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A Numerical Analysis on the Unsteady Flow of a Thermomagnetic Reactive Maxwell Nanofluid over a Stretching/Shrinking Sheet with Ohmic Dissipation and Brownian Motion

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Abstract: The major objective of this current investigation is to examine the unsteady flow of a thermomagnetic reactive Maxwell nanofluid flow over a stretching/shrinking sheet with Ohmic dissipation and Brownian motion. Suitable similarity transformations were used to reduce the governing non-linear partial differential equations of momentum, energy and species conservation into a set of coupled ordinary differential equations. The reduced similarity ordinary differential equations were solved numerically using the Spectral Quasi-Linearization Method. The influence of some pertinent physical parameters on the velocity, temperature and concentration distributions was studied and analysed graphically. Further investigations were made on the impact of the Eckert number, Prandtl number, Schmidt number, thermal radiation parameter, Brownian motion parameter, thermophoresis parameter and chemical reaction parameter on the skin friction coefficient, surface heat and mass transfer rates. The results were displayed in a tabular form. Obtained results reveal that the Maxwell parameter and the unsteadiness parameter reduce the Maxwell nanofluid velocity and the fluid temperature is increased with an increase in the Eckert number and thermal radiation parameter.

Keywords: maxwell nanofluid; stretching sheet; ohmic dissipation; brownian motion; spectral quasi-linearization; gauss–lobatto points



Citation: Shateyi, S.; Muzara, H. A Numerical Analysis on the Unsteady Flow of a Thermomagnetic Reactive Maxwell Nanofluid over a Stretching/Shrinking Sheet with Ohmic Dissipation and Brownian Motion. *Fluids* **2022**, *7*, 252. <https://doi.org/10.3390/fluids7080252>

Academic Editor: Silvia C. Hirata

Received: 27 May 2022

Accepted: 19 July 2022

Published: 22 July 2022

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1. Introduction

There is a growing demand for fluids with improved thermal conductivity and heat transfer properties in industrial applications which include micro-manufacturing, chemical and metallurgical, power generation, transportation, cooling, ventilation and air conditioning. The heat transfer rate achieved by commonly used cooling media such as water and air is not suitable for other industrial applications, hence the need to develop advanced heat transfer fluids. Some fluids with a high thermal conductivity can be engineered by suspending nano-sized particles in conventional heat transfer fluids such as engine oil, water, ethylene glycol, etc. The nano-sized particles (<100 nm in diameter) can be metals, carbides, oxides and carbon. The mixture of the nano-particles and the base fluid results in a new innovative fluid, coined a nanofluid by Choi and Eastman [1]. The most commonly used nanofluids are ZnO in ethylene glycol, TiO₂ in water, Al₂O₃ in water and CuO in water. Nanofluids have found applications in electronic cooling, lubrication, microelectronic fuel cells, oil recovery, drug delivery and magnetic cell separation, Jakati et al. [2].

A non-Newtonian fluid is a fluid which exhibits a non-linear relationship between the applied stress and shear rate. Typical examples of non-Newtonian fluids include biological stuff (synovial fluid, blood, syrups), chemical material (pharmaceutical chemicals,

paints, tooth pastes), food products (milk, ice creams, chocolates) [3]. In the mathematical modelling of non-Newtonian fluids, it is difficult to come up with a one constitutive relationship that captures all the properties of these fluids. As a result, researchers have suggested several non-Newtonian models. Amongst these models, is the rate-type Maxwell model [4], which represents the characteristics of the fluid relaxation times. A vast range of the applications of non-Newtonian nanofluids are found in technology and industrial applications such as in paints, asphalts, tars, melts of polymers, glues and biological solutions [5].

Crane [6] conducted the pioneering work on the study of flow over a sheet which is stretching. Studying the boundary layer flow over a sheet which is stretching has quite a number of engineering and industrial applications. In particular, the study is of significance in areas such as in paper production, wire drawing, extrusion processes, cooling of metallic plate in a bath, food processing and movement of biological fluids. Khan et al. [7] employed the homotopy analysis method to study heat and mass transfer of a Carreau fluid flowing past a stretching surface. Kudenatti and Misbah [8] reported on a unified computational approach of a hydrodynamic power-law fluid past a stretching sheet. A study on the effect of thermal radiation on a three dimensional MHD boundary layer flow of a Jeffrey nanofluid over a non-linearly stretching sheet was performed by Gireesha et al. [9]. Shahzad et al. [10] used the Keller box method to study viscous Jeffrey nanofluid flow past a stretching sheet embedded in a porous medium. Jayachandra and Sandeep [11] used the Runge–Kutta method to study cross-diffusion effects on MHD Williamson fluid past a stretching sheet. A model to consider boundary value problems in polymer flow due to a stretching sheet was studied by Baranovskii [12]. More studies on the non-Newtonian fluid flow past a stretching sheet have been performed by [13–16].

Quite a lot of research has been conducted on the Maxwell nanofluid flow past a stretching or shrinking sheet. Prasannakumara [17] used the Runge–Kutta–Fehlberg fourth-fifth order (RKF-45) method to solve ordinary differential equations modelling heat transport in a Maxwell nanofluid flowing a stretching sheet. Hussain et al. [18] used the RKF-45 algorithm to perform a numerical and statistical analysis of graphene Maxwell nanofluid flowing past a sheet which is stretched. The implicit finite difference method was used by Ibrahim and Negera [19] to investigate the slip effects and stagnation point flows of a Maxwell nanofluid flowing over a stretching sheet. Shafiq et al. [20] used Matlab's bvp4c to study the flow of Maxwell nanofluid over a sheet which is stretching exponentially. A semi-analytical technique homotopy analysis method was used to consider the unsteady thermal flow of the Maxwell power law nanofluid over a stretching sheet by Jawad et al. [21].

The system of partial differential equations modelling the flow of the Maxwell nanofluid flowing over a stretching sheet are solved using the Spectral Quasi-linearization Method (SQLM) [22]. It has been reported that this numerical technique needs a few seconds and iterations to provide accurate and converged results, respectively [23]. The SQLM has been used to analyse the flow behaviour of non-Newtonian fluid flow. Some of the studies include the works by Alharbey et al. [24], Magodora et al. [25], Mondal and Sibanda [26], Mondal and Bharti [27], Nayak et al. [28], Sahoo and Nandkeolyar [29]. The utilization of similarity transformations on the coupled system of non-linear partial differential equations governing the Maxwell nanofluid flow resulted in a system of non-linear ordinary differential equations which in turn were solved using SQLM. The influence of some pertinent parameters such as thermophoresis parameter, Eckert number, magnetic parameter, Prandtl number, thermal radiation parameter, Brownian motion parameter and the unsteadiness parameter on the velocity, temperature and concentration profiles were analysed. The results for the skin friction coefficient and the heat transfer rate obtained using SQLM were found to be in good agreement with those in the published literature. The reliability of the SQLM was also demonstrated by comparing the results obtained for the skin friction coefficient and the heat transfer rate with those from Matlab bvp4c [30]. An excellent agreement was established.

Based on research in the literature, it is to the best knowledge of the authors that no work has been conducted on the reactive thermomagnetic Maxwell nanofluid with Ohmic dissipation and Brownian motion. Significant merits of the recent Spectral Quasi-Linearization Method were noted in published work. This motivated the authors to use the numerical technique to analyse the current problem under study which is of utmost importance to mechanical and physical engineering applications.

