

Article

Magnetic-Field-Controlled Thermohydrodynamic Behavior of Fe_3O_4 /SWCNT Hybrid Ferrofluid in a Cavity with a Rotatable Elliptical Barrier

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Abstract

Magnetic regulation of ferrofluid transport can modulate heat and fluid motion without mechanical actuation, but the combined effect of field strength and an orientable internal obstacle remains insufficiently resolved. This study numerically examines laminar natural convection of a 1.3 vol% Fe_3O_4 /SWCNT (Single-Walled Carbon Nanotube)–water hybrid ferrofluid in a square cavity containing a rotatable elliptical barrier under a vertical magnetic field. Eight barrier orientations (0 – 315° in 45° increments) are evaluated at $\text{Ra} = 10^3$ and $\text{Ha} = 20$ and 80 using the finite element method in COMSOL Multiphysics. The governing formulation was corrected to use a symmetric viscous operator and the Lorentz term associated with a vertical magnetic field; the base-fluid Prandtl number calculated from the tabulated properties is 6.07 . An independent, differentially heated square-cavity benchmark reproduced the reference values for maximum horizontal and vertical velocities and the average Nusselt number with relative errors below 0.76% . The computed fields show that barrier orientation controls the locations and signs of the velocity extrema and the pressure range, whereas the reported temperature profiles change comparatively little between the two Hartmann numbers. The 90° orientation gives the minimum reported Bejan number, while 45° gives the maximum. Because Be is a ratio, these extrema identify changes in the relative contribution of thermal irreversibility and do not, by themselves, establish a minimum or maximum of total entropy generation. The results demonstrate a coupled geometric–magnetic redistribution of the local thermohydrodynamic fields and identify 90° as the preferred orientation only under the Bejan-number criterion adopted here.

Keywords: Fe_3O_4 nanoparticles; single-walled carbon nanotubes; hybrid ferrofluid; magnetic-field control; magnetohydrodynamics; natural convection; Bejan number



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1. Introduction

Ferrofluids combine liquid transport with field-responsive behavior, allowing flow and heat-transfer patterns to be adjusted by an external magnetic field rather than by moving mechanical components. This controllability is relevant to compact heat exchangers, electronic cooling chambers, thermal-energy-storage enclosures, magnetically assisted separation, lubrication and sealing, and biomedical transport systems. In a closed cavity, the final state results from competition among buoyancy, viscous diffusion, thermal diffusion,

the Lorentz force, and the restrictions imposed by the internal geometry. The interaction is especially sensitive at low Rayleigh number, where conduction remains important, and even a modest geometric redirection of the weak circulation can alter local velocity, pressure, and entropy-generation patterns. In such regimes, the presence of a solid obstacle or barrier inside the cavity introduces an additional length scale and modifies the pathways available to the recirculating flow, so that the orientation of the barrier becomes a meaningful control parameter rather than a mere geometric detail.

The Fe_3O_4 /SWCNT combination was selected to unite two complementary functions. Fe_3O_4 provides magnetic responsiveness and established compatibility with water-based ferrofluid formulations. The synthesis, structural characterization, and magnetic properties of Fe_3O_4 nanoparticles have been extensively documented [1], and surface-modified or carbon-coated Fe_3O_4 core-shell particles have been specifically developed for heat-transfer applications [2]. Doping and functionalization strategies have further extended the range of Fe_3O_4 properties, including antibacterial functionality [3], while classical studies have clarified the relationship between synthesis route, particle size, and magnetic response [4]. More recent reviews have consolidated the synthesis–property–application landscape of Fe_3O_4 and related nanoparticles [5], and controlled-size synthesis has reached the level of a chemical milestone [6]. Taken together, these studies [1–6] establish Fe_3O_4 as a reliable magnetic phase for water-based ferrofluids. Notably, the combination of Fe_3O_4 with carbon nanotubes has been specifically highlighted for biomedical and biotechnological applications, because the magnetite nanocrystals advance targeting efficiency and enable magnetically engaged delivery, while the CNT framework provides chemical stability and prevents agglomeration. This dual functionality, magnetic responsiveness plus enhanced thermal transport, forms the basis for the present hybrid formulation.

SWCNTs, on the other hand, possess very high intrinsic thermal conductivity and can improve the effective heat-conduction pathway at low loading. Heat-transfer analyses of SWCNT suspensions in non-Newtonian base fluids have been performed for uniform and non-uniform channels [7], and MHD Darcy–Forchheimer flow of SWCNT– H_2O nanofluid over a vertical porous sheet has been examined [8]. Natural convection of water/SWCNT nanofluid in a triangular cavity with cold fluid injection has been simulated numerically [9], while molecular-dynamics simulations have quantified the enhancement of thermal conductivity and viscosity in SWCNT nanofluids [10]. Comparative MHD oscillatory studies of SWCNT and MWCNT nanofluids have also been reported [11], and the thermophysical properties of SWCNT nanofluids have been reviewed [12]. In parallel, MHD buoyancy-driven hybrid nanofluid flow in partially heated porous square cavities [13], ternary-hybrid nanofluids in porous cavities with shape effects [14], and ternary nanofluid flow in porous square cavities using the marker-and-cell technique [15] have been investigated. Heat-transfer analysis of MHD hybrid nanofluids containing SWCNT/MWCNT has also been conducted [16]. Collectively, these studies [7–16] demonstrate that SWCNTs can serve as an effective thermal-conductivity enhancer at low volume fractions, although their influence is coupled to base-fluid rheology, flow regime, and channel or cavity geometry.

Experimental and numerical studies of Fe_3O_4 –carbon hybrid suspensions further show that temperature, particle ratio, and total volume fraction influence thermal conductivity and viscosity. The relative thermal conductivity of SWCNT(15%)– Fe_3O_4 (85%)/water hybrid ferro-nanofluid has been modeled with response-surface methodology, with a maximum relative thermal conductivity enhancement of 32.20% reported under optimized conditions [17]. More recent studies have extended this to ternary formulations: the combined effects of volume fraction and temperature on thermal conductivity have been examined for a SWCNT– Fe_3O_4 –CuO/ethylene glycol–water ternary hybrid nanofluid [18]. Thermal performance of Fe_3O_4 , SWCNT, and MWCNT in water under MHD flow past a

wedge, including Soret and Dufour effects, has been analyzed [19], while comparative heat-transfer studies of $\text{Fe}_3\text{O}_4\text{-H}_2\text{O}$ and $\text{Fe}_3\text{O}_4\text{-Cu-H}_2\text{O}$ inside a tilted square porous cavity with shape effects have been reported [20]. Unsteady hybrid nanofluid flow of $\text{Fe}_3\text{O}_4\text{-SWCNT-water}$ and its thermal performance have been investigated numerically [21], and the thermal and flow properties of graphene/ Fe_3O_4 water-based hybrid nanofluids have been characterized experimentally [22]. These works [17–22] collectively indicate that the $\text{Fe}_3\text{O}_4\text{/SWCNT}$ combination is not a simple superposition of its constituents; rather, temperature, composition ratio, and total loading jointly determine the effective transport properties.

The present total loading is 1.3 vol% (0.8 vol% Fe_3O_4 and 0.5 vol% SWCNT), which retains a dilute–mixture framework while combining magnetic and thermal functions. This selection is intended for proof-of-concept geometric control rather than as a universal optimum composition. The dilute regime is deliberately chosen so that the underlying MHD natural-convection physics remains interpretable, and so that the observed effects can be attributed primarily to the coupled magnetic and geometric forcing rather than to strong non-Newtonian or clustering effects.

Previous cavity studies have considered $\text{Fe}_3\text{O}_4\text{-water}$ ferrofluids under uniform or spatially varying magnetic fields. Numerical simulation of natural convection in a chamfered square cavity with $\text{Fe}_3\text{O}_4\text{-water}$ nanofluid under magnetic excitation has been performed [23], complementing the ferrofluid formulation and property studies already discussed [1–6]. In parallel, CNT-based nanofluids and hybrid CNT formulations have been investigated in a variety of configurations. These include SWCNT–MWCNT/blood pulsating micropolar hybrid nanofluid flow in a vertical channel with thermal radiation and heat source/sink [24], MHD forced convection of micropolar ferrofluid over double stationary cylinders inside a vented chamber [25], and free convection in a square wavy porous cavity with a partly applied magnetic field [26]. These studies [7–16,24–26] confirm that CNT-based and hybrid suspensions can alter both the intensity and the spatial distribution of convective heat transfer, but they also show that the response depends strongly on the imposed field configuration and on the geometry of the confinement.

Magnetic hybrid nanofluids in enclosures or porous domains have also been studied extensively. The flow and thermomagnetic convection of a power-law non-Newtonian ferrofluid within a circular cavity with a permanent magnet has been examined [27], and the effect of nanoparticle nature on MHD behavior in a square cavity with a metallic obstacle has been investigated [28]. The Bejan number has been redefined in a way that clarifies its interpretation as an irreversibility ratio [29], and numerical analysis of MHD nanofluid flow in a circular porous medium containing a Cassini oval under Lorentz and buoyancy forces has been reported [30]. These studies [17–22,27–30] establish that increasing Ra generally strengthens buoyancy-driven circulation, while a magnetic field can suppress the velocity component perpendicular to the field and redistribute the associated thermal boundary layers.

Recent trends in this field point in three directions. First, the shift from single-component nanofluids to hybrid and ternary formulations reflects a growing recognition that no single nanoparticle can simultaneously optimize thermal conductivity, magnetic response, and rheological stability; studies increasingly report that hybrid nanofluids outperform their mono counterparts in thermal conductivity and heat-transfer rate, and ternary-hybrid nanofluids are now being explored for advanced applications including biomedical transport and energy systems. Second, cavity geometry is increasingly treated not as a fixed constraint but as a design variable: rotating cylinders and baffles, wavy walls, and variable aspect ratios are now actively optimized rather than merely described, and the rotational speed and size of internal obstacles have been shown to significantly modulate

the Nusselt number. Third, irreversibility analysis via the Bejan number is moving from a supplementary metric to a primary evaluation criterion, reflecting a shift in emphasis from heat-transfer rate alone to thermodynamic quality. The present study aligns with all three directions: it employs a hybrid Fe_3O_4 /SWCNT ferrofluid, treats barrier orientation as a controllable geometric parameter, and reports Bejan-number comparisons alongside conventional thermal performance indicators.

The research gap is therefore the coupled influence of magnetic intensity and the orientation of a rotatable elliptical barrier in a dilute Fe_3O_4 /SWCNT–water hybrid ferrofluid. As summarized in Table 1, earlier studies provide either property data, fixed or different obstacle configurations, porous-domain results, or single-component nanofluids, but do not examine the present combination of eight barrier orientations, two Hartmann numbers, split thermal conditions on the barrier, and a Bejan-number comparison. The objectives are to (i) formulate the governing MHD natural-convection problem consistently for a vertical magnetic field, (ii) compare local velocity, temperature, and pressure responses across the eight orientations, and (iii) interpret the reported Bejan number as an irreversibility ratio rather than as a direct measure of total entropy generation. This coupled geometric–magnetic analysis is the main originality of the work.

Table 1. Comparison of the present work with the most directly related studies already cited in the manuscript.

Study	Working Medium and Configuration	Principal Scope	Difference Relative to the Present Study
[17]	Experimental SWCNT/ Fe_3O_4 –water hybrid ferrofluid	Thermal-conductivity correlation and property trends	No enclosure flow field, obstacle rotation, or cavity-scale MHD benchmark
[23]	Fe_3O_4 –water in a chamfered square cavity	Natural convection with magnetic excitation	Single-particle fluid and no rotatable internal ellipse
[26]	Nanofluid in a square wavy porous cavity with a partly applied field	Porous-medium heat transfer and entropy behavior	Different geometry, porosity model, and spatially localized field
[28]	Nanofluid in a square cavity with a metallic obstacle	Effect of nanoparticle type on MHD behavior	Fixed the obstacle and no Fe_3O_4 /SWCNT hybrid composition
[30]	Al_2O_3 –water in a circular porous enclosure containing a Cassini oval	Lorentz–buoyancy interaction in a porous domain	Different fluid, outer geometry, obstacle, and Darcy resistance
Present Study	1.3 vol% Fe_3O_4 /SWCNT–water in a square cavity	Eight ellipse orientations at $\text{Ra} = 10^3$ and $\text{Ha} = 20, 80$; velocity, temperature, pressure, and Be	Coupled orientation–magnetic-field analysis with corrected vertical-field formulation

2. Physical Model, Assumptions, and Mesh Assessment

The two-dimensional model consists of a square cavity containing a centrally located elliptical barrier. The cavity is filled at 25 °C with a water-based hybrid ferrofluid containing 0.8 vol% Fe_3O_4 and 0.5 vol% SWCNT (Figure 1). The thermal boundary conditions are shown in Figure 2: the right cavity wall and the left half of the barrier are cold, while the upper, lower, and left cavity walls and the right half of the barrier are hot. No-slip conditions are imposed on all solid surfaces. The analysis is restricted to steady, two-dimensional, laminar natural convection at $\text{Ra} = 10^3$ and to $\text{Ha} = 20$ and 80. The base-fluid Prandtl number calculated from Table 2 is $\text{Pr} = \mu\text{Cp}/k = 6.07$.

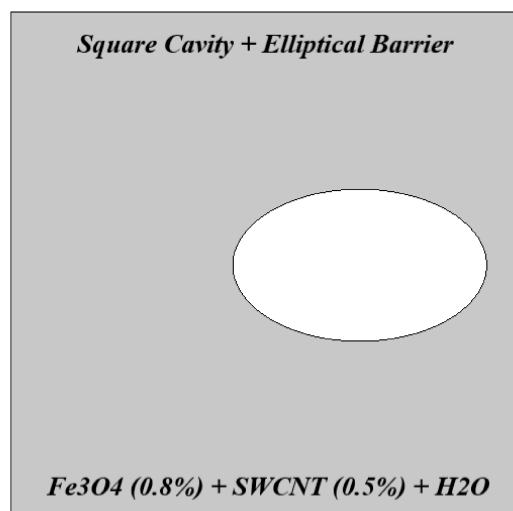


Figure 1. 2D model of the cavity and elliptical barrier.

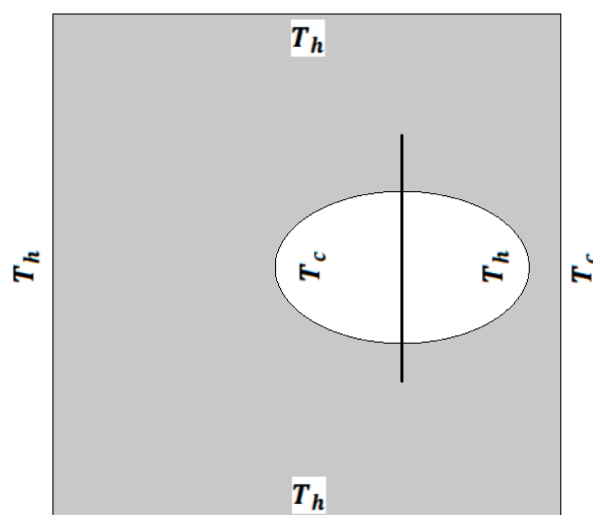


Figure 2. Boundary conditions.

Table 2. Thermophysical properties of Fe_3O_4 , SWCNTs, and water at 25 °C.

Properties	Fe_3O_4	SWCNT	H_2O
c_p (j/(kg.°k))	670	425	4179
ρ (kg/m ³)	5180	2600	997.1
k (w/(m.°k))	6	6600	0.613
β (1/(°k))	1.18×10^{-5}	1×10^{-6}	2.1×10^{-4}
μ (kg/ms)	2×10^{-3}	1.5×10^{-3}	0.89×10^{-3}
σ (S/m)	2.5×10^4	5×10^6	0.1

2.1. Homogeneous Single-Phase Assumption and Scope

The suspension is treated as incompressible and homogeneous, with constant effective properties evaluated from the constituent data and the specified volume fractions. This approximation is commonly used for dilute nanofluids when the characteristic particle slip is small relative to the carrier-fluid velocity and the dispersion remains stable over the modeled time scale. The total loading of 1.3 vol% limits particle crowding and supports the use of an effective-property description for the present parametric comparison. The model therefore resolves the continuum-scale Lorentz, buoyancy, viscous, and thermal-diffusion effects, rather than individual particle trajectories.

The assumption remains conditional. Sedimentation, field-induced chaining, agglomeration, Brownian migration, thermophoresis, and concentration polarization are not explicitly represented. The results should consequently be interpreted as the response of an ideally stable dilute suspension. Supplementary Section S4 quantifies how the effective properties and the coefficients entering the governing equations change when the total concentration is varied by $\pm 50\%$ at the same $\text{Fe}_3\text{O}_4\text{:SWCNT}$ ratio; it does not replace a two-phase or concentration-resolved flow simulation.

2.2. Mesh Assessment

Mesh sensitivity was assessed with the Finer, Extra Fine, and Extremely Fine meshes shown in Figure 3. The reported maximum and minimum values of the horizontal velocity component u at $\text{Ra} = 10^3$ and $\text{Ha} = 20$ were used as consistency indicators (Table 3). The change from the Finer to the Extremely Fine mesh is 3.93% for the reported positive extremum and 5.22% for the negative extremum. Because both extrema are of order 10^{-25} in the exported data, they are effectively at the numerical-zero level; the percentages are therefore not interpreted as evidence of a finite circulation strength. They only show that further refinement does not alter the near-zero magnitude or sign pattern at the displayed precision.

