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Supplementary Material: The Half-Range Moment Method in Harmonically Oscillating Rarefied Gas Flows

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Table S1: Eigenvalues r_j with non-zero coefficients for the oscillatory Stokes flow obtained by the HRMM $(N=1)$.

j	$\mathrm{Re}(r_j)$	$\mathrm{Im}(r_j)$
	$\theta=1$	
1	-2.0327176763	3.4627245906
2	-6.7178551881(-1)	8.8871560504(-1)
j	$\theta=5$	
1	-9.3928476024	3.5748867079
2	-2.2952945064	2.0043060836
j	$\theta=10$	
1	-1.8790793580(1)	3.5424141308
2	-3.2468692012	3.0161069804
j	$\theta=20$	
1	-3.7605182240(1)	3.5319397904
2	-4.5447646365	4.3778099098
j	$\theta=50$	
1	-9.4033696598(1)	3.5288409562
2	-7.1214381956	7.0152161209

Table S2: Velocity and shear stress coefficients g_j and h_j respectively for the oscillatory Stokes flow obtained by the HRMM $(N=1)$.

j	Velocity coefficients g_j		Shear stress coefficients h_j	
	$\mathrm{Re}(g_j)$	$\mathrm{Im}(g_j)$	$\mathrm{Re}(h_j)$	$\mathrm{Im}(h_j)$
	$\theta=1$			
1	3.8823945941(-1)	-3.4916175121(-2)	7.8982743278(-2)	-5.6448553268(-2)
2	1.5667068003(-1)	1.3360028859(-1)	1.8450112733(-1)	1.0864108820(-2)
j	$\theta=5$			
1	2.0071973223(-1)	-1.8474296343(-1)	-1.0075819587(-2)	-2.5204246440(-2)
2	5.1295808862(-1)	3.3399987499(-1)	1.9328307045(-1)	-5.4703069394(-2)
j	$\theta=10$			
1	1.1401666410(-1)	-1.2957347969(-1)	-5.5543149247(-3)	-7.1147792246(-3)
2	6.7326432380(-1)	2.6462534228(-1)	1.4714762132(-1)	-7.0668493791(-2)
j	$\theta=20$			
1	7.3867540696(-2)	-8.5986392747(-2)	-2.0836867848(-3)	-2.1599947700(-3)
2	7.7192081742(-1)	1.9810126095(-1)	1.0747450784(-1)	-6.6321993779(-2)
j	$\theta=50$			
1	4.5033148121(-2)	-5.0817241577(-2)	-5.2170846259(-4)	-4.9848273552(-4)

2	8.5596180251(-1)	1.3182219096(-1)	6.9485138795(-2)	-5.1746364506(-2)

Table S3: Eigenvalues r_j with non-zero coefficients for the oscillatory Stokes flow obtained by the HRMM $(N = 3)$.

j	$\mathrm{Re}(r_j)$	$\mathrm{Im}(r_j)$
	$\theta = 1$	
1	-6.0558957675	7.5233079739
2	-1.1395997523	1.7357647345
3	-6.8783301329(-1)	8.0294696729(-1)
4	-4.4057091064(-1)	4.4371050779(-1)
j	$\theta = 5$	
1	-3.0067131607(1)	7.5575969024
2	-4.9592479805	1.6251484910
3	-2.4493156129	2.0742661529
4	-2.2219799693	4.7517044974(-1)
j	$\theta = 10$	
1	-6.0115503286(1)	7.5583031753
2	-1.0133628645(1)	1.5838237646
3	-3.3301428396	3.0443066780
4	-4.4942628042	5.1247474879(-1)
j	$\theta = 20$	
1	-1.2022186604(2)	7.5584383530
2	-2.0370158105(2)	1.5754713163
3	-4.5921155712	4.3752040037
4	-9.0574019579	5.3557319551(-1)
j	$\theta = 50$	
1	-3.0054832165(2)	7.5584728232
2	-5.0996476848(1)	1.5732443916
3	-7.1445587313	7.0038536138
4	-2.2710410955(1)	5.4404923666(-1)

Table S4: Velocity and shear stress coefficients g_j and h_j respectively for the oscillatory Stokes flow obtained by the HRMM $(N=3)$.

j	Velocity coefficients g_j		Shear stress coefficients h_j	
	$\mathrm{Re}(g_j)$	$\mathrm{Im}(g_j)$	$\mathrm{Re}(h_j)$	$\mathrm{Im}(h_j)$
	$\theta=1$			
1	1.6215908981(-1)	-5.3745697825(-2)	9.5899616205(-3)	-1.4863342854(-2)
2	3.1580866577(-1)	4.8138469597(-2)	1.3986287576(-1)	-6.4092343140(-2)
3	6.4646227670(-2)	9.9706462557(-2)	1.0778742512(-1)	3.1841098084(-2)
4	2.2961561831(-3)	4.5848791391(-3)	7.7722120418(-3)	2.6158240155(-3)
j	$\theta=5$			
1	6.8773769170(-2)	-5.9876172308(-2)	-1.3323006423(-3)	-2.6222242084(-3)
2	5.1853021481(-2)	-1.9527621245(-1)	-3.2463603700(-2)	-2.1094165579(-2)
3	6.0832402591(-1)	3.9826681237(-1)	2.1717786240(-1)	-6.4442220493(-2)
4	-1.5272995709(-2)	6.1424839452(-3)	1.2378892900(-3)	7.1383200293(-3)
j	$\theta=10$			
1	4.5367870185(-2)	-4.4007613334(-2)	-6.2724997318(-4)	-8.3354231288(-4)
2	3.6734147385(-2)	-8.8908465796(-2)	-8.0113455747(-3)	-4.8770986803(-3)
3	7.2159071876(-1)	2.7838679095(-1)	1.5344651935(-1)	-7.6408871159(-2)
4	-1.6411748419(-2)	-1.0418849232(-2)	-2.6995540892(-3)	3.3438843632(-3)
j	$\theta=20$			
1	3.0921561293(-2)	-3.1215848444(-2)	-2.4252276102(-4)	-2.7245172372(-4)
2	2.8747309036(-2)	-5.0560158364(-2)	-2.3588112711(-3)	-1.5936817165(-3)
3	7.9012029413(-1)	2.0819614550(-1)	1.0969439006(-1)	-6.7547289429(-2)
4	-4.0008063426(-3)	-1.4305270483(-2)	-1.5999261366(-3)	3.4711154520(-4)
j	$\theta=50$			
1	1.9143242645(-2)	-1.9570761546(-2)		-6.5290717852(-5)
2	1.9436056994(-2)	-2.6948734380(-2)	-5.1619404242(-4)	-3.9705010282(-4)
3	8.6029891781(-1)	1.3579846585(-1)	6.9887309765(-2)	-5.1902216036(-2)
4	2.1167331857(-3)	-8.2740205351(-3)	-3.6188669914(-4)	-1.0187474690(-4)