2. Mathematical Formulation

In this study, the unsteady flow of a thermomagnetic reactive optically thick Maxwell nanofluid over a stretching/shrinking sheet with Ohmic dissipation and Brownian motion was analysed. The flow is in the upper region $y > 0$ and has a uniform magnetic field of strength $B = \frac{B_0}{\sqrt{1-\alpha t}}$ subjected to it, with an initial magnetic field strength denoted by B_0 . The magnetic field is applied perpendicular to the sheet. The unsteady fluid, heat and mass transfer flows are assumed to start at $t = 0$. The sheet emerges at point $(x, y) = (0, 0)$ moving with a non-uniform velocity $u_w(x, t) = \frac{ax}{1-\alpha t}$, where a and α are positive constants with inverse time dimension $(time)^{-1}$, and a is an initial stretching rate. The velocity of mass transfer is $v_w(x, t) = \frac{v_0}{\sqrt{1-\alpha t}}$, where v_0 is the constant mass flux velocity. Additionally, T_w and C_w are the surface temperature and concentration, respectively. Then T_∞ and C_∞ are their respective ambient values. In this problem, the induced magnetic field is considered negligible due to the magnetic Reynolds number in the flow which is assumed to be very small. Assumed to be negligible is the external electric field and the electric field which results from the polarization of charges. The flow diagram and coordinate system of the current problem is shown in Figure 1.

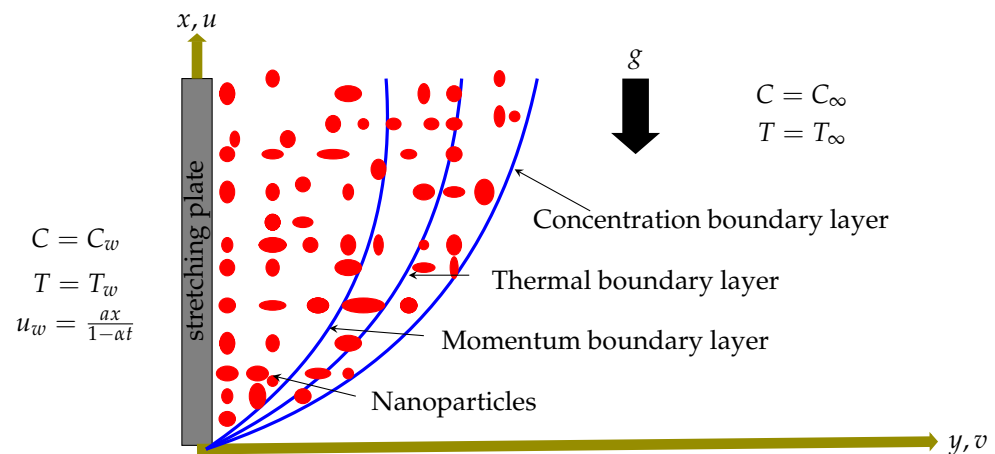


Figure 1. Schematic flow diagram and coordinate system.

Therefore, the governing equations of such a model flow, under the usual boundary layer approximations in the usual notations are (Madhu et al.): [31]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + \lambda \left(u^2 \frac{\partial^2 u}{\partial x^2} + v^2 \frac{\partial^2 u}{\partial y^2} + 2uv \frac{\partial^2 u}{\partial x \partial y} \right) = \nu_f \frac{\partial^2 u}{\partial y^2} + g\beta_f(T - T_\infty) + g\beta_f^*(C - C_\infty) - \frac{\sigma B^2}{\rho_f} u, \quad (2)$$

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k_f}{(\rho c)_f} \frac{\partial^2 T}{\partial y^2} - \frac{1}{(\rho c)_f} \frac{\partial q_r}{\partial y} + \frac{\mu_f}{(\rho c)_f} \left(\frac{\partial u}{\partial y} \right)^2 + \frac{\sigma B^2}{(\rho c)_f} u^2 + \tau \left\{ D_B \frac{\partial T}{\partial y} \frac{\partial C}{\partial y} + \frac{D_T}{T_\infty} \left(\frac{\partial T}{\partial y} \right)^2 \right\}, \quad (3)$$

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_B \frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty} \frac{\partial^2 T}{\partial y^2} - k_r(C - C_\infty), \quad (4)$$

where the velocities in the x and y directions are u and v respectively. $\lambda = \frac{\lambda_0}{1-\alpha t}$ is the relaxation parameter of the Maxwell fluid, g is the acceleration due to gravity, β_f is volumetric thermal expansion, β_f^* is volumetric solution expansion, T is the fluid temperature, C is the fluid concentration, D_B is the Brownian diffusion coefficient, c_f is the specific heat capacity of the fluid, k_r is the chemical reaction rate constant, μ_f is effective viscosity, q_r is the radiative heat flux, ρ_f is density of the nanoparticles, $\tau \left(= \frac{(\rho c)_p}{(\rho c)_f} \right)$ is the ratio of effective heat capacity of the nano-particles $(\rho c)_p$ to the heat capacity of the fluid $(\rho c)_f$. The formula for the radiative heat flux q_r , according to the Rosseland approximation has the form [32]:

$$q_r = -\frac{4\sigma^*}{3k^*} \frac{\partial T^4}{\partial y}. \quad (5)$$

Here, the mean absorption coefficient and the Stefan–Boltzmann constant are denoted by k^* and σ^* , respectively. When the temperature differences in the Maxwell fluid are assumed to be sufficiently small, expanding T^4 about T_∞ using linear Taylor's series expansion gives:

$$T^4 \simeq 4T_\infty^3 T - 3T_\infty^4.$$

From Equation (5),

$$\frac{\partial q_r}{\partial y} = -\frac{16\sigma^* T_\infty^3}{3k^*} \frac{\partial^2 T}{\partial y^2},$$

which reduces Equation (3) to the form:

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k_f}{(\rho c)_f} \frac{\partial^2 T}{\partial y^2} + \frac{16\sigma^* T_\infty^3}{3(\rho c)_f k^*} \frac{\partial^2 T}{\partial y^2} + \frac{\mu_f}{(\rho c)_f} \left(\frac{\partial u}{\partial y} \right)^2 + \frac{\sigma B_0^2}{(\rho c)_f} u^2 + \tau \left\{ D_B \frac{\partial T}{\partial y} \frac{\partial C}{\partial y} + \frac{D_T}{T_\infty} \left(\frac{\partial T}{\partial y} \right)^2 \right\}. \quad (6)$$

The appropriate boundary conditions for the model under consideration are given by:

$$u = u_w(x, t), \quad v = v_w(x, t), \quad T = T_w(x, t), \quad C = C_w(x, t), \quad D_B \frac{\partial C}{\partial y} + \frac{D_T}{T_\infty} \frac{\partial T}{\partial y} = 0 \quad \text{at } y = 0, \\ u \rightarrow 0, \quad C \rightarrow C_\infty, \quad T \rightarrow T_\infty, \quad \text{as } y \rightarrow \infty, \quad (7)$$

where $T_w(x, t) = T_\infty + aT_0(1 - \alpha t)^{-\frac{3}{2}}/(2\nu_f)$, $C_w(x, t) = C_\infty + aC_0(1 - \alpha t)^{-\frac{3}{2}}/(2\nu_f)$, with T_0 and C_0 being reference, temperature and concentration, respectively.

