

The following shows listed the equations and coefficients for hybrid formulas (existing formula + ANN model 2 and 3) used for the models of Chen [29], Schorle et al. [30], Barr and White [31], Models Sousa et al. [32], and Ofor and Alabi [33].

a. The hybrid model 2 for the formula of Chen [29]:

$$f_{H,Chen} = \left[-2\log \left[\frac{\varepsilon}{3.7065D} - \frac{5.0452}{Re} \log \left(\frac{1}{2.8257} \left(\frac{\varepsilon}{D} \right)^{1.1098} + \frac{5.8506}{Re^{0.8981}} \right) \right] \right]^{-2} \\ + 10^{-6} [E_1 \tanh(a_1 \zeta + b_1 \eta + c_1) + E_2 \tanh(a_2 \zeta + b_2 \eta + c_2) + E_3 \tanh(a_3 \zeta + b_3 \eta + c_3) \\ + E_4 \tanh(a_4 \zeta + b_4 \eta + c_4) + E_5 \tanh(a_5 \zeta + b_5 \eta + c_5) + E_6 \tanh(a_6 \zeta + b_6 \eta + c_6) \\ + E_7 \tanh(a_7 \zeta + b_7 \eta + c_7) + F] \quad (S.1)$$

b. The hybrid model 2 for the formula of Schorle et al. [30],

$$f_{H,S} = \left[-2\log \left[\frac{\varepsilon}{3.7D} - \frac{5.02}{Re} \log \left(\frac{\varepsilon}{3.7D} + \frac{14.5}{Re} \right) \right] \right]^{-2} \\ + 10^{-6} [E_1 \tanh(a_1 \zeta + b_1 \eta + c_1) + E_2 \tanh(a_2 \zeta + b_2 \eta + c_2) + E_3 \tanh(a_3 \zeta + b_3 \eta + c_3) \\ + E_4 \tanh(a_4 \zeta + b_4 \eta + c_4) + E_5 \tanh(a_5 \zeta + b_5 \eta + c_5) + E_6 \tanh(a_6 \zeta + b_6 \eta + c_6) \\ + E_7 \tanh(a_7 \zeta + b_7 \eta + c_7) + F] \quad (S.2)$$

b. The hybrid model 2 for the formula of Bar and White [31],

$$f_{H,BW} = \left[-2\log \left\{ \frac{\varepsilon}{3.7D} + \frac{4.518 \log \left(\frac{Re}{7} \right)}{Re \left[1 + \frac{Re^{0.02}}{29} \left(\frac{\varepsilon}{D} \right)^{0.7} \right]} \right\} \right]^{-2} \\ + 10^{-6} [E_1 \tanh(a_1 \zeta + b_1 \eta + c_1) + E_2 \tanh(a_2 \zeta + b_2 \eta + c_2) + E_3 \tanh(a_3 \zeta + b_3 \eta + c_3) \\ + E_4 \tanh(a_4 \zeta + b_4 \eta + c_4) + E_5 \tanh(a_5 \zeta + b_5 \eta + c_5) + E_6 \tanh(a_6 \zeta + b_6 \eta + c_6) \\ + E_7 \tanh(a_7 \zeta + b_7 \eta + c_7) + F] \quad (S.3)$$

d. The hybrid model 2 for the formula of Sousa et al., [32]

$$f_{H,So} = \left[-2\log \left[\frac{\varepsilon}{3.7D} - \frac{5.16}{Re} \left(\frac{\varepsilon}{3.7D} + \frac{5.09}{Re^{0.87}} \right) \right] \right]^{-2} \\ + 10^{-6} [E_1 \tanh(a_1 \zeta + b_1 \eta + c_1) + E_2 \tanh(a_2 \zeta + b_2 \eta + c_2) + E_3 \tanh(a_3 \zeta + b_3 \eta + c_3) \\ + E_4 \tanh(a_4 \zeta + b_4 \eta + c_4) + E_5 \tanh(a_5 \zeta + b_5 \eta + c_5) + E_6 \tanh(a_6 \zeta + b_6 \eta + c_6) \\ + E_7 \tanh(a_7 \zeta + b_7 \eta + c_7) + F] \quad (S.4)$$

e. The hybrid model 2 for the formula of Ofor and Alabi [33]

$$f_{H,of} = \left[-2\log \left\{ \frac{\varepsilon}{3.71D} - \frac{1.975}{Re} \left(\ln \left(\left(\frac{\varepsilon}{15.72R} \right)^{1.092} + \left(\frac{7.627}{Re + 395.9} \right) \right) \right) \right\} \right]^{-2} \\ + 10^{-6} [E_1 \tanh(a_1 \zeta + b_1 \eta + c_1) + E_2 \tanh(a_2 \zeta + b_2 \eta + c_2) + E_3 \tanh(a_3 \zeta + b_3 \eta + c_3) \\ + E_4 \tanh(a_4 \zeta + b_4 \eta + c_4) + E_5 \tanh(a_5 \zeta + b_5 \eta + c_5) + E_6 \tanh(a_6 \zeta + b_6 \eta + c_6) \\ + E_7 \tanh(a_7 \zeta + b_7 \eta + c_7) + F] \quad (S.5)$$

The Coefficients a , b , c , and E for the hybrid models Eq. (S.1) to (S.3)) are shown in Table S.1 and for hybrid models Eqs. (S4) and (S5) are shown in Table l S2.

