciences. The history of statistics is better known thanks to pioneering works about the modern nation-states (Desrosières 1993; Brian 1994). Still, there is much left to be done in connecting the use of mathematics in daily or administrative settings and neighboring fields, such as the history of metrology. In recent years, seminal works such as Witold Kula's *Measure and Men* Kula (1984) have come into renewed focus, more specifically from the perspective of the history of science.

4. The Renewal of Zilsel's Thesis: Mathematics and the Development of Knowledge

In order to better illustrate recent developments in the history of mathematics, we will finally discuss one specific issue, namely the current debates surrounding, and spanning from, the so-called 'Zilsel thesis'. While no case study can claim to be representative of the breadth and variety of existing methodologies, focusing on one debate will allow us to go into more details regarding some concrete issues in the field, notably the growing links with neighboring fields in the history of science, history of knowledge, as well as the philosophy of science.

Edgar Zilsel (1891–1944) was an eclectic intellectual who lived in the first half of the 20th century.⁵ Trained both in philosophy and mathematics at the university of Vienna, he wrote his *Dissertation* on the philosophical aspect of the Law of Large Numbers. While Zilsel indisputably had an interest in mathematics, and was influenced by the Vienna Circle, he was not a historian or philosopher of science in the modern, narrow sense of the word. His broad field of research would have been, if anything, what is now labeled as 'intellectual history', and his first major works dealt with the concept of "genius". When Austria was annexed by Nazi Germany during the *Anschluss*, Zilsel, who was both a Jew and a leftist, emigrated to the United States. After publishing a slate of insightful, yet scarcely developed articles, he committed suicide in 1944. His papers on the rise of science gradually became a major source for the development of modern historiography, and were collected in 2003 under the title *The Social Origins of Modern Science* (Zilsel 2000b).

Zilsel's fundamental intuition was that a loosely-defined group of mathematical practitioners were responsible for the birth of modern science. More precisely, these practitioners were "some groups of superior craftsmen who needed more knowledge for their work than their colleagues did. The most important of them may be called artist-engineers, for not only did they paint their pictures, cast their statues, and build their cathedrals, but also constructed lifting-gears, earthworks, canals and sluices, guns and fortresses, found new pigments, detected the geometrical rules of perspective, and invented new measuring tools for engineering and gunnery". The examples of Filippo Brunelleschi or Leonardo da Vinci show that their use of mathematics was not scientific *per se*: They devised formulas and tables of ratios which, however, were not considered as mathematical laws. They were nevertheless "the immediate predecessors of science", which was born with Galileo Galilei when "the experimental method of the craftsmen [overcame] the prejudice against manual work and [was] adopted by rationally trained university-scholars" (Zilsel 2000c). The "artist-engineers" were used to "expressing their results in empirical rules and quantitative terms", and this gradually led scholars to the modern concept of "physical law" (Zilsel 2000a).

The essays of Edgar Zilsel are brief and can sometimes be frustrating to read for current historians. While Zilsel touches on many key figures of early modern sciences, such as Nicolaus Copernicus, Johannes Kepler, William Harvey, René Descartes, and obviously Galileo, he adds some experimenters such as William Gilbert or Simon Stevin. Above all, his reasoning is often sketchy, hinting at general arguments that stay undeveloped. Zilsel was not exactly isolated, nor was he the only one dealing with the use of artisanal mathematics: some of his intuitions were shared by contemporary works such as Leonardo Olschky's studies on vernacular scientific literature, or Henryk Grossmann and Franz Borkenau's debates about the role of mathematics in the development of mechanics and machinery.⁶

These intuitions, however, only recently became widely discussed: “A debate over Edgar Zilsel’s stimulating proposals should finally commence”, urged a historian in 1985 (Nordmann 1985). It is only in the last two decades, as the separation between sociology and epistemology has been shrinking, that his seminal ideas have been collectively discussed, amended, and refined. Pamela Long has underlined that the distinction between “artisans” and “scholars” was not as clear as Zilsel thought, and adapted Peter Galison’s concept of “trading zone” to describe their interactions (Long 2011). Lesley Cormack has argued that more attention should be devoted to “mathematical practitioners”, who “provided an agent for the changing nature of the scientific enterprise in the early modern period”, while Floris Cohen claims that by the 17th-century “mathematics had impinged upon craftsmen’s activities only in a few exceptional cases” (Cormack 2017, p. 34; Cohen 2010, p. 309).