Similarity Transformations

The similarity transformations considered are [31]:

$$\eta = \sqrt{\frac{a}{\nu_f(1 - \alpha t)}} y, \quad \psi = \sqrt{\frac{a\nu_f}{1 - \alpha t}} x f(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}. \quad (8)$$

The definition of the stream function ψ is given by $u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$. Using the dimensionless quantities in (8), the momentum, energy and nanoparticle concentration Equations (3), (4) and (6) take the following dimensionless form, respectively.

$$A(f' + \frac{\eta}{2} f'') + f'^2 - f f'' + \beta(f^2 f''' - 2f f' f'') = f''' + \varepsilon(\theta + N^* \phi) - M f', \quad (9)$$

$$\frac{A}{2}(3\theta + \eta \theta') - f \theta' = \frac{1}{Pr} \left(1 + \frac{4}{3} R_d \right) \theta'' + Ec f'^2 + Ec M f'^2 - Nb \theta' \phi' - Nt \theta'^2, \quad (10)$$

$$\frac{A}{2}(\phi + \eta \phi') - f \phi' = \frac{1}{Sc} \phi'' + \frac{1}{Sc} \frac{Nt}{Nb} \theta'' - \gamma \phi, \quad (11)$$

subject to the boundary conditions

$$f'(0) = 1, \quad f(0) = S, \quad \theta(0) = 1, \quad Nb \phi'(0) + Nt \theta'(0) = 0, \quad f'(\infty) = 0, \quad \theta(\infty) = 0, \quad \phi(\infty) = 0, \quad (12)$$

where $A = \frac{\alpha}{a}$, $\beta = \lambda_0 a$, $\varepsilon = \frac{g\beta_f(T_w - T_\infty)x}{u_w^2}$, $N^* = \frac{\beta_f^*(C_w - C_\infty)}{\beta_f(T_w - T_\infty)}$, $M = \frac{\sigma B_0^2}{\rho_f a}$, $Pr = \frac{\nu_f}{\alpha_f}$, $Sc = \frac{\nu_f}{D_B}$, $Ec = \frac{u_w^2}{(c_p)_f(T_w - T_\infty)}$, $Nb = \frac{\tau D_B}{\nu_f}(C_w - C_\infty)$, $Nt = \frac{\tau D_T}{\nu_f T_\infty}(T_w - T_\infty)$, $R_d = \frac{4\sigma^* T_\infty^3}{\kappa k^*}$, $S = -\frac{v_0}{\sqrt{av_f}}$ and $\gamma = \frac{k_r(1-\alpha t)}{a}$.

The flow attributes of engineering importance are skin friction coefficient, the local Nusselt number and the local Sherwood number which are given by Daniel et al. [33] as:

$$C_f = \frac{\tau_w}{\rho u_w^2}, Nu_x = \frac{xq_w}{\kappa(T_w - T_\infty)}, Sh_x = \frac{xq_m}{D_B(C_w - C_\infty)},$$

where

$$\tau_w = \mu_f \left(\frac{\partial u}{\partial y} \right)_{y=0}, q_w = - \left(\left(\kappa + \frac{16\sigma^* T_\infty^3}{3k^*} \right) \frac{\partial T}{\partial y} \right)_{y=0}, q_m = D_B \left(\frac{\partial C}{\partial y} \right)_{y=0},$$

are the shear stress, the surface heat flux and mass flux, respectively. In non-dimensional form, the local skin friction coefficient, local Nusselt number and local Sherwood number are presented as:

$$Re_x^{\frac{1}{2}} C_f = f''(0), Re_x^{-\frac{1}{2}} Nu_x = -(1 + \frac{4}{3} R_d) \theta'(0), Re_x^{-\frac{1}{2}} Sh_x = -\phi'(0),$$

respectively, where $Re_x = \frac{u_w x}{\nu_f}$ is the local Reynold's number.

3. Numerical Solution Using the Spectral Quasi-Linearization Method

The system of Equations (9)–(11) subject to boundary conditions Equation (12) are solved using the Spectral Quasi-Linearization Method. In principle, the SQLM algorithm combines two main approaches namely: Quasi-linearization (QLM) and Chebyshev spectral collocation method. The QLM prescription which is used to solve non-linear boundary value problems was originally introduced by Bellman and Kalaba [34] as a generalization of the Newton–Raphson method. In deriving the QLM formula, it is assumed that the solutions of Equations (9)–(11) (and all their derivatives) at the current and previous iterations denoted by f_r, θ_r, ϕ_r and $f_{r+1}, \theta_{r+1}, \phi_{r+1}$ respectively, are sufficiently close enough so that multi-variable linear Taylor series yields the following iterative sequence of linear differential equations:

$$a_{0,r} f_{r+1}''' + a_{1,r} f_{r+1}'' + a_{2,r} f_{r+1}' + a_{3,r} f_{r+1} + a_{4,r} \theta_{r+1} + a_{5,r} \phi_{r+1} = R_{1,r}, \quad (13)$$

$$b_{0,r} \theta_{r+1}'' + b_{1,r} \theta_{r+1}' + b_{2,r} \theta_{r+1} + b_{3,r} f_{r+1}'' + b_{4,r} f_{r+1}' + b_{5,r} \phi_{r+1}' = R_{2,r}, \quad (14)$$

$$c_{0,r} \phi_{r+1}'' + c_{1,r} \phi_{r+1}' + c_{2,r} \phi_{r+1} + c_{3,r} f_{r+1} + c_{4,r} \theta_{r+1}' = R_{3,r}, \quad (15)$$

where the variable coefficients evaluated at the current iteration level are defined as:

$$a_{0,r} = 1 - \beta f_r^2, a_{1,r} = f_r + 2\beta f_r f_r' - A \frac{\eta}{2}, a_{2,r} = 2\beta f_r f_r'' - 2f_r' - A - M, a_{3,r} = f_r'' + 2\beta f_r' f_r'' - 2\beta f_r f_r''',$$

$$a_{4,r} = \varepsilon, a_{5,r} = \varepsilon N, b_{0,r} = 1 + \frac{4}{3} R, b_{1,r} = -Pr \left(Nb \phi_r' - 2Nt \theta_r' - \frac{\eta}{2} A \right), b_{2,r} = -\frac{3PrA}{2},$$

$$b_{3,r} = 2PrEc f_r'', b_{4,r} = 2PrEc M f_r', b_{5,r} = \theta_r', b_{6,r} = -PrNb \theta_r', c_{0,r} = 1, c_{1,r} = Sc f_r - \frac{ScA\eta}{2},$$

$$c_{3,r} = -\gamma Sc - \frac{ScA}{2}, c_{4,r} = Sc \phi_r', c_{5,r} = \frac{Nt}{Nb}, R_{1,r} = f_r f_r'' - (f_r')^2 + 4\beta f_r f_r' f_r'' - 2\beta (f_r')^2 f_r''',$$

$$R_{2,r} = PrEc((f_r'')^2 + M(f_r')^2 - Nb \theta_r' \phi_r' - Nt(\theta_r')^2 + f_r \theta_r'), R_{3,r} = Sc f_r \phi_r'.$$