Table S5: Eigenvalues r_j with non-zero coefficients for the oscillatory Stokes flow obtained by the HRMM $(N=5)$.

j	$\mathrm{Re}(r_j)$	$\mathrm{Im}(r_j)$
	$\theta=1$	
1	-1.1283872391(1)	1.2747533062(1)
2	-2.0053292402	2.6563493812
3	-9.4496559017(-1)	1.2709740290
4	-6.5594582842(-1)	7.1519264176(-1)
5	-4.5873930506(-1)	4.6224054296(-1)
6	-3.2927480267(-1)	3.2931123092(-1)
j	$\theta=5$	
1	-5.6310205035(1)	1.2766927871(1)
2	-9.8146013765	2.6216807354
3	-3.8818667161	1.1000413896
4	-2.4767236781	2.0979159189
5	-2.3167743287	4.9507702200(-1)
6	-1.6465135507	3.2973114504(-1)
j	$\theta=10$	
1	-1.1260876277(2)	1.2767851423(1)
2	-1.9667191861(1)	2.6105163531
3	-7.9820031221	1.0990765143
4	-3.3342763664	3.0433315084
5	-4.6837346776	5.2685152787(-1)
6	-3.2936353928	3.3064523324(-1)
j	$\theta=20$	
1	-2.2521159318(2)	1.2768083340(1)
2	-3.9356729378(1)	2.6075619759
3	-1.6057776458(1)	1.0982518434
4	-4.5922669417	4.3747330625
5	-9.4250192320	5.4337208273(-1)
6	-6.5892221183	3.3210798872(-1)
j	$\theta=50$	
1	-5.6302481151(2)	1.2768148316(1)
2	-9.8408067684(1)	2.6067239512
3	-4.0207904409(1)	1.0980000034
4	-7.1445412717	7.0038346569
5	-2.3614229787(1)	5.4902179385(-1)
6	-1.6477121938(1)	3.3332417356(-1)

Table S6: Velocity and shear stress coefficients g_j and h_j respectively for the oscillatory Stokes flow obtained by the HRMM $(N=5)$.

j	Velocity coefficients g_j		Shear stress coefficients h_j	
	$\mathrm{Re}(g_j)$	$\mathrm{Im}(g_j)$	$\mathrm{Re}(h_j)$	$\mathrm{Im}(h_j)$
	$\theta=1$			
1	9.1249792090(-2)	-3.5175347734(-2)	2.6439907327(-3)	-5.0997947171(-3)
2	2.1782374733(-1)	-4.7629728605(-2)	4.3611018556(-2)	-5.0853317816(-2)
3	2.0747402780(-1)	1.1562670710(-1)	1.4868722404(-1)	-1.9573651978(-2)
4	2.6036770736(-2)	6.0793688290(-2)	6.2115968902(-2)	2.8032975228(-2)
5	2.2819756405(-3)	5.0026671787(-3)	7.8983211226(-3)	2.9841537199(-3)
6	4.3825840211(-5)	6.6127239939(-5)	1.6695137864(-4)	3.3871780356(-5)
j	$\theta=5$			
1	3.8828646760(-2)	-3.3182046122(-2)	-4.1176778359(-4)	-7.8290704722(-4)
2	6.2952077709(-2)	-8.0310104047(-2)	-6.0385054555(-3)	-8.0271330552(-3)
3	-4.0636449727(-2)	-1.2674743803(-1)	-3.2970028164(-2)	1.1252457770(-3)
4	6.6870307653(-1)	3.8542936329(-1)	2.2376770820(-1)	-8.0451945895(-2)
5	-1.5994046636(-2)	3.8381368672(-3)	1.7350632629(-4)	6.9406611737(-3)
6	-1.7548378409(-4)	2.2899959885(-4)	1.1319826014(-4)	1.2924811699(-4)
j	$\theta=10$			
1	2.6085172638(-2)	-2.4598993158(-2)	-1.8974289401(-4)	-2.5315775624(-4)
2	3.9198117428(-2)	-5.0636161339(-2)	-2.2701060394(-3)	-2.2943929508(-3)
3	9.7053115886(-3)	-6.0202307644(-2)	-7.2376100563(-3)	-2.2124770626(-3)
4	7.2591512800(-1)	2.8180271703(-1)	1.5451040235(-1)	-7.6684930720(-2)
5	-1.3146191048(-2)	-1.1412613221(-2)	-2.7179787486(-3)	2.5010425650(-3)
6	-4.7655069572(-4)	9.9220919834(-5)	1.5444259530(-5)	1.4623879364(-4)
j	$\theta=20$			
1	1.7927207015(-2)	-1.7636576272(-2)	-7.3561785940(-5)	-8.3772108539(-5)
2	2.6755426231(-2)	-3.2834719103(-2)	-7.8579432565(-4)	-7.3188077593(-4)
3	1.3799831706(-2)	-3.3380078201(-2)	-2.0105669614(-3)	-9.9689646443(-4)
4	7.9004962423(-1)	2.0866264108(-1)	1.0973893804(-1)	-6.7498485532(-2)
5	-2.2645831228(-3)	-1.2362308647(-2)	-1.3211093278(-3)	1.6410886363(-4)
6	-4.7914794420(-4)	-3.3409065008(-4)	-5.4229887902(-5)	6.9983642518(-5)
j	$\theta=50$			
1	1.1142937400(-2)	-1.1178430941(-2)	-1.9395447075(-5)	-2.0231046860(-5)
2	1.6746713696(-2)	-1.9217914135(-2)	-1.9064643909(-4)	-1.7522624660(-4)
3	1.0794740679(-2)	-1.7122494796(-2)	-4.1820562392(-4)	-2.7989348415(-4)
4	8.6031223555(-1)	1.3578140237(-1)	6.9887196663(-2)	-5.1904503227(-2)
5	2.0509589160(-3)	-6.8297216349(-3)	-2.8704614435(-4)	-9.3526383244(-5)
6	-5.2635606801(-5)	-4.2789147761(-4)	-2.6022796980(-5)	2.6680375170(-6)

