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Numerical Study of the Effects of Asymmetric Velocity Profiles in a Curvilinear Channel on Migration of Neutral Buoyant Particle

Ryo Naito ¹ and Tomohiro Fukui ^{2,*}
¹ Department of Master's Program, Kyoto Institute of Technology, Kyoto 606-8585, Japan

² Department of Mechanical Engineering, Kyoto Institute of Technology, Kyoto 606-8585, Japan

* Correspondence: fukui@kit.ac.jp; Tel.: +81-75-724-7314

Abstract: The microstructure and suspended particle behavior should be considered when studying the flow properties exhibited by particle suspension. In addition, particle migration, also known as Segré–Silberberg effects, alters the microstructure of the suspension and significantly affects the viscosity properties of the suspension. Therefore, particle behavior with respect to the changes in mechanical factors should be considered to better understand suspension. In this study, we investigated the particle behavior in asymmetric velocity profiles with respect to the channel center numerically using the lattice Boltzmann method and a two-way coupling scheme. Our findings confirmed that the final equilibrium position of particles in asymmetric velocity profiles converged differently between the outer and inner wall sides with respect to the channel center. This indicates that the mechanical equilibrium position of particles can be changed by asymmetric velocity profiles. In addition, centrifugal force acting on the particles is also important in the study of equilibrium position. These results suggest that the microstructure and viscosity characteristics of a suspension in a pipe could be handled by changes in velocity profiles.

Keywords: regularized lattice Boltzmann method; two-way coupling; virtual flux method; particle migration; microfluids; microchannel; Segré–Silberberg effects



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1. Introduction

Rheology is a discipline that deals with the study of deformation and flow in all materials in general. Hence, it is essential to understand the rheological properties of materials, as rheology is closely related to the final properties of materials. Suspension systems (solid particles (solute) dispersed in a liquid (solvent)) are an important aspect of rheology and have wide applications in industrial products, food, medicine, cosmetics, blood works, etc. [1]. Therefore, studying the rheological properties of suspensions is essential to improving the quality of industrial products and facilitating advanced technological innovations. The most demanding factors for suspension properties include concentration [2], shape [3,4], interaction [5], and the spatial arrangement of the particles dispersed in suspension [6].

Examining changes in microstructure, generally defined by the relative position of suspended particles, is important to understanding the rheological properties of suspensions [7,8]. Einstein's viscosity formula [9] can be used to estimate the macroscopic viscosity of a suspension (Equation (1)).

$$\eta_{\text{eff}} = \eta_0(1 + [\eta]\varphi), \quad (1)$$

where η_{eff} is the effective viscosity, η_0 is the viscosity of the solvent, $[\eta]$ is the intrinsic viscosity, and φ is the volume fraction of suspended particles. Intrinsic viscosity $[\eta] = 2$ for two dimensions and $[\eta] = 2.5$ for three dimensions [9,10]. This viscosity equation indicates that the volume fraction of particles in a suspension can be used to estimate the relative viscosity of the suspension. However, this formula has three assumptions: low concentration, negligibly small particles, and a steady and uniform flow. So, when the

concentration, microstructure, and particle size of the particles in suspension deviate from these assumptions, it becomes difficult to apply Einstein's viscosity formula.

The microstructure of a suspension depends on the inertial migration of suspended particles in a pipe flow. In inertial migration, the particles flowing in a pipe move in a direction perpendicular to the flow direction due to the effects of inertia and converge at a certain position. Segré and Silberberg [11,12] were the first to report this phenomenon, in which particles flowing down with inertia moved to a position about 0.6 times the tube radius. This change in the movement of particles owing to inertial effects is called Segré–Silberberg effects or tubular pinch effects. Many experimental [13–15], numerical [16–18], and theoretical [19] studies have examined this inertial migration. In particular, Matas et al. [13,14] studied inertial migration in the Poiseuille flow of particles. They suggested that the equilibrium position of the particles approaches the wall surface and the center of the channel with the increase in Reynolds numbers and confinement, respectively. Here confinement is defined as the ratio of particle diameter to channel width. Schonberg and Hinch [20] studied the inertial migration of a sufficiently small sphere in a Poiseuille flow. They suggested that the equilibrium position shifts toward the wall as the channel Reynolds number increases. Di Carlo et al. [21] examined the finite size effect in the inertial focusing in microfluids and evaluated the equilibrium position of particles using the lift force based on the particle's lateral position. However, these changes in microstructure and particle–fluid interaction due to the changes in Reynolds number and confinement fail to evaluate suspension rheology using Einstein's viscosity formula and assume the relative viscosity of the suspension in inertial pipe flow [22].

As the previous study of the relationship between macroscopic rheological property and dispersed particles suggested, Doyeux et al. [2] suggested that the total effective viscosity of suspensions can be expressed as a summation of each particle's contribution. Okamura et al. [23] suggested that macroscopic viscosity can be estimated from particle size and spatial arrangements at limited low concentrations based on their study on the effects of circular particle interactions on macroscopic viscosity in a two-dimensional channel using a two-way coupling. These studies have greatly contributed to understanding the importance of the spatial arrangement of particles and particle–fluid interactions in the evaluation of the flow properties exhibited by suspensions. In addition, the viscosity characteristics of the suspension in pipe flow may be affected by the changes in the particle equilibrium position due to the Segré–Silberberg effect [24]. Therefore, the particle equilibrium position, which is strongly related to the microstructure, and the clarification of the relationship and mechanism with suspension properties play an important role in the evaluation of suspension properties or methods to handle suspension properties in the pipe flow.

The particle behavior-related macroscopic viscosity of suspension was changed by various factors such as particle shapes, solvent properties, dispersed conditions, environmental conditions, etc. Liu [25] and Kawaguchi et al. [26] investigated numerically the particle behavior of elliptical-shape particles. Additionally, Kawaguchi et al. [26] investigated the effects of particle behavior under various aspect ratios and suggested that the particle equilibrium position was changed by the aspect ratio of particles. Other studies focus on particle shapes. There are many studies focusing on not only particle shapes (e.g., capsule [27–29]) but also particle characteristics (e.g., red blood cells [30–32] and vesicles [33]). In studies focusing on solvent properties, Hu et al. [34] and Chrit et al. [35] suggested that the properties of non-Newtonian fluids as a solvent affect the particle equilibrium position. In studies focusing on dispersed conditions [36–38], Chun et al. [38] investigated the effects of bidispersity suspension on particle migration and suggested that the small particles were depleted from the midplane region by the accumulation of large particles there. In studies focusing on thermal fluids, Liu and Wu [39] assumed the thermal fluids as an environmental condition of the channel and suggested that the particle equilibrium position was changed by thermal convection. In studies focusing on body forces, Zhang et al. [40] assumed the centrifugal force and Coriolis force for environmental conditions and suggested that the particles shifted to the bottom wall because of these body forces.