Additionally, the boundary conditions given in Equation (12) at the current iteration level are:

$$\begin{aligned} f'_{r+1}(0) = 1, f_{r+1}(0) = S, \theta_{r+1}(0) = 1, Nb\phi'_{r+1}(0) + Nt\theta'_{r+1}(0) = 0, \\ f'_{r+1}(\infty) = 0, \theta_{r+1}(\infty) = 0, \phi_{r+1}(\infty) = 0. \end{aligned} \quad (16)$$

To apply the Chebyshev pseudo-spectral method, the physical domain $[0, \infty)$ on which Equations (9)–(11) are defined is moved to the computational domain $[-1, 1]$ using a linear transformation $\zeta = \frac{2}{L_\infty}\eta - 1$. The number L_∞ is a finite number chosen in such a way that the behaviour of flow properties far away from the sheet are represented. The derivatives of the unknown variables evaluated at Gauss–Lobatto collocation points $\zeta_j = \cos\left(\frac{\pi j}{N}\right), j = 0, 1, \dots, N$ are approximated by the $(N + 1) \times (N + 1)$ Chebyshev differentiation matrix $\mathbf{D} = \frac{2}{L_\infty}D$, Trefethen [35]. This far, the system of Equations (9)–(11) can written as a vector matrix:

$$\begin{bmatrix} \mathbf{a}_{0,r}\mathbf{D}^3 + \mathbf{a}_{1,r}\mathbf{D}^2 + \mathbf{a}_{2,r}\mathbf{D} + \mathbf{a}_{3,r} & \mathbf{a}_{4,r}\mathbf{I} & \mathbf{a}_{5,r}\mathbf{I} \\ \mathbf{b}_{3,r}\mathbf{D}^2 + \mathbf{b}_{4,r}\mathbf{D} & \mathbf{b}_{0,r}\mathbf{D}^2 + \mathbf{b}_{1,r}\mathbf{D} + \mathbf{b}_{2,r}\mathbf{D}^2 & \mathbf{b}_{5,r}\mathbf{D} \\ \mathbf{c}_{3,r} & \mathbf{c}_{4,r}\mathbf{D}^2 & \mathbf{c}_{0,r}\mathbf{D}^2 + \mathbf{c}_{1,r}\mathbf{D} + \mathbf{c}_{2,r}\mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{F}_{r+1} \\ \mathbf{\Theta}_{r+1} \\ \mathbf{\Phi}_{r+1} \end{bmatrix} = \begin{bmatrix} \mathbf{R}_{1,r} \\ \mathbf{R}_{2,r} \\ \mathbf{R}_{3,r} \end{bmatrix}, \quad (17)$$

subject to boundary conditions

$$\begin{aligned} \mathbf{F}'_{r+1}(\eta_N) = 1, \mathbf{F}_{r+1}(\eta_N) = S, \mathbf{\Theta}_{r+1}(\eta_n) = 1, Nb\mathbf{\Phi}'_{r+1}(\eta_N) + Nt\mathbf{\Theta}'_{r+1}(\eta_N) = 0, \\ \mathbf{F}'_{r+1}(\eta_0) = 0, \mathbf{\Theta}_{r+1}(\eta_0) = 0, \mathbf{\Phi}_{r+1}(\eta_0) = 0, \end{aligned} \quad (18)$$

where

$$\begin{aligned} \mathbf{F}_{r+1} &= [f_{r+1}(\eta_0), f_{r+1}(\eta_1), \dots, f_{r+1}(\eta_{N-1}), f_{r+1}(\eta_N)]^T, \\ \mathbf{\Theta}_{r+1} &= [\theta_{r+1}(\eta_0), \theta_{r+1}(\eta_1), \dots, \theta_{r+1}(\eta_{N-1}), \theta_{r+1}(\eta_N)]^T, \\ \mathbf{\Phi}_{r+1} &= [\phi_{r+1}(\eta_0), \phi_{r+1}(\eta_1), \dots, \phi_{r+1}(\eta_{N-1}), \phi_{r+1}(\eta_N)]^T, \\ \mathbf{a}_{q,r} &= \begin{pmatrix} a_{q,r}(\eta_0) & & & & \\ & a_{q,r}(\eta_1) & & & \\ & & \ddots & & \\ & & & a_{q,r}(\eta_{N-1}) & \\ & & & & a_{q,r}(\eta_N) \end{pmatrix}, q = 0, 1, \dots, 5. \end{aligned}$$

Diagonal matrices $\mathbf{b}_{q,r}, (q = 0, 1, \dots, 5)$ and $\mathbf{c}_{q,r}, (q = 0, 1, \dots, 4)$ have the same definition as the diagonal matrix $\mathbf{a}_{q,r}$ and $\mathbf{I}$ is an $(N + 1) \times (N + 1)$ identity matrix.

4. Results and Discussions

The system of equations in vector matrix form Equation (17) together with boundary conditions in Equation (18) is solved using the Spectral quasi-linearization method implemented on MATLAB R2020a. To ascertain the accuracy and the convergence of the numerical technique used, solution based error infinity norms were used. The default values of the parameters (unless otherwise stated) used to achieve the desired accuracy, convergence and for all the numerical simulations in this study are: $N = 60, L_\infty = 0.8, A = 0.4, \beta = 0.1, \epsilon = 0.1, M = 1.0, Pr = 0.71, R_d = 0.1, Ec = 0.1, Nb = 0.3, Nt = 0.3, Sc = 0.6$ and $\gamma = 1.0$. Figure 2 is plotted to reveal the relationship between the residual error infinity norms and increased iterations. The residual error approximates the true solution to about 10^{-14} after six iterations. This confirms that the SQLM algorithm is very accurate. Figure 3 portrays that the solution based errors defined as:

$$\text{Err}[f(\eta)] = \|\mathbf{F}_{r+1} - \mathbf{F}_r\|_\infty, \text{Err}[\Theta(\eta)] = \|\mathbf{\Theta}_{r+1} - \mathbf{\Theta}_r\|_\infty, \text{Err}[\Phi(\eta)] = \|\mathbf{\Phi}_{r+1} - \mathbf{\Phi}_r\|_\infty,$$

diminish as the number of iteration is increased. This proves that the method is convergent.

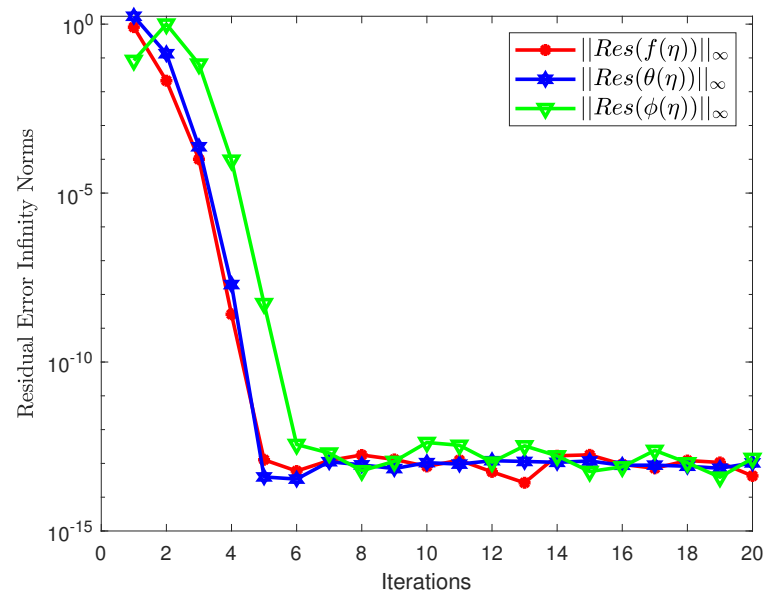


Figure 2. Accuracy graphs of f , θ and ϕ .