Table S7: Eigenvalues r_j for the oscillatory Couette flow obtained by the HRMM $(N=1)$.

j	$\mathrm{Re}\left[r_j\right]$	$\mathrm{Im}\left[r_j\right]$
	$\theta=1$	
1	-2.032718	3.462725
2	2.032718	-3.462725
3	-6.717855(-1)	8.887156(-1)
4	6.717855(-1)	-8.887156(-1)
j	$\theta=10$	
1	-1.879079(1)	3.542414
2	1.879079(1)	-3.542414
3	-3.246869	3.016107
4	3.246869	-3.016107
j	$\theta=20$	
1	-3.760518(1)	3.531940
2	3.760518(1)	-3.531940
3	-4.544765	4.377810
4	4.544765	-4.377810

Table S8: Velocity coefficients g_j for the oscillatory Couette flow obtained by the HRMM $(N=1)$.

$\theta=1$		$\theta=10$		$\theta=20$	
$\mathrm{Re}\left(g_j\right)$	$\mathrm{Im}\left(g_j\right)$	$\mathrm{Re}\left(g_j\right)$	$\mathrm{Im}\left(g_j\right)$	$\mathrm{Re}\left(g_j\right)$	$\mathrm{Im}\left(g_j\right)$

$\delta = 0.1$					
3.735028(-1)	-2.977571(-2)	1.959143(-1)	-6.421584(-2)	1.838937(-1)	-3.300852(-2)
1.093244(-2)	-7.038598(-2)	-1.486046(-1)	-2.783634(-2)	-1.490610(-1)	-1.336983(-2)
1.531789(-1)	1.313639(-1)	6.161561(-1)	4.928586(-1)	8.099274(-1)	6.323260(-1)
1.630053(-2)	-1.633054(-2)	-1.066542(-1)	-3.992745(-1)	-2.879261(-1)	-5.851816(-1)
$\delta = 1$					
3.888750(-1)	-3.538399(-2)	1.266566(-1)	-8.958682(-2)	1.112831(-1)	-4.568813(-2)
-2.916631(-3)	2.962401(-4)	-1.256518(-2)	-1.034760(-2)	-1.529349(-2)	-5.429550(-3)
1.569286(-1)	1.336011(-1)	5.980429(-1)	3.344549(-1)	7.131019(-1)	4.038709(-1)
1.472557(-3)	3.522454(-3)	3.626089(-2)	-1.874491(-1)	-5.468234(-2)	-3.287427(-1)
$\delta = 10$					
3.882395(-1)	-3.491617(-2)	1.141307(-1)	-1.295324(-1)	7.371469(-2)	-8.683396(-2)
-4.298974(-14)	-7.140516(-14)	-3.993124(-11)	-8.122746(-12)	7.843102(-11)	4.335439(-11)
1.566707(-1)	1.336003(-1)	6.732962(-1)	2.649292(-1)	7.744850(-1)	1.958121(-1)
-7.367530(-9)	-2.163417(-8)	-4.049161(-4)	-4.128684(-4)	-1.990900(-3)	5.020504(-3)
$\delta = 20$					
3.882395(-1)	-3.491618(-2)	1.140168(-1)	-1.295735(-1)	7.385957(-2)	-8.598194(-2)
-1.268040(-25)	-7.967170(-27)	-9.729686(-21)	-5.011605(-21)	-1.549716(-20)	-6.095000(-20)
1.566707(-1)	1.336003(-1)	6.732645(-1)	2.646258(-1)	7.718890(-1)	1.980834(-1)
-3.294612(-14)	-5.564588(-15)	-7.485447(-7)	-4.525512(-7)	5.716913(-5)	2.661123(-6)

Table S9: Shear stress coefficients h_j for the oscillatory Couette flow obtained by the HRMM $(N=1)$.

$\theta = 1$		$\theta = 10$		$\theta = 20$	
$\mathrm{Re}(h_j)$	$\mathrm{Im}(h_j)$	$\mathrm{Re}(h_j)$	$\mathrm{Im}(h_j)$	$\mathrm{Re}(h_j)$	$\mathrm{Im}(h_j)$

