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Communication

Gain Saturation Modified Quantum Noise Effect on Preparing a Continuous-Variable Entanglement

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Abstract: We examine the gain saturation effect in non-Hermitian systems of coupled gain–loss waveguides and whispering-gallery-mode microresonators, through which a continuous-variable (CV) entanglement of light fields is generated. Here, we consider squeezed vacuum inputs for coupled waveguide setup and coherent drive for coupled microresonators, and study the influence from the saturation of the used optical gain. Unlike the ideal situation without gain saturation, it is possible to generate stabilized entanglement measured by logarithmic negativity under gain saturation. Both types of setups realize steady CV entanglement, provided that the gain saturation is sufficiently quick. Particularly, with the coupled microresonators which are pumped by coherent drive, the created CV entanglement is actually out of the gain noise with a squeezing characteristic, under the condition of fast saturation of the initial optical gain.

Keywords: continuous-variable entanglement; non-Hermitian systems; quantum noise

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1. Introduction

Except for the purely linear optical schemes (see, e.g., [1–7]), the systems of quantum information processing mostly require various types of nonlinearity, but the ubiquitous existence of quantum noise [8] always reduces and even impairs their effectiveness. Gain saturation is one of the common types of nonlinearity in photonics, which is an inherent property of optical gain. If an optical gain is approximated to be constant, one will have the non-Hermitian optical systems, such as coupled gain–loss waveguides [9–12] and microcavities [13,14], to be categorized with a so-called parity-time (PT) symmetry [15] by balancing the involved gain and loss. Such structure can be generalized to those of multiple components (see, e.g., [16–19]). However, in reality, the optical gain drops at a certain pace due to its saturation, so that the amplified light intensity does not increase forever and finally reaches a steady state. Although the saturation effect has been investigated in several classical non-Hermitian optical systems [20–25], its dynamical features in quantum regimes for nonclassical light were less studied in the past. Especially, it was shown that continuous-variable (CV) entanglement is vulnerable to the quantum noise associated with amplification [26,27], and one thus estimates that the gain saturation can significantly influence the generation of entanglement by the relevant methods.

So far, there have been many studies about the non-Hermitian optical systems with balanced constant gain and loss, in which the light is treated as a classical electromagnetic field and all quantum dissipations are modeled in a non-Hermitian effective Hamiltonian [28]. In this paper, however, we adopt a fully quantum mechanical approach to examine the gain saturation effect on the CV entanglement in a coupled gain–loss systems in terms of the inevitable quantum noises [8]. To capture the main features, for the setup in Figure 1, we only consider the saturation of the gain medium and assume that the loss medium has a constant damping rate. Then, we study the time evolution of entanglement in situations where the initial (non-saturated) gain is greater/less than the loss rate or equal to it. In the presence of the associated quantum noise, a constant gain rate leads to entanglement sudden death [29], but its saturation can lead to a steady entangled state by reducing the detrimental quantum noise effect. Moreover, under a coherent drive, the CV entanglement can be created purely by the amplification noise under the condition of quick saturation of the optical gain.

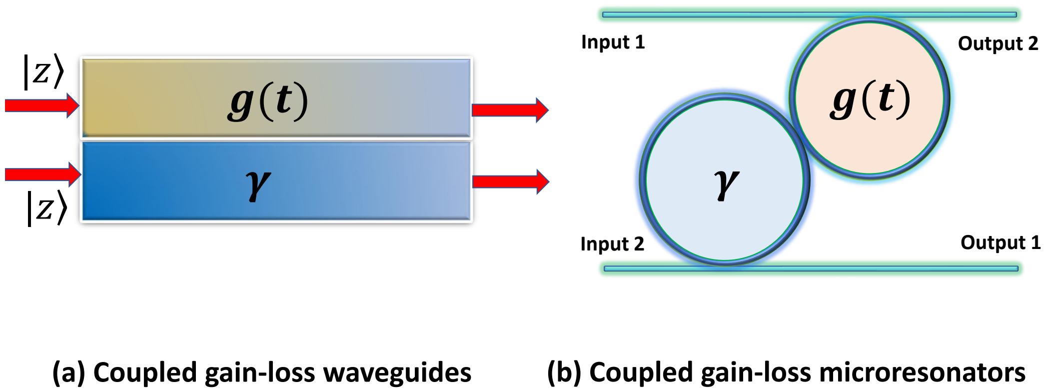

Figure 1. Exemplary setups, as the coupled waveguides and the coupled microresonators driven by a coherent field, which carry gain–loss media. The inputs $|z\rangle$ are squeezed vacuum or squeeze coherent states. The gain, loss, and coupling are denoted by g, γ, and J, respectively.

