

Article

Hydrodynamics and Thrust Efficiency of a Bio-Inspired Ray Model Under Prescribed Manta-like Pectoral-Fin Kinematics

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Abstract

Ray-like swimmers achieve propulsion through undulatory pectoral-fin motions, but the hydrodynamic consequences of these prescribed motions remain incompletely understood. We investigated a bio-inspired ray model that combines geometry reconstructed from a Javan cownose ray (*Rhinoptera javanica*) with pectoral-fin kinematics parameterized from published manta and mobuliform swimming studies. Pectoral-fin deformation was prescribed using analytical amplitude and phase functions, while fluid forces updated only streamwise rigid-body translation. The model yielded a maximum hydrodynamic Froude efficiency of 47.4% under the prescribed kinematics, and mean speed increased approximately linearly with flapping frequency over the retained 0.2–0.6 Hz range ($R^2 = 0.9890$). The surface-attached vortex became less continuous as frequency increased, while pressure evolution indicated that thrust arose mainly during downstrokes. These results characterize the hydrodynamic response of this hybrid, bio-inspired ray model to the imposed manta-like waveform; they are not species-specific predictions for manta rays or *R. javanica*.

Keywords: bio-inspired ray model; manta-like hydrodynamics; pectoral-fin propulsion; vortex dynamics; computational fluid dynamics (CFD); thrust efficiency

1. Introduction

The study of bio-inspired underwater propulsion offers profound insights into aquatic hydrodynamics and provides a foundation for high-efficiency robotic design. Aquatic animal propulsion is fundamentally driven by unsteady hydrodynamic phenomena, primarily governed by dynamic lift generation and vortex evolution [1]. A classic example is the reverse Bénard–von Kármán vortex street generated by oscillating fish caudal fins, where shed vortices form a downstream propulsive jet that markedly enhances efficiency [2].

Batoid fishes exhibit excellent maneuverability in fluid environments by virtue of their unique pectoral fin fluctuating propulsion patterns. The motions of their pectoral fins can be classified into two distinct categories: rajiform modes, which are characterized by multiple wave transmissions (e.g., stingrays), and mobuliform modes (e.g., manta rays), which are dominated by the overall beat of the pectoral fins [3]. The dynamic response of such flexible structures is highly coupled to fluid loading, and their energy transfer mechanisms need to be resolved through fluid–solid coupling (FSI) methods [4]. Although the vortex generation mechanism of the rajiform mode has been the subject of preliminary analysis through experiments and numerical simulations [5], the dynamical mechanism of the pectoral fins’ 3D motion and fluid interaction in the mobuliform mode remains to be elucidated. This is particularly true with regard to the effect of chordwise and spanwise



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flexible coupling on the generation of thrust. It is imperative that this issue be studied in depth.

Mobuliform swimmers, such as manta rays, feature broad pectoral fins and compound motions that may include spanwise flapping, chordwise bending, and localized twisting [6,7]. These published manta/mobuliform kinematic observations motivate the imposed waveform in the present bio-inspired ray model. Mechanistically, chordwise flexibility can mitigate dynamic stall by delaying flow separation, while spanwise twisting can optimize the local angle-of-attack distribution along the fin.

In recent years, significant progress has been made in the field of biomechanical modeling of mantas. The pectoral fins of these fish are composed of a flexible membrane supported by radial cartilage, and the cartilage units form a multi-stage hinge structure through synovial joints, resulting in a fin surface that combines overall stiffness with localized flexibility [8,9]. High-resolution CT scans revealed that the cartilage density at the leading edge of the pectoral fin was substantially higher than at the trailing edge. This gradient stiffness distribution may inhibit flow separation during flapping [5]. As demonstrated in the relevant literature, including the seminal study [10], kinematic studies have shown that the pectoral fin beat amplitude of manta increases nonlinearly with swimming speed. Consequently, researchers have developed a variety of bionic models. In their 2010 study, Shoele and Zhu simulated pectoral fin fluctuations through a discrete ray-membrane structure, yet no consideration was given to chordal flexibility [1]. In contrast, Russo et al. [8] constructed a rigid-flexible coupled multibody dynamics model that replicated joint motions, but simplified the dynamic feedback effects of fluid loading. The majority of these models presuppose that the fin surface moves solely in the direction of propulsion (x-axis); they do not take into account the potential modulation of vertical displacement (z-axis) on vortex reattachment.

Through theoretical analysis, numerical simulation, and experimental methodologies, research on the manta's fluid dynamics properties and propulsion mechanisms has gradually advanced. Early theoretical studies used slender-body theory and two-dimensional potential-flow analyses to examine thrust, power, and propulsive efficiency [11]. Subsequent numerical and experimental studies have examined the relationships between pectoral-fin kinematics, wake structures, and swimming performance in rays [5]. With the advent of high-precision motion capture and three-dimensional flow-field visualization technologies in recent years, researchers have used digital particle image velocimetry (DPIV) and numerical simulations (like the immersed boundary method) to systematically analyze the dynamic coupling process between the evolution of the vortex field and the flexible deformation of the manta's pectoral fins by numerical simulations [5,12] and experimental studies [5,13]. The joint verification of biomechanical tests and numerical simulations has been crucial in demonstrating its quick maneuverability and effective cruising. Currently, a key technique for investigating the low-energy, high-maneuverability propulsion strategy of the manta is the extensive use of three-dimensional reconstruction techniques based on live kinematic data and computational fluid dynamics (CFD) models to simulate its swimming process in detail (experimental data sources [5,14]). The spatiotemporal relationship between pectoral-fin vortex ring generation and propulsive force has been partially clarified by research; however, that study proves that rigid models exhibit energy losses up to 20% [1], while other studies have demonstrated that passive deformation of flexible pectoral fins can improve propulsive efficiency by reducing trailing-edge vortex intensity [15]. However, further research is needed to understand the mechanisms governing the formation, persistence, and shedding of pectoral-fin vortical structures, how fin membrane stiffness gradients affect energy transfer, and how the hydrodynamics of various movement modes (such as gliding and flapping) differ.

This work examines the hydrodynamic response of a bio-inspired ray model with deliberately separated geometric and kinematic inputs. The three-dimensional geometry was reconstructed from a Javan cownose ray (*Rhinoptera javanica*) collected in Hainan Province, China (Section 2.1). The pectoral-fin waveform was prescribed from published manta/mobuliform motion studies [12,15], using a spanwise displacement envelope with third-harmonic components (Section 2.2). We use “manta-like” only to identify the biological literature from which the imposed motion was parameterized. Within a CFD dynamic-mesh framework, fluid forces update only streamwise rigid-body translation and do not update fin deformation (Section 2.3). The simulations quantify the effects of this prescribed waveform on propulsion, wake vortices, pressure fields, thrust, and efficiency. All interpretations are restricted to the hybrid bio-inspired ray model and its imposed kinematics.

2. Materials and Methods

2.1. Measuring and Reconstructing the Geometry of the Javan Cownose Ray

A fresh specimen of the Javan cownose ray (*Rhinoptera javanica* from Sanya, Hainan, China; Figure 1) was morphometrically characterized before three-dimensional scanning. Disc length, disc width, and total length were measured as scaling references for the subsequent reconstruction. The species has a dorsoventrally flattened body and a rhomboidal disc, with a width-to-length ratio of approximately 1.9:1. Published morphometric data indicate that adult disc width ranges from 0.22 to 1.12 m [16]. Body thickness is approximately 6–8% of disc width and has been reported as 10–15 cm. The elongated, whip-like tail is approximately 1.2–1.5 times the disc width, tapering from 40–50 mm in diameter at its base to 2–3 mm at its tip [16]. Because the subsequent waveform is parameterized from manta/mobuliform studies rather than measured from this specimen, the reconstructed geometry is used as a bio-inspired ray-model geometry and not as a complete species-specific swimming model of *R. javanica*.

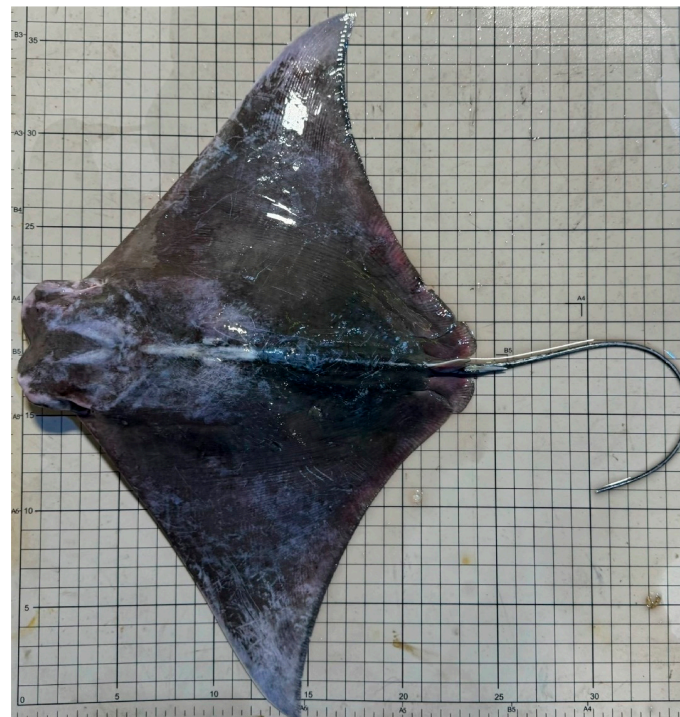


Figure 1. Live Javan cownose ray (*Rhinoptera javanica*) specimen used for geometric reconstruction; each grid cell measures 10 mm in both length and width.

To capture the specimen's surface geometry, multi-angle scans were acquired using a handheld Artec Eva 3D optical scanner (Artec 3D, Luxembourg). The individual scans were registered to generate a complete surface point cloud, following established three-dimensional model reconstruction procedures. Particular attention was given to geometrically informative structures, including the pectoral-fin sweep angle and cephalic fins. The reconstructed surface was subsequently converted into an STL model, with the measured morphometric parameters used to verify its scale. In the simulation, the model was scaled down by a factor of 2 (at a 1:2 scale).

The raw polygon mesh exhibited minor topological imperfections inherent to optical scanning, such as non-manifold edges at the gill slits and boundary discontinuities along the fin margins (Figure 2). These localized defects were repaired via local re-topology to yield a watertight, manifold surface representation while strictly preserving the biological contours of the specimen (Figure 3). The complete repaired model is shown in Figure 4.

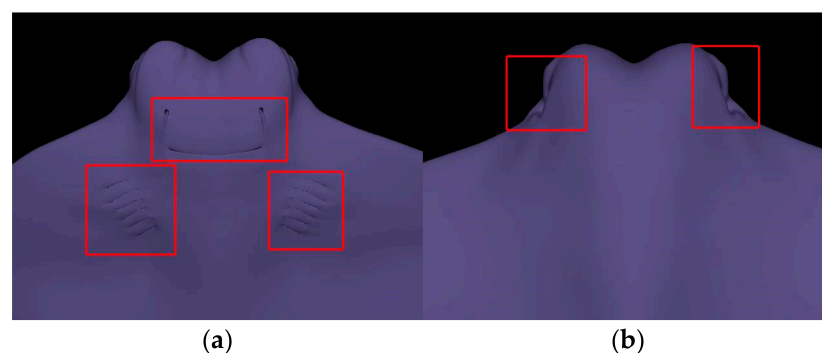


Figure 2. (a) Ventral surface of the reconstructed Javan cownose ray model, with the red bounding box highlighting non-streaming edges in the structural geometry; (b) dorsal view of the model, with the red-framed area highlighting non-streaming edges on the upper structural geometry.

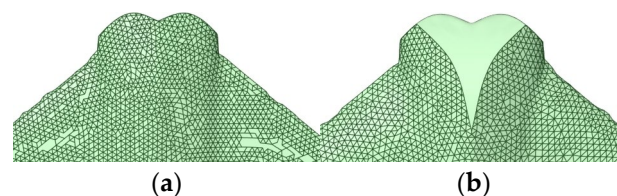


Figure 3. (a) Ventral surface of the model after SpaceClaim(2024R1) repair; (b) dorsal surface of the model after SpaceClaim repair.