However, most studies on these spatial arrangements, particle behavior, and particle-fluid interactions have assumed neutral flow fields or symmetric flow fields with respect to the channel center. Only a few studies have focused on the macroscopic flow field changes that dominantly determine the spatial arrangement and behavior of particles. A previous study examined the balance between the repulsive forces from the channel wall and the wall-directed forces on finite-sized particles due to velocity gradients so as to study the particle equilibrium position in a Poiseuille flow [41]. Therefore, studying the changes in the macroscopic flow field can help in understanding the microstructure of the particles flowing in the channel and the rheology of the suspension. In addition, two-dimensional analysis not only can reduce the computational costs but also can evaluate the factors that determine particle behavior in detail because it can consider factors without complex effects (the effects of three-dimensional rotation motion of particles, etc.). From among the several methods to reproduce asymmetric velocity fields, one simple method is to eccentricate the flow field using a curved channel. In this study, we investigate the effects of changes in inertia and particle size on single-particle behavior in an asymmetric velocity field using numerical analysis of pressure-driven flow in a two-dimensional curvilinear channel.

2. Methods

In this study, the regularized lattice Boltzmann method [42,43] with an incompressible formulation [44] is considered the fluid analysis method in a two-dimensional curved channel with pressure-driven flow. A two-way coupling scheme [45] was used to consider the particle-fluid interaction. A virtual flux method (VFM) [46–48] was employed to describe an arbitrarily shaped object, such as the particle. In addition, periodic boundary conditions [45] were applied to reduce computational costs.

2.1. Computational Models

The pressure-driven flow in a two-dimensional curvilinear channel was analyzed to investigate the behavior of particles in an eccentric velocity field. Figure 1 shows the computational model of a two-dimensional curvilinear channel with pressure-driven flow. If the channel width (characteristics length) is L_r , then the radius length of curvature R_c is set to $2.0L_r$. Confinement C is expressed as follows (Equation (2)).

$$C = \frac{d}{L_r}, \quad (2)$$

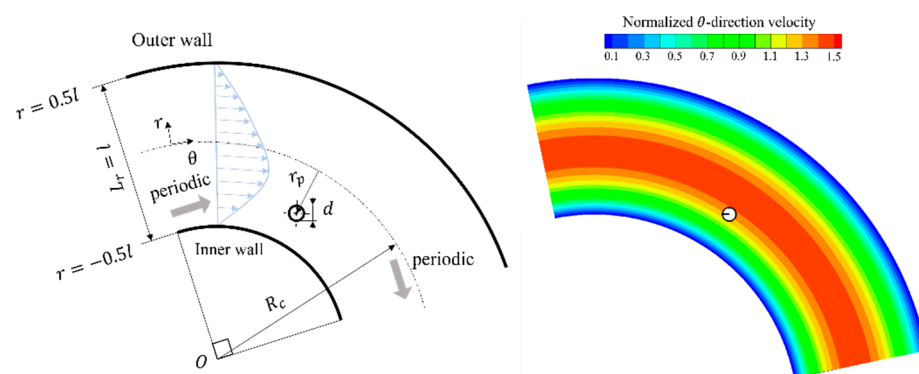


Figure 1. Schematic view of curvilinear channel.

The channel width L_r was set to 200 μm , the diameter of the particle d was set to 20–50 μm , and the Reynolds number Re was set to 16–64. Importantly, this analysis does not consider the effects of secondary flows caused by curvilinear flows because of two-dimensional geometry or collisions between particles because of single-particle flow. In addition, we employed the particle's center of gravity as the evaluation of particle position in the channel.

2.2. Governing Equations

For the equation of the flow, we used the regularized lattice Boltzmann method [42,43] (2D9V model), which is a modified version of the lattice Boltzmann method. The original method was modified to reduce its memory usage and increase its computing stability.

The advection and collision of virtual particles were considered to express the distribution function f_α due to the discrete velocity vector e_α of the lattice Boltzmann equation. When expanded up to the second-order moment, the distribution function can be evaluated as incompressible Navier-Stokes equations and is expressed as Equation (7). In this study, moments were used up to second order to obtain sufficient accuracy based on gas molecular kinetics.

$$f_\alpha \approx w_\alpha (a_0 + b_i e_{\alpha i} + c_{ij} e_{\alpha i} e_{\alpha j}), \quad (3)$$

where the parameters a_0 , b_i , and c_{ij} are moments in the distribution function f_α and satisfy the following relationships:

$$\sum_\alpha f_\alpha = \rho, \quad (4)$$

$$\sum_\alpha e_{\alpha i} f_\alpha = \rho u_i, \quad (5)$$

$$\sum_\alpha e_{\alpha i} e_{\alpha j} f_\alpha = \frac{c^2}{3} \rho \delta_{ij} + \rho u_i u_j + \Pi_{ij}^{\text{neq}}, \quad (6)$$

where ρ denotes fluid density, ρu_i denotes momentum, and Π_{ij}^{neq} denotes the nonequilibrium component of the stress tensor. As shown in Equation (8), the Taylor expansion of Maxwell's equilibrium distribution function to the second order series can be used to obtain the nonequilibrium distribution function f_α^{neq} [49,50].

$$f_\alpha^{\text{eq}} = w_\alpha \rho \left[1 + \frac{3(e_{\alpha i} u_i)}{c^2} + \frac{9(e_{\alpha i} u_i)^2}{2c^4} - \frac{3(u_i u_i)}{2c^2} \right], \quad (7)$$

$$f_\alpha^{\text{neq}} = \frac{9w_\alpha}{2c^2} \left(\frac{e_{\alpha i} e_{\alpha j}}{c^2} - \frac{1}{3} \delta_{ij} \right) \Pi_{ij}^{\text{neq}}, \quad (8)$$

Therefore, the time evolution equation for the lattice Boltzmann equation is given by

$$f_\alpha(t + \Delta t, \mathbf{x} + \mathbf{e}_\alpha \Delta t) = f_\alpha^{\text{eq}}(t, \mathbf{x}) + \left(1 - \frac{1}{\tau} \right) f_\alpha^{\text{neq}}, \quad (9)$$

where τ denotes the relaxation time, which was set to 0.80 for the simulations. The relaxation time τ is defined by the Chapman-Enskog expansion so as to be consistent with the Navier-Stokes equation, as follows:

$$\tau = \frac{3\nu}{c\delta x} + \frac{1}{2}, \quad (10)$$

where ν denotes kinematic viscosity.