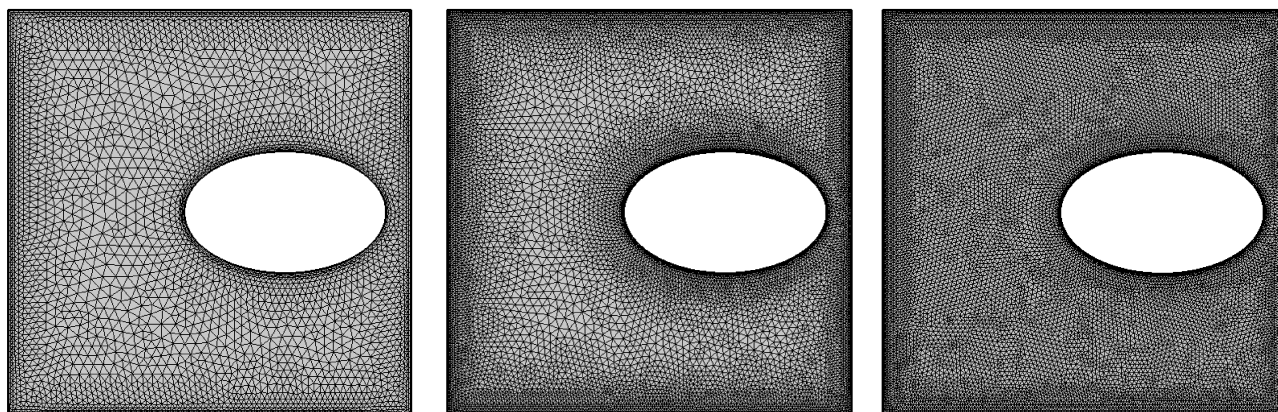


Figure 3. Computational meshes: Finer (left), Extra Fine (middle), and Extremely Fine (right).

Table 3. Mesh-sensitivity assessment at $\text{Ra} = 10^3$ and $\text{Ha} = 20$.

Mesh Indicator	Finer	Extra Fine	Extremely Fine	Deviation (%)
Domain elements	7422	20,610	26,692	—
Boundary elements	408	792	984	—
CPU time (s)	52	92	107	—
$u_{\max}$	9.66×10^{-26}	9.43×10^{-26}	9.28×10^{-26}	3.93
$u_{\min}$	-1.34×10^{-25}	-1.29×10^{-25}	-1.27×10^{-25}	5.22

3. Mathematical Formulation

Under the Boussinesq approximation, a negligible induced magnetic field, and a uniform vertical magnetic flux density $B_0 = B_0 e_y$, the dimensional conservation equations are the continuity equation, two momentum equations, and the energy equation [30,31]. The symmetric viscous Laplacian is retained in both momentum equations. For the adopted vertical field, the quasi-static Lorentz term opposes the horizontal velocity component u . Subscripts x and y denote partial derivatives, and $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$.

$$u_x + v_y = 0 \quad (1)$$

$$\rho_{\text{hff}}(uu_x + vu_y) = -p_x + \mu_{\text{hff}}\nabla^2 u - \sigma_{\text{hff}}B_0^2 u \quad (2)$$

$$\rho_{\text{hff}}(uv_x + vv_y) = -p_y + \mu_{\text{hff}}\nabla^2 v + (\rho\beta)_{\text{hff}}g(T - T_c) \quad (3)$$

$$uT_x + vT_y = \alpha_{\text{hff}}\nabla^2 T \quad (4)$$

The effective thermophysical properties are evaluated with the dilute homogeneous-mixture relations in Equation (5a–g) [17,20,30]. Here, φ_1 and φ_2 denote the Fe_3O_4 and SWCNT volume fractions, respectively, and $\varphi_t = \varphi_1 + \varphi_2$. The thermal-conductivity relation uses a Maxwell-type expression with the volume-weighted effective solid conductivity, while the viscosity is represented by the Brinkman relation.

$$\rho_{\text{hff}} = (1 - \varphi_t)\rho_{\text{bf}} + \varphi_1\rho_1 + \varphi_2\rho_2 \quad (5a)$$

$$(\rho C_p)_{\text{hff}} = (1 - \varphi_t)(\rho C_p)_{\text{bf}} + \varphi_1(\rho C_p)_1 + \varphi_2(\rho C_p)_2 \quad (5b)$$

$$(\rho\beta)_{\text{hff}} = (1 - \varphi_t)(\rho\beta)_{\text{bf}} + \varphi_1(\rho\beta)_1 + \varphi_2(\rho\beta)_2 \quad (5c)$$

$$\alpha_{\text{hff}} = \frac{k_{\text{hff}}}{(\rho C_p)_{\text{hff}}} \quad (5d)$$

$$\frac{k_{\text{hff}}}{k_{\text{bf}}} = \frac{k_{p,\text{eff}} + 2k_{\text{bf}} + 2\varphi_t(k_{p,\text{eff}} - k_{\text{bf}})}{k_{p,\text{eff}} + 2k_{\text{bf}} - \varphi_t(k_{p,\text{eff}} - k_{\text{bf}})} \quad (5e)$$

$$k_{p,\text{eff}} = \frac{\varphi_1 k_1 + \varphi_2 k_2}{\varphi_t} \quad (5f)$$

$$\mu_{\text{hff}} = \frac{\mu_{\text{bf}}}{(1 - \varphi_t)^{2.5}} \quad (5g)$$

The dimensional no-slip and thermal boundary conditions are:

$$u = v = 0 \text{ on all cavity and barrier surfaces} \quad (6a)$$

$$T = T_c \text{ on the right cavity wall} \quad (6b)$$

$$T = T_c \text{ on the left half of the barrier} \quad (6c)$$

$$T = T_h \text{ on the upper, lower, and left cavity walls} \quad (6d)$$

$$T = T_h \text{ on the right half of the barrier} \quad (6e)$$

The governing equations are nondimensionalized with the cavity length L , the base-fluid thermal-diffusion velocity α_{bf}/L , and the temperature difference $\Delta T = T_h - T_c$. The dimensionless pressure is scaled with $\rho_{\text{hff}}\alpha_{\text{bf}}^2/L^2$. The definitions are:

$$X = \frac{x}{L}, \quad Y = \frac{y}{L}, \quad U = \frac{uL}{\alpha_{\text{bf}}}, \quad V = \frac{vL}{\alpha_{\text{bf}}}, \quad \theta = \frac{T - T_c}{T_h - T_c}, \quad P = \frac{pL^2}{\rho_{\text{hff}}\alpha_{\text{bf}}^2} \quad (7a)$$

$$Pr = \frac{\nu_{\text{bf}}}{\alpha_{\text{bf}}}, \quad Ra = \frac{g\beta_{\text{bf}}(T_h - T_c)L^3}{\nu_{\text{bf}}\alpha_{\text{bf}}}, \quad Ha = B_0L\sqrt{\frac{\sigma_{\text{hff}}}{\rho_{\text{hff}}\nu_{\text{bf}}}} \quad (7b)$$

Substitution of Equation (7a,b) into Equations (1)–(4) gives Equations (8)–(11). Complete intermediate steps are provided in Supplementary Section S5.

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad (8)$$

$$UU_X + VU_Y = -P_X + Pr\frac{\nu_{\text{hff}}}{\nu_{\text{bf}}}\nabla^2 U - Ha^2 Pr U \quad (9)$$

$$UV_X + VV_Y = -P_Y + Pr \frac{\nu_{\text{hff}}}{\nu_{\text{bf}}} \nabla^2 V + \frac{(\rho\beta)_{\text{hff}}}{\rho_{\text{hff}}\beta_{\text{bf}}} Ra Pr \theta \quad (10)$$

$$U\theta_X + V\theta_Y = \frac{\alpha_{\text{hff}}}{\alpha_{\text{bf}}} \nabla^2 \theta \quad (11)$$

The corresponding dimensionless boundary conditions are:

$$U = V = 0 \text{ on all cavity and barrier surfaces} \quad (12a)$$

$$\theta = 0 \text{ on the right cavity wall} \quad (12b)$$

$$\theta = 0 \text{ on the left half of the barrier} \quad (12c)$$

$$\theta = 1 \text{ on the upper, lower, and left cavity walls} \quad (12d)$$

$$\theta = 1 \text{ on the right half of the barrier} \quad (12e)$$

Lorentz Force and Entropy-Generation Measures

For the low-magnetic-Reynolds-number approximation, the induced magnetic field is neglected, and the current density follows Ohm's law for a moving conductor [31]. With $\mathbf{B} = B_0 \mathbf{e}_y$ and no externally imposed electric field in the two-dimensional model, the Lorentz force acts opposite to the horizontal velocity. This orientation-specific result is the reason the magnetic damping term appears in Equations (2) and (9), rather than in the vertical momentum equation.

$$\mathbf{J} = \sigma_{\text{hff}}(-\nabla\phi_e + \mathbf{u} \times \mathbf{B}) \quad (13)$$

$$\mathbf{f}_L = \mathbf{J} \times \mathbf{B} = -\sigma_{\text{hff}} B_0^2 u \mathbf{e}_x \quad (14)$$

The local entropy-generation rate is decomposed into heat-transfer, fluid-friction, and magnetic contributions. With T_0 as the reference absolute temperature, the dimensional expressions and the Bejan number are [29]:

$$S_{\text{th}} = \frac{k_{\text{hff}}}{T_0^2} \left[\left(\frac{\partial T}{\partial x} \right)^2 + \left(\frac{\partial T}{\partial y} \right)^2 \right] \quad (15)$$

$$S_{\text{ff}} = \frac{\mu_{\text{hff}}}{T_0} \left\{ 2 \left[\left(\frac{\partial u}{\partial x} \right)^2 + \left(\frac{\partial v}{\partial y} \right)^2 \right] + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 \right\} \quad (16)$$

$$S_{\text{mag}} = \frac{\sigma_{\text{hff}} B_0^2}{T_0} u^2 \quad (17)$$

$$S_{\text{gen}} = S_{\text{th}} + S_{\text{ff}} + S_{\text{mag}} \quad (18)$$

$$Be = \frac{S_{\text{th}}}{S_{\text{gen}}} \quad (19)$$

Accordingly, $Be \rightarrow 1$ indicates dominance of heat-transfer irreversibility, whereas $Be \rightarrow 0$ indicates dominance of fluid-friction and/or magnetic irreversibility. A smaller Bejan number is not necessarily a smaller total entropy-generation rate; total entropy generation must be examined separately to make that claim.

4. Numerical Method and Verification

4.1. Finite-Element Solution and Deterministic Uncertainty

The coupled dimensionless equations were solved with the finite element method in COMSOL 6.2 Multiphysics (Burlington, MA, USA). The square cavity was discretized with an unstructured mesh refined near the elliptical barrier and the solid boundaries, where

velocity and temperature gradients are expected to be largest. The solved variables were velocity, pressure, and temperature. Mesh sensitivity was evaluated with the three grids in Table 3, and all reported orientation curves were obtained deterministically from one converged solution per parameter set. Consequently, statistical error bars are not physically defined for Figure 4; numerical reliability is instead addressed through mesh sensitivity, equation-level verification, and the independent benchmark described below.

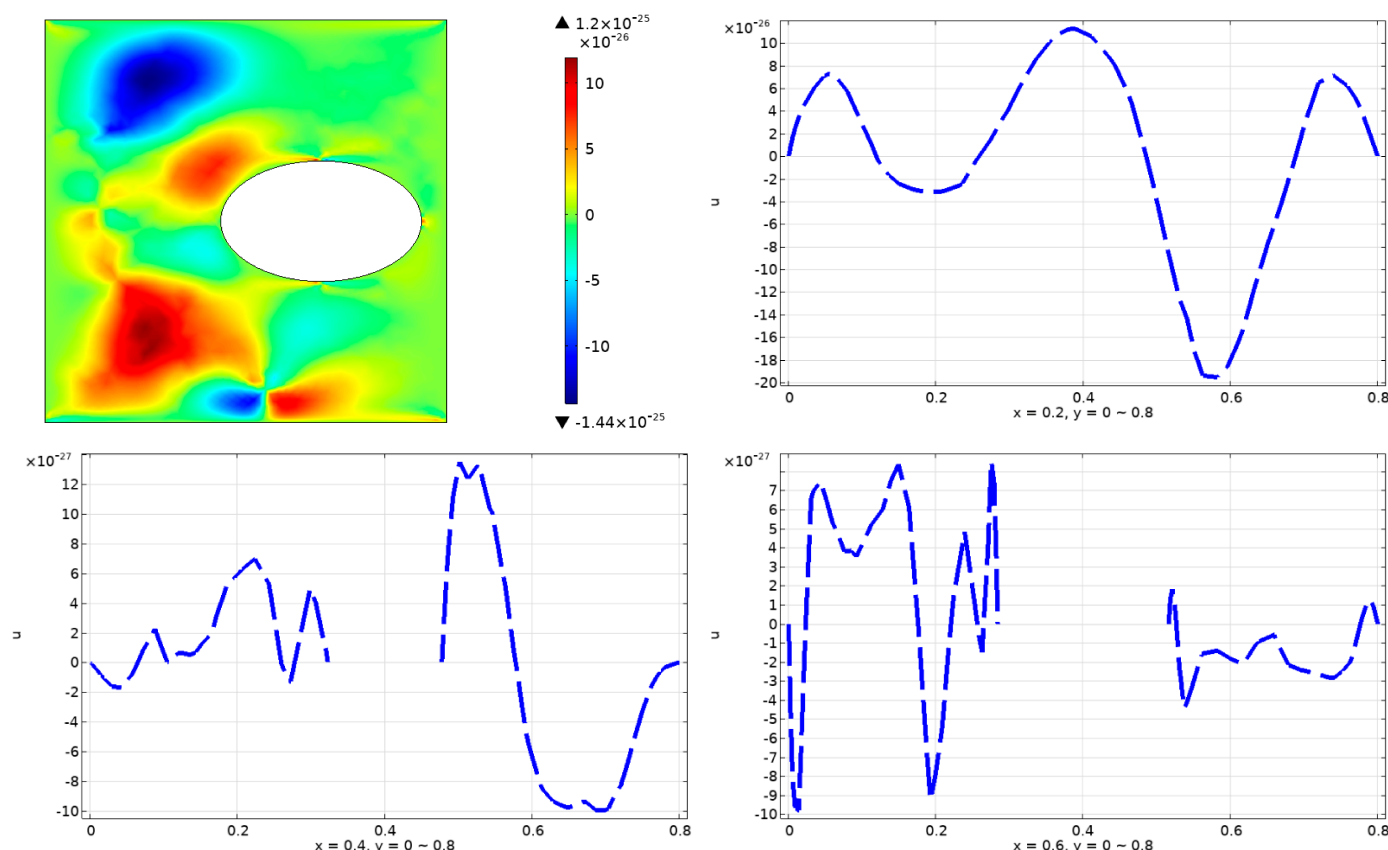


Figure 4. Contour and profiles of u at $Ha = 20$ and $Ra = 10^3$, horizontal elliptical-barrier orientation (0°).

4.2. Verification and Property Audit

The corrected equation set was independently checked with the classical differentially heated square-cavity benchmark of de Vahl Davis [32] at $Ra = 10^3$ and $Pr = 0.71$. An 81×81 second-order finite-difference streamfunction–vorticity calculation gave relative errors of 0.73% for the maximum horizontal velocity, 0.76% for the maximum vertical velocity, and 0.64% for the average Nusselt number. This benchmark verifies the corrected conservation equations and boundary-condition implementation independently; it is not presented as a reconstruction of the unavailable raw COMSOL project. The full comparison, grid refinement, and literature-compatibility matrix are reported in Supplementary Sections S1–S3.

At the nominal particle fractions, the revised mixture relations give $\rho_{hff} = 1038.58 \text{ kg m}^{-3}$, $k_{hff} = 0.6372 \text{ W m}^{-1} \text{ K}^{-1}$, and $\mu_{hff} = 9.196 \times 10^{-4} \text{ Pa s}$; the complete audit is provided in Supplementary Table S4.

5. Results and Discussion

This section examines how elliptical-barrier orientation and magnetic-field intensity redistribute the local velocity, temperature, and pressure fields of the $\text{Fe}_3\text{O}_4/\text{SWCNT}$ hybrid ferrofluid at $Ra = 10^3$. The flow remains within the steady laminar and conduction-

influenced regime. Results are compared for $Ha = 20$ and 80 , but the reported component extrema should not be interpreted as domain-integrated kinetic energy or as a direct measure of heat-transfer performance.

Detailed velocity and temperature profiles are retained for the principal orientations 0° , 90° , 180° , and 270° . Temperature contours for the diagonal orientations 45° , 135° , 225° , and 315° are collected in Section 5.5. Each contour retains its own annotated numerical scale because the field range changes from case to case; cross-case comparisons are therefore based on the printed extrema and profile values, not on color hue alone.

The reported temperature distributions at $Ha = 20$ and 80 are close for the examined cases, consistent with the low-Rayleigh-number regime in which thermal diffusion remains important. Detailed temperature contours are therefore shown at $Ha = 20$, whereas the velocity response is retained for both Hartmann numbers.

5.1. Elliptical Barrier at 0°

At $Ra = 10^3$, changing Ha from 20 to 80 modifies the positions and signs of the local u and v extrema (Figures 4–7). The vertical magnetic field directly damps cross-field horizontal motion, while the horizontal barrier redirects the continuity-coupled return flow through the spaces above and below the ellipse. Because the exported velocity extrema are extremely small in absolute magnitude, the differences are interpreted as a redistribution around near-zero motion rather than as evidence of a globally stronger circulation. At $Ha = 20$, the horizontal profile is spatially nonuniform at $x = 0.2$ and changes across $x = 0.4$ and 0.6 . The dimensionless temperature reaches a reported minimum of $\theta = 0.604$ at $y = 0.4$ on $x = 0.2$ and $\theta = 0.665$ on $x = 0.6$; along $y = 0.2$, $\theta = 0.687$ at $x = 0.6$ (Figures 8–11).