$\delta = 0.1$					
7.646575(-2)	-5.348649(-2)	-1.402071(-3)	-1.069039(-2)	-4.148183(-4)	-4.929076(-3)
6.526249(-3)	1.649567(-2)	2.870250(-3)	-7.367276(-3)	7.214580(-4)	-3.896082(-3)
1.807903(-1)	1.115275(-2)	1.761103(-1)	-2.617559(-2)	1.612121(-1)	-2.292124(-2)
-2.832877(-3)	2.051683(-2)	8.239052(-2)	4.368654(-2)	9.844269(-2)	3.147298(-2)
$\delta = 1$					
7.906025(-2)	-5.662915(-2)	-3.376896(-3)	-7.376962(-3)	-9.288117(-4)	-3.046485(-3)
5.890758(-4)	-4.313554(-4)	6.535087(-4)	-5.454898(-4)	1.809831(-4)	-3.896874(-4)
1.846863(-1)	1.072510(-2)	1.471401(-1)	-4.750814(-2)	1.244930(-1)	-3.698660(-2)
-2.961071(-3)	-1.725242(-3)	2.542158(-2)	3.478277(-2)	4.353195(-2)	2.990084(-2)
$\delta = 10$					
7.898274(-2)	-5.644855(-2)	-5.551099(-3)	-7.120243(-3)	-2.106407(-3)	-2.158064(-3)
1.823600(-14)	9.916041(-15)	8.042986(-13)	-1.973418(-12)	-1.336977(-12)	1.960073(-12)
1.845011(-1)	1.086411(-2)	1.472027(-1)	-7.062710(-2)	1.074951(-1)	-6.686632(-2)
1.698572(-8)	1.150358(-8)	1.304441(-4)	-3.536546(-6)	-3.541233(-4)	-7.791789(-4)
$\delta = 20$					
7.898274(-2)	-5.644855(-2)	-5.554312(-3)	-7.114788(-3)	-2.083589(-3)	-2.159774(-3)
3.727964(-27)	1.124176(-27)	3.518151(-22)	-4.514664(-22)	1.644982(-21)	-2.576024(-22)
1.845011(-1)	1.086411(-2)	1.471477(-1)	-7.066845(-2)	1.074690(-1)	-6.632033(-2)
2.660353(-14)	-1.384839(-14)	1.897782(-7)	-5.425331(-8)	-6.588876(-6)	6.232287(-6)

Table S10: Eigenvalues r_j for the oscillatory Couette flow obtained by the HRMM $(N=3)$.

j	$\mathrm{Re}[r_j]$	$\mathrm{Im}[r_j]$
	$\theta = 1$	
1	-6.055896	7.523308

2	6.055896	-7.523308
3	-1.139600	1.735765
4	1.139600	-1.735765
5	-6.878330(-1)	8.029470(-1)
6	-4.405709(-1)	4.437105(-1)
7	6.878330(-1)	-8.029470(-1)
8	4.405709(-1)	-4.437105(-1)
j	$\theta = 10$	
1	-6.011550(1)	7.558303
2	6.011550(1)	-7.558303
3	-1.013363(1)	1.583824
4	1.013363(1)	-1.583824
5	-3.330143	3.044307
6	-4.494263	5.124747(-1)
7	3.330143	-3.044307
8	4.494263	-5.124747(-1)
j	$\theta = 20$	
1	-1.202219(2)	7.558438
2	1.202219(2)	-7.558438
3	-2.037016(1)	1.575471
4	2.037016(1)	-1.575471
5	-4.592116	4.375204
6	-9.057402	5.355732(-1)
7	4.592116	-4.375204
8	9.057402	-5.355732(-1)

Table S11: Velocity coefficients g_j for the oscillatory Couette flow obtained by the HRMM $(N = 3)$.

$\theta = 1$		$\theta = 10$		$\theta = 20$	
$\mathrm{Re}(g_j)$	$\mathrm{Im}(g_j)$	$\mathrm{Re}(g_j)$	$\mathrm{Im}(g_j)$	$\mathrm{Re}(g_j)$	$\mathrm{Im}(g_j)$
$\delta = 0.1$					
1.552050(-1)	-5.008249(-2)	7.988220(-2)	-1.595531(-2)	7.817874(-2)	-8.092493(-3)
9.029035(-3)	-2.361331(-2)	-4.114850(-2)	-9.503042(-3)	-4.219017(-2)	-4.815017(-3)
3.073673(-1)	4.833040(-2)	7.960567(-2)	-6.805488(-2)	7.918314(-2)	-3.249505(-2)

1.192775(-2)	-4.554238(-2)	-7.659849(-2)	6.146759(-3)	-7.268503(-2)	3.075186(-3)
6.334264(-2)	9.801302(-2)	6.572036(-1)	5.236658(-1)	8.216973(-1)	6.560447(-1)
2.267117(-3)	4.527690(-3)	-1.158022(-2)	-1.524321(-2)	5.968018(-3)	-1.741679(-2)
1.134081(-2)	-5.828963(-3)	-1.151430(-1)	-4.296920(-1)	-2.890320(-1)	-6.046386(-1)
3.768544(-4)	-1.276679(-4)	-2.744953(-3)	1.144976(-2)	-1.157363(-2)	9.745889(-3)
$\delta = 1$					
1.627220(-1)	-5.468720(-2)	5.155613(-2)	-2.655708(-2)	4.792200(-2)	-1.359280(-2)
3.558271(-5)	2.251323(-5)	-5.245755(-5)	-9.928555(-5)	-9.750973(-5)	-5.885386(-5)
3.169124(-1)	4.739905(-2)	4.856851(-2)	-7.136492(-2)	4.933116(-2)	-3.403472(-2)
5.346608(-4)	7.060121(-3)	-1.867956(-2)	-4.369915(-3)	-1.809382(-2)	-2.048643(-3)
6.497626(-2)	9.981534(-2)	6.397504(-1)	3.603666(-1)	7.303572(-1)	4.284813(-1)
2.305554(-3)	4.589844(-3)	-1.342833(-2)	-1.124318(-2)	1.242938(-3)	-1.417935(-2)
3.104308(-4)	2.920848(-3)	3.808943(-2)	-2.054052(-1)	-5.559140(-2)	-3.447146(-1)
3.446892(-5)	1.106318(-4)	-2.126033(-3)	4.477002(-3)	-5.298297(-3)	3.580572(-3)
$\delta = 10$					
1.621591(-1)	-5.374570(-2)	4.541323(-2)	-4.398697(-2)	3.086582(-2)	-3.157894(-2)
3.392506(-32)	1.535274(-32)	8.214245(-30)	1.714050(-29)	-3.595502(-29)	2.483826(-29)
3.158087(-1)	4.813847(-2)	3.679154(-2)	-8.889960(-2)	2.863145(-2)	-5.094625(-2)
9.836668(-11)	-4.094330(-11)	4.516777(-8)	1.032875(-7)	1.913413(-7)	-1.291519(-7)
6.464623(-2)	9.970646(-2)	7.216057(-1)	2.787109(-1)	7.928246(-1)	2.058846(-1)
2.296156(-3)	4.584879(-3)	-1.640791(-2)	-1.042652(-2)	-4.066691(-3)	-1.435222(-2)
-2.049314(-9)	-1.659038(-9)	-3.751629(-4)	-4.256072(-4)	-2.067652(-3)	5.026816(-3)
8.690362(-10)	4.481048(-10)	1.812596(-6)	-4.318369(-6)	-2.673535(-6)	-1.298303(-5)
$\delta = 20$					
1.621591(-1)	-5.374570(-2)	4.536793(-2)	-4.400760(-2)	3.091827(-2)	-3.121410(-2)
1.582883(-60)	1.896096(-60)	3.484871(-57)	-3.961608(-57)	-2.091222(-56)	2.736901(-56)
3.158087(-1)	4.813847(-2)	3.673422(-2)	-8.890847(-2)	2.874400(-2)	-5.055776(-2)
4.009910(-18)	-1.846799(-17)	1.395888(-13)	-7.692203(-14)	-6.362198(-13)	6.017088(-13)
6.464623(-2)	9.970646(-2)	7.215908(-1)	2.783872(-1)	7.900891(-1)	2.081780(-1)
2.296156(-3)	4.584879(-3)	-1.641175(-2)	-1.041886(-2)	-4.001035(-3)	-1.430448(-2)
4.545572(-14)	-1.431001(-14)	-5.759647(-7)	-4.348958(-7)	5.490523(-5)	3.033014(-6)
1.090746(-13)	-2.003330(-13)	-1.343142(-9)	1.288310(-9)	1.114814(-8)	9.087348(-9)