In this study, the pressure distribution function proposed by He et al. [44] was adopted in order to treat the working fluid as an incompressible fluid, and an incompressible formulation was applied. Then, the local pressure distribution function p_α was defined using the density distribution function f_α , as follows:

$$p_\alpha = c_s^2 f_\alpha, \quad (11)$$

where c_s denotes the speed of sound, with $c_s = c/\sqrt{3}$ in this model. Therefore, the pressure p , velocity components u_i , and the nonequilibrium component of the stress tensor Π_{ij}^{neq} , are expressed as follows:

$$\sum_\alpha p_\alpha = p, \quad (12)$$

$$\frac{1}{\rho_0 c_s^2} \sum_{\alpha} p_{\alpha} e_{\alpha i} = u_i, \quad (13)$$

$$\sum_{\alpha} e_{\alpha i} e_{\alpha j} p_{\alpha} - \frac{c^2}{3} \rho \delta_{ij} - \frac{c^2}{3} \rho u_i u_j = \Pi_{ij}^{\text{neq}}. \quad (14)$$

Therefore, the time evolution equation is given as:

$$p_{\alpha}(t + \Delta t, \mathbf{x} + \mathbf{e}_{\alpha} \Delta t) = p_{\alpha}^{\text{eq}}(t, \mathbf{x}) + \left(1 - \frac{1}{\tau}\right) p_{\alpha}^{\text{neq}}. \quad (15)$$

2.3. Governing Equations for Particle Movements

In this study, we have discussed the relationship between the behavior of suspended small particles in the fluid and macroscopic flow characteristics by considering the interaction between the particles and the fluid. Therefore, the particle-fluid interaction was considered to use the two-way coupling scheme [45] due to the translational and rotational motions of the particles. The motions of finite-size particles obeyed Newton's second law for translation and the angular equation of motion for rotation. In addition, the particle conditions included neutrally buoyant particles and non-Brownian motions.

$$\mathbf{F}_p = m_p \frac{d^2 \mathbf{x}_p}{dt^2}, \quad (16)$$

$$T_p = I \frac{d^2 \theta_p}{dt^2}, \quad (17)$$

where $\mathbf{F}_p$ denotes the external force vector, m_p denotes particle mass, $\mathbf{x}_p$ denotes the position vector, T_p denotes torque, I denotes the moment of inertia, and θ_p denotes the particle angle. The external force vector $\mathbf{F}_p = (F_{px}, F_{py})$ acting on the particles was calculated from the hydrodynamic forces around the particle, as expressed by the third-order Adams-Bashforth method of the tertiary system. Therefore, the velocity vector $\mathbf{u}_p = (u_{px}, u_{py})$ and component of velocity for r - θ system were expressed as follows:

$$\mathbf{u}_p^{n+1} = \mathbf{u}_p^n + \Delta t \frac{23\mathbf{F}_p^n - 16\mathbf{F}_p^{n-1} + 5\mathbf{F}_p^{n-2}}{12\rho_p}, \quad (18)$$

$$\begin{pmatrix} u_{pr} \\ u_{p\theta} \end{pmatrix} = \begin{pmatrix} e_{rx} & e_{ry} \\ e_{\theta x} & e_{\theta y} \end{pmatrix} \begin{pmatrix} u_{px} \\ u_{py} \end{pmatrix}, \quad (19)$$

where ρ_p denotes particle density, and u_{pr} and $u_{p\theta}$ denote the velocity component of radial direction position and theta direction position, respectively. e_{rx} and e_{ry} denote the components of the radial direction unit vector, while $e_{\theta x}$ and $e_{\theta y}$ denote the components of the theta direction unit vector. u_{pr} was set to 0 when fixing the radial direction of the particle movement to evaluate the lift force acting on the particles [51]. In addition, the lift coefficient is expressed as follows:

$$C_L = \frac{F_L}{\frac{1}{2} \rho_p U_{64}^2 L_r}, \quad (20)$$

where U_{64} denotes the characteristic velocity for $Re = 64$, and L_r denotes the characteristic length.

2.4. Virtual Flux Method

In this study, the VFM [46–48] was used to represent arbitrarily shaped objects of particles and curved channel walls. This method analyzes the flow field around an arbitrarily shaped object on a cartesian grid. VFM can calculate the flow field around a particle and the flow phenomena inside the particle simultaneously. In addition, VFM is associated with

ease of implementation and high computational efficiency [48]. This method can capture the pressure field around the object with high accuracy, although the curves of a virtual object are difficult to represent on a cartesian grid. A schematic diagram of the virtual flux method for a particle is shown in Figure 2.

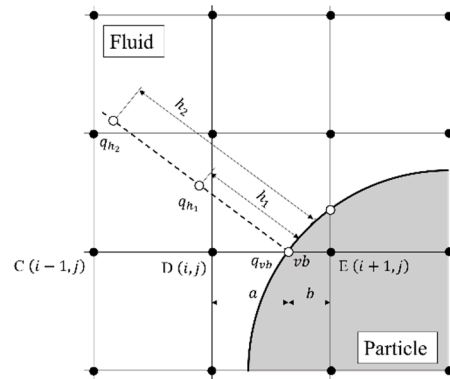


Figure 2. Schematic view of the VFM for a particle. a and b are internal division ratios across the virtual boundary. q_{vb} is the physical quantity on virtual boundary point.