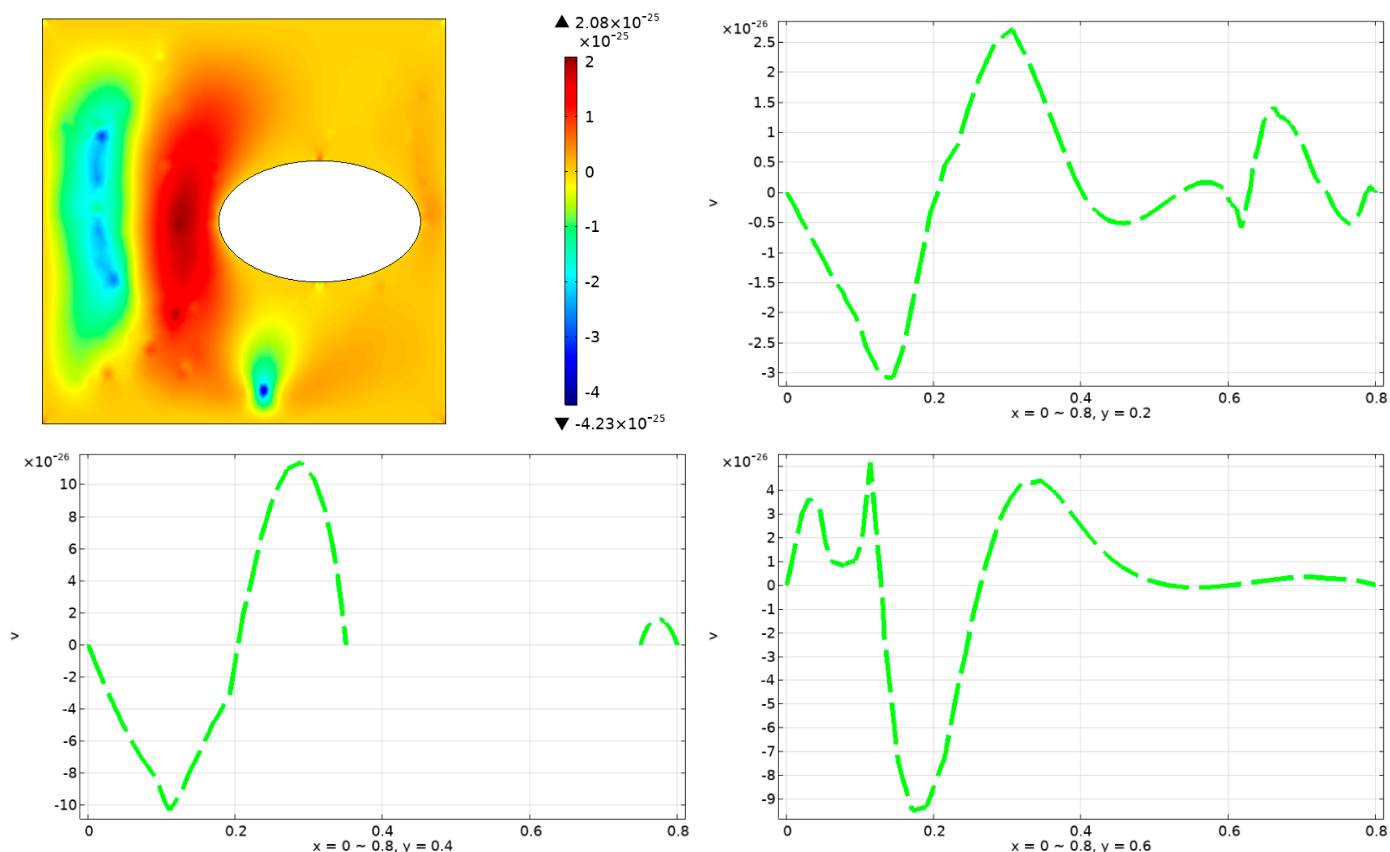


Figure 5. Contour and profiles of v at $Ha = 20$ and $Ra = 10^3$, horizontal elliptical-barrier orientation (0°).

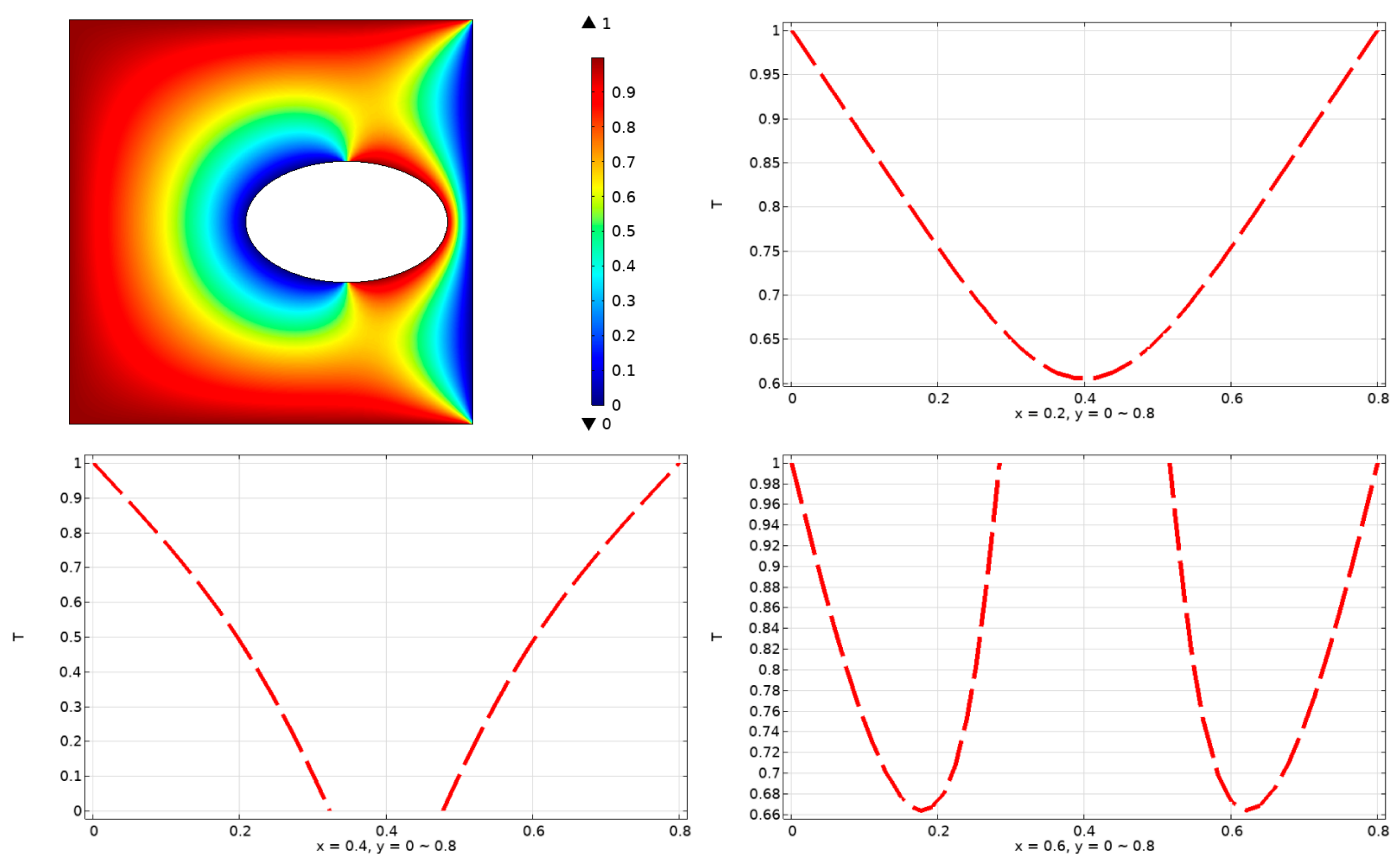


Figure 6. Temperature contour and profiles in the x-planes at $Ha = 20$ and $Ra = 10^3$, horizontal elliptical-barrier orientation (0°).

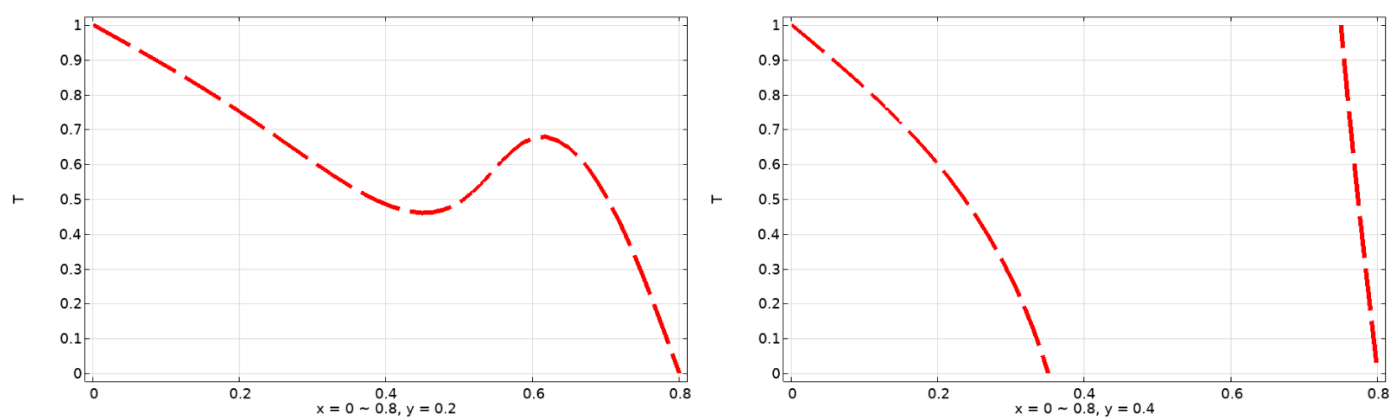


Figure 7. Temperature profiles in the y-planes at $Ha = 20$ and $Ra = 10^3$, horizontal elliptical-barrier orientation (0°).

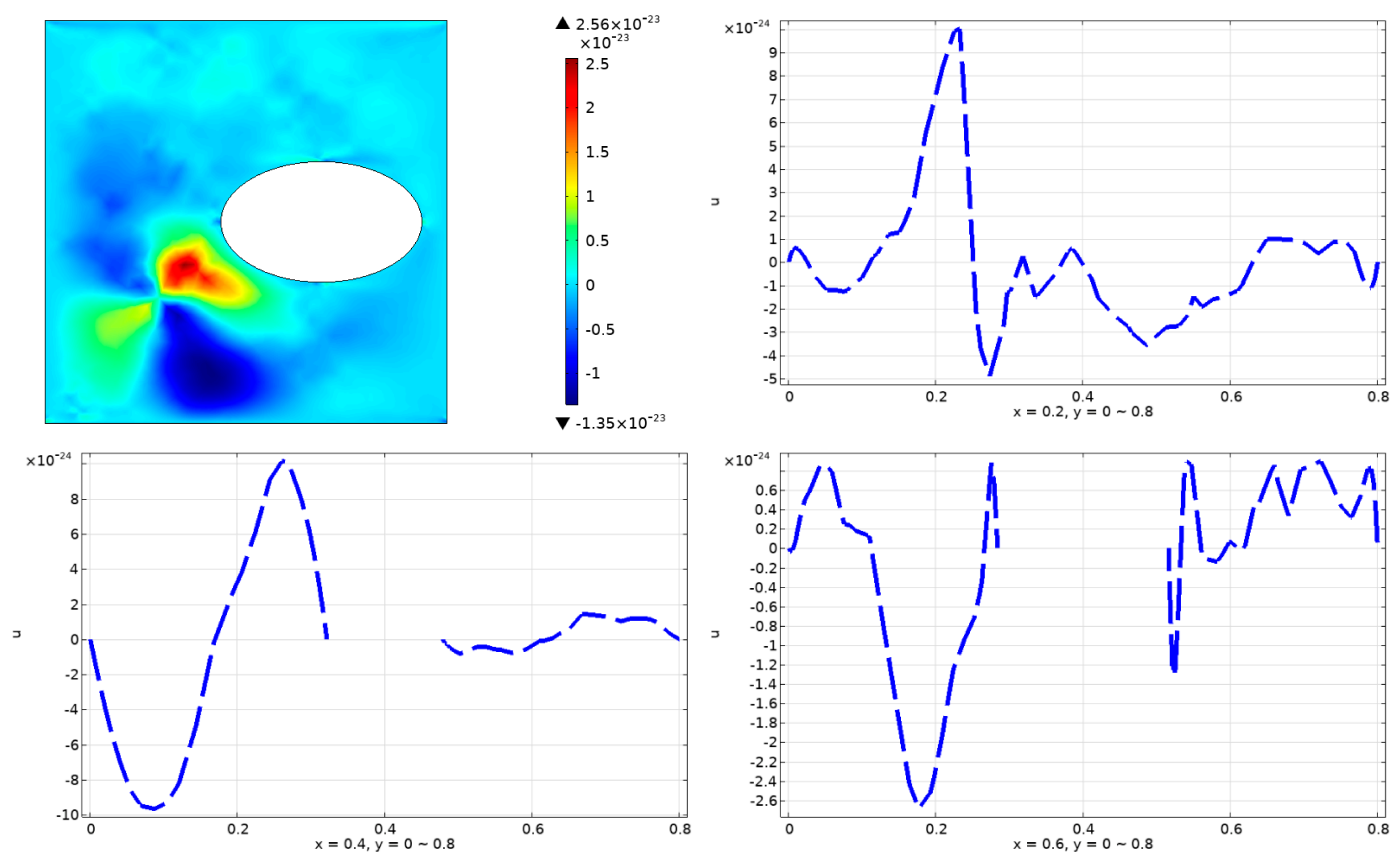


Figure 8. Contour and profiles of u at $Ha = 80$ and $Ra = 10^3$, horizontal elliptical-barrier orientation (0°).

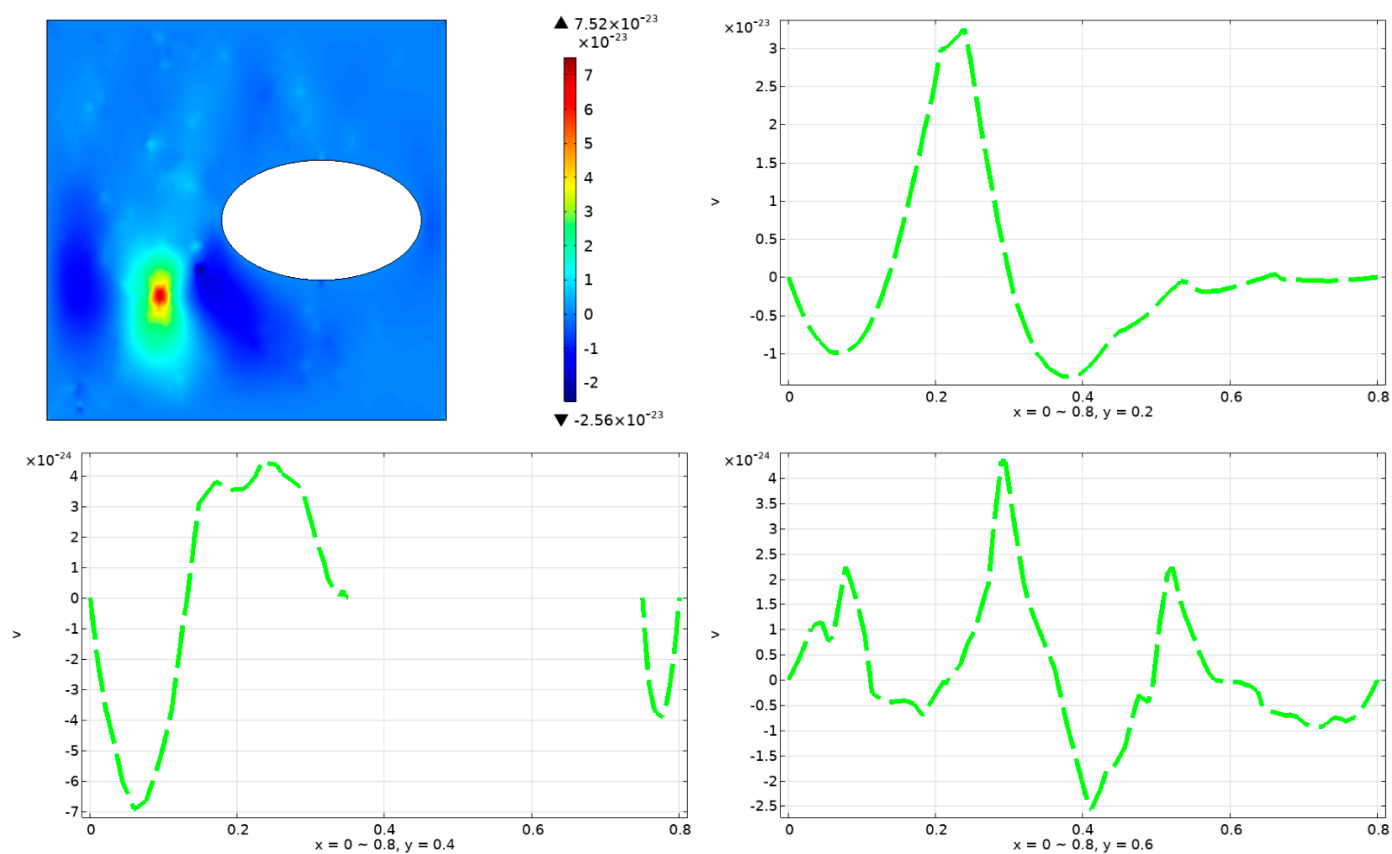


Figure 9. Contour and profiles of v at $Ha = 80$ and $Ra = 10^3$, horizontal elliptical-barrier orientation (0°).