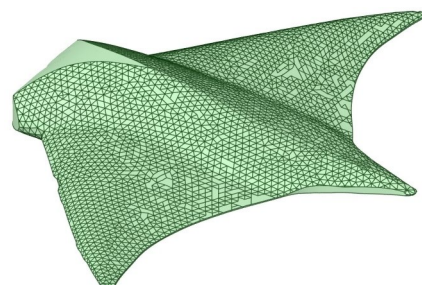


Figure 4. The full model repaired using SpaceClaim.

2.2. Prescribed Manta-like Pectoral-Fin Kinematics

Published studies of manta and other mobuliform swimmers describe pectoral-fin motions that combine spanwise flapping, chordwise bending, and travelling-wave-like surface motion [10]. We use these studies only as the source of a prescribed, manta-like

waveform for the bio-inspired ray model. In the present model, the motion is imposed kinematically rather than computed as deformation of a flexible structure.

To simplify the mathematical description of the complex three-dimensional flapping motion, a Decoupled Modal Kinematic (DMK) method is introduced. The motion imposed on the bio-inspired ray model is decomposed into two orthogonal components: a prescribed spanwise displacement envelope and chordwise phase-varying wave propagation.

Assuming the fin root ($y = 0$) remains fixed relative to the central body, the spanwise vertical displacement $z_{span}(y, t)$ is modeled using a quadratic polynomial function to reflect the exponential flexural rigidity gradient from the fin root to the fin tip ($y = b$):

$$A(y) = A_{tip} \cdot \left(\frac{y}{b}\right)^2$$

The function A_{tip} is the maximum flapping amplitude at the fin tip, and b represents the total pectoral fin span [10].

The fluid-propulsive force is primarily generated by a phase-lagged traveling wave propagating from the leading edge ($x = 0$) to the trailing edge ($x = c(y)$). The localized spatial-temporal wave function is expressed as:

$$z(x, y, t) = A(y) \cdot \sin\left(2\pi ft - k_c \frac{x}{c(y)} + \phi_0\right) \quad (1)$$

where f denotes the flapping frequency, $c(y)$ is the local chord length at spanwise location y , k_c is the dimensionless chordwise phase coefficient, and ϕ_0 is the initial phase angle [10].

To prevent unphysical surface stretching during large-amplitude flapping, a second-order arc-length correction is applied to the spanwise coordinate y :

$$y'(y, t) = y - \frac{1}{2} \int_0^y \left(\frac{\partial z(\xi, \eta, t)}{\partial \eta} \right)^2 d\eta \quad (2)$$

Thus, the complete 3D kinematic mapping from undeformed coordinates $(x, y, 0)$ to instantaneous deformed space (x', y', z') is explicitly established by Equations (1) and (2). This formulation provides a clear interpretation of each kinematic parameter and simplifies identification from high-speed tracking.

To check implementation of the prescribed waveform, the instantaneous vertical displacement at the outermost pectoral-fin boundary was compared with the reference high-speed video trajectory [10] (Figure 5b). The agreement indicates that the analytical function reproduces the selected reference fin-tip trajectory.

2.3. Numerical Method for Prescribed-Kinematics Self-Propulsion

This study combines three-dimensional viscous CFD with dynamic-mesh techniques to simulate self-propulsion of the bio-inspired ray model. The fin-surface motion is prescribed by Equations (1) and (2), and no structural deformation equation is solved. Hydrodynamic forces are integrated to update streamwise body translation, which changes the relative flow at the next time step. Accordingly, the method is one-way coupled at the fin and two-way coupled only for rigid-body translation.

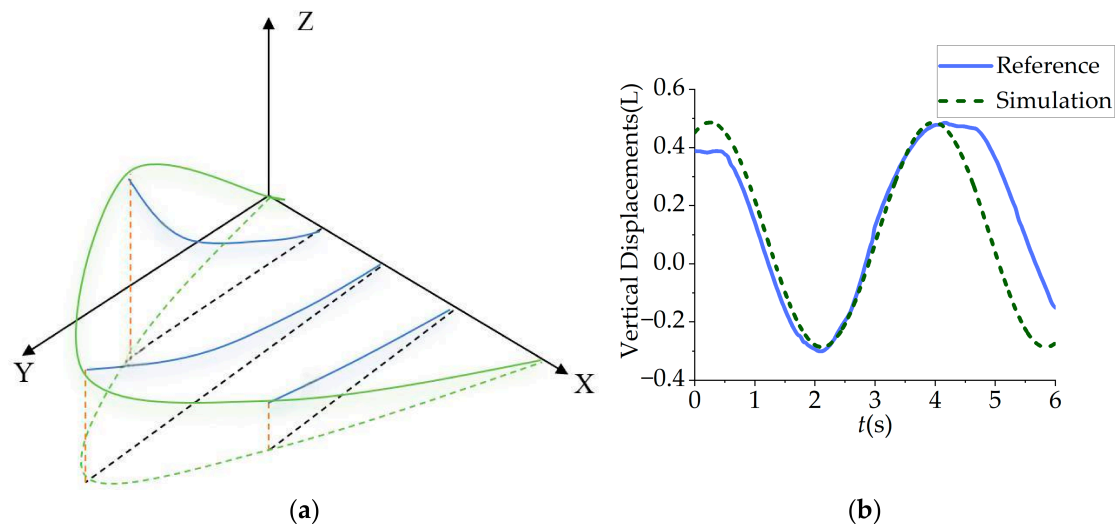


Figure 5. (a) Coordinate system and geometric features used to prescribe the fin motion: blue lines denote fin rays and green lines denote pectoral-fin margins, the dotted line represents the starting position. (b) Comparison of the prescribed fin-tip vertical displacement with reference data digitized from published manta/mobuliform literature. This comparison verifies reproduction of the selected reference trajectory only.

2.3.1. Governing Fluid Equations and Turbulence Model

We employ a three-dimensional incompressible viscous fluid dynamics approach to numerically simulate rays' self-propelled swimming. The governing equations of fluid motion are mass conservation (continuity equation) and the Navier–Stokes equation. The continuity equation is formulated as the zero dispersion of the velocity field, or the incompressible qualities of the fluid volume.

$$\frac{\partial u_i}{\partial x_i} = 0 \quad (3)$$

The momentum equations characterize the dynamic equilibrium of the fluid microelements' acceleration in terms of the pressure gradient, viscous forces, and turbulent pulsating stresses:

$$\frac{\partial(\rho u_i)}{\partial t} + \frac{\partial(\rho u_i u_j)}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} \left(\mu \frac{\partial u_i}{\partial x_j} - \rho \overline{u_i' u_j'} \right) + S_i \quad (4)$$

where u_i is the time-averaged velocity component in the Cartesian coordinate system, p is the pressure field, μ is the dynamic viscosity, ρ is the fluid density, $u_i' u_j'$ represents the Reynolds stress term due to turbulent pulsation, and S_i is the volumetric force source term.

We used the shear stress transport SST k - ω turbulence model to simulate flow separation and vortex shedding caused by large-amplitude oscillations of the prescribed pectoral fins. The model combines the high accuracy of the standard k - ω model in the near-wall region with the stability of the k - ε model in the main flow region using a mixing function. The governing equations contain the transport equations of turbulent kinetic energy (k) and specific dissipation rate (ω).

The turbulent kinetic energy equation describes the processes of turbulent energy creation, diffusion, and dissipation:

$$\frac{\partial(\rho k)}{\partial t} + \frac{\partial(\rho u_j k)}{\partial x_j} = \frac{\partial}{\partial x_j} \left(\Gamma k \frac{\partial k}{\partial x_j} \right) + G_k - Y_k \quad (5)$$

The specific dissipation rate equation then quantifies the turbulent vortex scale evolution properties:

$$\frac{\partial(\rho\omega)}{\partial t} + \frac{\partial(\rho u_j \omega)}{\partial x_j} = \frac{\partial}{\partial x_j} \left(\Gamma_\omega \frac{\partial \omega}{\partial x_j} \right) + G_\omega - Y_\omega + D_\omega \quad (6)$$

where G_k and G_ω are the generation terms for turbulent kinetic energy and specific dissipation rate, Y_k and Y_ω are the dissipation terms, and D_ω is the cross-diffusion correction term. The model uses a limiting function to manage turbulent viscosity, allowing it to accurately describe transient flow characteristics such as pectoral-fin LEV formation, vortex core migration, and vortex pairing in the wake.

To accommodate the prescribed time-varying pectoral-fin boundary, the computational mesh is updated using spring smoothing and local reconstruction. A user-defined function (UDF) applies the analytical amplitude envelope and phase distribution directly to the dynamic-grid boundary. The near-wall grid height is controlled at $y^+ < 1$ using adaptive refinement based on cell distortion. Grid orthogonality is maintained to resolve the boundary-layer velocity profile.

To solve the problem of hard coupling of the pressure–velocity field in the Navier–Stokes equation (Equations (3) and (4)) for incompressible fluids, this study uses ANSYS Fluent’s (2024R1) built-in pressure-based coupled algorithm (Coupled).

Pressure–velocity coupling was solved using ANSYS Fluent’s pressure-based coupled algorithm. Node-based gradient reconstruction was used, and the convective terms were discretized using a second-order upwind scheme. Dynamic-mesh deformation was handled by spring smoothing and local remeshing when the mesh-quality criteria were exceeded.

2.3.2. Swimming Dynamics

In the self-propelled simulation, boundary velocity is the sum of rigid-body translational velocity (V_r) and prescribed pectoral-fin deformation velocity (V_d):

$$V = V_r + V_d \quad (7)$$

where V_d is prescribed by Equations (1) and (2) and is not updated by fluid loading, whereas V_r is solved from the fluid-force-driven rigid-body dynamics. Only the x-direction degree of freedom is released, so translation is governed by the streamwise thrust–drag balance. This produces two-way coupling between the flow and body translation, while fin deformation remains one-way prescribed. The governing rigid-body equations are:

$$m \frac{dV_r}{dt} = F_x \quad (8)$$

where m is the mass assigned to the bio-inspired ray model and F_x represents the net fluid force in the x-direction (pressure–viscous stress integral). The velocity is updated using explicit time integration:

$$V_{r,x}^{t+\Delta t} = V_{r,x}^t + \frac{F_x^t}{m} \Delta t \quad (9)$$

Then the velocity is updated using the improved Euler method (second-order accuracy):

$$\begin{aligned} k_1 &= \frac{F_x^t}{m} \Delta t \\ k_2 &= \frac{F_x^{t+\Delta t}}{m} \Delta t \\ V_r^{t+\Delta t} &= V_r^t + \frac{1}{2}(k_1 + k_2) \end{aligned} \quad (10)$$

The streamwise velocity was advanced using the explicit second-order improved-Euler scheme given. This predictor–corrector update is second-order accurate in time. The time step was selected to provide adequate temporal resolution and numerical stability for the coupled flow, rigid-body translation, and dynamic-mesh calculation.

2.3.3. Dynamic-Mesh Method

To simulate self-propulsion of the bio-inspired ray model, this study employs a dynamic-mesh approach to deal with deformation of the flow-field domain caused by the moving boundary. The grid nodes in the computational domain are dynamically modified using Fluent spring smoothing and local reconstruction methods. The interior mesh is elastically deformed to account for periodic displacements caused by prescribed pectoral-fin motion.

$$X_i^{\rightarrow n+1} = X_i^{\rightarrow n} + \Delta X_i^{\rightarrow converged} \quad (11)$$

Here $X_i^{\rightarrow n+1}$ and $X_i^{\rightarrow n}$ represent the position of node i at the new and current time steps, respectively. The maximum deflection threshold is set to activate the local reconstruction mechanism in response to local mesh distortion induced by large amplitude fluctuation of the pectoral fin. When the grid cell deflection rate surpasses 30% or the aspect ratio exceeds 5, the system automatically executes Delaunay triangulation to reclassify the affected area while maintaining a near-wall grid orthogonality of at least 25 degrees.

DEFINE_ZONE_MOTION using the user-defined function (UDF) was used to perform displacement calculations, mesh updates, and flow-field variable interpolation at each time step of the moving mesh, ensuring the continuity and conservation of the coherent motion border.