In the 2D9V model of RLBM, a virtual boundary point must be placed at the intersection of the 8-directional discrete velocity and the particle surface when the 8-directional discrete velocity vector straddles the boundary of the object surface. In this analysis, the non-slip boundary condition was employed as the boundary condition of the body surface. Velocities on virtual boundary points are provided as ($\mathbf{u}_{vb} = \mathbf{u}_{wall}$) in order to satisfy the non-slip boundary condition on body surfaces, where $\mathbf{u}_{wall}$ is the velocity vector on the wall surface of the body surface. Pressures on virtual boundary point p_{vb} are calculated under the Nuemann boundary condition ($\partial p / \partial n|_{vb} = 0$), which is an approximate pressure condition on the body surface. In reference to Figure 2, the case where the distribution function at point E ($i+1, j$) moves to point D (i, j) across the virtual boundary is considered so as to determine the distribution function at point D (i, j) in the next time step. However, the distribution function at point E ($i+1, j$) cannot move to point D (i, j) because a virtual boundary separates the fluid from the particles. Therefore, we obtain the virtual distribution function at point E ($i+1, j$) from the distribution function at the virtual boundary point “vb”. The physical quantity q_{vb} on the virtual boundary point is used to obtain the equilibrium distribution function f_{α}^{eq} at the virtual boundary point vb (Equation (21)):

$$f_{\alpha}^{eq}(t, vb) = w_{\alpha} \rho \left[1 + \frac{3(\mathbf{e}_{\alpha} \cdot \mathbf{u}_{vb})}{c^2} + \frac{9(\mathbf{e}_{\alpha} \cdot \mathbf{u}_{vb})^2}{2c^4} - \frac{3\mathbf{u}_{vb}^2}{2c^2} \right]. \quad (21)$$

Therefore, the distribution function f_{α} on the virtual boundary point is expressed as follows:

$$f_{\alpha}(t, vb) = f_{\alpha}^{eq}(t, vb) + \left\{ f_{\alpha}(t, D) - f_{\alpha}^{eq}(t, D) \right\}. \quad (22)$$

Finally, the virtual distribution function $f_{\alpha}^{*}(t, E)$ and the equilibrium distribution function $f_{\alpha}^{eq*}(t, E)$ at point E ($i+1, j$) are calculated by linear extrapolation using the internal division ratios a and b .

$$f_{\alpha}^{*}(t, E) = \frac{a+b}{a} f_{\alpha}(t, vb) - \frac{b}{a} f_{\alpha}(t, D), \quad (23)$$

$$f_{\alpha}^{eq*}(t, E) = \frac{a+b}{a} f_{\alpha}^{eq}(t, vb) - \frac{b}{a} f_{\alpha}^{eq}(t, D). \quad (24)$$

Therefore, the fluid and particles shown in Figure 2 can be calculated independently.

The equilibrium distribution function at the object surface is expressed as shown below because this analysis adopts the pressure distribution function proposed by He and Luo [44]:

$$p_{\alpha}^{\text{eq}}(t, vb) = w_{\alpha} \left[p_{vb} + \rho \left\{ (\mathbf{e}_{\alpha} \cdot \mathbf{u}_{vb}) + \frac{3(\mathbf{e}_{\alpha} \cdot \mathbf{u}_{vb})^2}{2c^2} - \frac{1}{2} \mathbf{u}_{vb}^2 \right\} \right], \quad (25)$$

where p_{vb} denotes pressure on the virtual boundary point and is calculated by extrapolating from the pressures p_{h1} and p_{h2} at the points separated from the boundary point by distances $h1$ and $h2$ in the direction normal to the wall surface.

$$p_{vb} = \frac{h_2^2 P_{h1} - h_1^2 P_{h2}}{h_2^2 - h_1^2}, \quad (26)$$

where the pressures at points P_1 and P_2 are interpolated with weights from the surrounding four grid points, and $h1$ and $h2$ are set to $\sqrt{2}$ and $2\sqrt{2}$ times the grid width, respectively.

2.5. Periodic Boundary Condition

The particles need to flow a sufficient distance to reach the equilibrium position, when performing pressure-driven flow in two-dimensional curvilinear channels. Therefore, the periodic boundary condition [45] was applied to reduce the computational costs. In a flow field driven by pressure difference, the pressure difference Δp is caused by flowing pressure-driven over the channel length to be in kinetic balance with the hydrodynamic forces along the flow direction.

$$\Delta p = \frac{F_{fd}}{L_w}, \quad (27)$$

where F_{fd} denotes the flow direction component of the fluid force (θ -direction for a curvilinear channel), and L_w denotes the channel length. F_{fd} is determined by the viscous forces of flow direction acting on the channel walls and particle surfaces, as follows:

$$F_{fd} = \int_0^{L_{fd, \text{upper}}} |\tau_{w, \text{upper}}| dx + \int_0^{L_{fd, \text{lower}}} |\tau_{w, \text{lower}}| dx + \rho u^2 \frac{d}{2} \sum C_D, \quad (28)$$

where $\tau_{w, \text{upper}}$ and $\tau_{w, \text{lower}}$ denote the shear stress of the upper and lower walls of the channel (the outer and inner walls in a curvilinear channel, respectively), and C_D denotes the drag coefficient of each particle. The resulting pressure drop is considered to be in the flow direction in order to realize periodic boundary conditions. In this study, the pressure drop was calculated step by step by multiplying the pressure drop in the previous time step by a correction term using the flow rate ratio.

$$\Delta p^{n+1} = \Delta p^n \times \frac{Q_{th}}{Q_{ave}}, \quad (29)$$

where Δp^n denotes pressure drop caused by drag forces acting on the particle and wall shear stress, Q_{th} denotes the theoretical volume flow rate calculated using the characteristic velocity U and the channel width L_w , and Q_{ave} denotes the average volume flow rate obtained by numerical solution in the channel.

3. Results and Discussion

3.1. Velocity Profiles in Curvilinear Channel

First, the dependence of eccentricity and Reynolds number Re on the velocity distributions in a curvilinear channel was investigated. The characteristic velocity U was used to normalize the velocity profiles.

The velocity distribution in the channel approaches that for a straight channel with the increase in the radial length of curvature R_c (Figure 3a). Our findings showed that the normalized velocity distribution in the curvilinear channel was independent of Re (Figure 3b). In a two-dimensional straight parallel-plate flow, the mechanical equilibrium

position of a particle is generally considered to be three points: the channel center, where the shear rate $\gamma = 0$, and two points axisymmetric to the channel center. Therefore, the point of shear rate $\gamma = 0$ should be considered when examining the mechanical equilibrium position in a curvilinear channel. Table 1 shows the radial direction position where the velocity gradient $\gamma = 0$.