2. System

We study two different types of systems of gain–loss media—the first is coupled waveguides whose inputs are squeezed vacuum states, and the other is coupled whispering-gallery-mode microresonators driven by a coherent light (see Figure 1). The waveguide (microresonator) A (B) carries a saturable (non-saturable) gain (loss) medium with a gain (loss) coefficient $g(t)$ (γ), and the light field with its operator $\hat{a}(\hat{b})$ and frequency ω_0 propagates (circulates) inside. The waveguides (microresonators) are coupled via evanescent waves so that the coupling strength J can be adjusted by the gap distance. In the situations of coupled waveguides, we also neglect the effect of evanescent propagation [30,31] of the coupled fields.

The system dynamics is mostly described by the non-Hermitian effective Hamiltonian ($\hbar = 1$) [28]:

$$H_{\text{eff}} = ig(t)\hat{a}^\dagger\hat{a} - i\gamma\hat{b}^\dagger\hat{b} + J(\hat{a}\hat{b}^\dagger + \hat{a}^\dagger\hat{b}) + iE_0(\hat{a}^\dagger e^{i\Delta t} - \hat{a}e^{-i\Delta t}), \quad (1)$$

where the first two terms describe the amplification and dissipation of the light fields in the waveguides A and B, and the third term is from their coupling, and the last term represents the coherent drive field with amplitude E_0 and detuning Δ from the resonant frequency ω_0 (this term is zero for the waveguide setup). The corresponding Heisenberg equation of motion determines the time evolution of the fields $\hat{a}$ and $\hat{b}$. In this approach the involved quantum noises are neglected.

Since the concerned CV entanglement is highly sensitive to quantum noises, we will adopt a different approach that involves the system and environment interaction, and explicitly considers the quantum noises as the stochastic driving forces. Then, the total Hermitian Hamiltonian includes the system, reservoir, and the system-reservoir couplings. In an interaction picture, such a Hamiltonian reads

$$\begin{aligned} H = {} & J\left(\hat{a}^\dagger\hat{b} + \hat{a}\hat{b}^\dagger\right) + iE_0(\hat{a}^\dagger - \hat{a}) + i\sqrt{2g(t)}\left[\hat{f}_a^\dagger(t)\hat{a}^\dagger - \hat{f}_a(t)\hat{a}\right] + \\ & i\sqrt{2\gamma}\left[\hat{f}_b(t)\hat{b}^\dagger - \hat{f}_b^\dagger(t)\hat{b}\right], \end{aligned} \quad (2)$$

by using the rotating-wave and Markovian approximations with smooth system-reservoir coupling (the more detailed derivation of the similar Hamiltonian can be found in, e.g., [8,32]). Here, we assume the resonant pump drives, and $\hat{f}_c(t)$ $(c = a, b)$ is the noise operator satisfying $[\hat{f}_c(t), \hat{f}_c^{\dagger}(t')] = \delta(t - t')$. The associated Heisenberg equations of motion are, therefore, found as

$$\begin{aligned} \frac{d\hat{a}}{dt} &= g(t)\hat{a} - iJ\hat{b} + iE_0 + \sqrt{2g}\,\hat{f}_a^{\dagger}(t), \\ \frac{d\hat{b}}{dt} &= -iJ\hat{a} - \gamma\hat{b} + \sqrt{2\gamma}\,\hat{f}_b(t). \end{aligned} \tag{3}$$

by applying the appropriate Ito's laws for the noise operators [8]. For the coupled waveguides, the drive amplitude is $E_0 = 0$ and the input fields simply evolve under the coupling.

Gain saturation is a nonlinear phenomenon that stabilizes the light intensity after some quick growth. A frequently used gain medium is an erbium-doped amplifier [33], which we can model as an ensemble of two-level atoms. For a population inverted medium of erbium ions, an optical signal exponentially grows due to the stimulated emission of photons from dopant ions. In addition to the stimulated emission, an excited state can decay to a lower state by spontaneously emitting photons. In a waveguide, the light eventually reaches an intensity at a certain specific distance of propagation such that there are not enough excited states to sustain light field amplification. In other words, when the signal intensity increases to a certain value I_{sat} (saturation intensity), the population difference between the upper and lower levels and hence the gain rate decreases, reaching a gain saturation. The saturation intensity depends on the energy difference between the upper and lower levels, the stimulated cross-section, and the lifetime of the upper level [34]. The gain rate can be modeled as a function of time:

$$g(t) = \frac{g_0}{1 + I_a(t)/I_{\text{sat}}}, \tag{4}$$

where g_0 and $I_a(t)$ are the initial gain and the real-time light intensity inside the gain medium, respectively.