2.3.4. Numerical Procedure for Fluid-to-Body Coupling

The self-propelled motion of the bio-inspired ray model is solved using a three-dimensional viscous incompressible CFD method linked with Newton's second law. The specific solution procedure is as follows:

1. Initial stage: The fluid and bio-inspired ray model are initialized at rest. During subsequent time steps, only streamwise translation is solved from the integrated hydrodynamic force. Body pitch and dorsoventral heave are not included.
2. Time-step progression: At the commencement of each time step, the total translational displacement of the bio-inspired ray model is updated according to the solution outcomes from the preceding time step. The computational-domain mesh is updated via a dynamic-mesh technique that integrates spring smoothing with localized reconstruction.
3. Fluid solution: Resolve the Navier–Stokes equations and continuity equations, then use the SST $k-\omega$ turbulence model to accurately depict flow separation. The interconnected pressure–velocity solution is executed by a coupled algorithm to ascertain the pressure and velocity distribution inside the flow field.
4. Dynamics response calculation: The hydrodynamic force is obtained by surface integration, the streamwise acceleration is calculated from Newton's second law, and the velocity is updated using the explicit second-order improved-Euler scheme.
5. Kinematic boundary condition: At the moving body surface S , velocity continuity is imposed:

$$u_i = V_i = \frac{\partial x_i}{\partial t} \text{ at } S \quad (12)$$

Here V_i encompasses the components of advection and deformation velocity.

6. Iterative convergence: Repeat stages 2–5 until a quasi-steady state is attained, characterized by a balance between cycle-averaged push and drag, and stabilization of the average velocity.

The force-to-motion coupling procedure, including hydrodynamic-force integration, rigid-body acceleration, velocity updating, and dynamic-mesh motion, was independently validated using the gravity-driven settling of a sphere described in Appendixes A and B.

All production simulations used a time step of $\Delta t = T/1000$, corresponding to 1000 time steps per flapping cycle. The maximum Courant number was maintained below 0.5, and the continuity and momentum residuals were reduced below 10^{-6} at each time step.

2.4. Computational Domain and Mesh

The self-propelled ray-model simulation used a rectangular computational domain of $18L$ (length) $\times$ $4L$ (width) $\times$ $4L$ (height) (Figure 6). Throughout this study, L denotes the body-disc length of the CFD geometry and is distinct from disc width and total pectoral-fin span. Its dimensional value in the simulations is 0.228 m, and all computational-domain dimensions and mesh sizes are normalized by this reference length. The model was initially located at $10L$ along the X-axis with its centroid on the Y/Z midplanes. The computational mesh was unstructured, with a base mesh size of $0.05L$, local refinement in a $4L \times 2L \times 2L$ swimming region, and a model-surface mesh size of $0.0045L$ (Figure 7). The mesh then transitions gradually to the outer boundary, resulting in approximately 2.68 million cells.

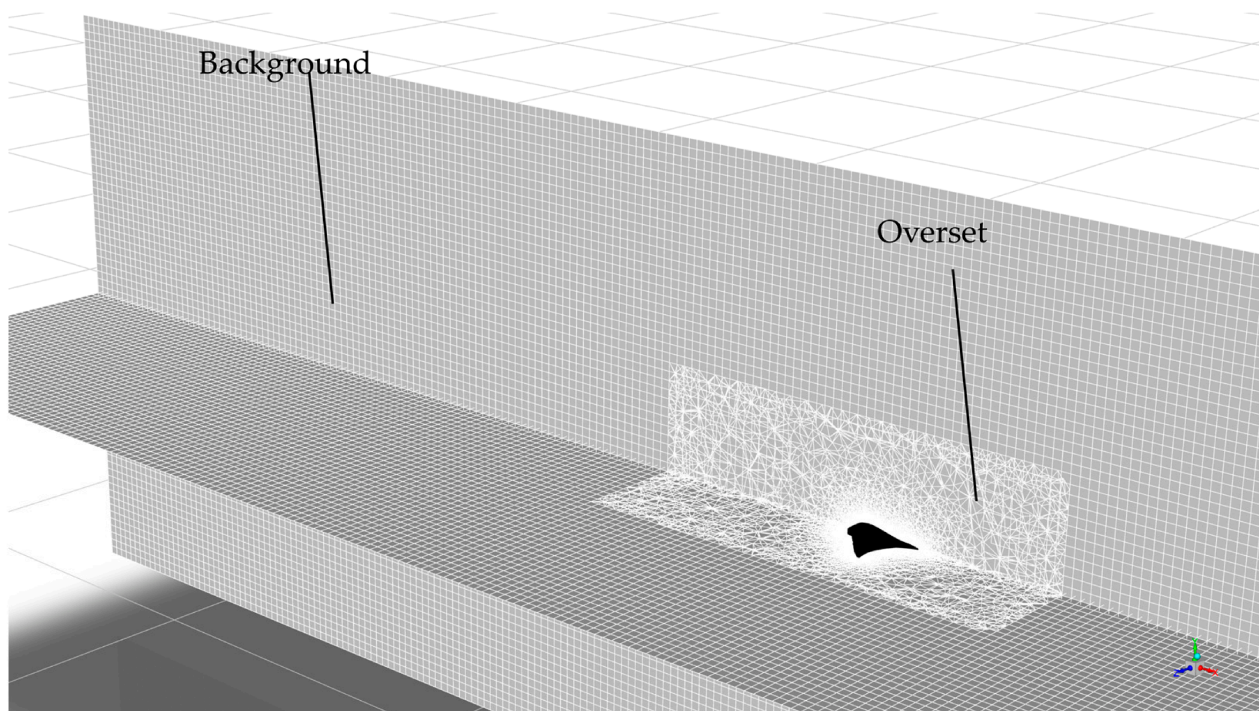


Figure 6. Computational domain.

For self-propulsion of the bio-inspired ray model, the computational-domain boundary employs a velocity-inlet condition derived from the Dirichlet boundary ($UBC = 0$) and the Neumann boundary condition. This mesh configuration and boundary-condition setup supports flow-field resolution for the prescribed-kinematics simulation.

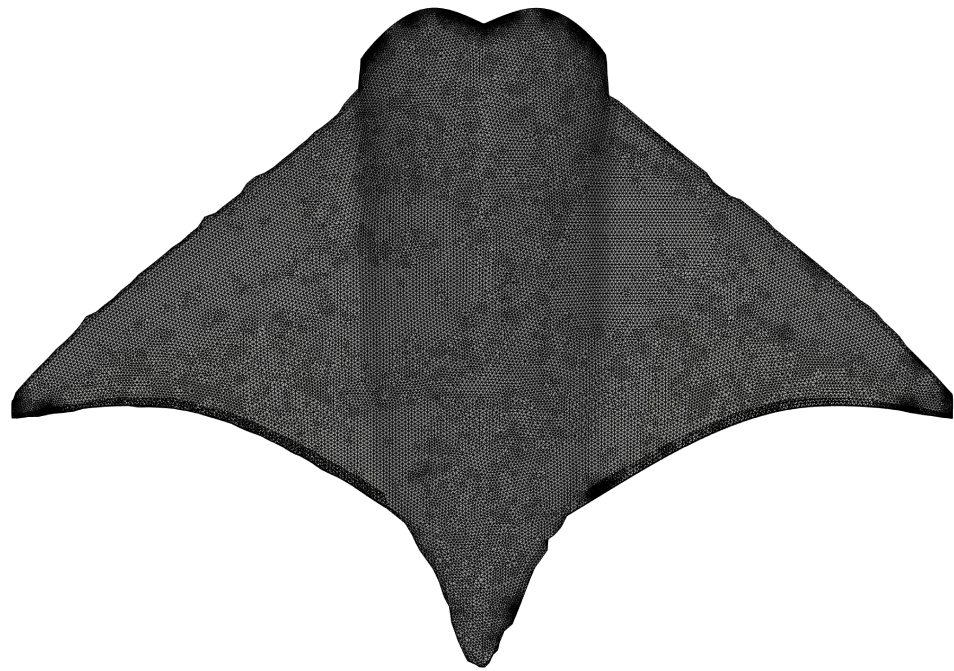


Figure 7. Mesh distribution for the bio-inspired ray model.

2.5. Calculation of Hydrodynamic Parameters

This study identified key hydrodynamic characteristics for the analysis of self-propelled swimming of the bio-inspired ray model: thrust, Strouhal number (St), Reynolds number (Re), and Froude propulsive efficiency. The methodology for determining each parameter is outlined as follows:

To quantitatively evaluate the propulsive performance associated with the prescribed pectoral-fin kinematics during quasi-steady swimming along the x direction, the hydrodynamic forces and energy metrics are defined [17,18]. The total instantaneous hydrodynamic force $F(t)$ in the swimming direction x is calculated by integrating the surface pressure p and viscous shear stress tensor τ_{xj} over the total surface area A of the body:

$$F(t) = \int \int_A (-pn_x + \tau_{xj}n_j)dA \quad (13)$$

where n_j denotes the j component of the unit normal vector pointing into the fluid.

To eliminate the numerical ambiguity during self-propelled swimming where the period-averaged net force vanishes ($\bar{F} \approx 0$), the thrust $T(t)$ is isolated by considering only the positive force components contributing to forward propulsion:

$$T(t) = \frac{1}{2} \left[\iint_A -pn_x dA + \left| \iint_A -pn_x dA \right| \right] + \frac{1}{2} \left[\iint_A \tau_{xj}n_j dA + \left| \iint_A \tau_{xj}n_j dA \right| \right] \quad (14)$$

The Strouhal number (St) [19–21] describes the link between swimming frequency and fluid inertia effect:

$$St = \frac{fA}{\bar{U}} \quad (15)$$

Here f is the frequency of prescribed pectoral-fin motion, A is the maximum pectoral-fin-tip displacement, and $\bar{U}$ is the average swimming speed of the bio-inspired ray model in the quasi-steady state.

P denotes the cycle-averaged hydrodynamic work rate associated with the prescribed lateral and vertical surface motion.

The two types of power loss (P) owing to the lateral and vertical undulations ($i = 1, 2$ for the Y and Z directions) were calculated as follows:

$$P = \sum_{i=2}^3 \left[\iint_A (-pn_i \dot{h}_i) dA + \iint_A (\tau_{ijn} n_j \dot{h}_i) dA \right] \quad (16)$$

where $\dot{h}_i$ represents the velocity component of the surface deformation in the i direction ($i = 2, 3$ for y and z directions, respectively).

Negative instantaneous P indicates that some energy is temporarily stored in fluid inertia, vortex structures, or body elasticity and is subsequently returned. The P used in the efficiency calculation should be the cycle-averaged value representing the net lateral energy expenditure; under normal dissipative conditions, this average loss should be positive.

To eliminate the scaling effect, the thrust and power loss are dimensionless:

$$T^* = \frac{T}{\rho \bar{U}^2 L^2}, \quad P^* = \frac{P}{\rho \bar{U}^3 L^2} \quad (17)$$

The average thrust and power figures were calculated by averaging the instantaneous measurements over multiple swimming cycles when all swimmers attained a steady state. The Froude propulsive efficiency, derived from the thrust force during the quasi-steady state of inline swimming, was defined as follows [17]:

$$\eta_f = \frac{\overline{TU}}{\overline{TU} + \overline{P}} = \frac{\overline{T^*}}{\overline{T^*} + \overline{P^*}} \quad (18)$$

Accordingly, the efficiency defined is interpreted as the Froude propulsive efficiency rather than an efficiency based on the net streamwise force. Its numerator represents the useful propulsive power required to overcome the mean hydrodynamic drag at the mean swimming speed. Although the mean net streamwise force vanishes during quasi-steady self-propelled swimming, the mean thrust is non-zero and is balanced by an equal mean drag. Therefore, the thrust-based term (T) has a clear physical interpretation as the resistance power that must be supplied to maintain the mean swimming speed. η_f represents how much power was used for thrust and how much was wasted. This metric is used here to quantify hydrodynamic propulsive performance and is evaluated only after quasi-steady self-propelled equilibrium has been reached; it is not applied during the initial acceleration phase.

Notably, the Froude efficiency (Equation (19)) is strictly valid for inline constant-speed swimming scenarios where thrust and drag forces are in equilibrium, resulting in a net mean force of zero on the fish body [17,18]. Consequently, this equation is employed to calculate efficiency exclusively during the quasi-steady state of the virtual swimmer, characterized by a constant mean velocity, as demonstrated in earlier studies [22].

The parameter set enables assessment of how the frequency of prescribed pectoral-fin motion influences the performance of the bio-inspired ray model. The roles of St and Re in vortex development and thrust production can then be examined through the numerical simulations.