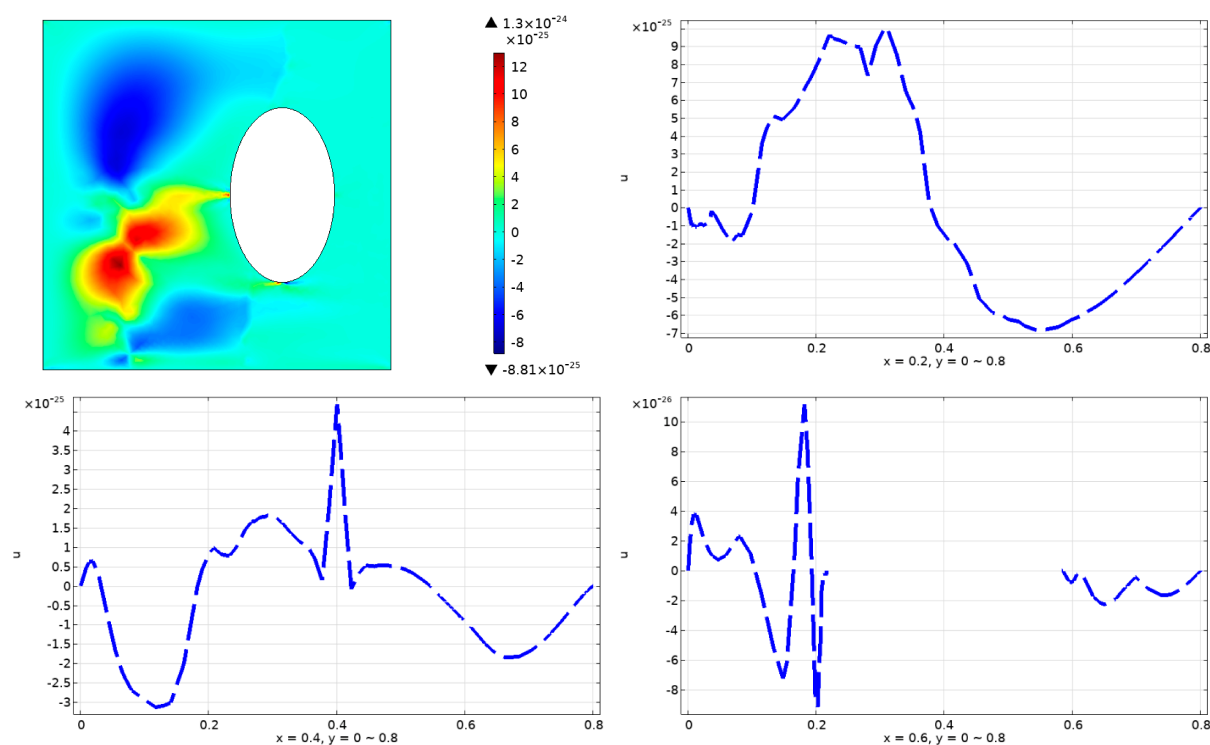


Figure 10. Contour and profiles of u at $Ha = 20$ and $Ra = 10^3$, 90° counterclockwise elliptical-barrier orientation.

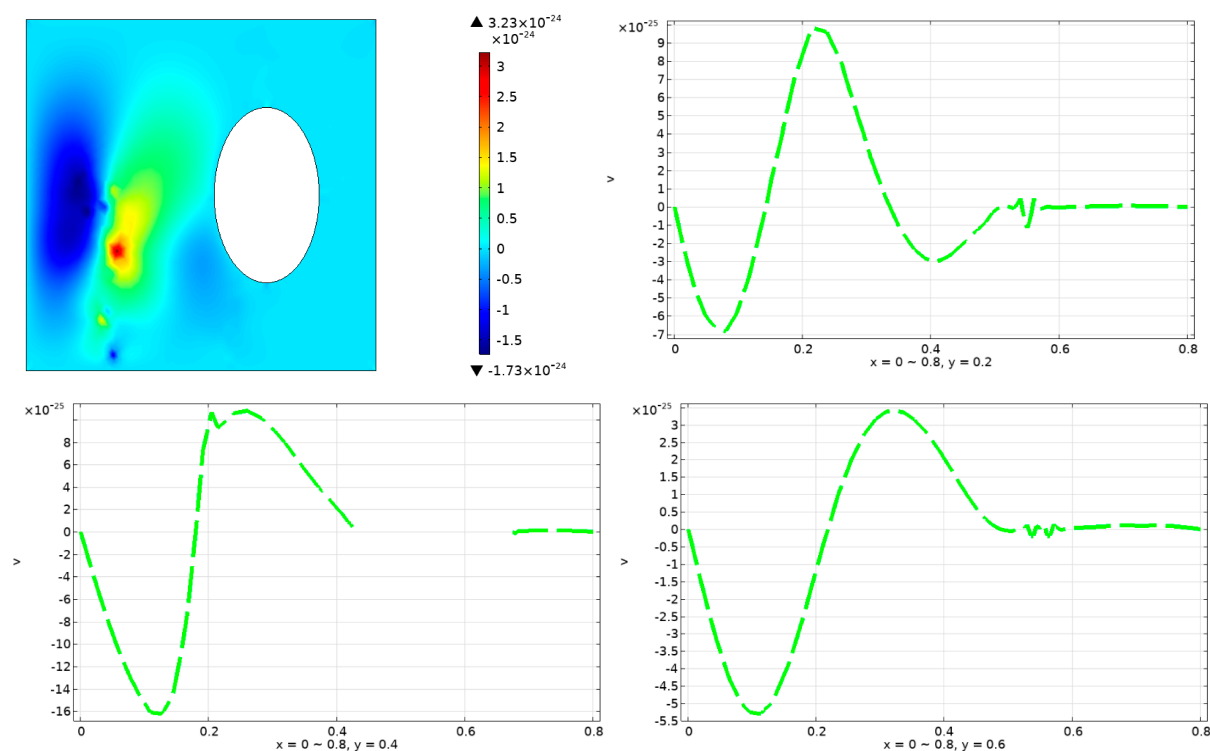


Figure 11. Contour and profiles of v at $Ha = 20$ and $Ra = 10^3$, 90° counterclockwise elliptical-barrier orientation.

5.2. Elliptical Barrier at 90°

Rotating the ellipse to 90° changes the available side passages and the locations at which the weak circulation meets the barrier. The resulting u and v profiles differ from the 0° case, but an increase in a single component extremum is a local kinematic effect and is

not sufficient to establish an increase in total circulation. The vertical long axis produces a different pressure redistribution and redirects the return flow through the narrower lateral gaps. The reported minimum temperature on $x = 0.2$ is $\theta = 0.802$, while $\theta = 0.08$ is reported at $x = 0.6$ along $y = 0.2$ (Figures 12–15). These localized values reflect the proximity of the cold boundary segments and the altered thermal path around the rotated barrier.

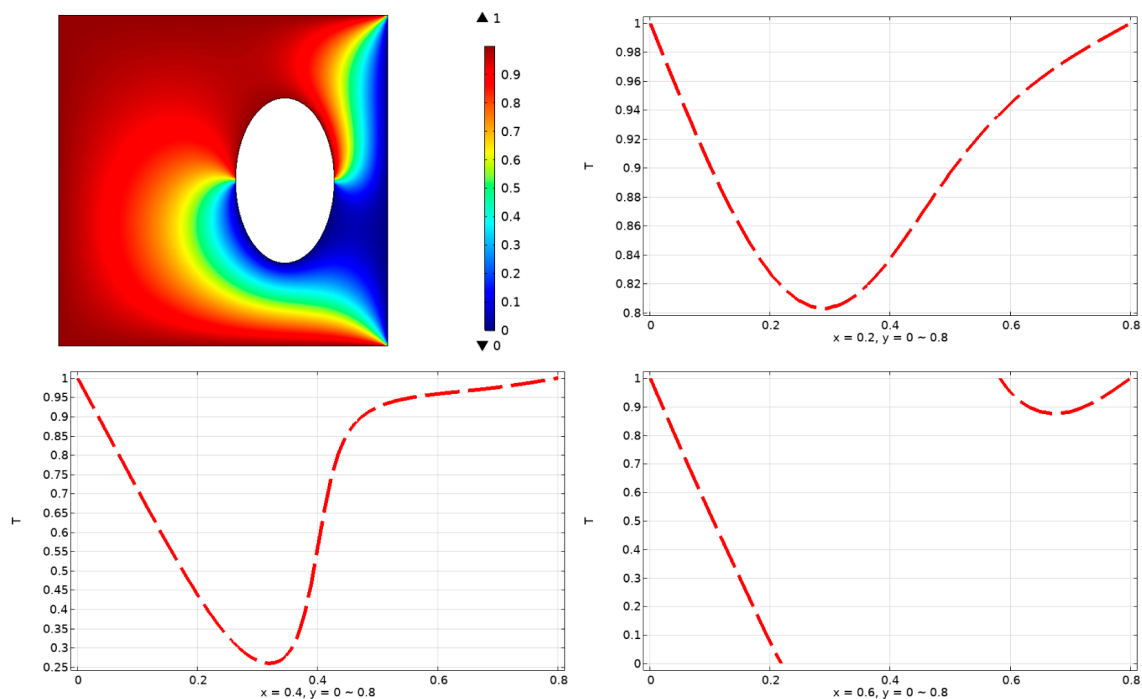


Figure 12. Temperature contour and profiles in the x-planes at $Ha = 20$ and $Ra = 10^3$, 90° counterclockwise elliptical-barrier orientation.

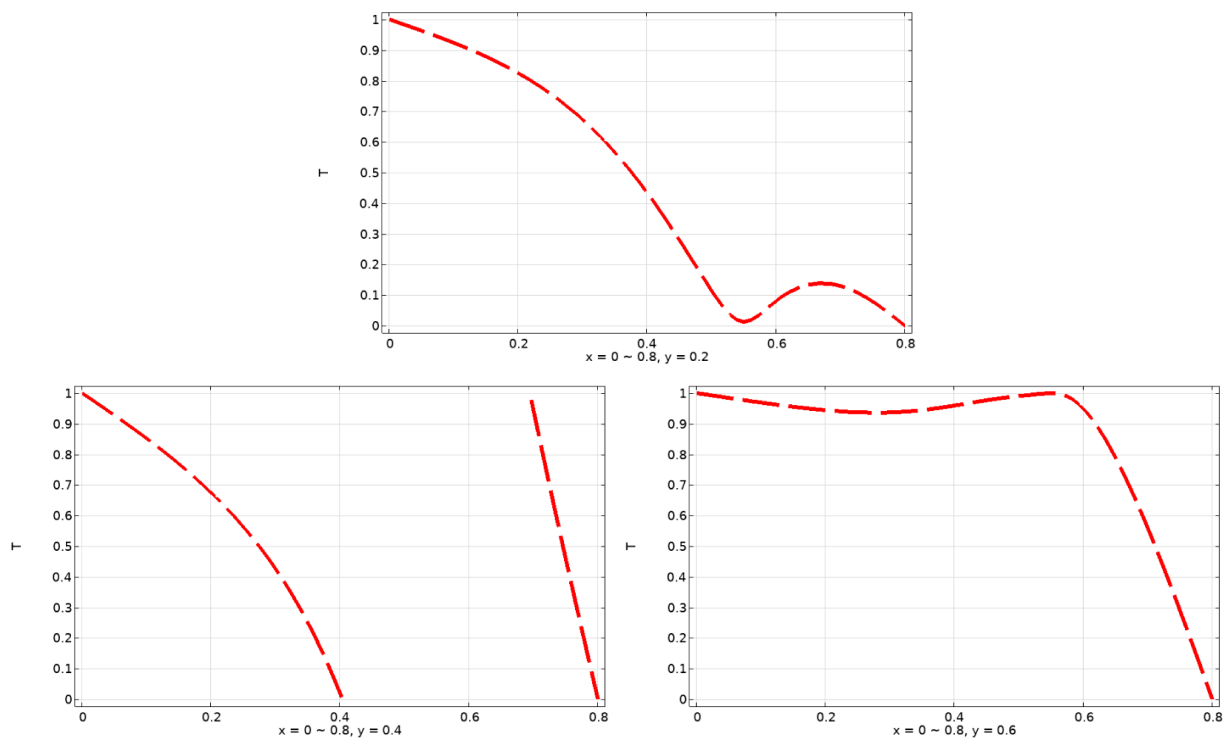


Figure 13. Temperature profiles in the y-planes at $Ha = 20$ and $Ra = 10^3$, 90° counterclockwise elliptical-barrier orientation.

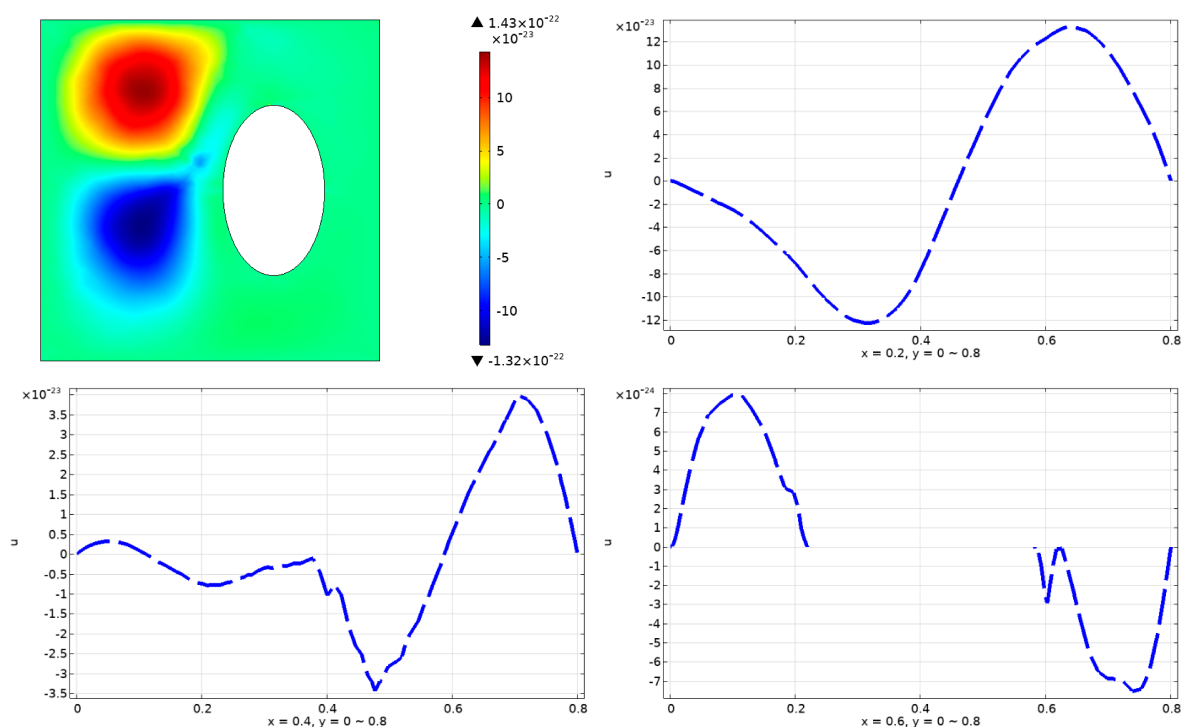


Figure 14. Contour and profiles of u at $Ha = 80$ and $Ra = 10^3$, 90° counterclockwise elliptical-barrier orientation.

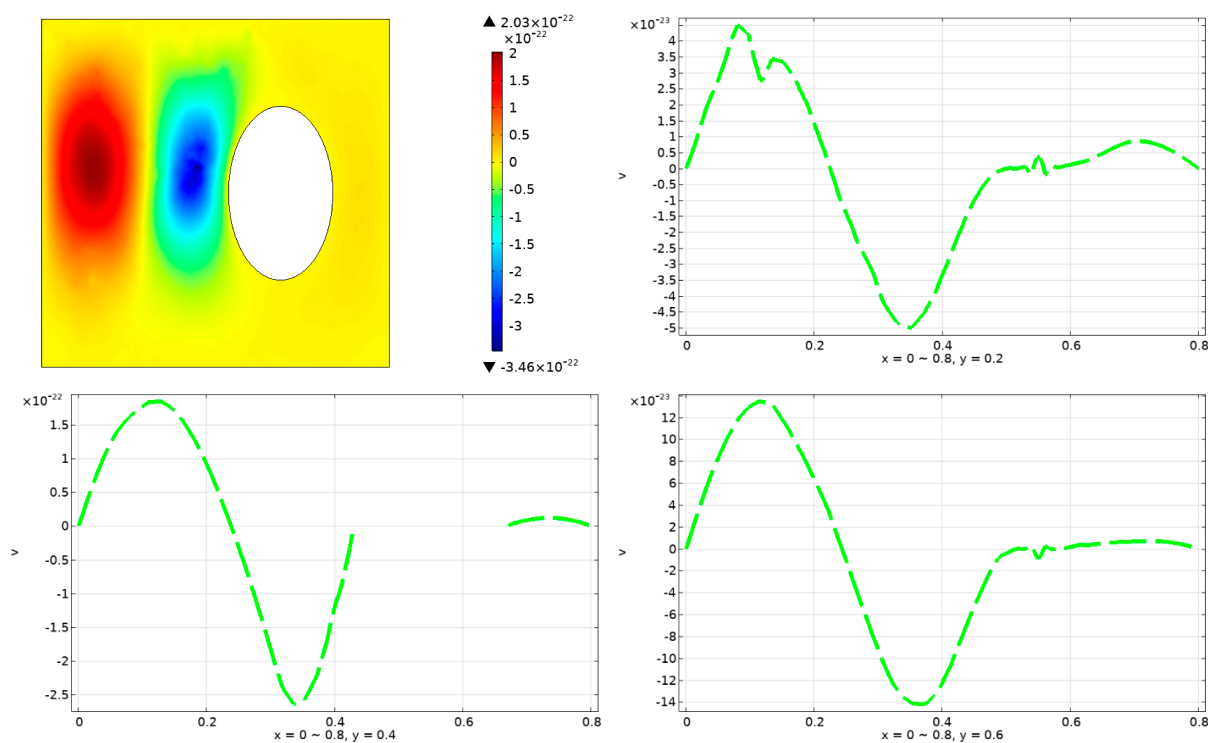


Figure 15. Contour and profiles of v at $Ha = 80$ and $Ra = 10^3$, 90° counterclockwise elliptical-barrier orientation.