Table S12: Shear stress coefficients h_j for the oscillatory Couette flow obtained by the HRMM $(N = 3)$.

$\theta = 1$		$\theta = 10$		$\theta = 20$	
$\mathrm{Re}(h_j)$	$\mathrm{Im}(h_j)$	$\mathrm{Re}(h_j)$	$\mathrm{Im}(h_j)$	$\mathrm{Re}(h_j)$	$\mathrm{Im}(h_j)$
$\delta = 0.1$					
9.266892(-3)	-1.411637(-2)	-9.680943(-5)	-1.340984(-3)	-2.632490(-5)	-6.519422(-4)
8.048491(-4)	2.490823(-3)	2.403412(-4)	-6.542726(-4)	6.187014(-5)	-3.470461(-4)
1.365152(-1)	-6.178392(-2)	-5.357103(-3)	-8.692876(-3)	-1.286885(-3)	-3.986743(-3)

7.235494(-3)	2.148725(-2)	5.611230(-4)	-7.471141(-3)	1.242645(-4)	-3.558601(-3)
1.058090(-1)	3.142682(-2)	1.839412(-1)	-2.919702(-2)	1.642490(-1)	-2.244596(-2)
7.674814(-3)	2.583644(-3)	-3.638212(-3)	2.161807(-3)	-1.877408(-3)	-7.699236(-4)
-4.559443(-3)	1.116524(-2)	8.750843(-2)	4.542133(-2)	1.004512(-1)	3.276537(-2)
-2.838176(-4)	5.695373(-4)	-2.446188(-3)	-8.897038(-4)	-9.969697(-4)	-1.336761(-3)
$\delta = 1$					
9.574240(-3)	-1.497583(-2)	-3.287427(-4)	-8.989506(-4)	-8.765669(-5)	-4.041240(-4)
-4.331725(-6)	4.943635(-7)	1.733884(-6)	-6.546119(-7)	5.384089(-7)	-7.772313(-7)
1.401118(-1)	-6.468175(-2)	-6.143234(-3)	-5.752955(-3)	-1.474689(-3)	-2.535792(-3)
-2.081322(-3)	-2.700969(-3)	7.021767(-4)	-1.733578(-3)	1.682636(-4)	-8.752377(-4)
1.080915(-1)	3.171623(-2)	1.546184(-1)	-5.076193(-2)	1.283397(-1)	-3.676842(-2)
7.788473(-3)	2.610869(-3)	-2.805894(-3)	2.667930(-3)	-1.551958(-3)	-2.289978(-4)
-2.020252(-3)	-1.907039(-3)	2.790462(-2)	3.694726(-2)	4.539388(-2)	3.114383(-2)
-1.637810(-4)	-8.671117(-5)	-9.301237(-4)	-5.791157(-4)	-3.594733(-4)	-6.062248(-4)
$\delta = 10$					
9.589962(-3)	-1.486334(-2)	-6.268184(-4)	-8.342426(-4)	-2.455601(-4)	-2.721790(-4)
-3.733121(-33)	9.642899(-34)	-2.976014(-31)	9.923710(-32)	-1.870612(-31)	-3.108329(-31)
1.398629(-1)	-6.409234(-2)	-8.009627(-3)	-4.882494(-3)	-2.378090(-3)	-1.589485(-3)
-2.877896(-11)	4.248260(-11)	-1.062953(-8)	2.795886(-9)	5.580380(-9)	9.824815(-9)
1.077874(-1)	3.184110(-2)	1.535018(-1)	-7.636286(-2)	1.097246(-1)	-6.810736(-2)
7.772212(-3)	2.615824(-3)	-2.701143(-3)	3.342849(-3)	-1.605521(-3)	3.540548(-4)
2.492876(-9)	-6.930038(-11)	1.257239(-4)	2.275941(-6)	-3.489272(-4)	-7.827068(-4)
-1.491176(-9)	4.707209(-10)	9.031304(-7)	5.062960(-7)	1.445816(-6)	-2.096843(-7)
$\delta = 20$					
9.589962(-3)	-1.486334(-2)	-6.272496(-4)	-8.335433(-4)	-2.425100(-4)	-2.724236(-4)
-2.507780(-61)	-5.016554(-62)	5.767853(-59)	6.538950(-59)	-2.158647(-58)	-1.875185(-58)
1.398629(-1)	-6.409234(-2)	-8.011345(-3)	-4.877106(-3)	-2.358707(-3)	-1.593511(-3)
3.266993(-18)	8.494774(-18)	5.308184(-15)	1.460445(-14)	-2.696184(-14)	-3.331822(-14)
1.077874(-1)	3.184110(-2)	1.534466(-1)	-7.640883(-2)	1.096889(-1)	-6.754571(-2)
7.772212(-3)	2.615824(-3)	-2.699556(-3)	3.343883(-3)	-1.599841(-3)	3.471419(-4)
-2.384570(-14)	3.824894(-14)	1.572715(-7)	-2.918257(-8)	-6.317423(-6)	5.937397(-6)
1.019570(-13)	3.502592(-13)	-2.493364(-10)	-3.272885(-10)	-1.072337(-9)	1.167424(-9)