Vortex identification protocol: Vortices were visualized using the Q criterion; a common Q^* threshold must be used for all cross-case comparisons.

$$Q^* = \frac{QL^2}{\bar{U}} \quad (19)$$

To provide a quantitative vortex metric, we calculated the total normalized vortex volume V_{tot}^* , the largest-component volume V_M^* , and the connectivity index C_A .

$$V_{\text{tot}}^* = \frac{V_{\text{tot}}}{L^3} \quad (20)$$

$$C_A = \frac{V_M^*}{V_{\text{tot}}^*} \quad (21)$$

2.6. Mesh and Domain Sensitivity

We tested grid independence of the numerical simulation results for the self-propelled bio-inspired ray model. Three grid configurations were developed from the baseline scheme: coarse grid (cell size 0.1L), standard grid (0.05L), and fine grid (0.025L). The surface-grid size was 0.0045L, resulting in approximately 2.68 million cells for the standard mesh. The refined grid decreased the grid size by 50% within the overset region, elevating the overall count to 20.94 million cells while preserving the grid-transition mechanism.

The impact of grid density was assessed by comparing mean swimming speed, thrust coefficient, and other critical metrics across the three mesh configurations. Under standard working conditions ($St = 0.306$, $Re = 5.08 \times 10^4$), the discrepancy in mean swimming speed between the standard and fine grids was about 3.4%. The coarse grid produced an 8.3% difference relative to the standard grid, consistent with delayed boundary-layer-separation prediction at inadequate near-wall resolution.

To assess the impact of the computational domain scale, the initial dimensions of the computational domain were increased from $18L \times 4L \times 4L$ to $24L \times 8L \times 8L$. The comparison results indicate that the mean swimming speed of the extended domain scheme deviates by less than 2.3% from the standard domain, and the properties of vortex structure growth in the wake region remain remarkably consistent. A thorough sensitivity study of grid density and computational domain size verified that the standard grid design can efficiently reconcile computational resource usage with the demands of physical field resolution while maintaining computational correctness. The corresponding grid-independence and computational-cost results are summarized in Table 1.

Table 1. Validation of grid independence and computational cost analysis.

Grid Scheme	Mean Swimming Speed (L/s)	Speed Deviation	Computational Cost (Core-Hours)
Coarse grid (1.22 million)	1.059	+8.3%	8
Baseline grid (2.68 million)	0.9775	Reference	22
Fine grid (20.94 million)	1.011	+3.4%	68
Extended domain (13.67 million)	1.000	+2.3%	34

3. Results

3.1. Swimming Performance

This research performed numerical simulations of the self-propelled bio-inspired ray model under prescribed manta-like pectoral-fin kinematics. Using a three-dimensional incompressible viscous CFD approach, two baseline conditions were established: high frequency ($f = 0.6$ Hz) and low frequency ($f = 0.3$ Hz).

Numerical simulation results (Figure 8) demonstrate that, from an initial motionless condition, the bio-inspired ray model produces thrust under the imposed pectoral-fin waveform and its speed approaches a quasi-steady state. Under the high-frequency condition, the model attains a quasi-steady state following seven motion cycles ($T = 1.67$ s), with a mean swimming speed of 0.9775 L/s. The low-frequency condition similarly reaches

dynamic equilibrium after seven cycles ($T = 3.33$ s), yielding a quasi-steady mean speed of 0.3079 L/s.

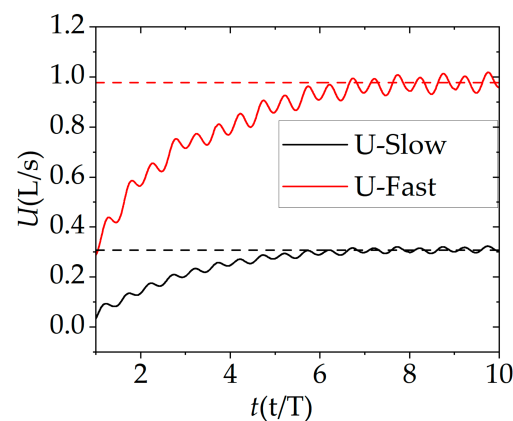


Figure 8. Calculated swimming speed U and steady speed for slow ($Re = 1.60 \times 10^4$, $St = 0.487$) and fast ($Re = 5.08 \times 10^4$, $St = 0.306$) cases. For a portion of the iteration steps in the first cycle, the iteration error did not fall below 1×10^{-6} ; hence, the first cycle data is not displayed. The red and black dashed lines indicate the quasi-steady mean speeds for the fast and slow cases, respectively.

While the bio-inspired ray model reaches a quasi-steady state, its instantaneous speed exhibits periodic variations around the mean. Figure 9 shows the normalized speed fluctuation over dimensionless time (t/T). The high-frequency condition (0.6 Hz) exhibits a fluctuation amplitude of $\pm 4.32\%$ (peak-to-peak 8.54%), whereas the low-frequency condition (0.3 Hz) exhibits $\pm 3.37\%$ (peak-to-peak 6.60%). Both cases show periodic speed oscillations synchronized with the prescribed pectoral-fin motion.

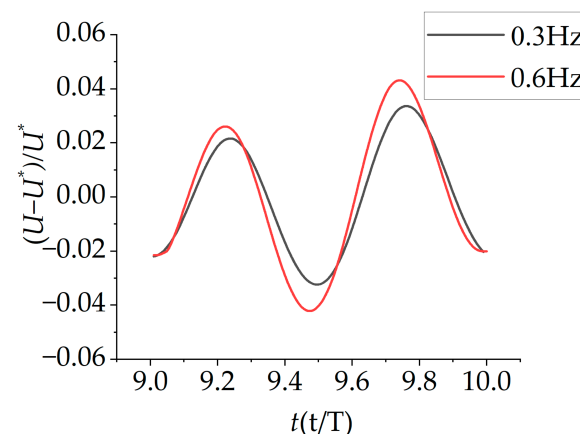


Figure 9. Velocity fluctuations for slow ($Re = 1.60 \times 10^4$, $St = 0.487$) and fast ($Re = 5.08 \times 10^4$, $St = 0.306$) cases in the last cycle.

The fluctuation morphology has specific periodic characteristics; during swimming, each motion cycle reveals a bimodal shape. Velocity escalates at $t/T = 9.00$ – 9.25 and $t/T = 9.50$ – 9.75 , but progressively diminishes during $t/T = 9.25$ – 9.50 and $t/T = 9.75$ – 10.00 . The principal peak ($t/T = 9.75$) and secondary peak ($t/T = 9.25$) align with moments of maximum transient thrust production, when the downstroke and upstroke of the pectoral fins transmit to the mid-body disc and trailing edge, respectively, illustrating complete synchronization with fin kinematics. Moreover, the principal peak magnitude is roughly 1.25 times that of the secondary peak, signifying enhanced velocity increase during the downstroke. Both situations demonstrate peak speed loss at $t/T = 9.4$ – 9.6 , a phase that aligns with the terminal upstroke during which instantaneous drag intensifies.

In the self-propelled simulation of the bio-inspired ray model, hydrodynamic parameters exhibit periodic fluctuations during the quasi-steady phase. Subsequent analyses are based on the final complete motion cycle, as illustrated in Figure 10:

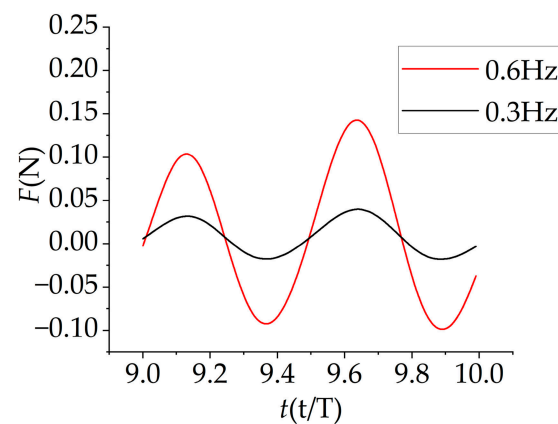


Figure 10. Force for the low-frequency ($Re = 1.60 \times 10^4$, $St = 0.487$) and high-frequency ($Re = 5.08 \times 10^4$, $St = 0.306$) bio-inspired ray-model cases in the last cycle.

Figure 10 shows the temporal variation in pectoral-fin force during the final cycle at 0.3 and 0.6 Hz. The 0.6 Hz condition (red curve, high-speed swimming) exhibits a pronounced bimodal waveform, with positive peaks near $t/T = 9.12$ and 9.62 and negative minima near $t/T = 9.35$ and 9.92 . Positive forces, corresponding to the propulsive direction, occur approximately over $t/T = 9.0$ – 9.22 and 9.50 – 9.74 , whereas negative forces, corresponding to the resistive direction, occur during the intervening intervals. The 0.3 Hz condition (black curve, low-speed swimming) follows the same phase sequence but has markedly smaller force excursions. Overall, the 0.6 Hz case reaches approximately 0.14 N in the positive direction and -0.10 N in the negative direction, while the 0.3 Hz response remains within roughly ± 0.03 N. These results indicate that increasing swimming frequency preserves the bimodal force pattern while substantially increasing instantaneous force magnitude. Table 2 summarizes the corresponding performance metrics for both swimming conditions.

Table 2. Bio-inspired ray-model performance for low- and high-frequency cases.

Case	f (Hz)	Re	St	$\bar{U}$	$\bar{T}^*$
Slow	0.3	16,005.9	0.487	0.3079	0.000937
Fast	0.6	50,814.3	0.306	0.9775	0.003975

3.2. Wake Structure and Pressure Field

Vortex structures were examined using Q-criterion isosurfaces. The revised visualizations should be plotted at a common dimensionless threshold, $Q^* = 41.9$. Figures 11 and 12 show the flow structures induced by undulatory pectoral-fin motion together with surface-pressure contours. The instantaneous force coefficient in Figure 10 provides the corresponding quantitative loading measure at the sampled phases. The vortex visualizations identify the locations and continuity of vortical structures, whereas the force coefficient quantifies the net hydrodynamic loading.