5.3. Elliptical Barrier at 180°

At 180°, the ellipse has the same geometric major-axis alignment as at 0°, but the hot and cold halves of the barrier are exchanged by the rotation. This changes the buoyancy source distribution even though the projected obstacle shape is unchanged. Figures 16–19 show backflow near the central region and the barrier, with changes in the local u and v

extrema as Ha increases. The temperature profiles are comparatively symmetric in the reported x -planes; the stated minima are $\theta = 0.9866$ on $x = 0.2$ and $\theta = 0.9256$ on $x = 0.4$ (Figures 20 and 21). The near-unity values indicate that much of these sampled lines remains influenced by the hot boundaries under the selected low- Ra condition.

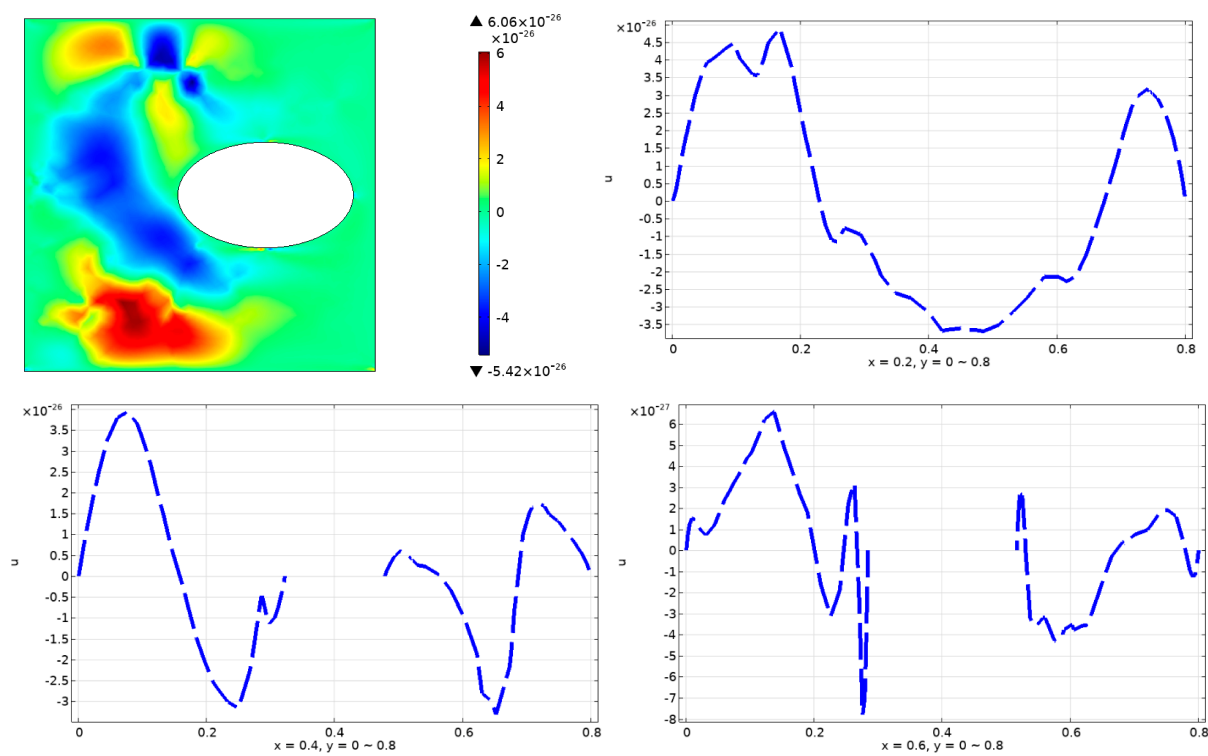


Figure 16. Contour and profiles of u at $Ha = 20$ and $Ra = 10^3$, 180° counterclockwise elliptical-barrier orientation.

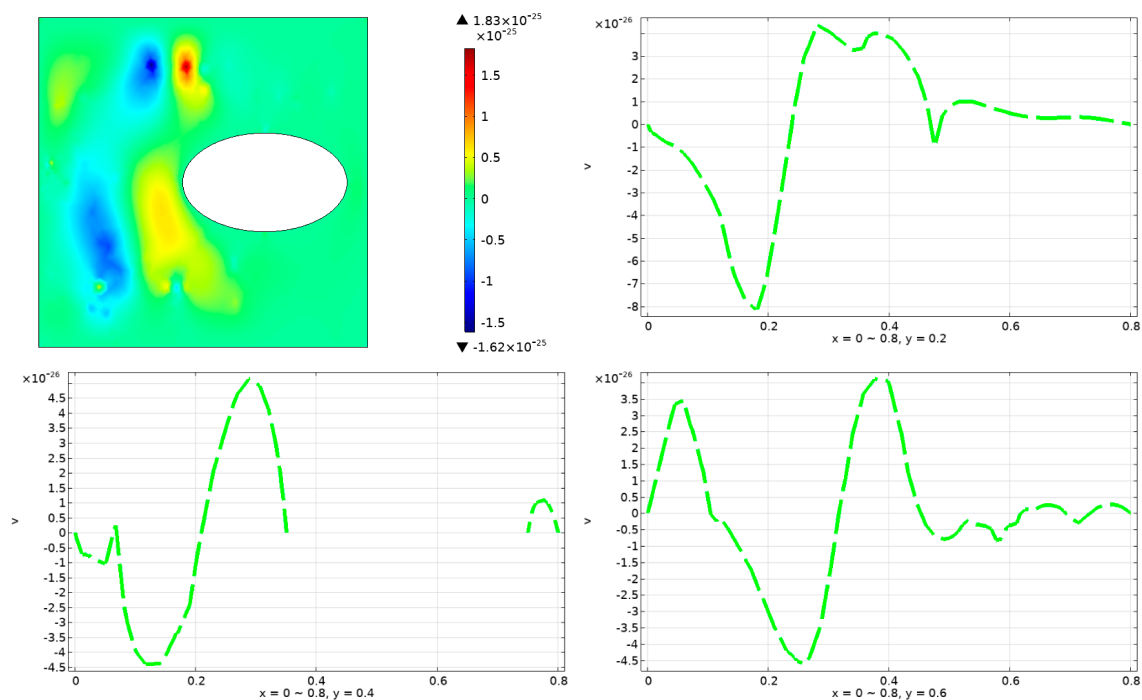


Figure 17. Contour and profiles of v at $Ha = 20$ and $Ra = 10^3$, 180° counterclockwise elliptical-barrier orientation.

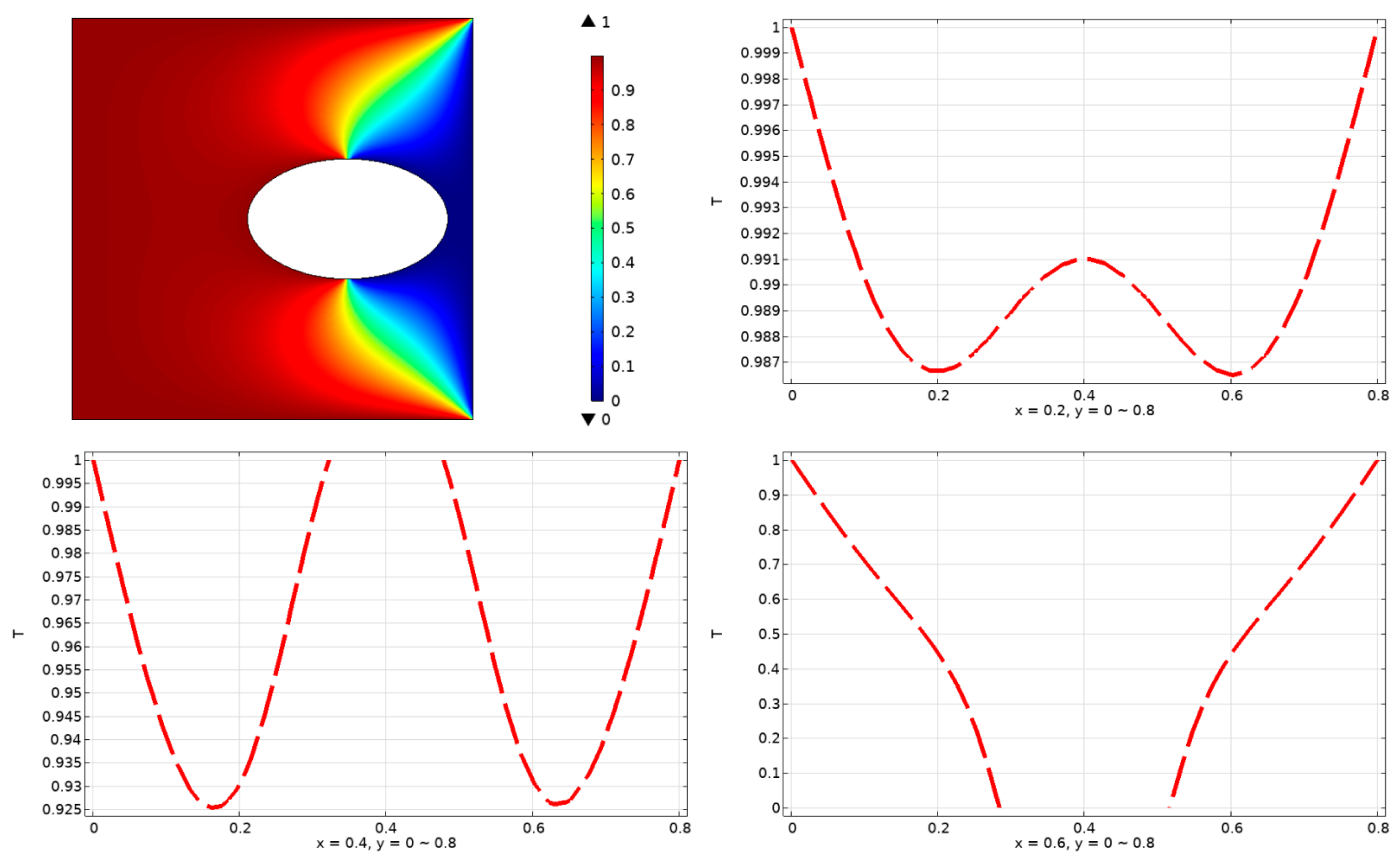


Figure 18. Temperature contour and profiles in the x-planes at $Ha = 20$ and $Ra = 10^3$, 180° counterclockwise elliptical-barrier orientation.

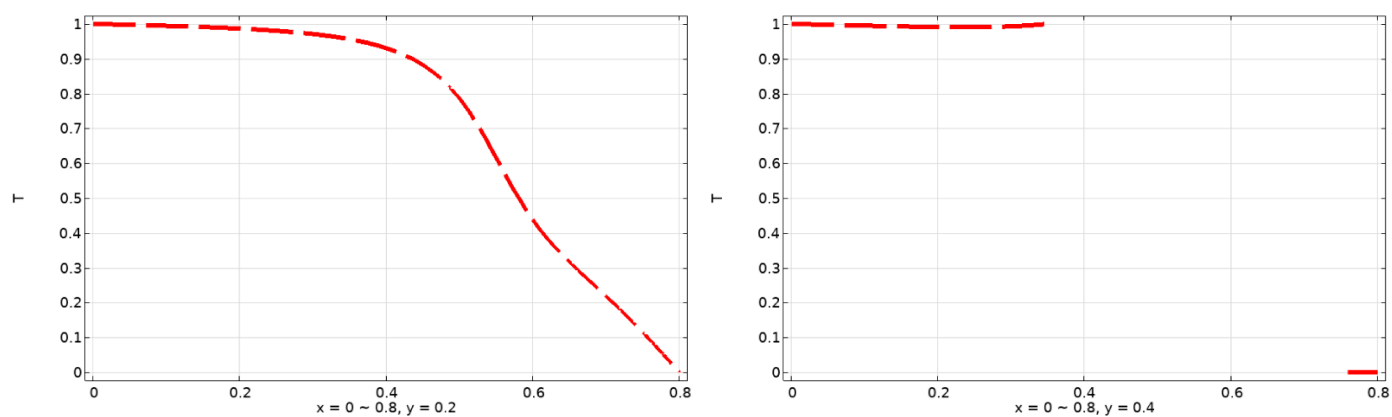


Figure 19. Temperature profiles in the y-planes at $Ha = 20$ and $Ra = 10^3$, 180° counterclockwise elliptical-barrier orientation.

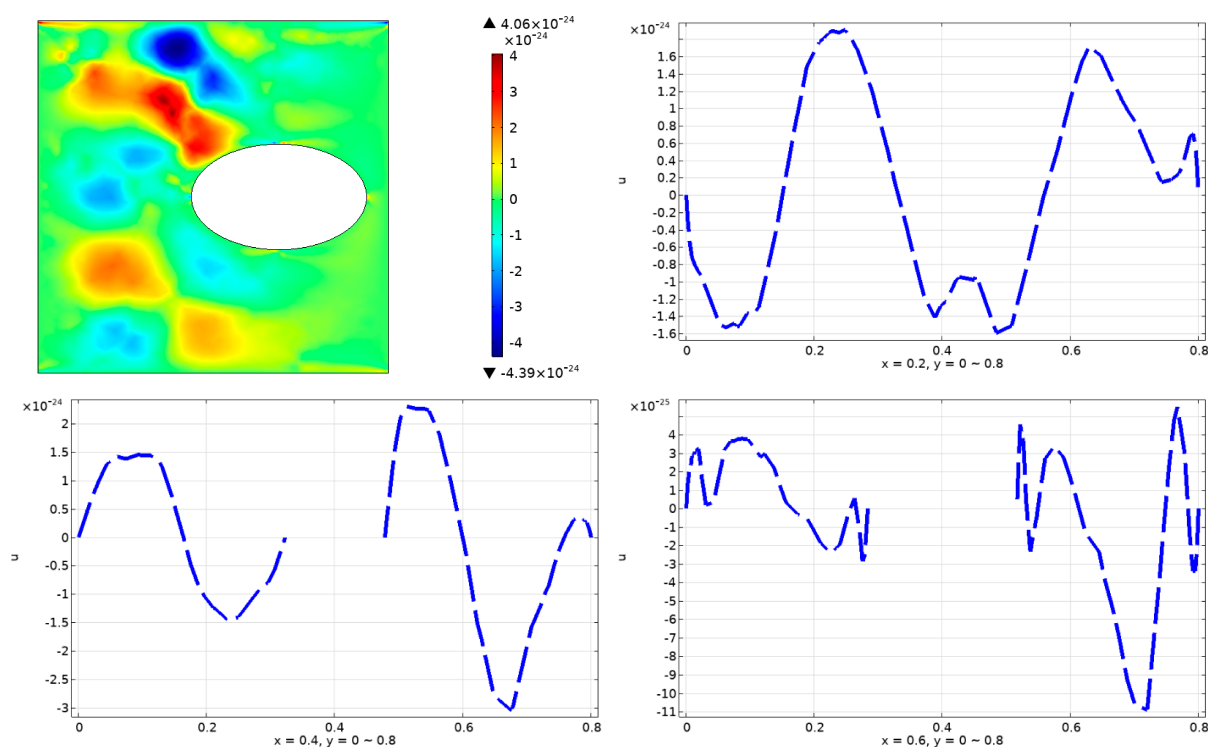


Figure 20. Contour and profiles of u at $Ha = 80$ and $Ra = 10^3$, 180° counterclockwise elliptical-barrier orientation.