Table S13: Eigenvalues r_j for the oscillatory Couette flow obtained by the HRMM $(N = 5)$.

j	$\mathrm{Re}[r_j]$	$\mathrm{Im}[r_j]$
	$\theta = 1$	
1	-1.128387(1)	1.274753(1)
2	1.128387(1)	-1.274753(1)
3	-2.005329	2.656349
4	2.005329	-2.656349

5	-9.449656(-1)	1.270974
6	9.449656(-1)	-1.270974
7	-6.559458(-1)	7.151926(-1)
8	-4.587393(-1)	4.622405(-1)
9	-3.292748(-1)	3.293112(-1)
10	6.559458(-1)	-7.151926(-1)
11	4.587393(-1)	-4.622405(-1)
12	3.292748(-1)	-3.293112(-1)
j	$\theta = 10$	
1	-1.126088(2)	1.276785(1)
2	1.126088(2)	-1.276785(1)
3	-1.966719(1)	2.610516
4	1.966719(1)	-2.610516
5	-7.982003	1.099077
6	7.982003	-1.099077
7	-3.334276	3.043332
8	3.334276	-3.043332
9	-4.683735	5.268515(-1)
10	-3.293635	3.306452(-1)
11	4.683735	-5.268515(-1)
12	3.293635	-3.306452(-1)
j	$\theta = 20$	
1	-2.252116(2)	1.276808(1)
2	2.252116(2)	-1.276808(1)
3	-3.935673(1)	2.607562
4	3.935673(1)	-2.607562
5	-1.605778(1)	1.098252
6	1.605778(1)	-1.098252
7	-4.592267	4.374733
8	-9.425019	5.433721(-1)
9	-6.589222	3.321080(-1)
10	4.592267	-4.374733
11	9.425019	-5.433721(-1)
12	6.589222	-3.321080(-1)

Table S14: Velocity coefficients g_j for the oscillatory Couette flow obtained by the HRMM $(N = 5)$.

$\theta = 1$		$\theta = 10$		$\theta = 20$	
$\mathrm{Re}(g_j)$	$\mathrm{Im}(g_j)$	$\mathrm{Re}(g_j)$	$\mathrm{Im}(g_j)$	$\mathrm{Re}(g_j)$	$\mathrm{Im}(g_j)$
$\delta = 0.1$					
8.714579(-2)	-3.261453(-2)	4.678324(-2)	-8.210587(-3)	4.603062(-2)	-4.150313(-3)
6.813906(-3)	-6.292792(-3)	-1.421106(-2)	-3.926208(-3)	-1.468530(-2)	-1.993975(-3)