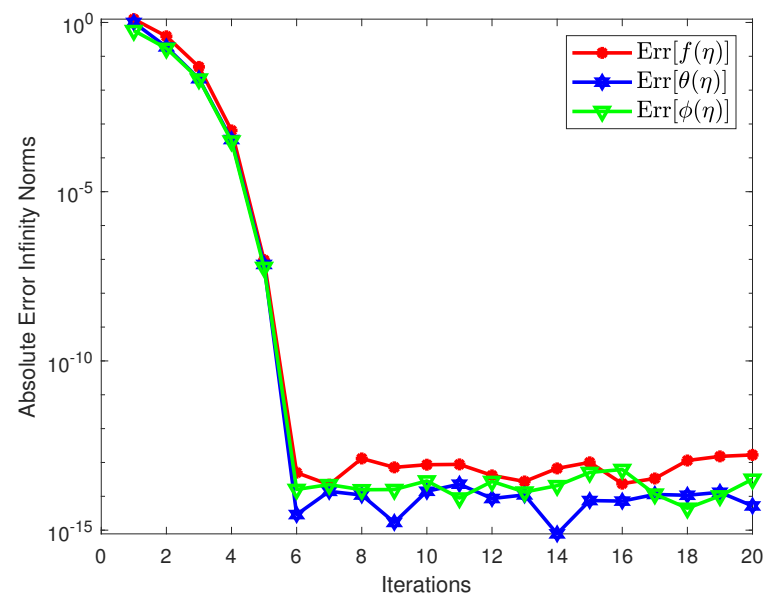


Figure 3. Convergence graphs of f , θ and ϕ .

To validate the SQLM results obtained in this study, Table 1 displays the values of the skin friction coefficient ($-f''(0)$) when $\beta = M = \epsilon = 0$ for different values of the unsteadiness parameter (A) compared against the results obtained earlier by Hussain et al. [18] and Elbashbeshy and Bazid [36]. There is a good agreement of the results obtained in the present study with those in the mentioned published literature. Table 2 depicts the numerical values of $-f''(0)$ and $-(1 + \frac{4}{3}R_d)\theta'(0)$ when $A = \epsilon = 0$ and varying the values of the magnetic parameter (M) and suction-injection parameter (S). The results obtained are in excellent agreement with those obtained by Ibrahim and Shankar [37] and Matlab bvp4c.

Table 1. The values of $-f''(0)$ compared with published results for selected values of A .

A	Elbashbeshy and Bzid [36]	Hussain et al. [18]	Present Study
0.8	1.3321	1.3333	1.331497109737629
1.2	1.4691	1.4684	1.469781692104270
2.0	1.7087	1.7090	1.713344535151333

Table 2. The values of $-f''(0)$ and $-(1 + \frac{4}{3}R_d)\theta'(0)$ compared with published results and Matlab bvp4c for selected values of M and S when $A = 0 = \beta$, $M = 1.0 = \gamma$, $Nt = Nb = Ec = R_d = 0.1$, $Sc = 0.6$ and $Pr = 0.71$.

M	S	$-f''(0)$			$-(1 + \frac{4}{3}R_d)\theta'(0)$	
		Ibrahim and Shankar [37]	Matlab Bvp4c	SQLM	Matlab Bvp4c	SQLM
0	0.5	1.2808	1.28077620	1.28077641	0.60136784	0.60136784
0.5		1.5000	1.50000000	1.50000000	0.55812915	0.55813015
1.0		1.6861	1.68614066	1.68614066	0.52446025	0.52446152
1.5		1.8508	1.85078106	1.85078106	0.49672490	0.49672670
2.0		2.0000	2.00000000	2.00000000	0.47305633	0.47305881
1.0	0	1.4142	1.41421356	1.41421356	0.29167318	0.29167374
	0.2	1.5177	1.51774469	1.51774469	0.38145187	0.38145259
	0.7	1.8069	1.80688023	1.80688023	0.62431224	0.62431389
	1.0	2.0000	2.00000000	2.00000000	0.77943236	0.77942860

The impact of the magnetic parameter (M) on the fluid velocity is portrayed in Figure 4. It is noted that an increase in the magnetic parameter significantly reduces the velocity of the nanofluid throughout the boundary layer. Physically, the application of a transverse magnetic field results in a Lorentz force that has a tendency to create a resistance to the fluid flow. Figures 5–7 reflect the Maxwell parameter (β), mixed convection parameter (ϵ) and the unsteadiness parameter (A) on the velocity profiles. It is apparent in Figure 5 that the velocity profiles of the Maxwell nanofluid decline as β is enhanced. Physically, at high values of β , the fluid demonstrates a solid-like behaviour hence a reduced velocity. It is observed in Figure 6 that the velocity distribution and the momentum boundary layer are enhanced as the values of ϵ are increased. Figure 7 shows that increasing the unsteadiness parameter reduces the velocity profiles and the momentum boundary layer thickness. This shows that the flow rate as a result of the stretching sheet is reduced as the unsteadiness parameter is increased.

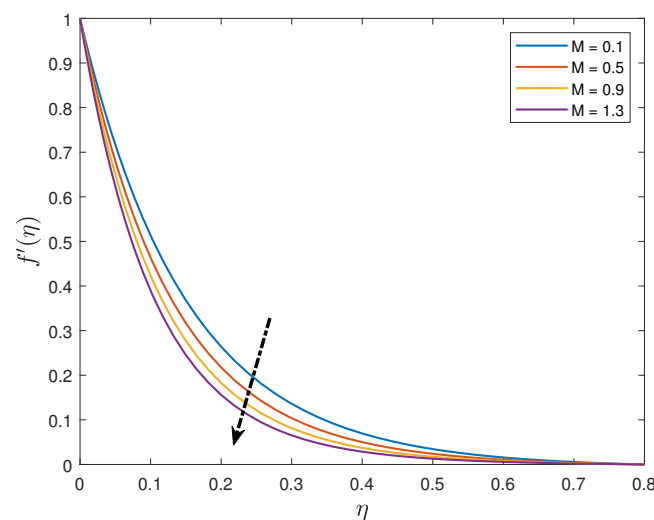


Figure 4. Influence of M on the nanofluid velocity.

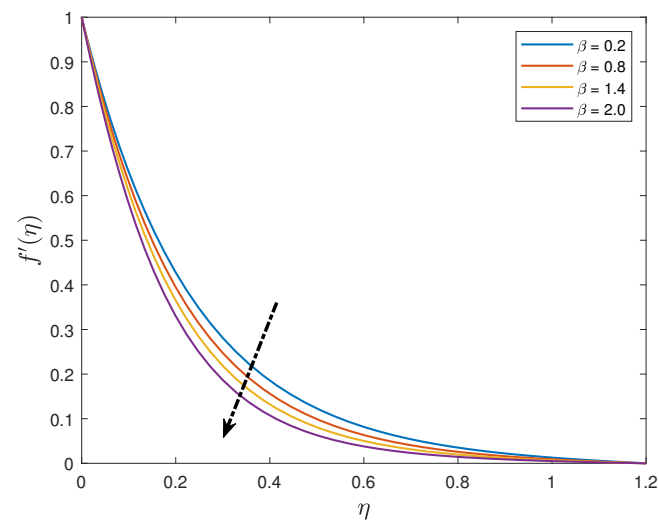


Figure 5. Influence of β on the nanofluid velocity.