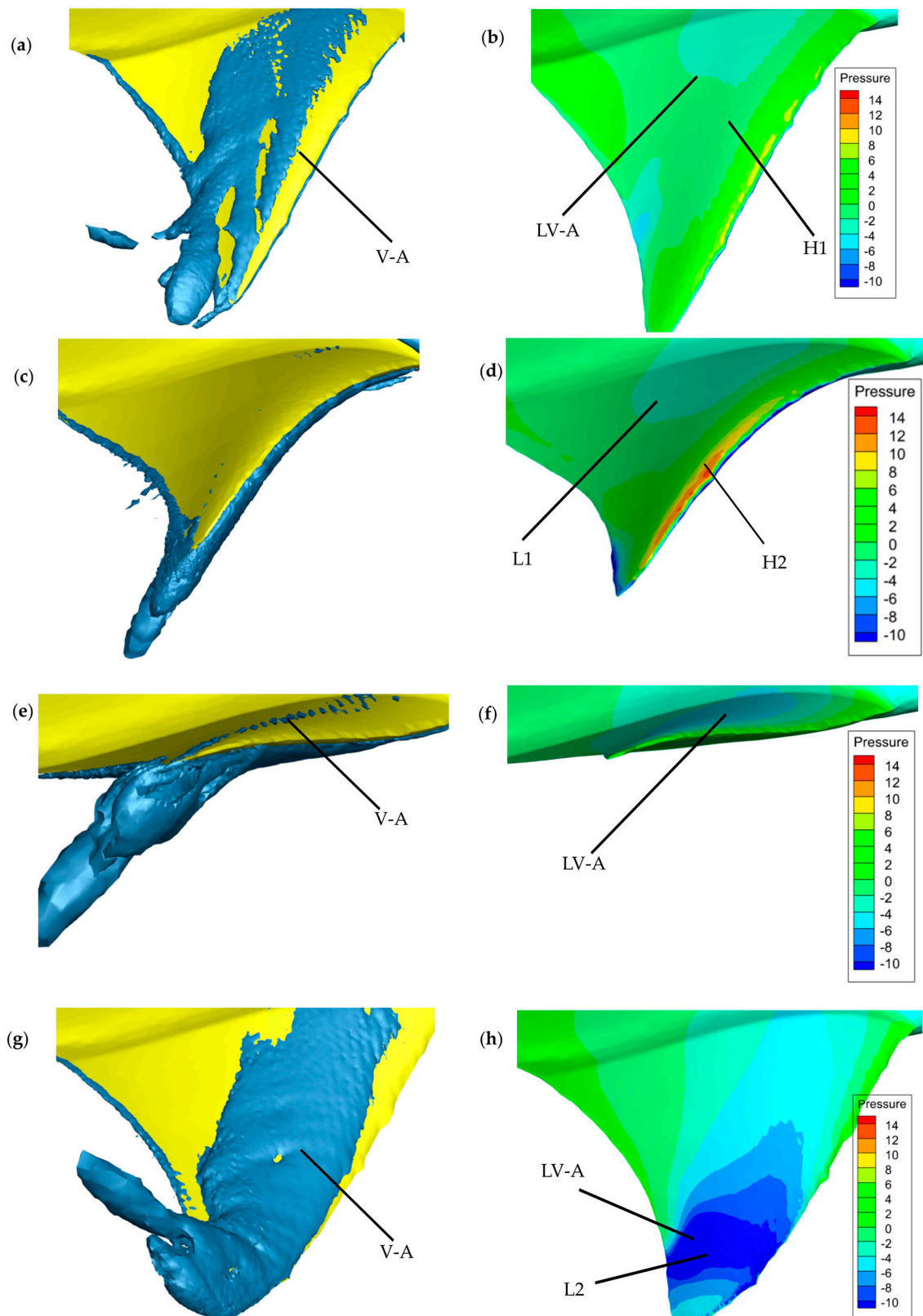


Figure 11. Flow visualization for the fast case at $t/T = 9.0, 9.2, 9.4$, and 9.8 : (a,c,e,g) three-dimensional vortex structures visualized using Q-criterion isosurfaces at the common dimensionless threshold $Q^* = 41.9$; (b,d,f,h) corresponding surface-pressure contours viewed from below. V-A denotes the dorsal surface-attached vortex.

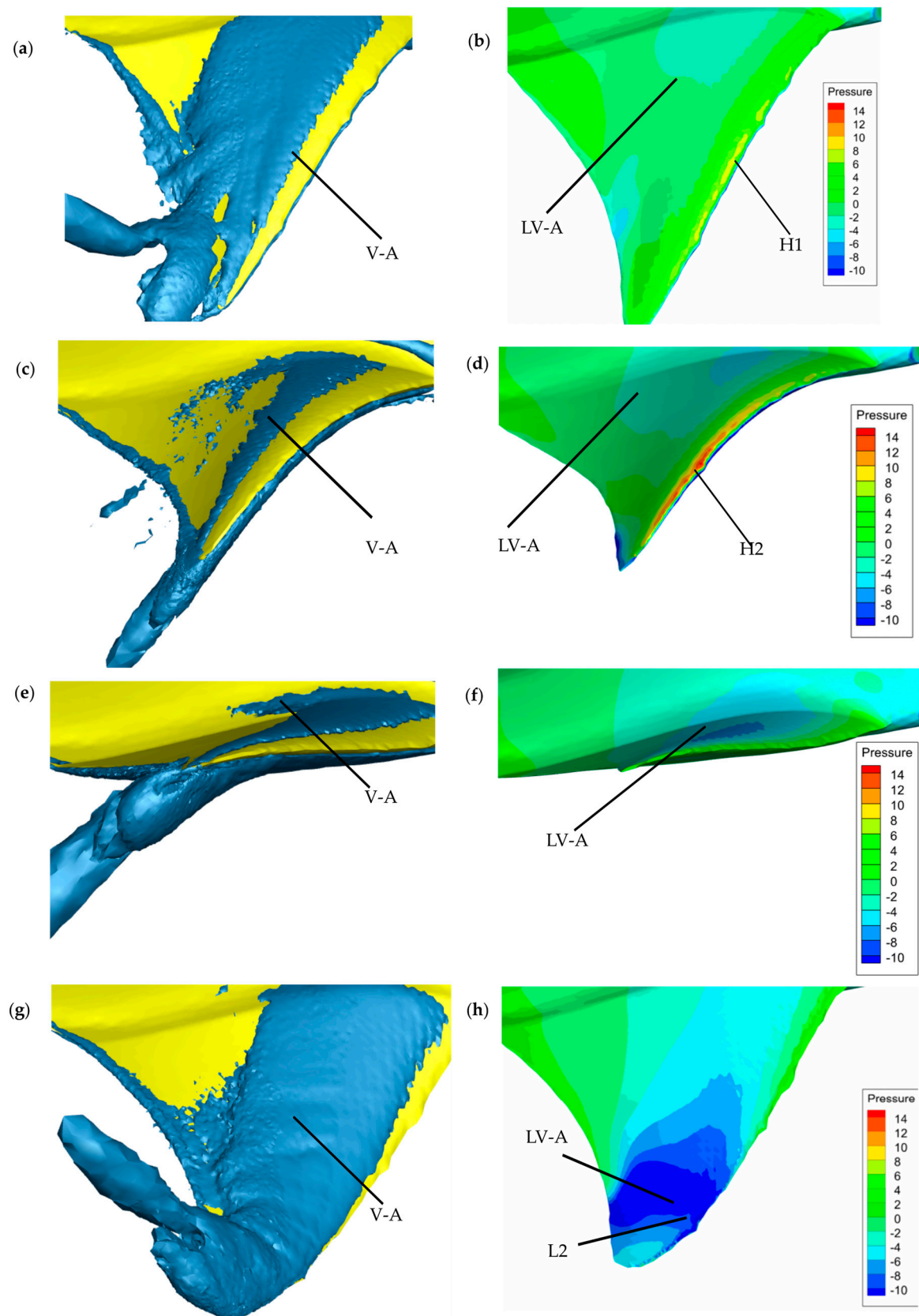


Figure 12. Flow visualization for the slow case at $t/T = 9.0, 9.2, 9.4,$ and 9.8 : (a,c,e,g) three-dimensional vortex structures visualized using Q-criterion isosurfaces at the common dimensionless threshold $Q^* = 41.9$; (b,d,f,h) corresponding surface-pressure contours viewed from below. V-A denotes the dorsal surface-attached vortex.

Figures 11 and 12 identify V-A as a dorsal surface-attached vortex. Its core lies over the dorsal pectoral-fin surface, rather than at the leading edge or tip. The swept, approximately triangular planform motivates only a qualitative comparison with surface-adjacent vortices over swept wings [23]. Because the fin undergoes spanwise flapping and a chordwise travelling wave, the sweep, local incidence, and curvature vary throughout each cycle. V-A is therefore not interpreted as a canonical delta-wing vortex, and its frequency-dependent change in appearance is not equated directly with delta-wing breakdown.

The pressure contours and Q-criterion fields were interpreted together. During high-speed swimming (Figure 11c,e), no continuous V-A structure is visible over the dorsal fin at the selected threshold. During low-speed swimming (Figure 12c,e), a distinct V-A structure and its associated LV-A low-pressure region are visible. These observations distinguish a surface-attached vortical signature from curvature-associated pressure reduction.

At motion cycle commencement ($t/T = 9.0$) marking the transition from the prior downstroke to the present upstroke, (Figures 11a and 12a) demonstrate persistent V-A vortices over the pectoral fin's upper surface shedding into the wake. Simultaneously, Figures 11b and 12b reveal low-pressure zone LV-A growing beneath V-A at the trailing edge, while high-pressure region H1 dominates the leading edge. Under high-speed conditions, the net force vector aligns with the thrust direction; despite substantial inclination angles between LV-A/H1 surfaces and propulsion direction, their restricted spatial extent and intensity yield negligible net thrust. Similarly in low-speed settings, whereas the net force vector opposes propulsion (drag direction), drag magnitude remains minimally consequential.

At motion-cycle phase $t/T = 9.2$, the two conditions differ markedly. Under high-frequency operation, V-A attenuates nearly to extinction, with the resulting low-pressure zone L1 primarily associated with surface curvature. Under low-frequency operation, V-A is associated with a localized low-pressure zone LV-A. In both cases, a high-pressure region denoted H2 is visible in the pressure contours (Figures 11d and 12d). Because an added-mass force decomposition was not performed, the physical origin of H2 cannot be assigned from the present simulations. For the high-frequency case, the resultant force vector aligns with the propulsive direction. Under low-frequency conditions, LV-A is associated with a low-pressure region that increases pressure drag relative to the high-frequency case. Nonetheless, the net resultant force acting on the bio-inspired ray model remains directed in the propulsive direction.

At $t/T = 9.4$, V-A regains intensity under both conditions and forms an associated low-pressure zone LV-A. At high frequency, LV-A forms over the posterior pectoral fin. Its orientation relative to the swimming direction creates a force component in the resistive direction. At low frequency, V-A and the associated LV-A are larger. The anterior LV-A is at a relatively small angle to the swimming direction and has limited effect on thrust generation, whereas the posterior LV-A produces a resistive force component. Consequently, the resultant force at this instant is resistive in both cases (Figures 11f and 12f).

At the time instant ($t/T = 9.6$) during the motion cycle, the pectoral fin initiates the downstroke. Since the vorticity distributions and pressure contours at this instant are similar to those at $t/T = 9.8$, only the instant $t/T = 9.8$ is presented, while $t/T = 9.6$ is omitted from the analysis.

At the time instant ($t/T = 9.8$) inside the motion cycle, the pectoral fin motion is nearing the finish of the downstroke. Under both situations, the V-A vortex begins to progressively migrate backward and starts to shed. An additional low-pressure region, denoted L2, is visible near the fin tip. Its origin cannot be resolved from the present pressure and Q-criterion visualizations alone; therefore, no added-mass attribution is made. At

this phase, the total integrated hydrodynamic force is directed in the propulsive direction (Figures 11h and 12h).

This section has analyzed the vortex formations, pressure distributions, and related forces on the pectoral fin over a whole flapping cycle. The next section examines the influence of prescribed pectoral-fin frequency on the hydrodynamic performance of the bio-inspired ray model.

3.3. Influence of Pectoral Fin Motion Frequency on Hydrodynamic Performance

This study examined the effect of flapping frequency on self-propelled performance using a frequency-only parametric sweep. The prescribed amplitude envelope, chordwise phase distribution, and all other kinematic parameters were held constant across cases; only the flapping frequency was varied. The original low- and high-frequency baseline cases ($f = 0.3$ and 0.6 Hz) were supplemented with three additional cases ($f = 0.2, 0.4$, and 0.5 Hz). The resulting Reynolds and Strouhal numbers are summarized in Table 3. The fluid's kinematic viscosity was sustained at $\nu = 1.0 \times 10^{-6} \text{ m}^2/\text{s}$.

Table 3. Reynolds number and Strouhal number for five self-propelled simulations.

Case	f (Hz)	Re	St	$\bar{U}$
1	0.2	10,032.9	0.518	0.1930
2	0.3	16,005.9	0.487	0.3079
3	0.4	27,795.8	0.374	0.5347
4	0.5	39,398.6	0.329	0.7579
5	0.6	50,814.3	0.306	0.9775

Upon reaching quasi-steady self-propulsion, the bio-inspired ray model shows a pronounced linear correlation between mean swimming speed and prescribed undulatory frequency f , as illustrated in Figure 13. The mean swimming speed is represented as a function of frequency as follows:

$$\bar{U} = a_0 + a_1 f \quad (22)$$

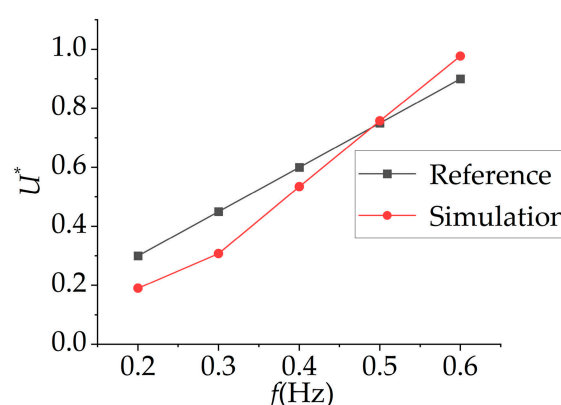


Figure 13. Average swimming velocities of the bio-inspired ray model under different prescribed undulatory frequencies. Published manta data from Fish et al. [10] are shown as a qualitative literature comparison only and are not used for species-specific validation.