We investigate the gain saturation effect on the time evolution of CV entanglement by assuming that the quantum state of the inputs are single-mode squeezed vacuum states, $|z\rangle = S(z)|0\rangle$, where $S(z) = \exp\left(\frac{1}{2}z(\hat{c}^{\dagger})^2 - \frac{1}{2}z^*\hat{c}^2\right)$ for $\hat{c} = \hat{a}, \hat{b}$ (waveguides), or coherent state. The squeezing parameter is defined as $z = r\exp(i\theta)$ where r and θ are its magnitude and phase, respectively [35]. The time evolution from these input states will lead to an entanglement through the couplers (waveguides and microresonators).

3. Method

To quantify the degree of entanglement of a field in the Gaussian state, we first define $X \equiv (Q_A, P_A; Q_B, P_B)$, where

$$Q_m = \frac{1}{\sqrt{2}}(\hat{c} + \hat{c}^{\dagger}); \quad P_m = -\frac{i}{\sqrt{2}}(\hat{c} - \hat{c}^{\dagger}), \tag{5}$$

for $m = A, B$. Then we define a 4×4 covariance matrix [36] V as

$$\begin{pmatrix} E & F \\ F^T & G \end{pmatrix}, \tag{6}$$

where E, F, and G are 2×2 matrices and

$$V_{i,j} = \frac{1}{2}\langle X_i X_j + X_j X_i \rangle - \langle X_i \rangle \langle X_j \rangle. \tag{7}$$

Then, the logarithmic negativity [36]:

$$E_N = \max[0, -\ln 2\eta], \tag{8}$$

quantifies the degree of CV entanglement, where

$$\eta = \frac{1}{\sqrt{2}}\sqrt{\sigma - \sqrt{\sigma^2 - 4\det V}} \tag{9}$$

and

$$\sigma = \det E + \det F - 2\det G. \tag{10}$$

For the waveguide setups, the covariant matrix elements $V_{i,j}$ are calculated with respect to the input state. In the case of coupled microresonators, the entanglement is out of the coherent drive and $V_{i,j}$ are found by the evolved system operators with respect to the initial vacuum state before pumping.

4. Results

In a coupled gain–loss system, a light field goes from one medium to another and, given a suitable input, an entangled output can be generated [27]. The advantage of this system is that one can manipulate the light transmission simply by adjusting the coupling between the resonators or controlling the loss. Depending on the ratio of loss over gain, the light field intensity will experience oscillations or exponential growth [26] and, therefore, one might expect to realize high-intensity entangled output fields. However, the amplification process adds more photons to the propagating light fields, and the dissipation process subtracts photons. Thus, even if the input is shot-noise-limited, the output will be noisy [35]. It is also essential to include the noise terms in the dynamical evolution to preserve the canonical commutation relation. Since quantum noises significantly impact non-classical light fields, we expect that the noise from optical gain is highly relevant to the degree of generated entanglement. We will focus on its effects in what follows.

4.1. Entanglement Evolution without Gain Saturation

As it is known that quantum noise impairs the entanglement, we first take a look how the quantum noises associated with amplification and dissipation will influence on the entanglement when the optical gain can be approximated as a constant. For this purpose, we present the time evolution of the logarithmic negativity versus the normalized time Jt for squeezed vacuum and squeezed coherent states in Figure 2a,b, respectively, in the waveguide systems. In Figure 2a, we choose three different ratios of loss and gain. For all these three cases, entanglement sudden death occurs due to the quantum noise effect. Obviously, when the ratio of loss over gain is higher, the sudden death occurs later, and a higher degree of entanglement is also achieved transiently. This phenomenon indicates that the quantum noise associated with amplification impairs the entanglement significantly in this ideal situation of constant gain rate. The continued action of the gain noise will always kill the entanglement in the end. We also observe the same behavior in Figure 2b for the input as squeezed coherent states.