Table S1. The coefficient of hybrid model 2 of Chen [29], Schorle et al [30], and Barr and White [31], represented by Equations (S1), (S2), and (S3), respectively.

Hybrid Model.	Coefficients				
	k	a _k	b _k	c _k	E _k & F
Chen, [9] + ANN (2-7-1), Eq. (S1)	1	-0.22895	-1.92941	2.48445	46.983
	2	-1.65093	-3.29912	-2.00805	-21.328
	3	2.68661	-3.75265	-5.86613	5.027
	4	2.76529	2.21235	1.29194	1281.312
	5	-3.02927	-2.62628	-1.54637	2264.769
	6	3.39868	3.12021	1.85289	976.187
	7	6.12918	0.20538	6.43163	-453.501
	8			F =	397.428
Schorle et al., [25] + ANN (2-7-1), Eq. (S2)	1	0.72198	-0.06450	1.50837	711.242
	2	-0.03236	-0.53817	1.76410	5639.952
	3	-0.00181	0.74607	-0.77848	844.718
	4	2.40883	1.81017	0.28952	233.901
	5	3.67332	-0.01913	5.66030	-32971.849
	6	-2.40733	-1.65760	-0.07438	206.935
	7	-3.20530	1.71559	-5.65000	-2062.784
	8			F =	25432.108
Barr and White [2] + ANN (2-7-1), Eq. (S3)	1	1.96283	0.28360	2.98361	-434789.0
	2	0.98364	-1.07863	1.79173	125578.4
	3	0.98645	-1.07247	1.75365	-116870.0
	4	-1.63008	0.58571	-2.63955	516012.9
	5	-1.68764	0.40038	-2.66833	-1257888.2
	6	-1.78750	0.13192	-2.75933	1450592.4
	7	1.89515	0.12706	2.88146	1119738.4
	8			F =	15057.6

Table S2. The coefficient of hybrid model 2 of Sousa et al. [32] and Offor and Alabi [33], represented by Equations (S4) and (S5), respectively.

Hybrid Model	Coefficients				
	k	a _k	b _k	c _k	E _k & F
Sousa et al., [28] + ANN (2-7-1), Eq. (S4)	1	1.09341	3.13224	-4.02045	149.020
	2	0.49380	2.31649	-3.61171	-841.302
	3	5.91829	-0.00693	6.53159	-529.318
	4	2.18236	1.20692	1.23765	307.234
	5	2.88144	-3.53466	6.70302	224.023
	6	2.50932	1.71597	1.49374	-517.202
	7	-2.94989	-2.25659	-1.83661	-233.899
	8			F =	-414.143
Offor and Alabi [19] +	1	2.12823	0.54601	2.54974	73.003
	2	4.24131	2.64972	-0.72078	-1.246
	3	-4.31415	-3.81400	-1.92845	-231.692

4	4.28357	3.83633	1.95012	-230.451
5	4.19568	0.58768	3.74202	-62.931
6	14.97284	-3.96590	18.31466	-86.335
7	8.77714	-0.08176	9.92507	929.391
8			$F =$	-852.616

a. The hybrid model 3 for the formula of Chen [24],

$$f_{H,Chen} = \left[-2 \log \left[\frac{\varepsilon}{3.71D} - \frac{5.0452}{Re} \log \left(\frac{1}{2.8257} \left(\frac{\varepsilon}{D} \right)^{1.1098} + \frac{5.8506}{Re^{0.8981}} \right) \right] \right]^{-2} \\ + 10^{-6} [E_1 \tanh(a_1 \zeta + b_1 \eta + c_1) + E_2 \tanh(a_2 \zeta + b_2 \eta + c_2) + E_3 \tanh(a_3 \zeta + b_3 \eta + c_3) \\ + E_4 \tanh(a_4 \zeta + b_4 \eta + c_4) + E_5 \tanh(a_5 \zeta + b_5 \eta + c_5) + E_6 \tanh(a_6 \zeta + b_6 \eta + c_6) \\ + E_7 \tanh(a_7 \zeta + b_7 \eta + c_7) + E_8 \tanh(a_8 \zeta + b_8 \eta + c_8) + E_9 \tanh(a_9 \zeta + b_9 \eta + c_9) \\ + E_{10} \tanh(a_{10} \zeta + b_{10} \eta + c_{10}) + F] \quad (S.6)$$