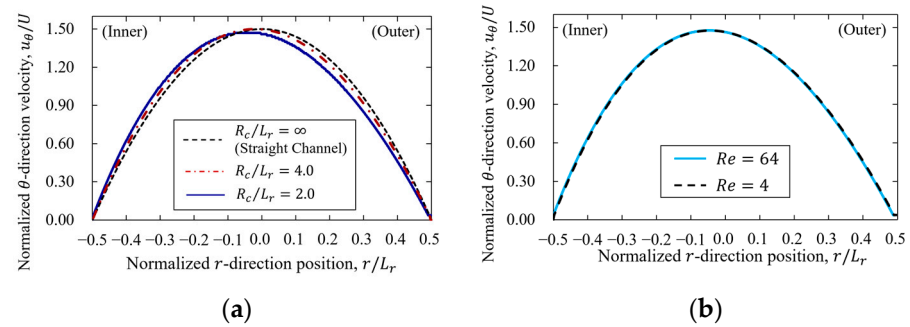


Figure 3. Velocity profiles of curvilinear channel with $l = 200$ cells. (a) Difference in velocity profiles between curvilinear channel and straight channel with $Re = 32$. (b) Reynolds number Re dependence using $R_c/L_r = 2.0$.

Table 1. Shear rate at each radial length of curvature and location where $\gamma = 0$.

Radial Length of Curvature R_c/L_r	Normalized r -Direction Position for $\gamma=0$
4.0	−0.020
2.0	−0.043

3.2. Validation

In this study, the dependence of the particle equilibrium position in a curvilinear channel on the radial length of curvature was investigated to physically validate the calculation results. As suggested in Section 3.1, the flow field in a curvilinear channel approached the straight channel with the increase in the radial length of curvature R_c . Thus, the equilibrium position of the particle in a curvilinear channel is also expected to approach that of the straight channel. Accordingly, we compared the final particle equilibrium position with increasing radial length of curvature to the equilibrium position in a straight channel and discussed the same in Section 3.3 and thereafter.

3.2.1. Equilibrium Position of Particle in Straight Channel

The analysis model of a straight channel is shown in Figure 4, and the equilibrium position of the particle and the results for others are shown in Table 2.

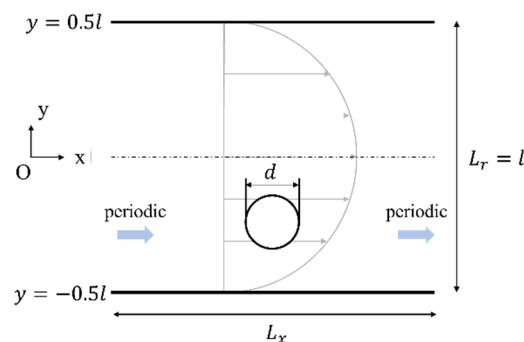


Figure 4. Computational model of straight channel.

Table 2. Equilibrium position $y_{p,eq}$ in straight channel with $L_r \times L_x = 200 \times 200$ cells for $Re = 12.73, 27.54, 54.91$ and $C = 0.25$.

Re	Present	Inamuro et al. [52]
12.73	0.274	0.2745
27.54	0.272	0.2733
54.91	0.271	0.2723

Here, L_x denotes the periodic length. The computational domain is $L_x \times L_r = 200 \times 200$. The confinement $C = 0.25$, and the Reynolds number $Re = 12.73, 27.54, 54.91$. Table 2 shows the equilibrium positions of the particle and the results for others with respect to the varying values of Re . Table 2 shows that the equilibrium positions in the straight channel are consistent with the results of Inamuro [52]. Thus, it enabled the evaluation of the physical validity of the analysis of the straight channel.

The appropriate periodic length needs to be considered when using periodic boundary conditions to evaluate the particle equilibrium position. The periodic length is required to be sufficient for analysis as a single particle because the periodic length in the periodic boundary condition is related to the particle-particle interaction. Therefore, we investigated the dependence of the equilibrium position on the periodic length. The relationships between the periodic length and particle-particle interaction are shown in Figure 5. In this case, the initial position of the particle in the straight channel, $y_{p,ini}/L_r$ is set to -0.30 in order to compare the results of the equilibrium position in the straight channel with that in the curvilinear channel. In addition, the periodic length, which is expressed as the ratio of channel width to particle diameter L_x/d , was varied, i.e., $L_x/d = 4, 8, 16, 32$.

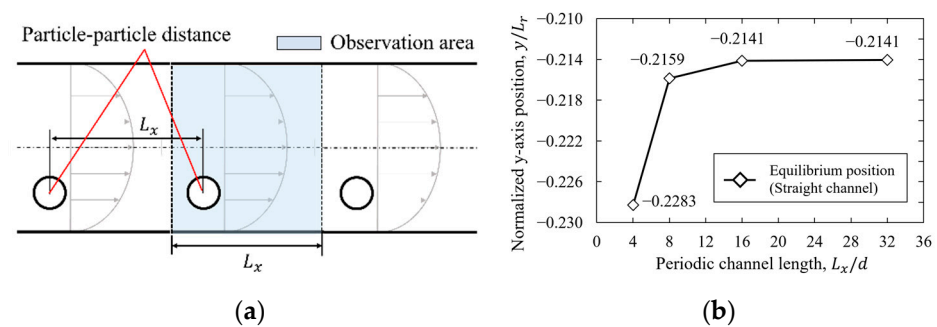


Figure 5. Examination of periodic length. (a) Schematic of relationship between particle-particle distance and periodic length. (b) Relationship between equilibrium position of particle and periodic length with $l = 200$ cells, $Re = 27.54$, and $C = 0.25$.

Figure 5b shows that the equilibrium position of the particle was constant after $L_x/d = 16$. Therefore, the periodic boundary condition requires a periodic length of about 16 times as long as the diameter d of the particle to consider it a single particle. In addition, the equilibrium position of particle $y_{p,eq}/L_r$ in the straight channel for $Re = 27.54$ and $C = 0.25$ was equal to -0.214 .