Here, $\bar{U}$ is defined as the mean swimming speed, and f denotes the frequency of undulatory motion.

Least-squares fitting produced coefficients $a_0 = -0.2534$ and $a_1 = 2.0190$, with $R^2 = 0.9890$, indicating that each 0.1 Hz increment in the prescribed undulatory frequency

increases model speed by approximately 0.2019 body lengths per second. Existing empirical equations were derived from manta specimens with different geometries and kinematics, whereas the present model uses geometry reconstructed from *R. javanica* and an imposed manta-like waveform. Figure 14 delineates the relationship between the mean thrust coefficient T^* and frequency f . Strouhal number (St) values manifest tightly clustered distributions; the Strouhal number is a reliable statistic for defining the average thrust coefficient (Table 3, Figure 15).

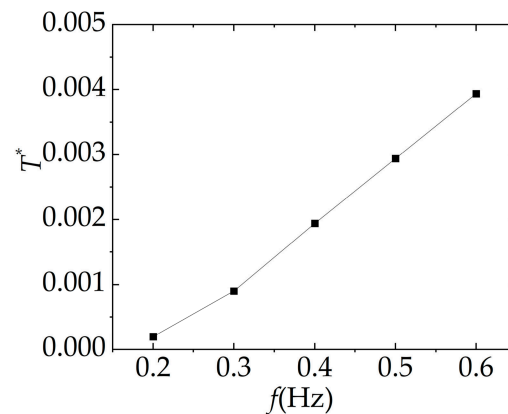


Figure 14. Average thrust coefficients under different undulatory frequencies.

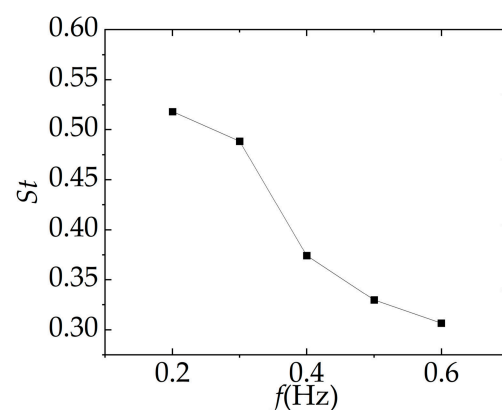


Figure 15. Strouhal number under different undulatory frequencies.

Figure 16 shows that the mean dimensionless power loss coefficient increased with flapping frequency, from 0.00035 at 0.2 Hz to 0.00437 at 0.6 Hz. This increase reflects the greater hydrodynamic work required to impose the higher-frequency fin motion. In contrast, Froude efficiency η_f exhibits an ascent from 45.3% (0.2 Hz) to 47.4% (0.6 Hz), yielding a marginal cumulative gain of 2.1%.

To assess numerical behavior under the imposed waveform, self-propelled kinematic and hydrodynamic metrics were compared with published empirical, biological, and numerical data (Table 4). These cross-study comparisons provide order-of-magnitude context for the simulated flow and rigid-body response.

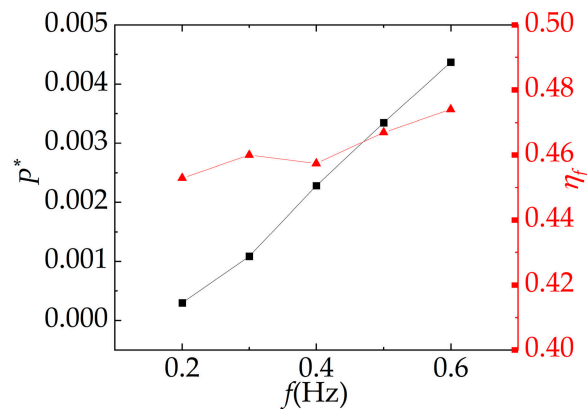


Figure 16. Average power-loss coefficient Froude efficiency under different undulatory frequencies.

Table 4. Comparison of the present model with published biological, experimental, and numerical data.

Parameter/Metric	Present Simulation (This Work)	Published Biological/Experimental Data	Reference
Swimming speed vs. frequency $\bar{U}$	$\bar{U} = -0.2534 + 2.0190f$	Figure 13 black line	Fish et al. [10]
Strouhal number (St)	0.306–0.374 for $f = 0.4$ – 0.6 Hz	0.20–0.40	Triantafyllou et al. [19], Eloy [2]
Froude propulsive efficiency (η_f)	47.4% (high frequency)	48% for a carangiform swimmer; 22.9% and 34.1% for slow- and fast-swimming stingrays, respectively	Borazjani & Sotiropoulos [17], Bottom et al. [5]
Role of (LEV)	Enhances thrust on the anterior pectoral fin at high speeds/frequencies	Intensifies low-pressure zones and total thrust during high-speed swimming	Bottom et al. [5]

3.4. Influence of Pectoral Fin Motion Frequency on Vortex Structures

The same dimensionless Q-criterion visualization protocol was used to examine the effect of undulatory frequency on vortex structure (Figure 17). Table 5 shows a qualitative change in the dorsal surface-attached vortex V-A with increasing frequency. The structure appears more continuous at low frequency and less continuous at high frequency. As summarized in Table 5, C_A decreased monotonically from 75.3% at 0.3 Hz to 2.1% at 0.6 Hz, indicating a progressive loss of dominance of the largest connected V-A structure and increased fragmentation at higher frequencies.

Table 5. Quantitative connected-component metrics of the surface-attached V-A vortex at the matched phase $t/T = 9.2$.

Frequency, f (Hz)	0.3	0.4	0.5	0.6
Largest-component volume, V_{tot}^*	5.11×10^{-6}	4.92×10^{-6}	2.15×10^{-6}	0.98×10^{-6}
Component dominance ratio, C_A	75.3%	65.2%	32.4%	2.1%

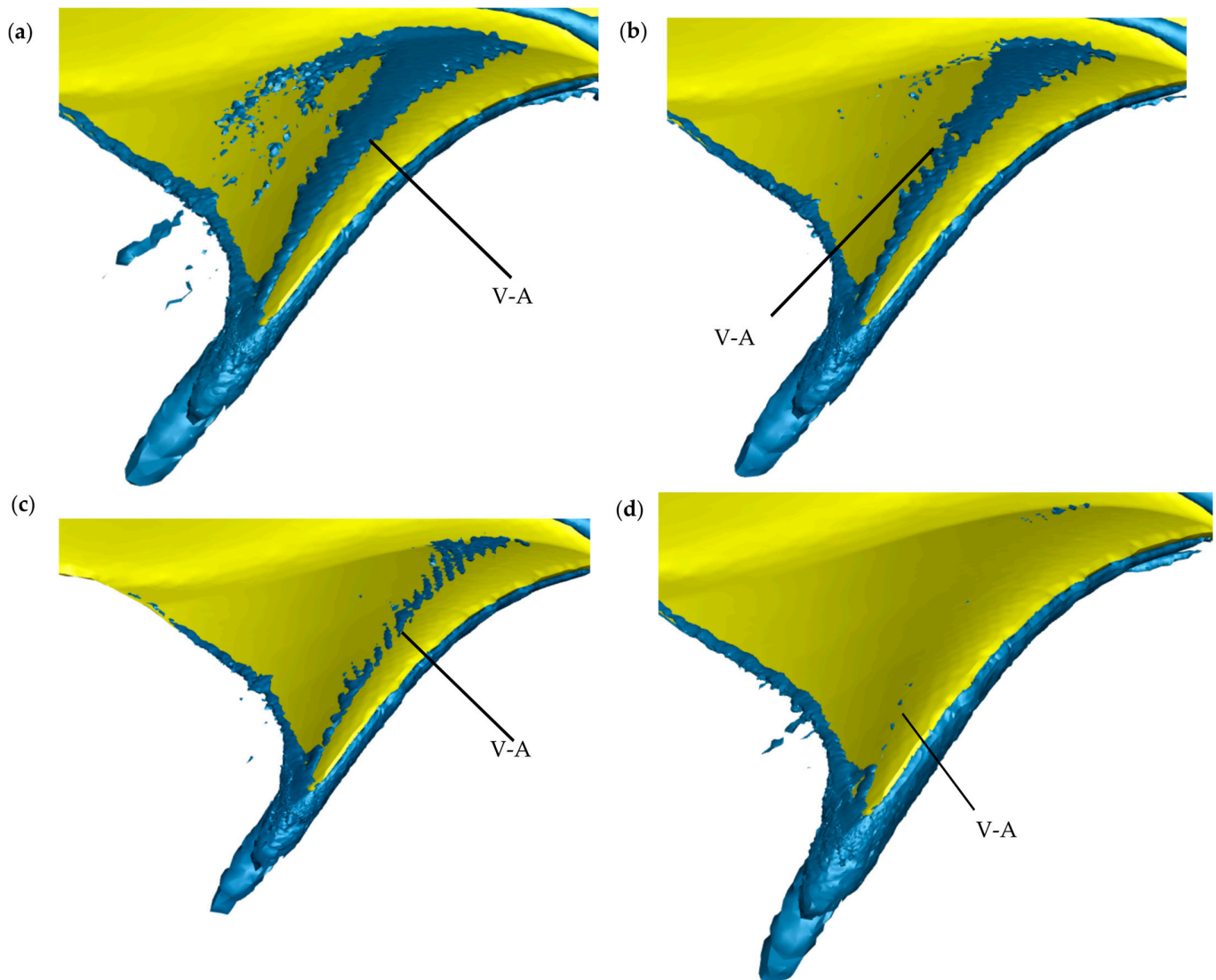


Figure 17. Three-dimensional vortex structures visualized using Q-criterion isosurfaces at a common dimensionless threshold, viewed from above the bio-inspired ray model at $t/T = 9.2$: (a) 0.3 Hz; (b) 0.4 Hz; (c) 0.5 Hz; (d) 0.6 Hz. V-A denotes the dorsal surface-attached vortex.

The imposed manta-like waveform and the reconstructed ray-like planform govern the simulated flow-field development. The prescribed motion has minimal amplitude near the anterior body section, waves that propagate toward the trailing edge, and progressively amplified oscillations from mid-fin to trailing edge. This waveform, together with the planform geometry, is associated with the surface-attached V-A vortex in the proximal fin region.

High-frequency undulation substantially enhances surface hydrodynamic characteristics of manta. Figure 18a–d validate that elevated frequencies drastically reduce dimensionless surface shear stress in V-A vortex domains, consequently amplifying net thrust with frequency. The scale transition of V-A vortices from macro- to micro-scale configurations induces dichotomous energy transfer: although enhanced drag reduction increases thrust, vortex miniaturization raises the hydrodynamic power loss associated with a given fin motion, ultimately causing gains in Froude efficiency to plateau—even as the oscillation frequency increases.