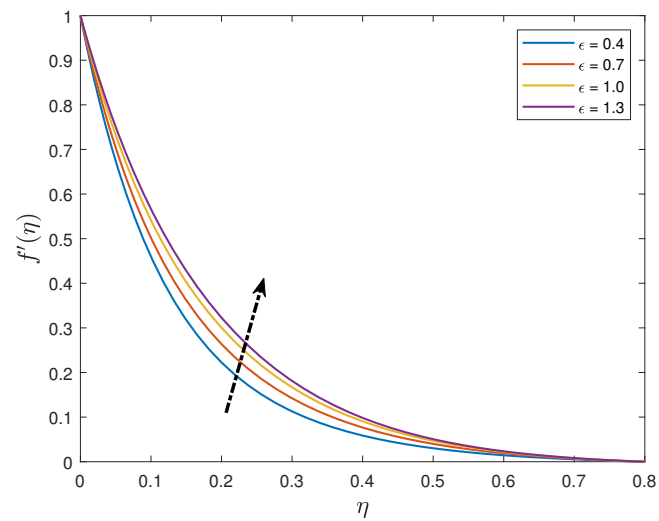


Figure 6. Influence of ϵ on the nanofluid velocity.

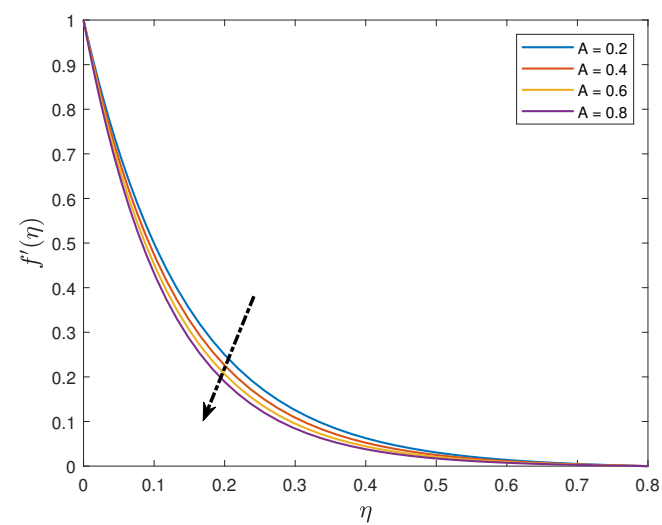


Figure 7. Influence of A on the nanofluid velocity.

Figures 8–11 display the impacts of Eckert number (Ec), thermal radiation parameter (R_d), Prandtl number (Pr) and the thermophoresis parameter (Nt) on the temperature distributions of the Maxwell nanofluid. Figure 8 reflects clearly that increasing the Ec values causes an upsurge in the temperature profiles. The Eckert number, which is used to characterise the impact of self-heating of the Maxwell nanofluid fluid due to dissipation effects, is a dimensionless number that represents the relationship between kinetic energy and enthalpy. The heat energy that is stored in the nanofluid due to frictional heating has the effect of enhancing the temperature of the fluid. It is clearly illustrated in Figure 9 that the fluid temperature is suppressed as the Prandtl number increases. Physically, increasing Pr entails a decrease in the thermal diffusivity that will also cause a decrease of the energy transfer ability. As a result, both the temperature and thermal boundary layer decrease. Figure 10 reveals that the fluid temperature is an increasing function of the thermal radiation parameter. For larger values of R_d , more heat is provided to the working fluid which in turn enhances the temperature and the thermal boundary layer thickness. The influence of the thermophoresis parameter is depicted in Figure 11. It is noted that increasing the values of Nt causes a decrease in the fluid temperature profiles.

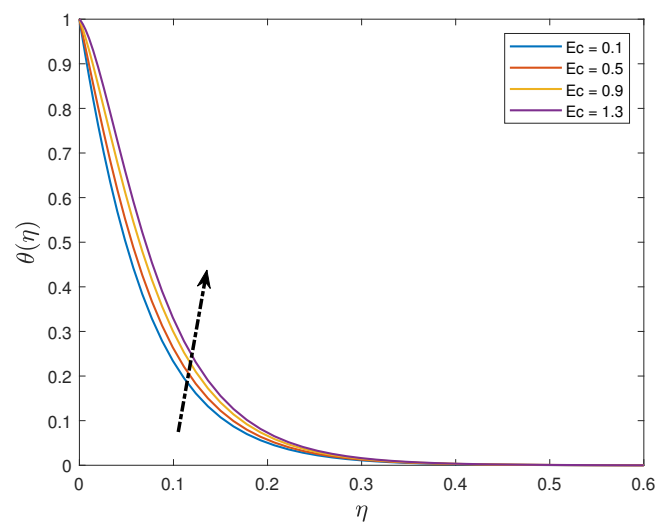


Figure 8. Influence of Ec on the nanofluid temperature.

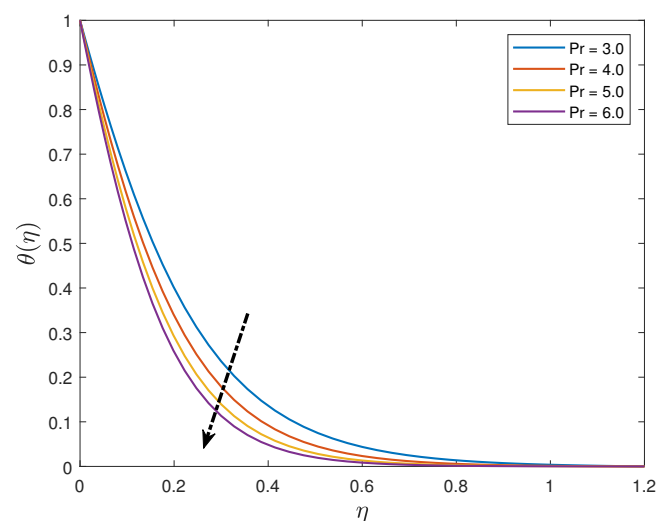


Figure 9. Influence of Pr on the nanofluid temperature.

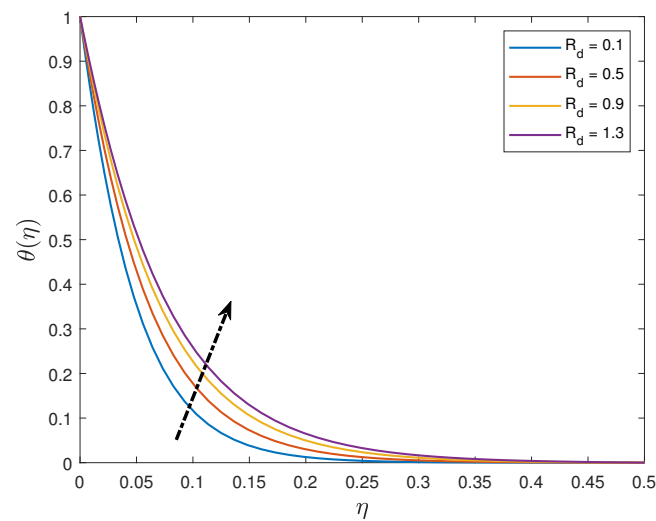


Figure 10. Influence of R_d on the nanofluid temperature.