Another remarkable feature of the debates surrounding Zilsel’s thesis is that they concern not only historians of mathematics, but more broadly historians of science, technology, and knowledge. His influence can be felt in works on the history of arts and architecture (Dubourg Glatigny and Vérin 2008), or in studies about technical and practical uses of quantification (Morel 2020; Préveraud 2020). Confronting the formulas, methods, and instruments used by craftsmen with the theories and problems tackled by scholars has gradually become self-evident. In that sense, the numerous conversations around Zilsel’s thesis illustrate the broadening of interests among historians (and philosophers) of mathematics. As Sophie Roux aptly puts it, “the grand narrative about mathematization of nature has to be enriched with the dense spectrum of various mathematical practices” (Roux 2010).⁷ In a way, recent studies about a wide array of mathematical practices can be seen as renewing the discussions about the “idea of the mathematization of nature being key to the Scientific Revolution”, an idea that had gradually become, as Floris Cohen regrets, “hopelessly obsolete” (Cohen 2016, p. 143). Instead of focusing on a handful of prominent scholars, current studies have tried to recast this process as a multitude of efforts, often uncoordinated, with wide-ranging implications. More importantly, many actors such as instrument-makers, military, and civil architects or assayers are now studied for their own mathematical culture, and not only in relation with the rise of the new sciences (Bennett 2011; Cormack et al. 2017). This is a multifaceted approach, acknowledging and trying to account for the growing importance of mathematics, seen as a set of methods and practices, in many areas of civil and scientific life during the early modern period.

In the last decades, the scope of the history of mathematics has widened in many directions. Traditional issues have been addressed anew, while the extent of what constitutes “mathematical activities” worth studying has considerably broadened. Material and visual aspects have been emphasized, new actors have been found to be legitimate subjects of inquiry, from school teachers to artisans and practitioners. Given these multifarious uses and applications of mathematics, it is now commonly acknowledged that its history cannot be restricted to modern research, and methodologies have evolved accordingly. While the specificity of mathematical sciences endures to some extent, the field has not stayed insulated from general debates and evolution of its neighboring disciplines. The variety of research directions has been sustained by hybridization and inspiration taken from the other historical and sociological sciences.

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Notes

- ¹ Interested readers will turn to detailed work such as (Dauben and Scriba 2002; Remmert et al. 2016a).
- ² We focus here on algebra of the modern era. However, it should be noted that several works published in the last three decades have provided new insights into the history of algebra in earlier periods. Some examples among others: Roshdi Rashed, in (Al-Khwarizmi and Rashed 2007), showed how the original form of al-Khwarizmi's treatise, often referred to as a seminal treatise on algebra, can be explained by social and cultural factors in 9th-century Baghdadi society. The various chapters of Rommevaux et al. (2012) clearly show the "plurality of algebra in the Renaissance", whether from the point of view of the status of algebra, its goals, its applications, or its forms according to the geographical areas and the target audiences.
- ³ Another account of the Unguru/Weil controversy, as well as a no-less vehement defense of the "traditional approach" of the history of mathematics, can be found in (Bläsjö 2014).
- ⁴ Nevertheless, roman numerals were still in use well into the 16th century, and understanding the complex dynamics, as well as the different issues and uses of numbers and arithmetic by social groups is a work in progress (Otis 2017).
- ⁵ A detailed biography of E. Zilsel can be found in (Krohn and Raven 2000).
- ⁶ (Olschki 1919, 1927). This literature, was notably not brought to the foreground during the 1960s, when the publication of Thomas Kuhn's *Structure of Scientific Revolution* deeply transformed the field of the history of science. It might have seemed handy, but was simply forgotten, as the history of mathematics was a rather singular endeavor.
- ⁷ For a set of more philosophical approaches to the issue of the mathematization of nature, see (Gorham et al. 2016).

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