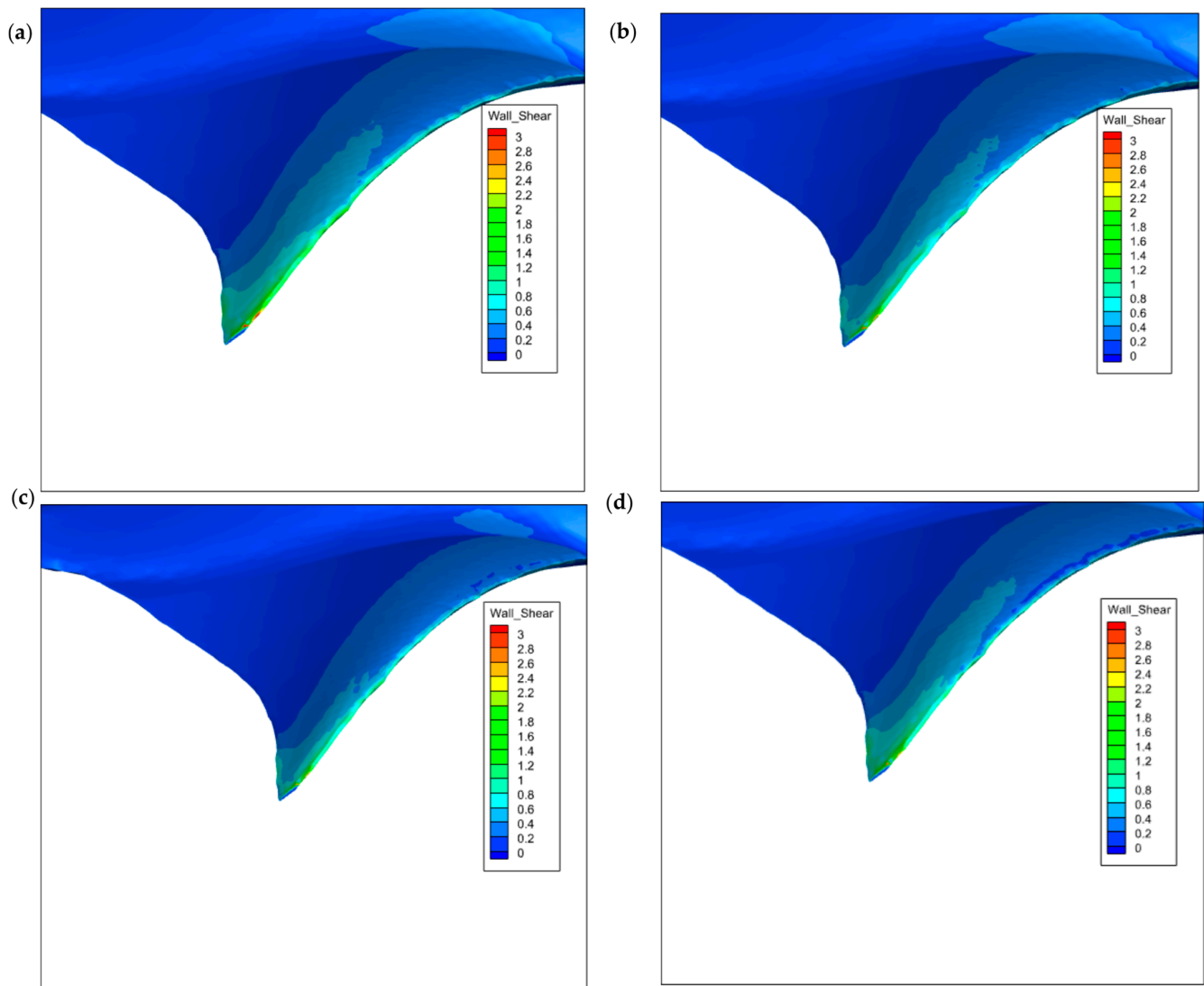


Figure 18. Instantaneous wall shear for four cases: surface shear stress contours viewed on the bio-inspired ray-model surface at $t/T = 9.2$; (a) 0.3 Hz; (b) 0.4 Hz; (c) 0.5 Hz; (d) 0.6 Hz.

4. Discussion

Previous biological observations suggest that free-swimming manta rays may co-vary flapping frequency and heave amplitude, and they often operate near the commonly cited Strouhal number (St) interval of 0.2–0.4 [20,24]. Accordingly, the simulated St values span 0.306–0.518, with the two lowest-frequency cases lying outside the 0.2–0.4 interval.

For consistency, the present value is compared only with prescribed-kinematics numerical studies that use the same hydrodynamic Froude efficiency definition. Borazjani and Sotiropoulos [17] reported a Froude efficiency of approximately 48% for a carangiform swimmer. In a separate stingray study, Bottom et al. [5] reported Froude efficiencies of 22.9% and 34.1% for the slow- and fast-swimming cases, respectively. The maximum value obtained here, 47.4%, is therefore comparable at the level of hydrodynamic energy transfer under imposed kinematics. It is not a whole-animal or robotic propulsive efficiency, because the denominator excludes muscle or actuator power, transmission losses, internal damping, and elastic energy storage and recovery. Differences in morphology, Reynolds number, Strouhal number, and imposed kinematics further limit quantitative cross-study comparisons. A coupled structural and actuator model with measured viscoelastic properties would be required to determine overall propulsive efficiency.

Species and kinematic transferability. The CFD geometry was reconstructed from a Javan cownose ray (*Rhinoptera javanica*), whereas the imposed fin waveform was parameterized from published manta/mobuliform observations. These inputs come from different batoid groups and may differ in planform, stiffness distribution, mass distribution, and natural fin motion. We therefore interpret the present calculations only as the hydrodynamic response of a hybrid bio-inspired ray model. They do not support species-specific conclusions for manta rays or *R. javanica*. Species-specific predictions would require geometry and free-swimming kinematics measured from the same target species.

The body was constrained to streamwise translation, so the simulations exclude dorsoventral heave and pitch. These motions would change the instantaneous relative velocity and effective angle of attack of the pectoral fins. They could therefore alter the location, magnitude, and phase of the surface-pressure extrema and the resulting thrust and power. This omission may be particularly relevant for the lower-Strouhal-number cases, although the present simulations do not support a quantitative error bound. The reported pressure distributions, thrust, and Froude efficiencies should therefore be interpreted as conditional on zero body pitch and zero dorsoventral heave. Simulations incorporating biologically measured pitch and heave amplitudes and phases are required to determine the magnitude and direction of this effect.

The reconstructed cownose-ray geometry and prescribed manta-like waveform provide a useful starting point for bio-inspired robotic design. Future work should combine species-matched geometry and kinematics with motion capture and a deformable structural solver [25–27].

Although the present calculations consider quiescent water, batoid fishes and bio-inspired vehicles operate in environments with ambient currents and unsteady waves. Numerical studies of wave-current and focused-wave loading show that changes in the surrounding flow can substantially modify unsteady hydrodynamic loads and body motions [28–31]. For a moving fin, the associated changes in local relative velocity are expected to modify the surface-pressure distribution from which thrust is inferred. Incoming-flow disturbances may also modify the formation, convection, and surface-pressure signature of the dorsal V-A vortex, and may consequently shift the instantaneous thrust-drag balance and power demand. These effects cannot be quantified from the present quiescent-water simulations. Future work should therefore couple the prescribed fin motion to controlled uniform-current, wave-current, and transient-wave inflows to determine the robustness of V-A dynamics and propulsive performance under representative operating conditions.

5. Conclusions

This study uses self-propelled numerical simulations to analyze the hydrodynamic response of a bio-inspired ray model. The CFD geometry was reconstructed from a Javan cownose ray (*Rhinoptera javanica*), whereas the prescribed pectoral-fin waveform was parameterized from published manta/mobuliform studies. The simulations predict quasi-steady speed, thrust, power loss, and Froude efficiency for this specific combination of geometry and imposed motion. Both baseline conditions showed similar temporal changes in velocity, thrust, and instantaneous power loss, while their Froude efficiencies differed by approximately 1%.

For bio-inspired robotic design, the dimensionless metrics may be mapped to physical scales corresponding to the scanned cownose-ray specimen. Such values are design references for the present hybrid model. Species-specific robotic design would require geometry, mass, and motion measurements from the target species.

These conclusions are limited to the prescribed manta-like kinematics and streamwise rigid-body translation of the bio-inspired ray model. The use of cownose-ray geometry

with manta/mobuliform kinematics is an additional transferability limitation. Future species-specific conclusions require matched morphology and kinematic measurements, together with structural and fluid–structure-interaction modeling.

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Abbreviations

The following abbreviations are used in this manuscript:

FSI	Fluid–Solid Coupling
DPIV	Digital Particle Image Velocimetry
CFD	Computational Fluid Dynamics

Appendix A. Validation Using a Flexible Plate in Uniform Flow

To assess the capability of the numerical framework for handling a large deforming fluid–solid interface, the flexible-plate benchmark of Luhar and Nepf [32] was considered. This experiment describes the flow-induced reconfiguration of a buoyant, vertically mounted flexible plate in a uniform water flow. The configuration was selected because it provides experimental measurements of the deformed plate shape, free-end deflection, and hydrodynamic drag, and therefore permits simultaneous assessment of the fluid solution, structural response, interface data transfer, and mesh deformation procedure.

The benchmark consists of a rectangular elastic plate clamped at its lower end and exposed to a uniform incoming flow in the streamwise x direction. The y direction is spanwise and the z direction is vertical. All parameters reproduce the reference configuration used in the experimental study and in the three-dimensional numerical replication [32].

The fluid motion (Figure A1) was solved using the incompressible Navier–Stokes equations, while the plate deformation was obtained from the structural momentum equation for an elastic solid. The same dynamic-mesh settings and numerical procedure as those used for the bio-inspired ray-model simulations were employed.

The numerical results were compared with the experimental data of Luhar and Nepf [32] using three quantities: the streamwise free-end deflection D_x/b , the vertical free-end displacement magnitude D_z/b , and the drag coefficient C_D .

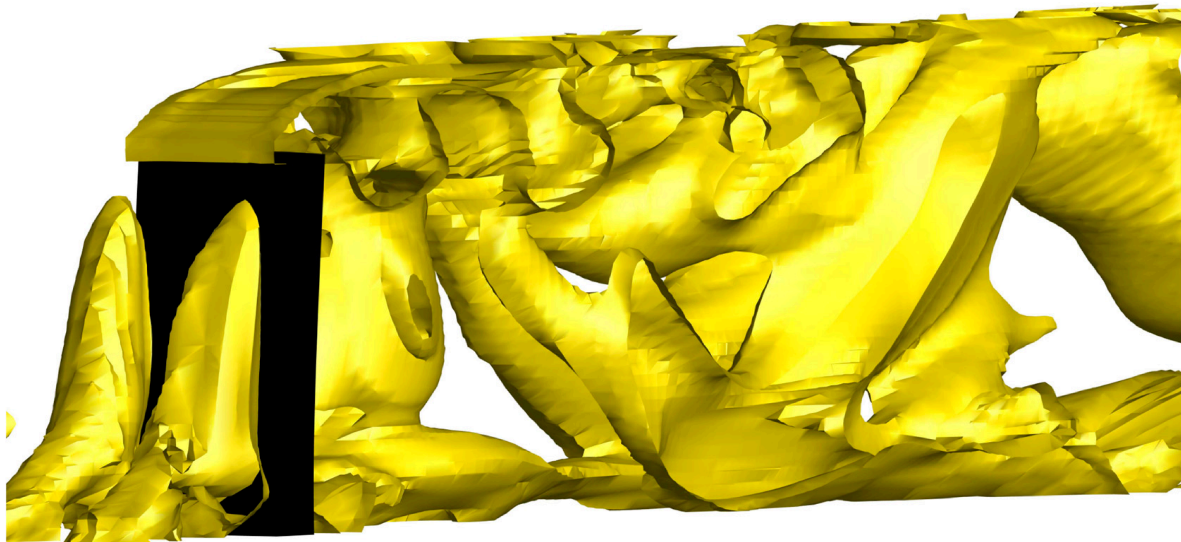


Figure A1. Bending of a flexible plate (black) in cross flow: instantaneous vortical structures (yellow).

Table A1. Validation quantitative comparison between the present simulation and the experimental measurements of Luhar and Nepf for the flexible plate in uniform flow.

Quantity	Present Simulation (This Work)	Experimental Data	Relative Deviation
D_x/b	2.01	2.14	6.1%
D_z/b	0.57	0.59	3.4%
C_D	1.08	1.15	6.1%

Table A1 presents a quantitative comparison between the present numerical results and the experimental measurements of Luhar and Nepf for a flexible plate in uniform flow. The predicted normalized streamwise free-end deflection, vertical free-end displacement, and drag coefficient are 2.01, 0.57, and 1.08, respectively, compared with the experimental values of 2.14, 0.59, and 1.15. The corresponding relative deviations are 6.1%, 3.4%, and 6.1%, respectively.

All numerical predictions are slightly lower than the experimental values, indicating a small underprediction of both the flow-induced deformation and hydrodynamic load. Nevertheless, the maximum deviation is only 6.1%, while the deviation in drag coefficient is 6.1%. The numerical predictions are therefore in good agreement with the experimental measurements.

Appendix B. Gravity-Driven Settling of a Sphere

To validate the force-to-motion coupling used in the self-propelled simulations, two gravity-driven sphere-settling cases were considered. The benchmark configuration followed the experiments of Ten Cate et al. [33]. A rigid sphere with a diameter of 0.015 m and a density of 1120 kg/m³ was released from rest along the centerline of a rectangular container measuring 0.10 m × 0.10 m × 0.16 m. The initial sphere position was 0.12 m above the bottom of the container, and gravitational acceleration was set to 9.81 m/s².