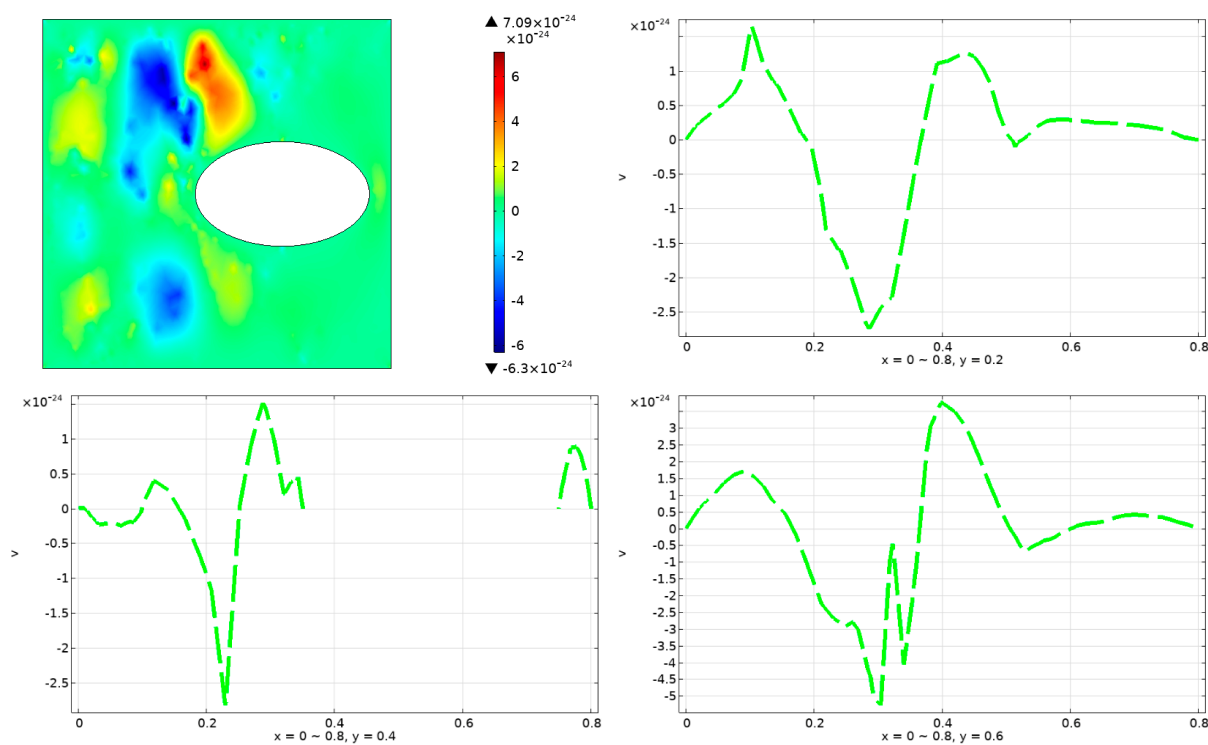


Figure 21. Contour and profiles of v at $Ha = 80$ and $Ra = 10^3$, 180° counterclockwise elliptical-barrier orientation.

5.4. Elliptical Barrier at 270°

At 270° , the heated half of the barrier is positioned in the lower part of the cavity, modifying the alignment between buoyant rise and the narrow passages beside the ellipse. Figures 22–25 show that the local u and v patterns are redistributed between $Ha = 20$ and 80 ,

with backflow concentrated near the barrier and in the left part of the cavity. This should again be interpreted as a change in local component structure rather than an unqualified enhancement of global flow by the magnetic field. The reported dimensionless temperature minima are $\theta = 0.801$ and 0.25 on $x = 0.2$ and 0.4 , respectively, and $\theta = 0.08$ on $x = 0.6$ (Figures 26 and 27).

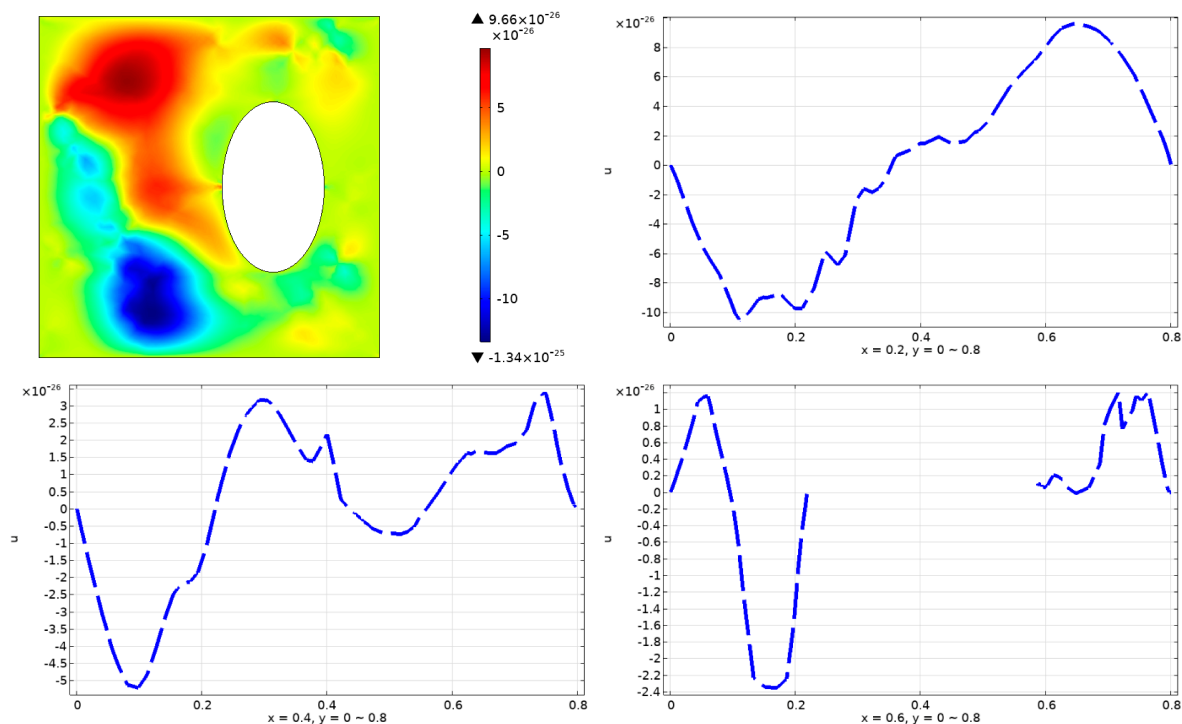


Figure 22. Contour and profiles of u at $Ha = 20$ and $Ra = 10^3$, 270° counterclockwise elliptical-barrier orientation.

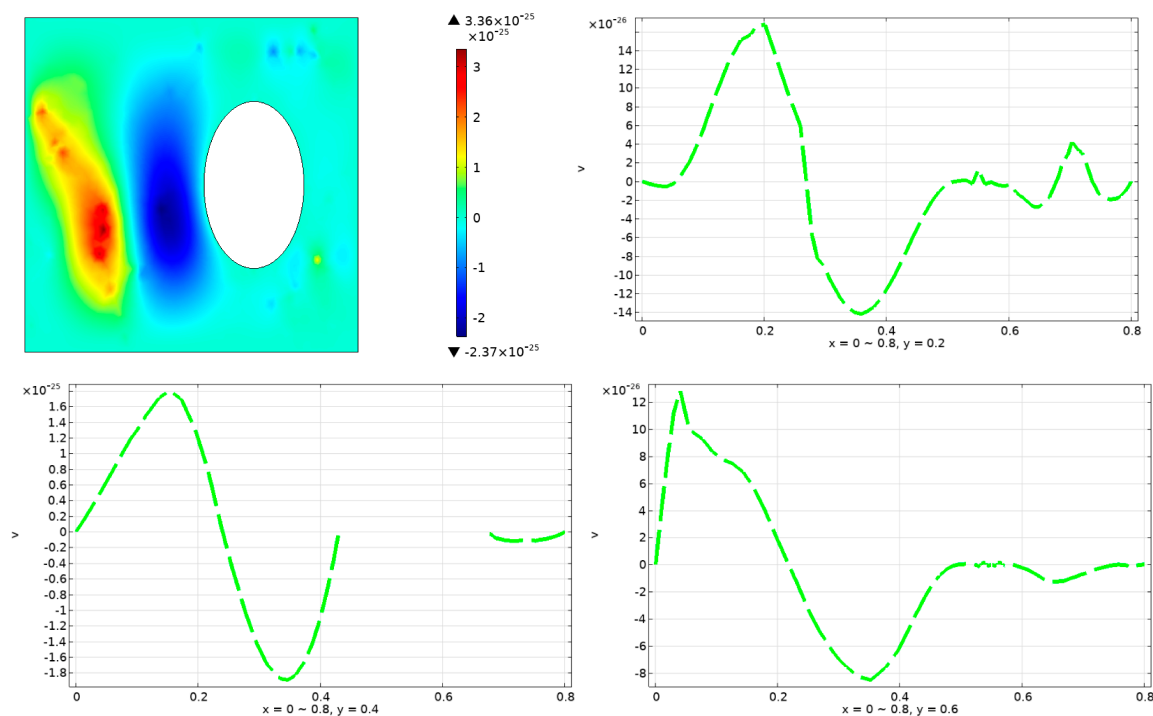


Figure 23. Contour and profiles of v at $Ha = 20$ and $Ra = 10^3$, 270° counterclockwise elliptical-barrier orientation.

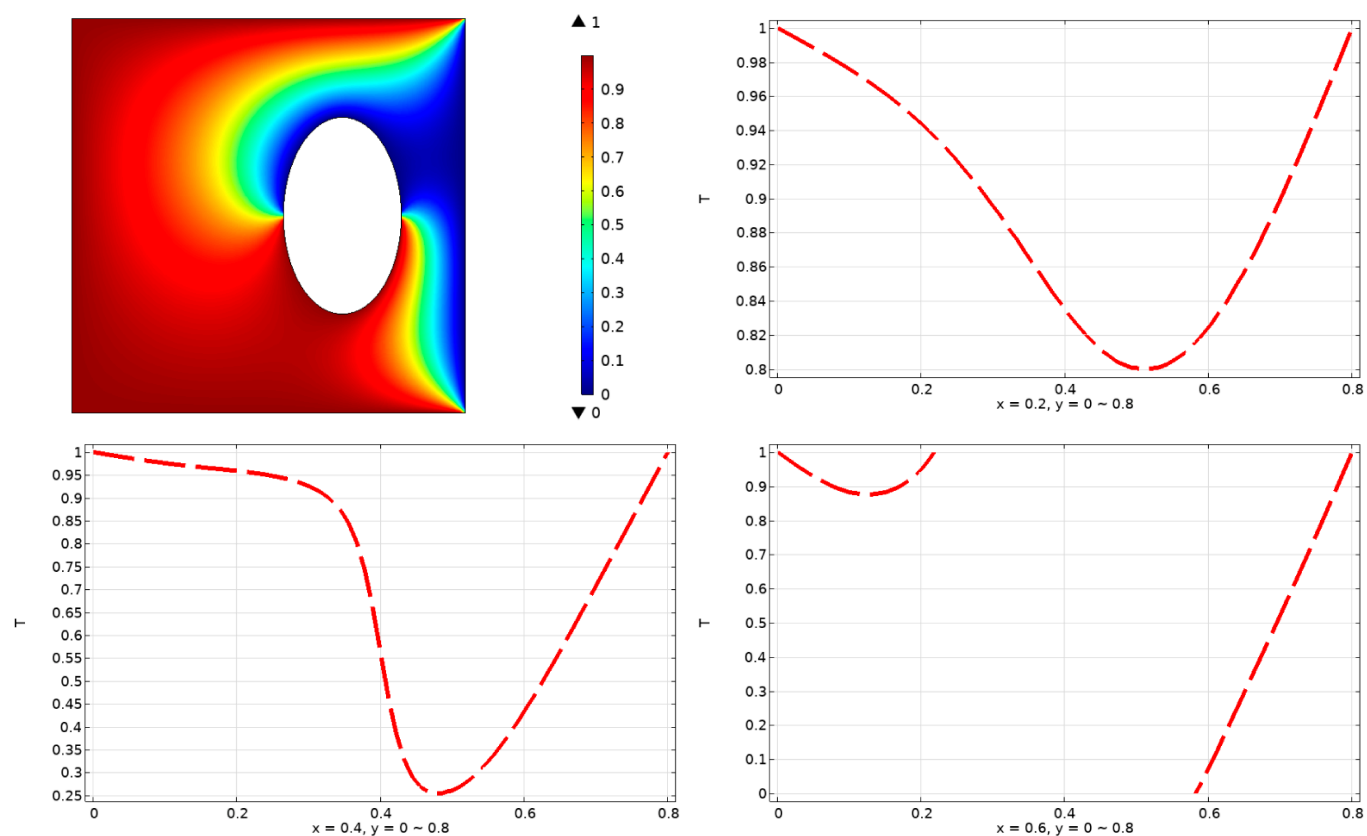


Figure 24. Temperature contour and profiles in the x-planes at $Ha = 20$ and $Ra = 10^3$, 270° counterclockwise elliptical-barrier orientation.

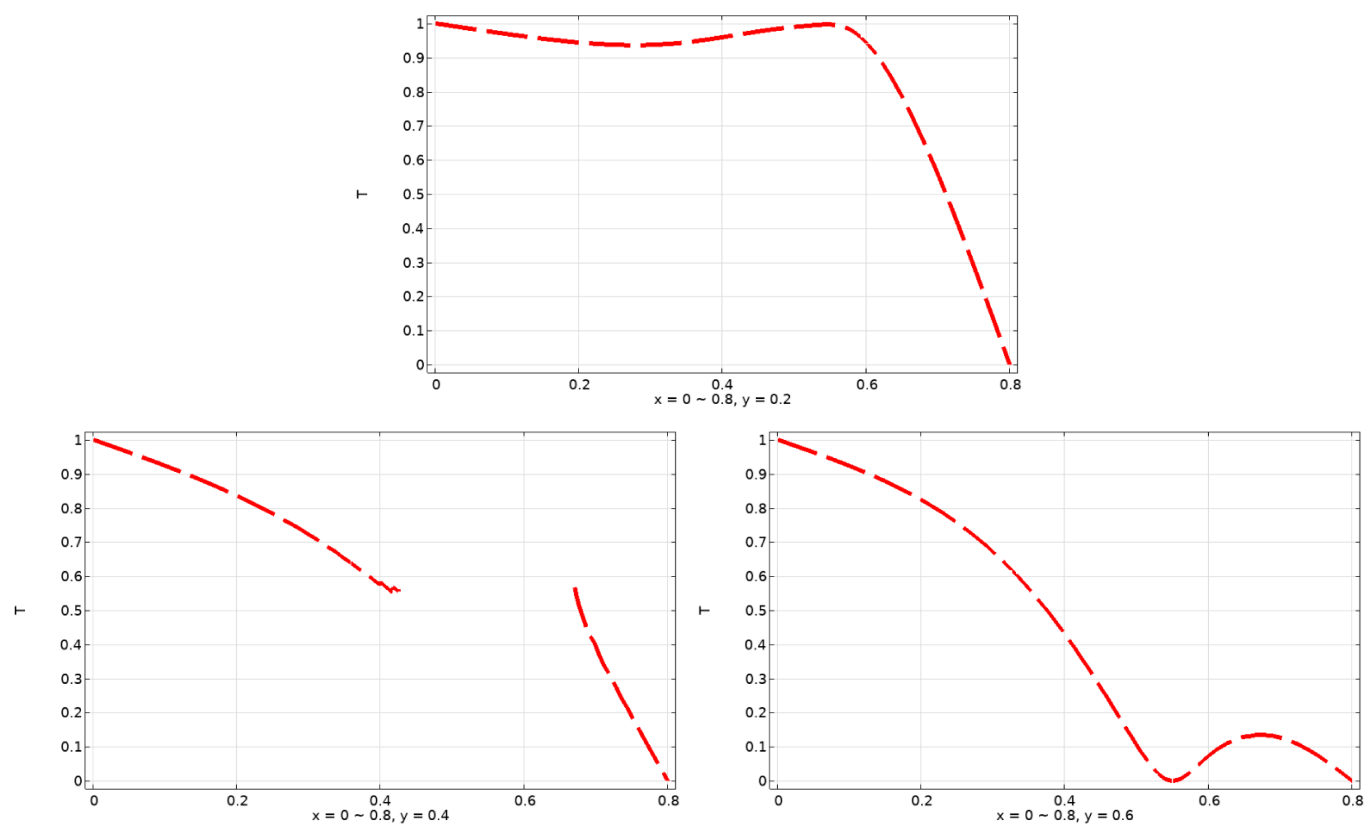


Figure 25. Temperature profiles in the y-planes at $Ha = 20$ and $Ra = 10^3$, 270° counterclockwise elliptical-barrier orientation.

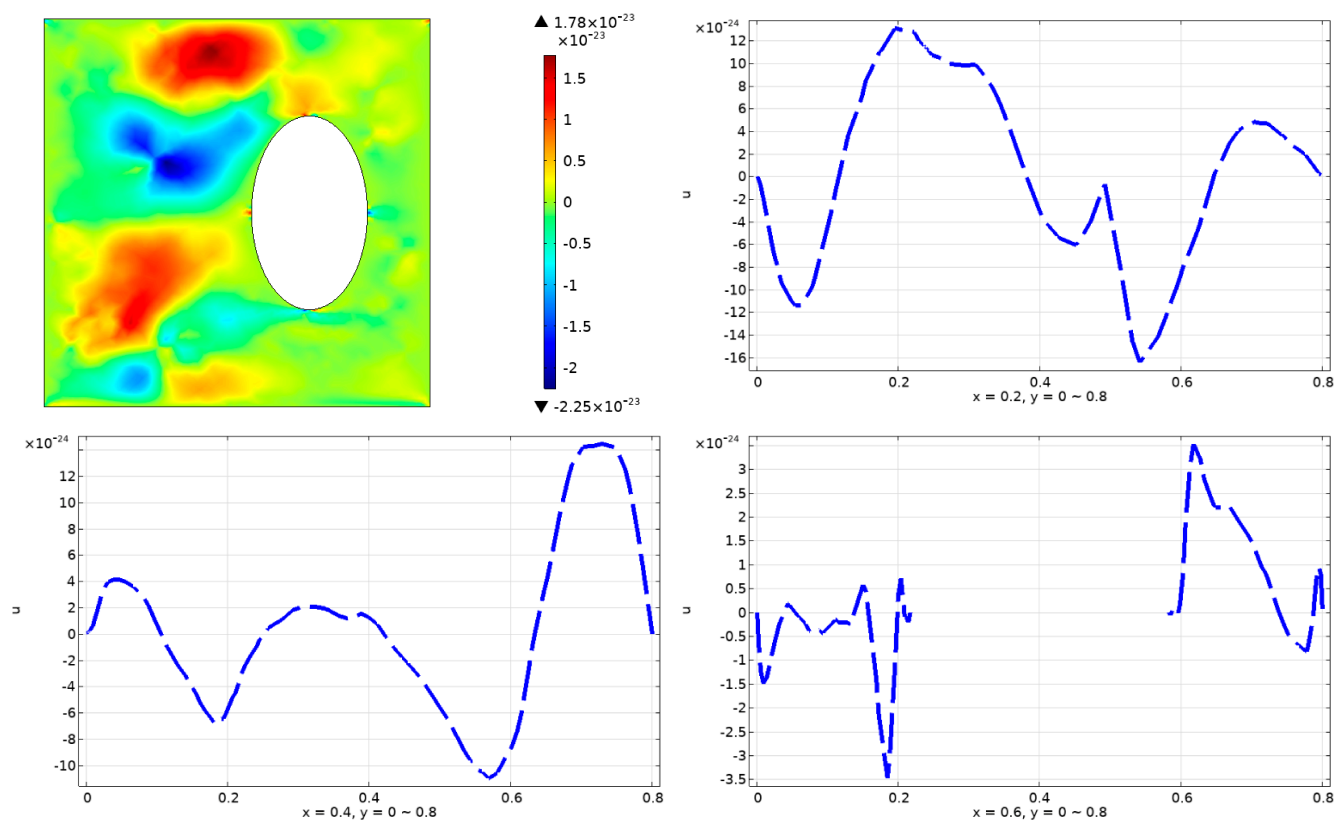


Figure 26. Contour and profiles of u at $Ha = 80$ and $Ra = 10^3$, 270° counterclockwise elliptical-barrier orientation.