3.2.2. Radial Length of Curvature Dependence

Next, the relationship between the final equilibrium position of the particles and the radius length of curvature R_c was investigated based on the equilibrium position in the straight channel $y_{p,eq}$ obtained in Section 3.2.1. Then, the initial position of the particles was set below the channel center (inner wall side). Figure 6 shows the trends of the equilibrium positions of particles in the curvilinear channel with varying radial lengths of curvature.

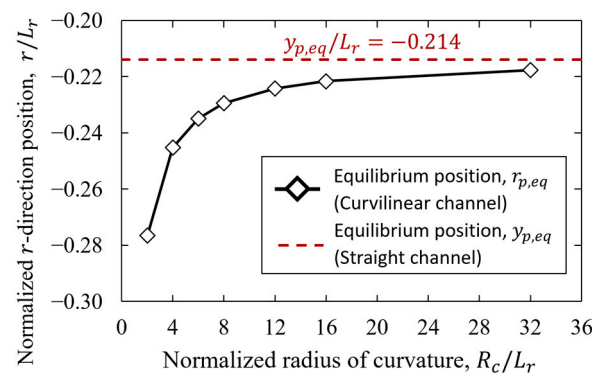


Figure 6. Segré–Silberberg effects at each radial length of curvature for $l = 200$ cells, $Re = 27.54$ and $C = 0.25$ and equilibrium position of particle in straight channel $y_{p,eq}$.

Table 3 shows the values of the equilibrium positions of particles in the curvilinear channel with varying radial lengths of curvature. As shown in Section 3.2.1, the equilibrium position in the curvilinear channel $r_{p,eq}$ approached the equilibrium position in the straight channel $y_{p,eq}$ when the radial length of curvature increased. In addition, the particle equilibrium positions $r_{p,eq}$ in the curvilinear channel and $y_{p,eq}$ in the straight channel were sufficiently small for $R_c/L_r = 32$. These results showed that the mechanical environments, such as velocity profiles and lift forces, acting on particles in the curvilinear channel approached those in the straight channel by increasing R_c/L_r .

Table 3. Equilibrium position at each radial length of curvature and periodic channel length.

R_c/L_r	$r_{p,eq}/L_r$	$(r_{p,eq} - y_{p,eq})/L_r$	Periodic Channel Length
2	−0.277	−0.063	12.57 <i>d</i>
4	−0.245	−0.031	25.13 <i>d</i>
6	−0.235	−0.021	37.70 <i>d</i>
8	−0.229	−0.015	50.27 <i>d</i>
12	−0.224	−0.010	75.40 <i>d</i>
16	−0.222	−0.008	100.53 <i>d</i>
32	−0.218	−0.004	201.06 <i>d</i>

3.3. Verification

In this study, we investigated the resolution dependence of the particle equilibrium positions for the numerical validity of the calculation results. The resolution was verified for $Re = 64$ and $C = 0.10$, which required the highest resolution among the parameters used in this study. The relationships between the equilibrium position of the particle and the resolution are shown in Figure 7, and the values of the final equilibrium position for each resolution are shown in Table 4.

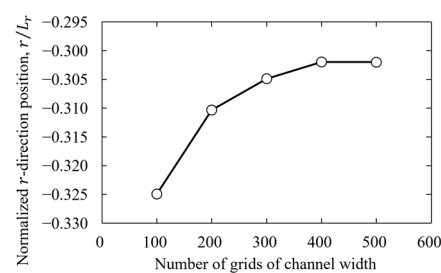


Figure 7. Relationships between the equilibrium position of the particle and the resolution at inner wall side for $Re = 64$ and $C = 0.10$.

Table 4. Value of equilibrium position of particle at each number of grids of channel width and particle diameter at inner wall side for $Re = 64$ and $C = 0.10$.

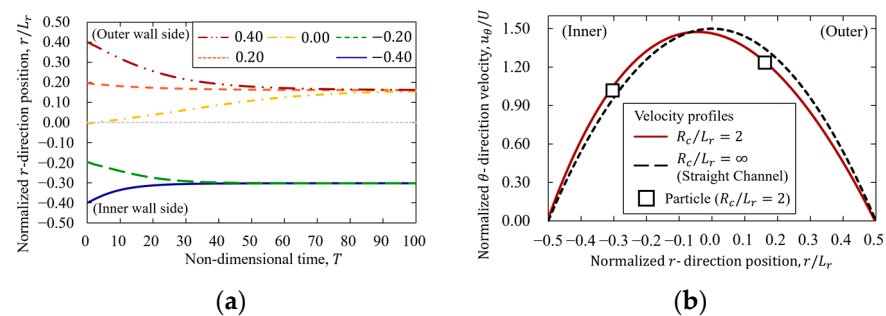
Channel Width l	Particle Diameter d	Equilibrium Position $r_{p,eq}/L_r$
100	10	−0.325
200	20	−0.310
300	30	−0.304
400	40	−0.302
500	50	−0.302

Table 4 shows that the equilibrium position with increasing resolution was constant after $l = 400$. In this study, as an appropriate resolution, we adopted $l = 400$ cells for the evaluation of the equilibrium position.

3.4. Segré–Silberberg Effects

3.4.1. Segré–Silberberg Effects at Different Initial Positions

Figure 8 shows the particle behavior flowing from different initial positions in a curvilinear channel, as well as the position and velocity of particles flowing at $T = 100$. In addition, the values of the initial position of the particle and the particle position at time $T = 100$ are shown in Table 5.

**Figure 8.** Segré–Silberberg effects at different initial positions for $l = 400$ cells, $Re = 32$ and $C = 0.10$. (a) Time history of particle behavior for 100 non-dimensional time. (b) Particle position at $T = 100$ in asymmetric velocity profiles, and theoretical velocity profiles of straight channel ($R_c/L_r = \infty$).**Table 5.** Value of initial particle position and particle position at $T = 100$.

Initial Position	Particle Position ($T=100$)
$r_{p,ini}/L_r$	$r_{p,eq}/L_r$
0.40	0.162
0.20	0.161
0.00	0.156
−0.20	−0.303
−0.40	−0.303

The equilibrium position of particles in axisymmetric flow profiles, such as flow between parallel plates, is symmetric with respect to the channel center, i.e., the equilibrium position is at a point of equal distance from the channel center [11,12]. However, the final equilibrium radial position of particles in asymmetric velocity profiles converged differently between the outer wall and inner wall sides with respect to the channel center (Figure 8 and Table 5). In addition, the particle was observed to move to the outer equilibrium position due to eccentricity when the channel center was taken as the initial position. These results suggest that the microstructural changes in a curvilinear channel are asymmetric with respect to the channel center. The particle-wall distance of the dispersing particles is suggested to play a key role in determining the macroscopic viscosity properties [26].