b. The hybrid model 3 for the formula of Schorle et al. [26],

$$f_{H,S} = \left[-2 \log \left[\frac{\varepsilon}{3.7D} - \frac{5.02}{Re} \log \left(\frac{\varepsilon}{3.7D} + \frac{14.5}{Re} \right) \right] \right]^{-2} \\ + 10^{-6} [E_1 \tanh(a_1 \zeta + b_1 \eta + c_1) + E_2 \tanh(a_2 \zeta + b_2 \eta + c_2) + E_3 \tanh(a_3 \zeta + b_3 \eta + c_3) \\ + E_4 \tanh(a_4 \zeta + b_4 \eta + c_4) + E_5 \tanh(a_5 \zeta + b_5 \eta + c_5) + E_6 \tanh(a_6 \zeta + b_6 \eta + c_6) \\ + E_7 \tanh(a_7 \zeta + b_7 \eta + c_7) + E_8 \tanh(a_8 \zeta + b_8 \eta + c_8) + E_9 \tanh(a_9 \zeta + b_9 \eta + c_9) \\ + E_{10} \tanh(a_{10} \zeta + b_{10} \eta + c_{10}) + F] \quad (S.7)$$

c. The hybrid model 3 for the formula of Bar and White [39],

$$f_{H,BW} = \left[-2 \log \left\{ \frac{\varepsilon}{3.7D} + \frac{4.518 \log \left(\frac{Re}{7} \right)}{Re \left[1 + \frac{Re^{0.02}}{29} \left(\frac{\varepsilon}{D} \right)^{0.7} \right]} \right\} \right]^{-2} \\ + 10^{-6} [E_1 \tanh(a_1 \zeta + b_1 \eta + c_1) + E_2 \tanh(a_2 \zeta + b_2 \eta + c_2) + E_3 \tanh(a_3 \zeta + b_3 \eta + c_3) \\ + E_4 \tanh(a_4 \zeta + b_4 \eta + c_4) + E_5 \tanh(a_5 \zeta + b_5 \eta + c_5) + E_6 \tanh(a_6 \zeta + b_6 \eta + c_6) \\ + E_7 \tanh(a_7 \zeta + b_7 \eta + c_7) + E_8 \tanh(a_8 \zeta + b_8 \eta + c_8) + E_9 \tanh(a_9 \zeta + b_9 \eta + c_9) \\ + E_{10} \tanh(a_{10} \zeta + b_{10} \eta + c_{10}) + F] \quad (S.8)$$

d. The hybrid model 3 for the formula of 4 Sousa et al. [40],

$$f_{H,So} = \left[-2 \log \left[\frac{\varepsilon}{3.7D} - \frac{5.16}{Re} \left(\frac{\varepsilon}{3.7D} + \frac{5.09}{Re^{0.87}} \right) \right] \right]^{-2} \\ + 10^{-6} [E_1 \tanh(a_1 \zeta + b_1 \eta + c_1) + E_2 \tanh(a_2 \zeta + b_2 \eta + c_2) + E_3 \tanh(a_3 \zeta + b_3 \eta + c_3) \\ + E_4 \tanh(a_4 \zeta + b_4 \eta + c_4) + E_5 \tanh(a_5 \zeta + b_5 \eta + c_5) + E_6 \tanh(a_6 \zeta + b_6 \eta + c_6) \\ + E_7 \tanh(a_7 \zeta + b_7 \eta + c_7) + E_8 \tanh(a_8 \zeta + b_8 \eta + c_8) + E_9 \tanh(a_9 \zeta + b_9 \eta + c_9) \\ + E_{10} \tanh(a_{10} \zeta + b_{10} \eta + c_{10}) + F] \quad (S.9)$$

e. The hybrid model 3 for the formula of Offor and Alabi [41]

$$f_{H,of} = \left[-2 \log \left\{ \frac{\varepsilon}{3.71D} - \frac{1.975}{Re} \left(\ln \left(\left(\frac{\varepsilon}{15.72R} \right)^{1.092} + \left(\frac{7.627}{Re + 395.9} \right) \right) \right) \right\} \right]^{-2} \\ + 10^{-6} [E_1 \tanh(a_1 \zeta + b_1 \eta + c_1) + E_2 \tanh(a_2 \zeta + b_2 \eta + c_2) + E_3 \tanh(a_3 \zeta + b_3 \eta + c_3) \\ + E_4 \tanh(a_4 \zeta + b_4 \eta + c_4) + E_5 \tanh(a_5 \zeta + b_5 \eta + c_5) + E_6 \tanh(a_6 \zeta + b_6 \eta + c_6) \\ + E_7 \tanh(a_7 \zeta + b_7 \eta + c_7) + E_8 \tanh(a_8 \zeta + b_8 \eta + c_8) + E_9 \tanh(a_9 \zeta + b_9 \eta + c_9) \\ + E_{10} \tanh(a_{10} \zeta + b_{10} \eta + c_{10}) + F] \quad (S.10)$$

The Coefficients a , b , c , E , and F for hybrid models Eq. (S.6) to (S.8) are shown in Table S.3 and for hybrid models Eqs. (S9) and (S10) are given in Table S4.