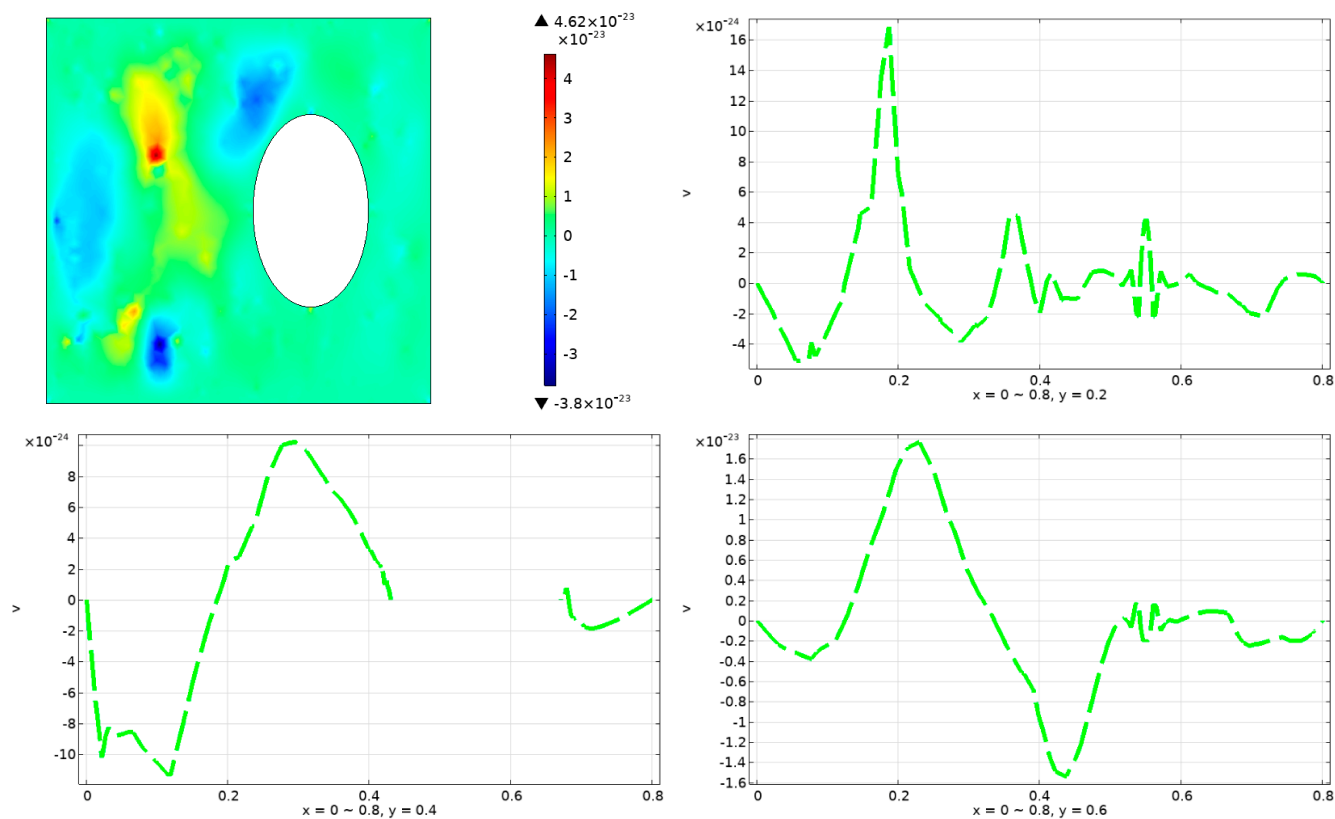


Figure 27. Contour and profiles of v at $Ha = 80$ and $Ra = 10^3$, 270° counterclockwise elliptical-barrier orientation.

5.5. Barrier Orientations at 45° , 135° , 225° , and 315°

Figure 28 compares the temperature contours for the four diagonal orientations. Rotating the barrier away from the principal axes breaks the reflection patterns present in the $0^\circ/180^\circ$ and $90^\circ/270^\circ$ pairs and moves the regions of strongest temperature gradient toward different barrier tips and wall passages. At $Ra = 10^3$, these changes are mainly local because thermal diffusion remains significant; nevertheless, the diagonal configurations alter the thermal-irreversibility share used in the Bejan-number comparison.

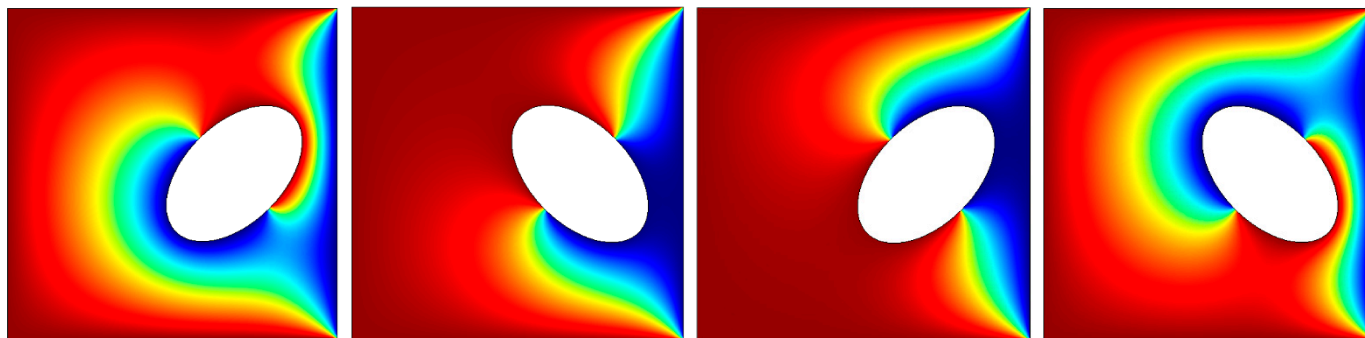


Figure 28. Temperature distribution for the 45° , 135° , 225° , and 315° barrier orientations.

5.6. Coupled Lorentz–Buoyancy–Geometry Mechanism

The vertical magnetic field and buoyancy act on different components of the two-dimensional flow. Buoyancy enters the vertical momentum equation, whereas the Lorentz force associated with $B = B_0 e_y$ opposes horizontal motion. The no-slip boundaries and incompressibility then require a coupled return flow. Increasing Ha can therefore reduce cross-field motion while simultaneously shifting the location of a vertical or horizontal component extremum. A larger plotted extremum in one component does not contradict magnetic damping if the overall circulation is being redirected into a narrower passage; domain-integrated kinetic energy or an average Nusselt number would be required to claim a global enhancement.

Barrier orientation controls this redistribution by changing the hydraulic width and the alignment of the split hot/cold barrier surfaces. At 90° , the lateral passages and thermal segments produce a Bejan-number minimum, which means that the thermal contribution is the smallest fraction of the total irreversibility calculated by Equation (19). At 45° , the diagonal barrier places its tips closer to different wall regions and produces the largest reported Be , indicating a larger relative contribution from temperature-gradient irreversibility. These statements concern the composition of irreversibility; without the separately integrated S_{gen} values, neither orientation can be claimed to minimize total entropy generation.

6. Comparative Performance Analysis

6.1. Horizontal Velocity at $Ha = 20$

At $Ha = 20$, the 90° orientation has the widest reported span between the positive and negative horizontal velocity extrema, while 135° and 225° lie closest to zero (Figure 29). This comparison identifies the orientation that most strongly redistributes the horizontal component in the exported profiles; it does not rank overall thermal performance or domain-integrated flow energy. The 45° , 270° , and 315° values are intermediate at the displayed scale.

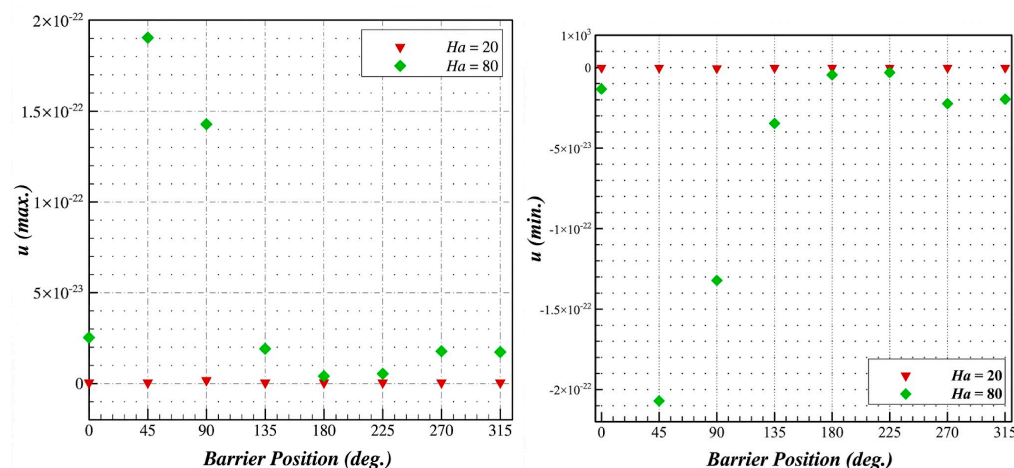


Figure 29. Maximum and minimum horizontal velocity component u at $Ra = 10^3$ for $Ha = 20$ and 80 . Symbols are deterministic extrema; statistical error bars are not applicable.

6.2. Vertical Velocity at $Ha = 20$

The vertical-velocity extrema at $Ha = 20$ are also widest for the 90° orientation, indicating stronger local ascending and descending branches in the sampled field (Figure 30). The 135° and 225° orientations remain close to numerical zero, while the other angles show intermediate and asymmetric component ranges. These ranges are used only as local kinematic descriptors.

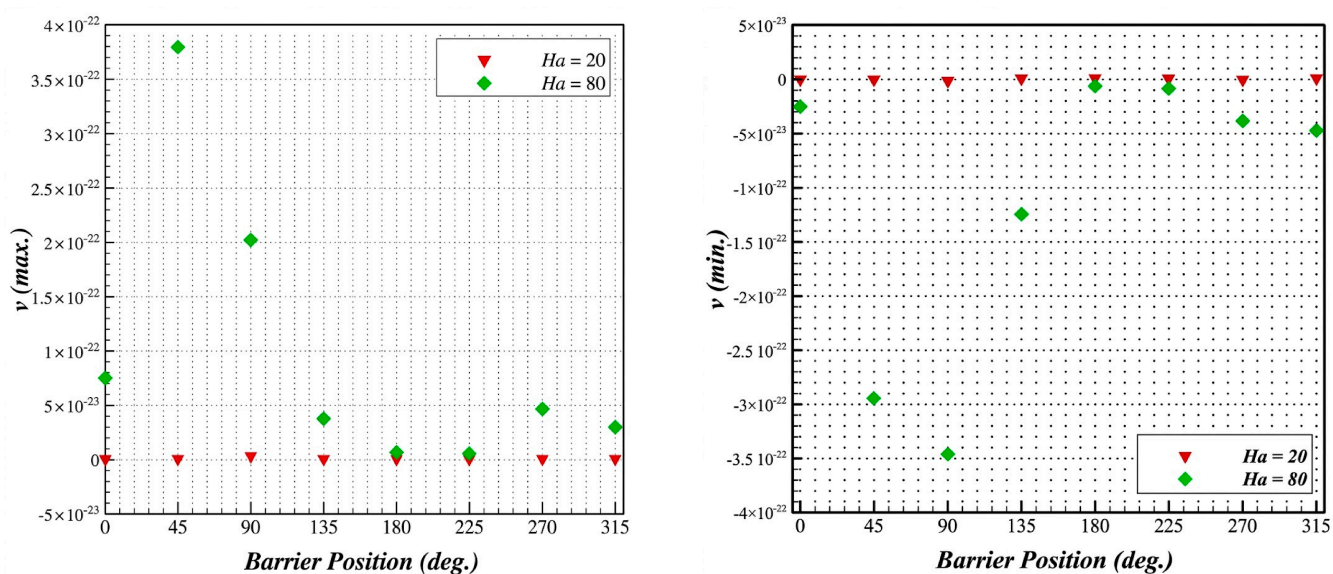


Figure 30. Maximum and minimum vertical velocity component v at $Ra = 10^3$ for $Ha = 20$ and 80 . Symbols are deterministic extrema; statistical error bars are not applicable.

6.3. Pressure at $Ha = 20$

At $Ha = 20$, the 90° orientation exhibits the largest reported pressure range, consistent with stronger local redirection through the lateral passages. The 0° configuration has a high positive pressure but a different negative extremum, whereas 135° and 225° show the smallest pressure ranges. The 315° case has a relatively large negative pressure extremum (Figure 31). Pressure is reported relative to the numerical reference level; the negative values denote local pressure deficits and should not be interpreted as absolute cavitation or suction pressures.

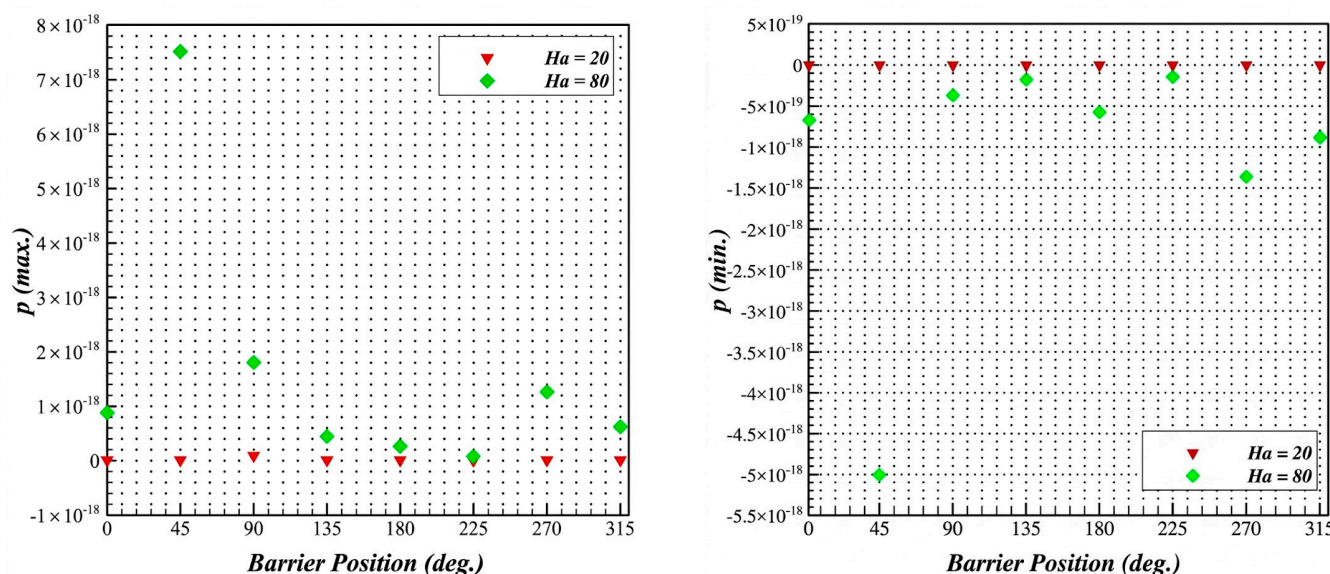


Figure 31. Maximum and minimum relative pressure at $Ra = 10^3$ for $Ha = 20$ and 80 . Symbols are deterministic extrema; statistical error bars are not applicable.

6.4. Horizontal Velocity at $Ha = 80$

At $Ha = 80$, the 45° orientation has the widest reported horizontal-velocity range, followed by 90° , while 180° and 225° remain comparatively weak (Figure 29). The apparent increase in selected component extrema between $Ha = 20$ and 80 is attributed to geometric redistribution and the position of the sampled extrema. It must not be interpreted as proof that a stronger magnetic field increases the global circulation, because the vertical field introduces the damping term $-Ha^2PrU$ in Equation (9).