Therefore, an asymmetric velocity profile changes the equilibrium position, and consequently, an asymmetric microstructure with respect to the channel axis may change the macroscopic suspension viscosity.

3.4.2. Lift Force Acting on the Particle Which Reached Equilibrium Position

Figure 9 shows the lift force acting on a particle for its inner wall side equilibrium position and the centrifugal force estimated using particle mass, radial position, and orbital angular velocity.

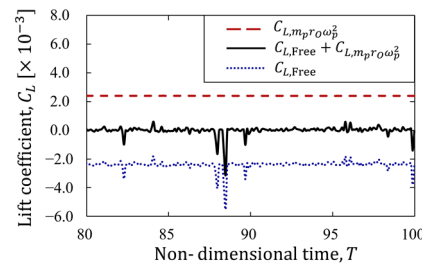


Figure 9. Lift force acting on the particle for its inner wall side equilibrium position for $l = 400$ cells, $Re = 32$ and $C = 0.10$ from 80 to 100 non-dimensional time.

The centrifugal force F_c approximated with a particle as the mass point is expressed as:

$$F_c = m_p r_O \omega_p^2, \quad (30)$$

where m_p denotes particle mass as a mass point, r_O denotes radial distance from center of revolution, and ω_p denotes orbital speed. In Figure 9, $C_{L,free}$ denotes the lift coefficient acting on a buoyant-free particle, and $C_{L, m_p r_O \omega_p^2}$ denotes the lift coefficient of centrifugal force calculated at equilibrium position using Equation (30). In general, the particle in its equilibrium position is expected to have C_L equal to 0 due to the force balance. However, a certain negative lift force acted on particles in the curvilinear channel. Therefore, we compared the value with the centrifugal force calculated from Equation (30) and found that it was equal to the lift force acting on the particles (Figure 9). This suggested that the particles in the curvilinear channel were subjected to a centripetal force equal to the centrifugal force.

3.4.3. Evaluation of Equilibrium Position by Lift Force Considering Centrifugal Force

The lift force acting on the particles at each radial position was calculated to quantitatively evaluate the mechanical equilibrium position of the particles. The distribution of the lift force acting on the particles at each radial position is shown in Figure 10.

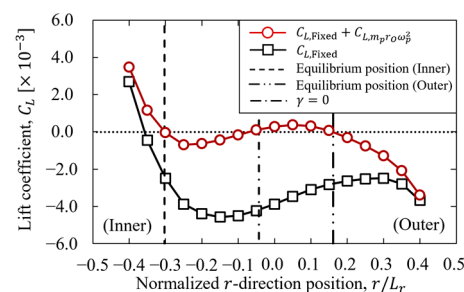


Figure 10. Lift coefficient acting on the particle for $l = 400$ cells, $Re = 32$ and $C = 0.10$.

In Figure 10, $C_{L,Fixed}$ denotes the lift coefficient acting on the particle obtained by fixing its radial direction movement at each radial direction position, while $C_{L, m_p r_O \omega_p^2}$ denotes the lift coefficient of centrifugal force calculated using Equation (30). As shown in the figure, there are three points where the mechanical equilibrium position of the particle

($C_L = 0$) is reached when correcting the centrifugal force, considering the centripetal force subjected to the particle. These three mechanical equilibrium positions matched the two equilibrium positions at the inner and outer walls (presented in Section 3.4.2) and the shear rate γ of 0 (presented in Section 3.1). These results suggest that the equilibrium position using lift forces can be quantitatively evaluated in terms of a mechanical equilibrium position by considering centrifugal forces. In addition, the centrifugal forces acting on the particle should be considered when evaluating the equilibrium position for particle flow in a curvilinear channel.

3.5. Particle Migration

As shown in the previous Section 3.4.1, particles flowing in the curvilinear channel move to asymmetric equilibrium positions on the inner and outer wall sides. This finding suggests that the microstructure of the suspension flowing through the curvilinear channel is asymmetric with respect to the channel center. In addition, the inertial forces and particle-fluid interactions in the curvilinear channel are asymmetric on the inner and outer wall sides in the case of asymmetric mechanical equilibrium positions. We investigated the changes in equilibrium position due to changes in particle inertia and particle size so as to consider the mechanical factors that determine the equilibrium position of particles. Particle behavior and final particle position for varying confinement C are shown in Figure 11 and Table 6, and those for varying Reynolds numbers Re are shown in Figure 12 and Table 7. The lift force acting on the particle at each radial position is presented in Figure 13.

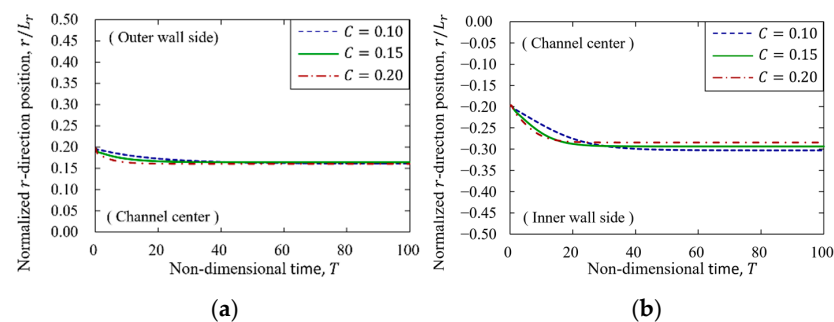


Figure 11. Particle migration for varying confinement C with $Re = 32$. (a) Particle migration on outer wall side. (b) Particle migration on inner wall side.

Table 6. Value of particle position at $T = 100$ for varying confinement $C = 0.10 \sim 0.20$ with $Re = 32$.