Table S3. The coefficient of hybrid model 3 of Chen [29], Schorle et al [30], and Barr and White [31], represented by Equations (S6), (S7), and (S8), respectively.

Hybrid Model	k	Coefficients			
		a_k	b_k	c_k	$E_k \& F$
Eq.2 of Chen, [9] + ANN (2-10-1)	1	1.58069	2.42043	0.78809	72.356
	2	0.16718	2.11576	-3.58089	-189.251
	3	2.60078	2.73782	0.30017	436.979
	4	2.95878	1.64840	1.75059	-303.888
	5	5.63389	0.29719	6.43170	-1231.992
	6	0.31999	5.27171	0.80008	-0.593
	7	2.41782	2.68432	0.41542	-512.617
	8	-8.15449	-1.14541	-8.97223	-212.087
	9	-2.86657	-1.46605	-1.59411	-322.293
	10	0.07378	-0.74006	3.03964	314.358
	11			$F =$	501.393
Eq.3 of Schorle et al. [25], + ANN (2-10-1)	1	-1.41636	-2.10024	-1.79082	28.533
	2	0.14778	-2.13326	2.30507	95.065
	3	3.11490	0.94558	1.60287	57.759
	4	1.09514	-3.21836	-0.46434	1.332
	5	-2.59735	1.55933	-5.37528	-5534.003
	6	4.18265	2.73811	1.26745	29.939
	7	-2.14519	0.34764	-3.81018	5445.459
	8	-5.10230	-4.99745	-1.31512	-16.979
	9	-5.31892	-0.11907	-7.24517	23475.301
	10	10.17892	0.39903	10.77632	383.005
	11			$F =$	22825.679
Eq.4 of Barr and White [2], ANN (2-10-1)	1	-2.89777	-2.40777	-1.21183	-1533.345
	2	-0.98338	1.19041	-1.86288	-5646.552
	3	3.79875	-1.64147	-1.19354	-8.470
	4	-3.36534	-2.77242	-1.37971	-2969.495
	5	-3.10932	-2.32403	-1.14439	-4893.810
	6	3.15546	2.48195	1.22979	-9379.429
	7	2.61564	-0.83774	3.59803	11898.594
	8	-1.07501	1.17273	-1.73326	4483.876
	9	-2.48780	1.21623	-3.50648	5901.480
	10	-3.37116	0.44887	-4.34176	6984.953
	11			$F =$	-180.736

Table S4. The coefficient of hybrid model 3 of Sousa et al., [32] and Offor and Alabi [33], represented by Equations (S9) and (S10), respectively.

Hybrid Model	k	Coefficients			
		a_k	b_k	c_k	$E_k \& F$
Eq.5 of Sousa et al., [28] + ANN (2-10-1)	1	0.00195	2.07449	-3.47762	-701.550
	2	0.54763	-1.66809	-0.42752	2.707
	3	-2.62691	-0.85109	-1.62278	79.879
	4	1.66562	-1.73416	3.77934	-490.130
	5	-4.51639	-4.03650	-0.52404	238.471
	6	-4.51031	-4.01228	-0.49318	-234.117
	7	-2.47356	-0.63194	-1.50239	-112.400
	8	4.64807	2.69375	5.30983	-2.483
	9	-1.92863	1.98379	-4.69849	-1284.642
	10	5.22836	-0.00411	6.26241	-1092.298
	11			$F =$	-434.577
Eq.2 of Offor and Alabi [19] ANN (2-10-1)	1	-1.98690	1.60710	2.23040	11.912
	2	4.74080	2.41210	-0.13435	-3.453
	3	3.98380	4.07240	1.47710	107.168
	4	-0.62084	1.52900	1.39420	-81.669
	5	-0.37651	1.52880	1.28730	73.016
	6	3.21360	0.18841	1.80680	10.154
	7	14.05300	-3.14920	16.92260	-117.138
	8	4.12450	0.54640	3.42460	-40.374
	9	-3.86810	-4.11380	-1.53690	105.705
	10	-9.67580	0.05652	-10.50220	-533.297
	11			$F =$	-387.171