6.5. Vertical Velocity at $Ha = 80$

At $Ha = 80$, the vertical-component range is largest for the diagonal 45° and 135° orientations, whereas 180° and 225° remain relatively weak (Figure 30). Since the Lorentz term acts directly on U rather than V , continuity and pressure can redirect the residual motion into the vertical component. This provides a mechanistic explanation for local V extrema that increase even while cross-field motion is magnetically resisted.

6.6. Pressure at $Ha = 80$

At $Ha = 80$, the pressure extrema depend strongly on orientation because the magnetic damping and the available passages redistribute the pressure required to satisfy continuity. The 270° and 315° cases show large negative relative-pressure extrema, while other orientations have smaller ranges (Figure 31). These patterns are consistent with localized recirculation and barrier-induced pressure deficits; they do not establish a separate suction-performance metric.

6.7. Bejan Number

The reported Bejan number depends strongly on barrier orientation (Figure 32). The maximum occurs at 45° and the minimum at 90° , with 270° also relatively high. From Equation (19), the 45° maximum means that heat-transfer irreversibility constitutes the largest share of the calculated total, whereas the 90° minimum means that fluid-friction and/or magnetic irreversibility constitute a larger relative share. Accordingly, 90° can be identified as the preferred configuration only under the adopted minimum-Be criterion. A conclusion about minimum total entropy generation would require the separately integrated S_{gen} values, which are not available in the exported dataset.

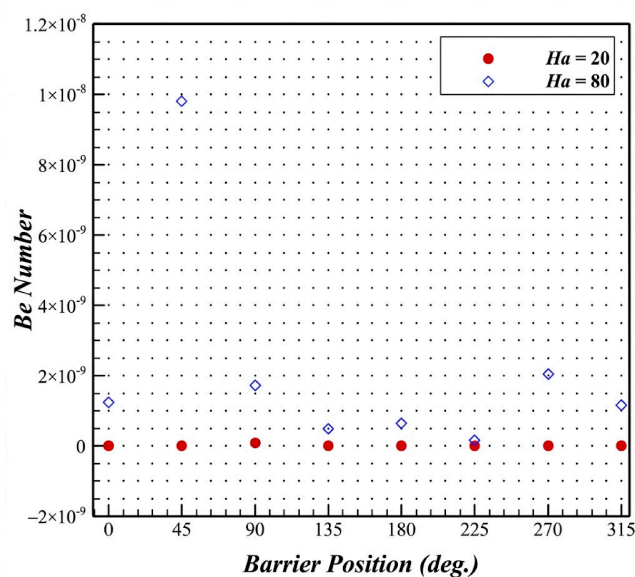


Figure 32. Bejan number at $Ra = 10^3$ for $Ha = 20$ and 80 . Be is the thermal-irreversibility fraction defined by Equation (19).

7. Conclusions

A corrected two-dimensional MHD formulation was used to examine natural convection of a dilute Fe_3O_4 /SWCNT–water hybrid ferrofluid in a square cavity with a rotatable elliptical barrier. Eight orientations were compared at $Ra = 10^3$ and $Ha = 20$ and 80 . The corrected model contains a symmetric viscous operator, places the vertical-field Lorentz damping in the horizontal momentum equation, and uses $Pr = 6.07$ from the tabulated water properties. An independent $Ra = 10^3$ square-cavity benchmark reproduced the reference velocity and average-Nusselt metrics with errors below 0.76%. The principal conclusions are:

- Barrier orientation is a primary control on the locations and signs of the local velocity extrema and on the relative-pressure range. At the selected low Rayleigh number, temperature changes are more subdued because thermal diffusion remains important.
- The 90° orientation gives the widest local velocity-component ranges at $Ha = 20$, whereas the 45° orientation gives the widest horizontal-velocity range at $Ha = 80$. These extrema describe local redistribution and do not establish an increase in domain-integrated circulation by the magnetic field.
- The vertical magnetic field directly resists horizontal motion. Geometry and continuity can redirect the residual motion into the vertical component, explaining why a local component extremum may increase even though the Lorentz term is dissipative.
- The minimum reported Bejan number occurs at 90° and the maximum at 45° . This ranks the relative share of thermal irreversibility under the adopted Be criterion; it does not prove that 90° minimizes total entropy generation.
- The conclusions are limited to a steady homogeneous single-phase model, one dilute composition, $Ra = 10^3$, and the available exported fields. Future work should examine higher Rayleigh numbers, concentration-dependent two-phase transport, sedimentation and magnetic chaining, domain-integrated Nusselt and entropy-generation rates, transient behavior, and experimental validation.

Supplementary Materials: The following supporting information can be downloaded at: <https://www.mdpi.com/article/10.3390/magnetochemistry12100110/s1>, The supporting file contains the validation-compatibility matrix, independent square-cavity benchmark, property audit, concentration

sensitivity of the effective coefficients, complete nondimensional derivation, and the single-phase model limitations. Table S1. Compatibility matrix for the literature sources named in the validation request; Table S2. Independent benchmark and grid-refinement results at $Ra = 10^3$ and $Pr = 0.71$; Table S3. Relative errors for the selected 81×81 benchmark solution; Table S4. Effective-property audit at 0.8 vol% Fe_3O_4 and 0.5 vol% SWCNT; Table S5. Effective-property sensitivity to total particle concentration at a fixed Fe_3O_4 :SWCNT ratio; Figure S1. Dimensionless temperature field from the independent 81×81 square-cavity benchmark at $Ra = 10^3$ and $Pr = 0.71$; Figure S2. Streamfunction contours from the independent 81×81 square-cavity benchmark at $Ra = 10^3$ and $Pr = 0.71$.

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Nomenclature

U, V	Dimensional Velocity Components ($m\ s^{-1}$)
X, Y	Cartesian coordinates (m)
T	Temperature (K)
P	Pressure (Pa)
B0	Magnetic flux density (T)
C_p	Specific heat capacity ($J\ kg^{-1}\ K^{-1}$)
HA	Hartmann number
L	Cavity characteristic length (m)
PR	Prandtl number
SGEN	Total entropy-generation rate
SFF	Fluid-friction entropy-generation rate
T0	Reference absolute temperature (K)
B	Thermal-expansion coefficient (K^{-1})
N	Kinematic viscosity ($m^2\ s^{-1}$)
Σ	Electrical conductivity ($S\ m^{-1}$)
Subscripts	bf: base fluid; hff: hybrid ferrofluid
1, 2	Fe_3O_4 and SWCNT constituents
U, V	Dimensionless Velocity Components
X, Y	Dimensionless Cartesian coordinates
θ	Dimensionless temperature
P	Dimensionless pressure
Be	Bejan number
g	Gravitational acceleration ($m\ s^{-2}$)
k	Thermal conductivity ($W\ m^{-1}\ K^{-1}$)
Nu	Nusselt number
Ra	Rayleigh number
Sth	Thermal entropy-generation rate
Smag	Magnetic entropy-generation rate
α	Thermal diffusivity ($m^2\ s^{-1}$)
μ	Dynamic viscosity (Pa s)
ρ	Density ($kg\ m^{-3}$)

φ	Particle volume fraction
c, h	Cold and hot conditions

References

1. Nguyen, M.D. Fe₃O₄ Nanoparticles: Structures, Synthesis, Magnetic Properties, Surface Functionalization, and Emerging Applications. *Appl. Sci.* **2021**, *11*, 11301. [\[CrossRef\]](#) [\[PubMed\]](#)
2. Imran, M.; Zouli, N.; Ahamad, T.; Alshehri, S.M.; Chandan, M.R.; Hussain, S.; Aziz, A.; Dar, M.A.; Khan, A. Carbon-coated Fe₃O₄ Core–Shell Super-paramagnetic Nanoparticle-Based Ferrofluid for Heat Transfer Applications. *Nanoscale Adv.* **2021**, *3*, 1962–1975. [\[CrossRef\]](#) [\[PubMed\]](#)
3. Ahghari, M.R.; Mollah, M.R.K.; Islam, M.R.; Hossain, M.A. Synthesis and characterization of Se doped Fe₃O₄ nanoparticles for catalytic and biological properties. *Sci. Rep.* **2023**, *13*, 1007. [\[CrossRef\]](#) [\[PubMed\]](#)
4. Yan, W.; Han, B.; Hu, X.; Lin, Y.; Wang, X.; Deng, X. Synthesis of Fe₃O₄ nanoparticles and their magnetic properties. *Procedia Eng.* **2012**, *27*, 632–637. [\[CrossRef\]](#)
5. Shah, S.I.A.; Ahmad, W.; Anwar, M.; Shah, R.; Khan, J.A.; Shah, N.S.; Al-Anazi, A.; Han, C.S. Synthesis, properties, and applications of Fe₃O₄ and Fe₃O₄-based nanocomposites: A review. *Appl. Catal. O-OPEN* **2025**, *203*, 207045. [\[CrossRef\]](#)
6. Castellanos-Rubio, I.; García, M.A.; López, M.A. A Milestone in the Chemical Synthesis of Fe₃O₄ Nanoparticles with Controlled Size and Magnetic Properties. *Chem. Mater.* **2021**, *33*, 8693–8704. [\[CrossRef\]](#) [\[PubMed\]](#)
7. Irfan, M.; Siddique, I.; Nazeer, M.; Saleem, S.; Radwan, N. Heat transfer analysis of single-walled carbon nanotubes in Ellis’s fluid model: Comparative study of uniform and non-uniform channels. *Case Stud. Therm. Eng.* **2024**, *54*, 104036. [\[CrossRef\]](#)
8. Kumar, S.; Choudhary, S.; Kumari, K.; Sharma, A.; Choudhary, P. MHD darcy-forchheimer flow of SWCNT-H₂O nanofluid over a porous stretching sheet. *Int. J. Thermofluids* **2025**, *26*, 101064. [\[CrossRef\]](#)
9. Sarlak, R.; Abed, A.M.; Akbari, O.A.; Marzban, A.; Baghaei, S.; Bayat, M. Numerical Investigation of Natural Convection Heat Transfer of Water/SWCNT Nanofluid Flow in a Triangular Cavity with Cold Fluid Injection. *Prog. Nucl. Energy* **2023**, *155*, 104513. [\[CrossRef\]](#)
10. Sarkar, S.; Pal, P.; Ghosh, N.K. Enhancing the thermal conductivity and viscosity of ethylene glycol-based single-walled carbon nanotube (SWCNT) nanofluid: An investigation utilizing equilibrium molecular dynamics simulation. *Chem. Thermodyn. Therm. Anal.* **2024**, *16*, 100142. [\[CrossRef\]](#)
11. Malviya, K.; Naganagowda, H.B.; Matada, P.K.; Varma, S.V.K.; Raju, C.S.K.; Sharma, R. Heat transfer analysis of MHD oscillatory SWCNT/MWCNT-H₂O hybrid nanofluid flow in a channel. *ZAMM-J. Appl. Math. Mech./Z. Angew. Math. Mech.* **2024**, *104*, e202300366. [\[CrossRef\]](#)
12. Das, S. Thermophysical Properties of SWCNT Nanofluid. *Macromol. Symp.* **2024**, *413*, 2200206. [\[CrossRef\]](#)
13. Thirumalaisamy, K.; Lee, Y.D.; Kim, H. Magnetohydrodynamic buoyancy-driven hybrid nanofluid flow and heat transfer within a partially heated porous square cavity. *Phys. Fluids* **2025**, *37*, 042011. [\[CrossRef\]](#)
14. Kim, H.; Do, Y.; Ramachandran, S.; Sankar, M.; Thirumalaisamy, K. Computational analysis of magnetohydrodynamic ternary-hybrid nanofluid flow and heat transfer inside a porous cavity with shape effects. *Phys. Fluids* **2024**, *36*, 082008. [\[CrossRef\]](#)
15. Thirumalaisamy, K.; Reddy, A.S. Numerical computation on MHD natural convective ternary nanofluid flow and heat transfer in a porous square cavity: Marker-and-cell technique. *Int. J. Numer. Methods Heat Fluid Flow* **2023**, *33*, 3425–3466. [\[CrossRef\]](#)
16. Gupta, T.; Kumar, M.; Yaseen, M.; Rawat, S.K. Heat transfer of MHD flow of hybrid nanofluid (SWCNT- MWCNT/C₃H₈O₂) over a permeable surface with Cattaneo–Christov model. *Numer. Heat Transf. B* **2025**, *86*, 436–451. [\[CrossRef\]](#)
17. Alidoust, S.; AmoozadKhalili, F.; Hamed, S. Investigation of Effective Parameters on Relative Thermal Conductivity of SWCNT (15%)–Fe₃O₄ (85%)/Water Hybrid Ferro-Nanofluid and Presenting a New Correlation with Response Surface Methodology. *Colloids Surf. A Physicochem. Eng. Asp.* **2022**, *645*, 128625. [\[CrossRef\]](#)
18. Esfe, M.H.; Hatami, H.; Alidoust, S.; Toghraye, D.; Karajabad, M.S. The Volume Fraction of Nanoparticles and Temperature Effects on Thermal Conductivity of SWCNT–Fe₃O₄–CuO/Ethylene glycol–Water Ternary Hybrid Nanofluid: Experimental, Numerical, and Statistical Investigations. *J. Therm. Anal. Calorim.* **2024**, *149*, 7773–7781. [\[CrossRef\]](#)
19. Vinutha, K. Thermal Performance of Fe₃O₄, SWCNT, MWCNT and H₂O based on Magnetohydrodynamic Nanofluid Flow across a Wedge with Significant Impacts of Soret and Dufour. *Nanoscale Adv.* **2023**, *5*, 5952–5964. [\[CrossRef\]](#) [\[PubMed\]](#)
20. Thirumalaisamy, K.; Ramachandran, S. Comparative heat transfer analysis on Fe₃O₄–H₂O and Fe₃O₄–Cu–H₂O flow inside a tilted square porous cavity with shape effects. *Phys. Fluids* **2023**, *35*, 022007. [\[CrossRef\]](#)
21. Khan, M.S.; Mei, S.; Shabnam; Fernandez-Gamiz, U.; Noeiaghdam, S.; Shah, S.A.; Khan, A. Numerical Analysis of Unsteady Hybrid Nanofluid Flow Comprising CNTs–Ferrous oxide/Water with Variable Magnetic Field. *Nanomaterials* **2022**, *12*, 180. [\[CrossRef\]](#) [\[PubMed\]](#)
22. Gal, S.; Cabaleiro, D.; Hassen, W.; Ba, H.; Estellé, P. Thermal and Flow Properties of Graphene/Fe₃O₄ Water-Based Hybrid Nanofluids. *Int. J. Thermophys.* **2025**, *46*, 126. [\[CrossRef\]](#)

23. Nciri, R.; Awouda, A.E.A.; Musa, A.A.; Alshomrani, H.G.; Nasri, F. Numerical Simulation of Natural Convection in a Chamfered Square Cavity with Fe₃O₄-Water Nanofluid and Magnetic Excitation. *Eng. Technol. Appl. Sci. Res.* **2025**, *15*, 20523–20528. [[CrossRef](#)]
24. Rajkumar, D.; Palanivel, S.; Murali, C. *Numerical Investigation on SWCNT-MWCNT/Blood Based Pulsating Micropolar Hybrid Nanofluid Flow Dynamic in a Vertical Channel with Thermal Radiation and Heat Source/Sink*; Sathyabama Institute of Science and Technology: Chennai, India, 2025.
25. Touki, H.D.; Zubly, N.J.; Saha, S. Magnetohydrodynamics Forced Convective Flow of Micropolar Ferrofluid over Double Stationary Cylinders inside a Vented Chamber. *S. Afr. J. Chem. Eng.* **2025**, *53*, 176–192. [[CrossRef](#)]
26. Mirzaei, A.; Jalili, B.; Jalili, P.; Ganji, D.D. Free Convection in a Square Wavy Porous Cavity with Partly Magnetic Field: A Numerical Investigation. *Sci. Rep.* **2024**, *14*, 14152. [[CrossRef](#)] [[PubMed](#)]
27. Khedher, N.B.; Shahabadi, M.; Alghawli, A.S.; Hulme, C.N.; Mehryan, S.A.M. Numerical Study of the Flow and Thermomagnetic Convection Heat Transfer of a Power Law Non-Newtonian Ferrofluid within a Circular Cavity with a Permanent Magnet. *Mathematics* **2002**, *10*, 2612. [[CrossRef](#)]
28. Bouzit, F.; Bouzit, M.; Mokhefi, A. Numerical Investigation of the Nanoparticles Nature Effect on the MHD Behavior in a Square Cavity with a Metallic Obstacle. *Metall. Mater. Eng.* **2022**, *28*, 223–244. [[CrossRef](#)] [[PubMed](#)]
29. Awad, M.M. A New Definition of Bejan Number. *Therm. Sci.* **2012**, *16*, 1251–1253. [[CrossRef](#)]
30. Jalili, B.; Sadighi, S.; Jalili, P.; Ganji, D.D. Numerical Analysis of MHD Nanofluid Flow and Heat Transfer in a Circular Porous Medium Containing a Cassini Oval under the Influence of the Lorentz and Buoyancy Forces. *Heat Transf.* **2022**, *51*, 6122–6138. [[CrossRef](#)]
31. Currie, I.G. *Fundamental Mechanics of Fluids*, 4th ed.; Taylor & Francis Group: Boca Raton, FL, USA, 2012; ISBN 978-1-4665-1700-4.
32. de Vahl Davis, G. Natural Convection of Air in a Square Cavity: A Bench Mark Numerical Solution. *Int. J. Numer. Methods Fluids* **1983**, *3*, 249–264. [[CrossRef](#)]

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