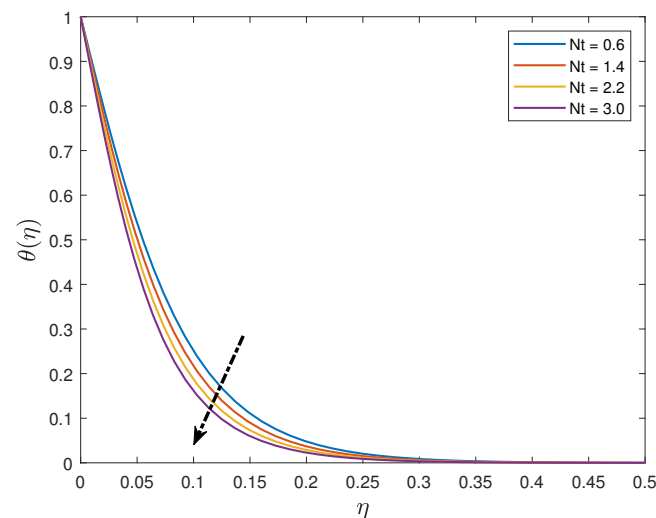


Figure 11. Influence of Nt on the nanofluid temperature.

Figures 12–15 present the effects of the chemical reaction parameter (γ), Brownian motion parameter (Nb), the Schmidt number (Sc) and the thermophoresis parameter (Nt) on the nano-particle volume fraction profiles. The influence of the chemical reaction parameter on the concentration distributions of the Maxwell nanofluid is reflected in Figure 12. It is noted that an increase in the destructive chemical reaction parameter decreases the concentration field. Physically, when the chemical reaction parameter is enhanced, more solute molecules will undergo chemical reaction and hence the concentration and solutal boundary layer thickness is reduced. Figure 13 reveals that increasing the Sc values suppresses the concentration profiles. In effect, Sc correlates the momentum transport diffusion in the hydrodynamic boundary layer and the species diffusion in the concentration boundary layer. Larger values of the Schmidt number are equivalent to decreasing chemical molecular diffusivity and vice versa. The concentration boundary layer thickness is reduced with increased values of Sc . Figures 14 and 15 highlight the significance of the Brownian motion parameter and the thermophoresis parameter on the concentration fields, respectively. The nano-particle concentration is a decreasing function of Nb whilst the opposite trend is observed for Nt . Due to temperature gradient, increasing the values of Nt enhances a thermophoresis force which in turn causes diffusion of the nano-particles in the ambient fluid and hence thickens the concentration boundary layer.

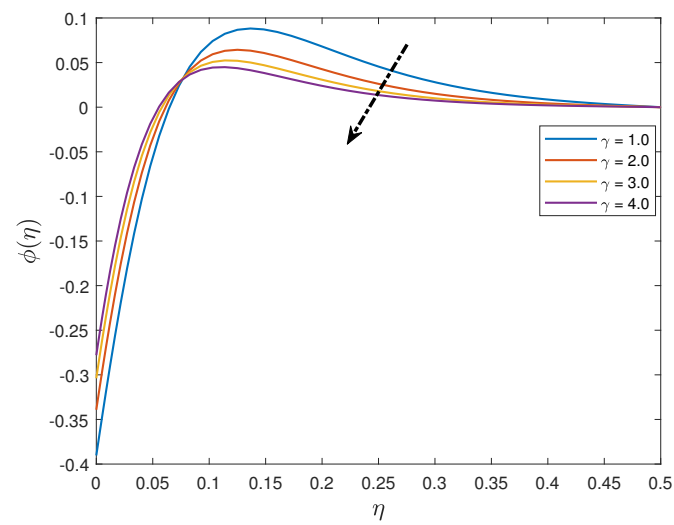


Figure 12. Influence of γ on the nanofluid concentration.

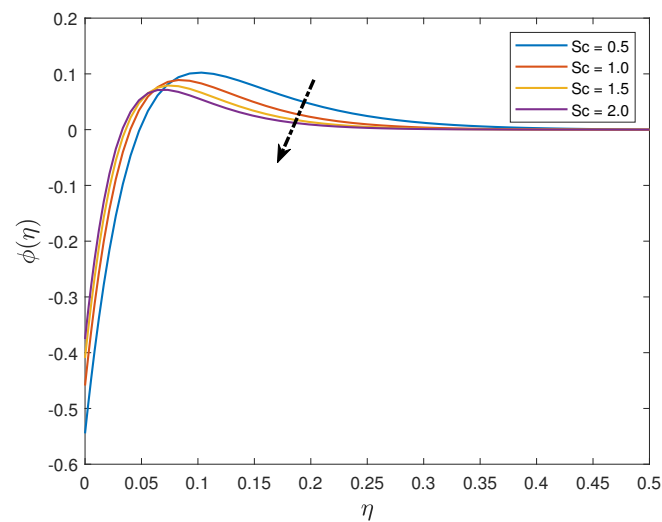


Figure 13. Influence of Sc on the nanofluid concentration.

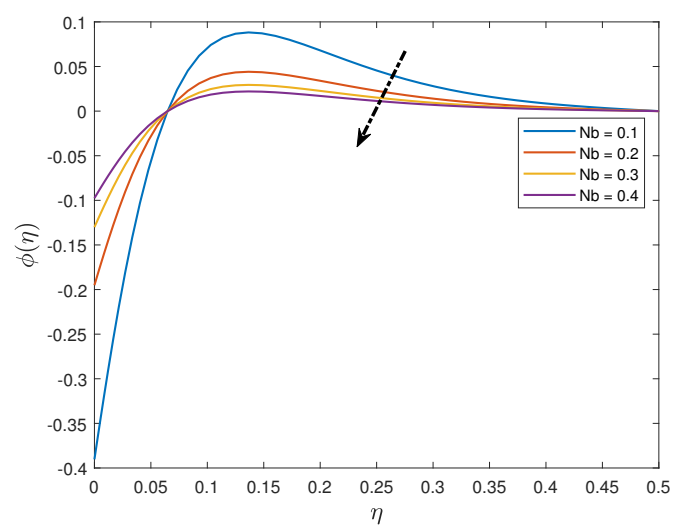


Figure 14. Influence of Nb on the nanofluid concentration.