C	Normalized Particle Position ($T=100$)	
	Outer Wall Side	Inner Wall Side
0.10	0.161	−0.303
0.15	0.164	−0.293
0.20	0.160	−0.284

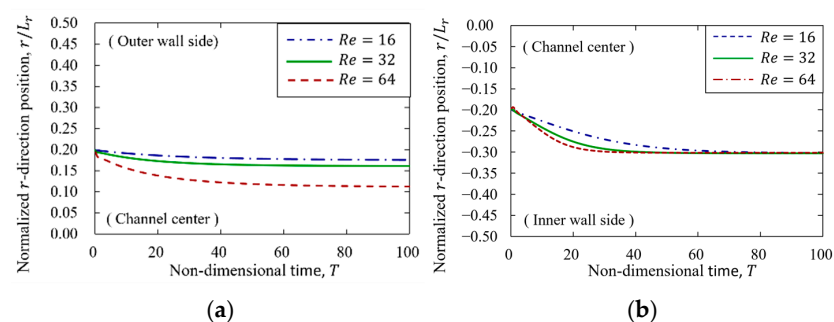
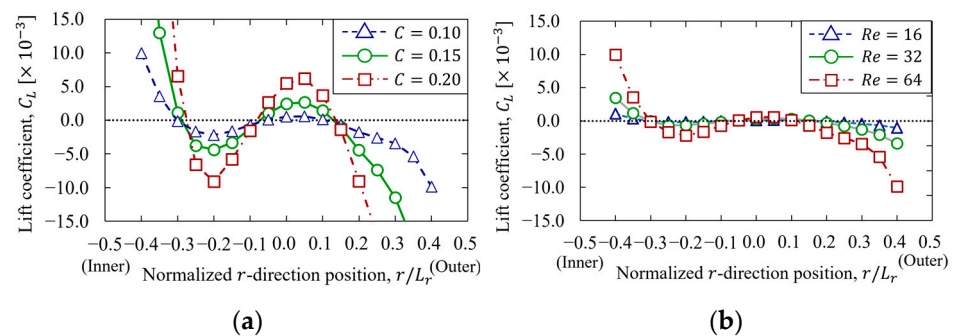


Figure 12. Particle migration for varying Reynolds number Re with $C = 0.10$. (a) Particle migration on outer wall side. (b) Particle migration on inner wall side.

Table 7. Value of particle position at $T = 100$ for varying Reynolds number $Re = 16 \sim 64$ with $C = 0.10$.

Re	Normalized Particle Position ($T=100$)	
	Outer Wall Side	Inner Wall Side
16	0.175	−0.302
32	0.161	−0.303
64	0.113	−0.302

**Figure 13.** Inertial lift force acting on the particle at each r -direction position. (a) confinement C dependence. (b) Reynolds number Re dependence.

In general, the equilibrium position of particles in a straight channel is discussed in terms of the balance of mechanical forces, mainly the lift force acting on finite-size particles due to the velocity gradient and the repulsive force between the particle and the wall. Previous studies have suggested that the particle equilibrium position moves to the center of the channel with the increase in the repulsive force from the wall surface due to the increase in particle size [52]. In addition, the particles move in the direction of the wall surface when the Reynolds number increases because of the increase in the lift force in the direction of the wall acting on finite-size particles caused by the increase in velocity gradient [13]. However, the inner and outer walls of a two-dimensional curvilinear channel were found to yield asymmetric equilibrium positions with respect to the channel center. These results suggested that there are other factors in addition to the velocity gradient and the repulsive force from the wall that determine the equilibrium position. This is because the equilibrium position on the outside of the channel does not move toward the channel center (Figure 11a).

Figure 11a shows the times history of particle migration on the outer wall for varying confinement C . Due to the increased repulsive force from the wall surface, the equilibrium positions were expected to shift toward the channel center. However, there was no significant change in particle equilibrium position with respect to the confinement effect. The equilibrium position did not change because the increase in wall repulsive force due to the increase in confinement was offset by the centrifugal force in the opposite direction. In Figure 11b, the equilibrium position was expected to move toward channel center with the increase in confinement. Then, the direction of the centrifugal force was also channeled centerward, as was the repulsive force from the channel wall. Therefore, the equilibrium position of particle shifted to the channel center. In Figure 12b, the particle was expected to move to the channel wall side (inner wall side) with the increase in Re . However, the particle was expected to move to channel center due to the centrifugal force because the direction of centrifugal force was channel center-ward. In this case, we considered that the equilibrium position had not changed significantly since the forces for wall surface direction caused by increasing inertial force and centrifugal force acting on the particle were offset.

In Figure 12a, it is difficult to consider the equilibrium position due to centrifugal forces. The changes in equilibrium position are expected to shift to the channel wall

direction (outer wall), given the assumption that the equilibrium position was determined predominantly by centrifugal force. In this simulation, the change in equilibrium position caused by increasing Re was also in the channel wall direction (outer wall). Therefore, the equilibrium position of particles was expected to shift in the channel wall direction. However, Figure 12a shows that the particle equilibrium position shifted to the center of the channel when Re increased, which was the opposite of the above expectation. This is because of the change in velocity gradient due to the eccentricity of the flow field.

Figure 13b shows the lift distribution acting on the particles at each radius direction position when centrifugal force is considered, using the same method as in Section 3.4.3. As shown in Figure 13, the change in lift force with increasing Re might be attributed to the repulsive force from the wall and the inertia force due to the velocity gradient, taking centrifugal forces into account. Near the center of the channel on the outer wall side, the lift force acting on the particles in the direction of the wall (outer wall) was independent of the increase in Re . This was expected because the velocity gradient on the outer wall was relatively gentle, which reduced the amount of increase in the wall direction force as the Reynolds number changed. In contrast, the lift force near the outer wall in the direction of the channel center increased with the increase in Re . We considered that the repulsive force from the channel became dominant since the repulsive force from the channel wall was increased due to the increase in Re . Therefore, the change in these two mechanical factors may change the mechanical balance between the velocity gradient and the repulsive force from the wall, causing the particles to shift to the channel center. This indicates the importance of the effect of changes in the velocity field in the determination of the equilibrium position of particles at the outer wall over that due to centrifugal forces.

4. Conclusions

In this study, particle migration in an asymmetric velocity field was investigated using a two-dimensional pressure-driven flow in a curvilinear channel. The particles took asymmetric equilibrium positions with respect to the channel center. These results suggested that an asymmetric velocity profile changes the equilibrium position of the particles, and consequently, an asymmetric microstructure may change the macroscopic suspension viscosity.

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