2.111253(-1)	-4.484520(-2)	6.991451(-2)	-2.804255(-2)	6.736293(-2)	-1.404616(-2)
-3.867033(-4)	-3.674281(-2)	-5.483886(-2)	-6.540357(-3)	-5.470109(-2)	-3.189845(-3)
2.023862(-1)	1.137689(-1)	3.658994(-2)	-5.291575(-2)	4.404566(-2)	-2.545081(-2)
1.751484(-2)	-2.586707(-2)	-4.427889(-2)	1.245551(-2)	-4.308475(-2)	5.779991(-3)
2.551913(-2)	5.979699(-2)	6.607514(-1)	5.292730(-1)	8.218808(-1)	6.573035(-1)
2.250962(-3)	4.939100(-3)	-1.152228(-1)	-4.335370(-1)	6.527965(-3)	-1.431795(-2)
4.338547(-5)	6.547258(-5)	-8.135610(-3)	-1.517180(-2)	-2.661499(-4)	-6.174747(-4)
6.474309(-3)	-1.955956(-3)	-4.652299(-4)	-4.395601(-5)	-2.892081(-1)	-6.055444(-1)
4.198721(-4)	-1.277006(-4)	-3.973091(-3)	1.000800(-2)	-1.062842(-2)	7.510224(-3)
4.504210(-6)	-2.161101(-6)	1.148929(-4)	2.235545(-4)	-1.264347(-4)	5.055088(-4)
		$\delta = 1$			
9.158771(-2)	-3.564658(-2)	2.975666(-2)	-1.438750(-2)	2.790444(-2)	-7.350671(-3)
1.138732(-7)	-5.920874(-8)	1.034048(-8)	-3.445288(-7)	-2.431630(-7)	-2.509032(-7)
2.184690(-1)	-4.834124(-2)	4.561933(-2)	-3.542577(-2)	4.217110(-2)	-1.777595(-2)
-1.199551(-3)	1.982760(-3)	-4.966080(-3)	-2.888005(-3)	-5.540173(-3)	-1.489072(-3)
2.082208(-1)	1.155267(-1)	1.843398(-2)	-5.138350(-2)	2.664381(-2)	-2.479320(-2)
7.324387(-4)	5.261656(-3)	-1.397074(-2)	1.174280(-4)	-1.324197(-2)	7.038492(-7)
2.617089(-2)	6.088051(-2)	6.431041(-1)	3.638868(-1)	7.298905(-1)	4.286688(-1)
2.290553(-3)	5.008263(-3)	3.899629(-2)	-2.065550(-1)	2.214607(-3)	-1.186708(-2)
4.393132(-5)	6.617508(-5)	-1.033954(-2)	-1.176278(-2)	-3.317072(-4)	-4.348329(-4)
-1.904318(-5)	1.515718(-3)	-4.399239(-4)	4.512877(-5)	-5.521804(-2)	-3.445844(-1)
2.796430(-5)	1.156996(-4)	-2.466645(-3)	3.695959(-3)	-4.711698(-3)	2.627135(-3)
7.675002(-7)	1.333450(-6)	3.307574(-5)	1.134340(-4)	-8.252233(-5)	2.324774(-4)
		$\delta = 10$			
9.124979(-2)	-3.517535(-2)	2.611171(-2)	-2.458699(-2)	1.789432(-2)	-1.784835(-2)
3.931718(-55)	2.044320(-55)	1.766579(-52)	8.348409(-54)	1.682624(-52)	-3.690828(-52)
2.178237(-1)	-4.762973(-2)	3.924032(-2)	-5.062052(-2)	2.669363(-2)	-3.315573(-2)
1.546143(-14)	3.094157(-15)	-4.600753(-12)	4.331381(-12)	1.410057(-11)	3.335608(-13)
2.074740(-1)	1.156267(-1)	9.739520(-3)	-6.020365(-2)	1.371139(-2)	-3.359998(-2)
4.774469(-10)	-2.152029(-10)	5.062571(-7)	2.639057(-7)	7.071088(-7)	-9.349605(-7)
2.603677(-2)	6.079369(-2)	7.259291(-1)	2.821296(-1)	7.927567(-1)	2.063519(-1)
2.281976(-3)	5.002667(-3)	-3.755871(-4)	-4.278908(-4)	-2.317603(-3)	-1.240736(-2)
4.382584(-5)	6.612724(-5)	-1.314170(-2)	-1.141906(-2)	-4.819154(-4)	-3.340683(-4)
1.156512(-9)	1.675778(-9)	-4.765954(-4)	9.904444(-5)	-2.069962(-3)	5.026783(-3)
-2.507443(-11)	-9.486077(-10)	1.870171(-6)	-3.010635(-6)	-1.079016(-6)	-9.422083(-6)
-3.950345(-11)	-1.175695(-12)	-1.572455(-7)	-2.899775(-7)	-1.250366(-6)	-9.283251(-7)
		$\delta = 20$			
9.124979(-2)	-3.517535(-2)	2.608521(-2)	-2.459899(-2)	1.792529(-2)	-1.763555(-2)
2.977453(-107)	-3.548178(-106)	-7.735751(-103)	-1.118757(-103)	2.087410(-102)	4.638454(-102)
2.178237(-1)	-4.762973(-2)	3.919817(-2)	-5.063615(-2)	2.675256(-2)	-3.283306(-2)
-1.397641(-25)	-2.334391(-25)	-1.154342(-22)	6.349037(-22)	-3.749805(-21)	-1.479902(-21)
2.074740(-1)	1.156267(-1)	9.705353(-3)	-6.020232(-2)	1.379802(-2)	-3.337850(-2)
-2.641096(-16)	2.911889(-16)	-6.583642(-13)	-6.890489(-12)	-9.315305(-12)	3.725155(-11)
2.603677(-2)	6.079369(-2)	7.259152(-1)	2.818031(-1)	7.900185(-1)	2.086445(-1)
2.281976(-3)	5.002667(-3)	-5.737059(-7)	-4.342886(-7)	-2.264837(-3)	-1.236165(-2)
4.382584(-5)	6.612724(-5)	-1.314619(-2)	-1.141262(-2)	-4.791385(-4)	-3.340643(-4)
-7.723088(-15)	-2.851475(-14)	-4.765508(-4)	9.922071(-5)	5.489640(-5)	3.038796(-6)

8.560789(-14)	-6.043596(-14)	-9.648562(-10)	6.484229(-10)	6.104860(-9)	6.000877(-9)
-1.596132(-14)	4.675196(-15)	1.156591(-10)	4.186173(-10)	5.740135(-9)	-8.498071(-10)

Table S15: Shear stress coefficients h_j for the oscillatory Couette flow obtained by the HRMM $(N = 5)$.

$\theta = 1$		$\theta = 10$		$\theta = 20$	
$\mathrm{Re}(h_j)$	$\mathrm{Im}(h_j)$	$\mathrm{Re}(h_j)$	$\mathrm{Im}(h_j)$	$\mathrm{Re}(h_j)$	$\mathrm{Im}(h_j)$
$\delta = 0.1$					
2.563184(-3)	-4.827378(-3)	-2.548029(-5)	-4.183384(-4)	-6.819053(-6)	-2.047749(-4)
-5.469993(-5)	5.420672(-4)	4.855047(-5)	-1.206938(-4)	1.251038(-5)	-6.449740(-5)
4.250884(-2)	-4.897301(-2)	-9.374815(-4)	-3.679316(-3)	-2.424281(-4)	-1.727661(-3)