Cases E1 and E4 of Ten Cate et al. [33] were selected to examine the coupling method under two particle Reynolds numbers. In Case E1, the fluid density and dynamic viscosity were 970 kg/m³ and 0.373 Pa · s, respectively. In Case E4, the corresponding values were 960 kg/m³ and 0.058 Pa · s. The reference settling velocities in an unbounded fluid were 0.038 and 0.128 m/s, corresponding to particle Reynolds numbers of 1.5 and 31.9,

respectively. No-slip boundary conditions were imposed on all container walls. The flow was treated as laminar in both cases.

At each time step, the hydrodynamic force was obtained by integrating the pressure and viscous stresses over the sphere surface. The resulting acceleration was calculated from Newton's second law. The sphere velocity and position were then updated using the same time-integration and dynamic-mesh procedures employed in the self-propelled bio-inspired ray-model simulations. Only vertical translation was permitted. Table A2 summarizes the physical and numerical parameters of the two validation cases.

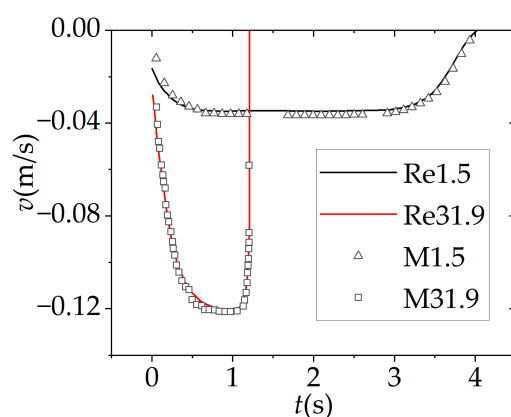


Figure A2. Comparison of the present numerical predictions with the experimental measurements of Ten Cate et al. [33]. Symbols denote the experimental measurements digitized from Figure 8 of Ten Cate et al., while solid lines denote the present numerical results.

Table A2. Parameters and validation results for the gravity-driven settling-sphere benchmark.

Case	ρ_f (kg/m ³)	μ_f (Pa · s)	$v_{t,num}$ (m/s)	$v_{t,exp}$ (m/s)	Re	E_{v_t} (%)
Case1	970	0.373	−0.0352	−0.0360	1.5	2.22
Case4	960	0.058	−0.1209	−0.1222	31.9	1.06

Figure A2 compares the calculated sphere trajectories and settling-velocity histories with the experimental measurements of Ten Cate et al. [33]. The experimental data were digitized from Figure 8 of the reference. Only the measured data, denoted by M in the original figure, were used for quantitative validation.

For Case 1, the calculated maximum settling velocity was −0.0352 m/s, compared with the experimental value of −0.0360 m/s. The corresponding relative error was 2.22%. For Case 4, the calculated maximum settling velocity was −0.1209 m/s, compared with the experimental value of −0.1222 m/s. The corresponding relative error was 1.06%.

The calculated velocity histories reproduced the experimentally observed acceleration, maximum-velocity stage, and deceleration during the approach to the bottom wall.

References

- Shoele, K.; Zhu, Q. Numerical Simulation of a Pectoral Fin during Labriform Swimming. *J. Exp. Biol.* **2010**, *213*, 2038–2047. [CrossRef] [PubMed]
- Eloy, C. Optimal Strouhal Number for Swimming Animals. *J. Fluids Struct.* **2012**, *30*, 205–218. [CrossRef]
- Rosenberger, L.J. Pectoral Fin Locomotion in Batoid Fishes: Undulation Versus Oscillation. *J. Exp. Biol.* **2001**, *204*, 379–394. [CrossRef] [PubMed]
- Javanmardi, N.; Ghadimi, P. Hydroelastic Analysis of Surface-Piercing Propeller through One-Way and Two-Way Coupling Approaches. *Proc. Inst. Mech. Eng. Part M J. Eng. Marit. Environ.* **2019**, *233*, 844–856. [CrossRef]
- Bottom, R.G., II; Borazjani, I.; Blevins, E.L.; Lauder, G.V. Hydrodynamics of Swimming in Stingrays: Numerical Simulations and the Role of the Leading-Edge Vortex. *J. Fluid Mech.* **2016**, *788*, 407–443. [CrossRef]

6. Marshall, A.D.; Compagno, L.J.V.; Bennett, M.B. Redescription of the Genus *Manta* with Resurrection of *Manta Alfredi* (Krefft, 1868) (Chondrichthyes; Myliobatoidei; Mobulidae). *Zootaxa* **2009**, *2301*, 1–28. [\[CrossRef\]](#)
7. Fontanella, J.E.; Fish, F.E.; Barchi, E.I.; Campbell-Malone, R.; Nichols, R.H.; DiNenno, N.K.; Beneski, J.T. Two- and Three-Dimensional Geometries of Batoids in Relation to Locomotor Mode. *J. Exp. Mar. Biol. Ecol.* **2013**, *446*, 273–281. [\[CrossRef\]](#)
8. Russo, R.S.; Blemker, S.S.; Fish, F.E.; Bart-Smith, H. Biomechanical Model of Batoid (Skates and Rays) Pectoral Fins Predicts the Influence of Skeletal Structure on Fin Kinematics: Implications for Bio-Inspired Design. *Bioinspir. Biomim.* **2015**, *10*, 046002. [\[CrossRef\]](#) [\[PubMed\]](#)
9. Pazzaglia, U.E.; Reguzzoni, M.; Terova, G.; Serena, F.; Mancusi, C.; Zarattini, G.; Zecca, P.A. Growth and Mechanical Correlations of Calcified Cartilage in BATOIDEA: A Histomorphological Study Using the *Raja asterias* Model. *J. Fish Biol.* **2025**, *107*, 23–33. [\[CrossRef\]](#) [\[PubMed\]](#)
10. Fish, F.; Schreiber, C.; Moored, K.; Liu, G.; Dong, H.; Bart-Smith, H. Hydrodynamic Performance of Aquatic Flapping: Efficiency of Underwater Flight in the Manta. *Aerospace* **2016**, *3*, 20. [\[CrossRef\]](#)
11. Lighthill, J. Aerodynamic Aspects of Animal Flight. In *Swimming and Flying in Nature*; Wu, T.Y.-T., Brokaw, C.J., Brennen, C., Eds.; Springer: Boston, MA, USA, 1975; pp. 423–491.
12. Fish, F.E.; Dong, H.; Zhu, J.J.; Bart-Smith, H. Kinematics and Hydrodynamics of Mobuliform Swimming: Oscillatory Winged Propulsion by Large Pelagic Batoids. *Mar. Technol. Soc. J.* **2017**, *51*, 35–47. [\[CrossRef\]](#)
13. Su, G.; Shen, H.; Li, N.; Zhu, Y.; Su, Y. Numerical Investigation of the Hydrodynamics of Stingray Swimming under Self-Propulsion. *J. Fluids Struct.* **2021**, *106*, 103383. [\[CrossRef\]](#)
14. Bozkurttas, M.; Mittal, R.; Dong, H.; Lauder, G.V.; Madden, P. Low-Dimensional Models and Performance Scaling of a Highly Deformable Fish Pectoral Fin. *J. Fluid Mech.* **2009**, *631*, 311–342. [\[CrossRef\]](#)
15. Menzer, A.; Gong, Y.; Fish, F.E.; Dong, H. Bio-Inspired Propulsion: Towards Understanding the Role of Pectoral Fin Kinematics in Manta-like Swimming. *Biomimetics* **2022**, *7*, 45. [\[CrossRef\]](#) [\[PubMed\]](#)
16. Gladston, Y.; Akhilesh, K.V.; Thakurdas, C.; Ravi, O.P.K.; Ajina, S.M.; Shenoy, L. Length–Weight Relationship of Selected Elasmobranch Species from North-eastern Arabian Sea, India. *J. Appl. Ichthyol.* **2018**, *34*, 753–757. [\[CrossRef\]](#)
17. Borazjani, I.; Sotiropoulos, F. Numerical Investigation of the Hydrodynamics of Anguilliform Swimming in the Transitional and Inertial Flow Regimes. *J. Exp. Biol.* **2009**, *212*, 576–592. [\[CrossRef\]](#) [\[PubMed\]](#)
18. Triantafyllou, G.S.; Triantafyllou, M.S.; Grosenbaugh, M.A. Optimal Thrust Development in Oscillating Foils with Application to Fish Propulsion. *J. Fluids Struct.* **1993**, *7*, 205–224. [\[CrossRef\]](#)
19. Taylor, G.K.; Nudds, R.L.; Thomas, A.L.R. Flying and Swimming Animals Cruise at a Strouhal Number Tuned for High Power Efficiency. *Nature* **2003**, *425*, 707–711. [\[CrossRef\]](#) [\[PubMed\]](#)
20. Rohr, J.J.; Fish, F.E. Strouhal Numbers and Optimization of Swimming by Odontocete Cetaceans. *J. Exp. Biol.* **2004**, *207*, 1633–1642. [\[CrossRef\]](#) [\[PubMed\]](#)
21. Borazjani, I.; Sotiropoulos, F. On the Role of Form and Kinematics on the Hydrodynamics of Self-Propelled Body/Caudal Fin Swimming. *J. Exp. Biol.* **2010**, *213*, 89–107. [\[CrossRef\]](#) [\[PubMed\]](#)
22. Triantafyllou, M.S.; Triantafyllou, G.S. An Efficient Swimming Machine. *Sci. Am.* **1995**, *272*, 64–70. [\[CrossRef\]](#)
23. Hu, T.; Zhao, Y.; Liu, P.; Qu, Q.; Guo, H.; Akkermans, R.A.D. Investigation on Lift Characteristics of a Double-Delta Wing Pitching in Various Reduced Frequencies. *Proc. Inst. Mech. Eng. Part G J. Aerosp. Eng.* **2021**, *235*, 2081–2094. [\[CrossRef\]](#)
24. Tytell, E.D.; Lauder, G.V. The Hydrodynamics of Eel Swimming: I. Wake Structure. *J. Exp. Biol.* **2004**, *207*, 1825–1841. [\[CrossRef\]](#) [\[PubMed\]](#)
25. Cardoso De Brito, M.; Sutherland, L.S.; Pereira, J.M.C.; Arruda, M.R. Fluid-Structure Interaction Analyses for Hydro-Elastic Tailoring of a Windsurfer Fin. *J. Mar. Sci. Eng.* **2022**, *10*, 1371. [\[CrossRef\]](#)
26. Kern, S.; Koumoutsakos, P. Simulations of Optimized Anguilliform Swimming. *J. Exp. Biol.* **2006**, *209*, 4841–4857. [\[CrossRef\]](#) [\[PubMed\]](#)
27. Kravchenko, A.G.; Moin, P. Numerical studies of flow over a circular cylinder at $Re = 3900$. *Phys. Fluids* **2000**, *12*, 403–417. [\[CrossRef\]](#)
28. Gao, J.; Mi, C.; Song, Z.; Ma, X. Hydrodynamics of transient gap resonance between a heaving box and a vertical wall excited by focused wave groups. *Ocean Eng.* **2026**, *357*, 125617. [\[CrossRef\]](#)
29. Gao, J.; Song, Z.; He, M. Influences of low velocity uniform current on characteristics of gap resonance occurring between two adjacent fixed bodies. *Mar. Struct.* **2026**, *106*, 103961. [\[CrossRef\]](#)
30. Mi, C.; Gao, J.; Song, Z.; Liu, Y. Hydrodynamic wave forces on two side-by-side barges subjected to nonlinear focused wave groups. *Ocean Eng.* **2025**, *317*, 120056. [\[CrossRef\]](#)
31. Gong, S.; Gao, J.; Song, Z.; Shi, H.; Liu, Y. Hydrodynamics of fluid resonance in a narrow gap between two boxes with different breadths. *Ocean Eng.* **2024**, *311*, 118986. [\[CrossRef\]](#)

32. Luhar, M.; Nepf, H.M. Flow-Induced Reconfiguration of Buoyant and Flexible Aquatic Vegetation. *Limnol. Oceanogr.* **2011**, *56*, 2003–2017. [[CrossRef](#)]
33. Ten Cate, A.; Nieuwstad, C.H.; Derksen, J.J.; Van den Akker, H.E.A. Particle imaging velocimetry experiments and lattice-Boltzmann simulations on a single sphere settling under gravity. *Phys. Fluids* **2002**, *14*, 4012–4025. [[CrossRef](#)]

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