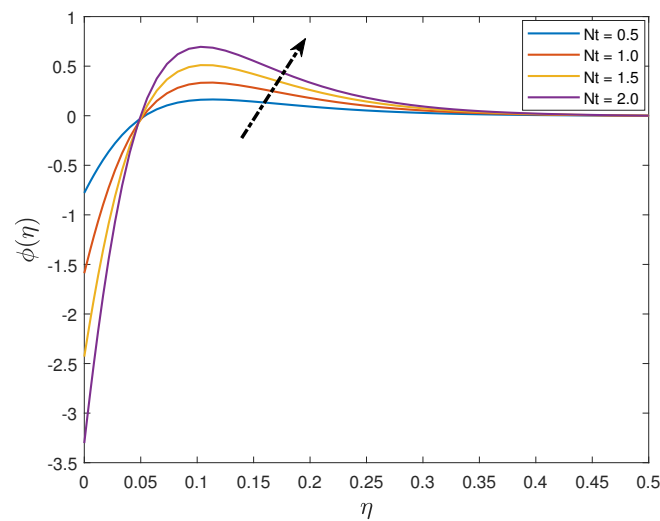


Figure 15. Influence of Nt on the nanofluid concentration.

Table 3 displays the values the skin friction coefficient ($-f''(0)$), the local Nusselt number ($-\theta'(0)$) and the local Sherwood number ($-\phi'(0)$) for different values of the Eckert number, Prandtl number, thermal radiation parameter, Brownian motion parameter, thermophoresis parameter, Schmidt number and the chemical reaction parameter. It is noted that the $-f''(0)$ is enhanced when the values of Pr and Nt are increased and is suppressed with increased value of Ec , R_d , Nb , Sc and γ . The local Nusselt number is an increasing function of Pr , Nt , Sc , and γ whilst it decreases when the values of Ec , R_d and Nb are increased. It is observed that there is an upsurge in the local Sherwood number with increasing Pr , Nt , Sc and γ whilst an opposite trend is realised when Ec , R_d and Nb are increased.

Table 3. Computation of the values of skin friction coefficient ($-f''(0)$), the local Nusselt number ($-\theta'(0)$) and the local Sherwood number $-\phi'(0)$ for different values of Ec , Pr , R_d , Nb , Nt , Sc , γ when $\epsilon = 0.1$, $\beta = 0.1$, $M = 0.1$, $A = 0.4$.

Ec	Pr	R_d	Nb	Nt	Sc	γ	$-f''(0)$	$-(1 + \frac{4}{3}R_d)\theta'(0)$	$-\phi'(0)$
0.1	0.71	0.1	0.3	0.3	0.6	1.0	1.516610	0.686537	-0.686537
0.3							1.515505	0.574942	-0.574942
0.5							1.514401	0.463533	-0.463533
	1.0						1.519931	0.819431	-0.819431
	2.0						1.526872	1.171820	-1.171820
	3.0						1.530777	1.439856	-1.439856
		1.0					1.510498	0.479179	-0.479179
		1.5					1.508763	0.426789	-0.426789
		2.0					1.507540	0.391166	-0.391166
			0.5				1.516226	0.686539	-0.411924
			0.7				1.516061	0.686540	-0.294231
			0.9				1.515969	0.686540	-0.228846
				0.5			1.517497	0.695090	-1.158483
				0.7			1.518385	0.703747	-1.642076
				0.9			1.519272	0.712507	-2.137521
					0.8		1.516506	0.688506	-0.688506
					1.0		1.516438	0.690138	-0.690138
					1.2		1.516391	0.691529	-0.691528
						2.0	1.516512	0.690097	-0.690097
						3.0	1.516449	0.692653	-0.692653
						4.0	1.516405	0.694631	-0.694631

5. Conclusions

An investigation of the flow of a thermomagnetic Maxwell nanofluid flowing past a stretching or shrinking plate in the presence of Ohmic dissipation and Brownian motion was carried out. The governing non-linear partial differential equations were reduced to non-linear ordinary differential equations which were then solved using the Spectral Quasi-Linearization Method. Solution based errors showed that the numerical technique used is very accurate and converges after a few iterations. Results obtained in this study were found to be in agreement with those existing in the literature. The impact of some chosen important physical parameters on the velocity, temperature and concentration boundary layers of the Maxwell nanofluid were discussed in detail. Core points from the study are:

1. The velocity distribution increases with an enhancement of the mixed convection parameter whilst it reduces when the values of the magnetic parameter, unsteadiness parameter and the Maxwell parameter are increased;
2. Increasing the values of the Eckert number and thermal radiation parameter enhances the temperature profiles whilst the profiles are depressed when the Prandtl number and the thermophoresis parameter are increased;
3. The nanoparticle concentration distribution is an increasing function of the thermophoresis parameter and a decreasing function of thermal radiation parameter, Schmidt number and the Brownian motion parameter.

Author Contributions: Conceptualization, S.S.; Data curation, H.M.; Formal analysis, H.M.; Funding acquisition, S.S.; Investigation, H.M.; Methodology, H.M.; Project administration, S.S.; Supervision, S.S.; Validation, H.M.; Writing—original draft, H.M.; Writing—review & editing, S.S. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Not applicable.

Conflicts of Interest: The authors declare no conflict of interest.

Nomenclature

The following abbreviations are used in this manuscript:

a	stretching rate [s^{-1}]
A	unsteadiness parameter
B	uniform magnetic field [T]
B_0	initial magnetic strength
C	species concentration [$mol\ m^{-3}$]
C_f	local skin friction coefficient
c_p	specific heat capacity [$J\ kg^{-1}\ K^{-1}$]
C_w	wall species concentration [$mol\ m^{-3}$]
C_∞	free stream concentration [$mol\ m^{-3}$]
D_B	mass diffusivity [$m^2\ s^{-1}$]
Ec	Eckert number
f	dimensionless stream function
g	acceleration due to gravity [$m\ s^{-2}$]
k_r	chemical reaction rate
k^*	mean absorption rate

M	magnetic parameter
N^*	concentration-thermal buoyancy ratio
Nb	Brownian motion parameter
Nt	thermophoresis parameter
Nu_x	local Nusselt number
Pr	Prandtl number
q_m	surface mass flux [$\text{kg s}^{-1} \text{m}^{-2}$]
q_w	surface heat flux [W m^{-2}]
Re_x	local Reynolds number
Rd	thermal radiation parameter
S	suction/injection parameter
Sc	Schmidt number
Sh_x	local Sherwood number
t	time [s]
T	temperature of the Maxwell fluid [K]
T_w	wall temperature [K]
T_∞	ambient temperature [K]
u	horizontal velocity component [m s^{-1}]
u_w	sheet velocity component [m s^{-1}]
v	vertical velocity component [m s^{-1}]
x	streamline coordinate [m]
y	transverse coordinate [m]
<i>Greek Symbols</i>	
α	constant [s^{-1}]
α_f	thermal diffusivity [$\text{m}^2 \text{s}^{-1}$]
β	Maxwell parameter
β_f	thermal expansion coefficient
β_f^*	solutal expansion coefficient
λ	relaxation parameter
λ_0	retardation time
η	dimensionless radial coordinate
μ_f	dynamic viscosity [$\text{kg m}^{-1} \text{s}^{-1}$]
ν_f	kinematic viscosity [$\text{m}^2 \text{s}^{-1}$]
σ	electrical conductivity [S m^{-1}]
σ^*	Stefan-Boltzmann constant [$\text{W m}^{-2} \text{K}^{-4}$]
θ	dimensionless temperature
ε	mixed convection parameter
ϕ	dimensionless concentration
ρ	density [kg m^{-3}]
ψ	dimensionless stream function
τ_w	wall shear stress [$\text{kg m}^{-1} \text{s}^{-2}$]
<i>Subscripts</i>	
w	wall conditions
∞	conditions far away from the plate

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