6.744157(-3)	8.740776(-3)	6.904956(-4)	-2.696689(-3)	1.723787(-4)	-1.378458(-3)
1.454093(-1)	-1.859825(-2)	-5.886576(-3)	-5.394603(-3)	-1.390846(-3)	-2.838074(-3)
8.701337(-4)	1.970522(-2)	-7.817896(-4)	-5.654989(-3)	-1.756203(-4)	-2.695119(-3)
6.102865(-2)	2.763661(-2)	1.852681(-1)	-2.906751(-2)	1.644167(-1)	-2.234231(-2)
7.795761(-3)	2.948417(-3)	8.813827(-2)	4.589038(-2)	-1.474311(-3)	-7.776180(-4)
1.652887(-4)	3.354631(-5)	-3.391724(-3)	1.355473(-3)	-9.550300(-5)	3.557819(-5)
-3.554364(-3)	5.994785(-3)	-2.725119(-5)	1.385155(-4)	1.005794(-1)	3.283784(-2)
-3.194955(-4)	5.933399(-4)	-2.015832(-3)	-1.075026(-3)	-7.294016(-4)	-1.169733(-3)
-3.558347(-6)	1.012044(-5)	-7.066443(-5)	2.778935(-5)	-7.555847(-5)	-2.299638(-5)
$\delta = 1$					
2.640507(-3)	-5.133677(-3)	-9.656291(-5)	-2.751967(-4)	-2.553235(-5)	-1.253507(-4)
-2.703348(-9)	7.037674(-9)	3.010408(-9)	4.331539(-10)	1.171525(-9)	-1.013291(-9)
4.363694(-2)	-5.114073(-2)	-1.467521(-3)	-2.514356(-3)	-3.790061(-4)	-1.096620(-3)
-7.128479(-5)	-6.926088(-4)	1.772374(-4)	-2.289803(-4)	4.695567(-5)	-1.376571(-4)
1.490279(-1)	-1.990568(-2)	-6.005558(-3)	-3.136374(-3)	-1.423857(-3)	-1.756629(-3)
-2.353352(-3)	-2.390151(-3)	2.220814(-4)	-1.719701(-3)	5.609439(-5)	-8.208090(-4)
6.227829(-2)	2.800549(-2)	1.555737(-1)	-5.087812(-2)	1.283121(-1)	-3.670502(-2)
7.913723(-3)	2.980974(-3)	2.797121(-2)	3.722606(-2)	-1.241432(-3)	-3.065422(-4)
1.671842(-4)	3.378427(-5)	-2.725244(-3)	1.900992(-3)	-6.835516(-5)	4.689566(-5)
-1.041246(-3)	-1.164326(-3)	2.901207(-7)	1.335970(-4)	4.534215(-2)	3.117017(-2)
-1.556259(-4)	-9.585465(-5)	-7.207462(-4)	-6.077140(-4)	-2.490916(-4)	-5.142745(-4)
-3.190044(-6)	-8.595166(-7)	-3.509482(-5)	6.519180(-6)	-3.456244(-5)	-1.426584(-5)
$\delta = 10$					
2.643991(-3)	-5.099795(-3)	-1.896113(-4)	-2.533785(-4)	-7.450737(-5)	-8.367971(-5)
-2.525229(-56)	6.315857(-57)	-2.488094(-55)	1.540565(-54)	1.591354(-54)	8.373501(-55)
4.361102(-2)	-5.085332(-2)	-2.269045(-3)	-2.296398(-3)	-7.940187(-4)	-7.308554(-4)
-4.267714(-15)	2.056964(-15)	-1.859078(-13)	-2.586067(-13)	-3.207194(-14)	3.561510(-13)
1.486872(-1)	-1.957365(-2)	-7.237196(-3)	-2.216706(-3)	-2.024573(-3)	-9.923468(-4)
-1.608490(-10)	2.889120(-10)	-4.101814(-8)	5.777685(-8)	5.495598(-8)	4.779393(-8)
6.211597(-2)	2.803298(-2)	1.545660(-1)	-7.663839(-2)	1.097695(-1)	-6.805882(-2)
7.898321(-3)	2.984154(-3)	1.260959(-4)	2.448676(-6)	-1.326197(-3)	1.694409(-4)
1.669514(-4)	3.387178(-5)	-2.719231(-3)	2.499943(-3)	-5.424762(-5)	7.040274(-5)
-2.045465(-9)	-4.670959(-10)	1.538986(-5)	1.462469(-4)	-3.487390(-4)	-7.829690(-4)
1.053397(-9)	1.006777(-9)	5.904005(-7)	4.657019(-7)	1.002955(-6)	-5.666179(-8)
6.177061(-11)	-5.819362(-11)	9.190833(-8)	-3.851562(-8)	1.500684(-7)	-1.821956(-7)
$\delta = 20$					
2.643991(-3)	-5.099795(-3)	-1.897428(-4)	-2.531581(-4)	-7.355774(-5)	-8.376337(-5)
1.250466(-107)	1.676535(-107)	1.749884(-105)	-6.671175(-105)	-2.105379(-104)	8.075045(-105)
4.361102(-2)	-5.085332(-2)	-2.270105(-3)	-2.294396(-3)	-7.857570(-4)	-7.318055(-4)
7.577355(-27)	3.067674(-27)	-3.095787(-23)	-9.978558(-24)	4.372290(-23)	-9.238052(-23)
1.486872(-1)	-1.957365(-2)	-7.237611(-3)	-2.212482(-3)	-2.010477(-3)	-9.967777(-4)
2.412476(-17)	-2.470436(-16)	8.583364(-13)	3.570697(-14)	-2.269552(-12)	-7.353350(-13)
6.211597(-2)	2.803298(-2)	1.545105(-1)	-7.668489(-2)	1.097335(-1)	-6.749690(-2)
7.898321(-3)	2.984154(-3)	1.567287(-7)	-2.901035(-8)	-1.321041(-3)	1.641397(-4)
1.669514(-4)	3.387178(-5)	-2.717981(-3)	2.501042(-3)	-5.422582(-5)	6.998241(-5)
2.572575(-14)	1.627540(-14)	1.544419(-5)	1.462388(-4)	-6.316920(-6)	5.936405(-6)
-2.793413(-14)	1.584682(-13)	-1.138290(-10)	-2.188056(-10)	-6.718066(-10)	6.089981(-10)

1.713864(-14)	-3.133364(-14)	-1.293208(-10)	2.213353(-11)	8.484679(-11)	8.754165(-10)