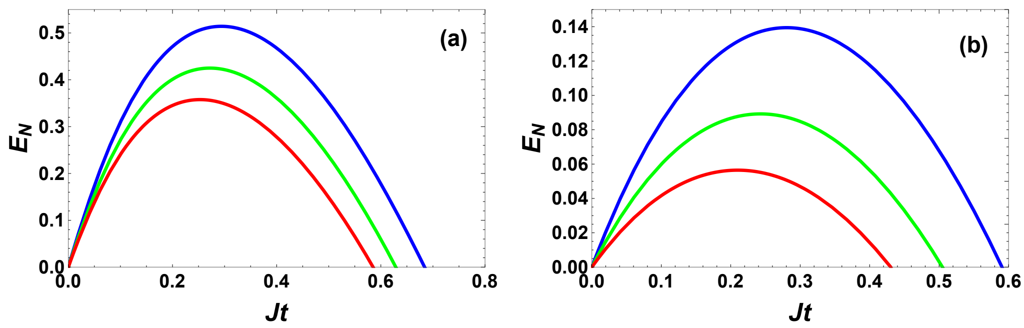

Figure 2. Entanglement generation with coupled gain–loss waveguides without the saturation effect. Panel (**a**) shows the time evolution of the entanglement when the input states are single-mode squeezed vacuum with squeezing parameters $r_1 = r_2 = 0.5$ and phase factors $\theta_1 = \theta_2 = 0$. We choose three different ratios of loss over gain: $g = 1.3\gamma$ (red), $g = \gamma$ (green), and $g = 0.7\gamma$ (blue). In (**b**), we assumed that the inputs are squeezed coherent states. Here, $\alpha = \beta = 10\exp(i\pi/4)$. The other parameters are like that of the panel (**a**).

4.2. Special Role of Gain Saturation

In Figure 3, we consider the gain saturation and discuss how different saturation intensities affect the entanglement in steady state. Here, the input fields for the waveguide setups are single-mode squeezed vacuum. In Figure 3a, we set the squeezing parameters to be $r_1 = r_2 = 0.5$ with phase factors $\theta_1 = \theta_2 = 0$. For the given loss, gain, initial gain, and saturation intensity, we convert them to the dimensionless parameters γ/J, $G_0 = g_0/J$, and $G(t) = G_0/(1 + I_a(t)/I_{sat})$. When we include the saturation effect, the gain rate decreases quickly with time and eventually reaches a steady value less than the loss rate. Therefore, unlike the constant gain that definitely leads to an entanglement sudden death, the gain saturation reduces the associated quantum noise effect. Consequently, it is possible to preserve the entanglement in the long-time limit, and the oscillation amplitude of E_N decreases with time but finally approaches a constant value. Figure 3 shows that higher degrees of entanglement are achievable given the lower gain saturation rate I_{sat}. We can explain this by the fact that, with a lower I_{sat}, the effective gain rate $g(t)$ drops more quickly with the ratio $I(t)/I_{sat}$ in the denominator of Equation (4). Intuitively, one can say that a high saturation intensity I_{sat} giving a slower gain saturation allows more photons in a different quantum state from the input state, so that the input state will be decohered more rapidly. The relevant noise term in the Hamiltonian has a factor $\sqrt{g(t)}$, and the decrease of this magnitude lowers the noise effect indeed. After increasing the initial gain rate while keeping all other parameters unchanged as in Figure 3b, we see that E_N also becomes lower due to the stronger quantum noise associated with the amplification. In this case, even the lower saturation intensities, such as $I_{sat} = 0.1$ and 0.8, cannot avoid entanglement sudden death.

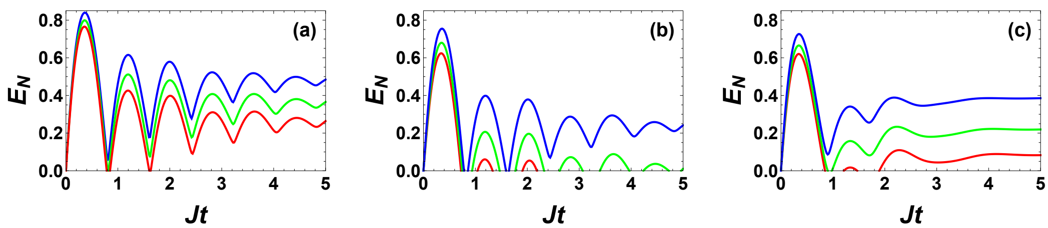

Figure 3. Time evolution of the logarithmic negativity for various saturation intensities in a coupled gain–loss waveguide system. We use the input states as single-mode squeezed vacuum with squeezing parameters r_1 and r_2 and phase factors θ_1 and θ_2. We set the parameters as $r_1 = r_2 = 0.5$, $\theta_1 = \theta_2 = 0$, $G_0 = g_0/J = 0.8$, $E_0/J = 0$, $G(t) = G_0/(1 + I_a(t)/I_{sat})$, and $I_{sat} = 0.1$ (red), 0.07 (green), and 0.04 (blue). The loss rate is set to $\gamma/J = 0.4$ in (**a**,**b**) and $\gamma/J = 0.8$ in (**c**). In (**b**,**c**), we change the initial gain rate to $G_0 = 2$.

In Figure 3c, while keeping the initial value of the gain as in Figure 3b, we increase the loss rate to $\gamma/J = 0.8$. We observe that, for a higher ratio γ/g_0, a steady value of E_N will be possible. Here, one can obtain a steady entanglement even with those saturation intensities that lead to entanglement sudden death in Figure 3b. It shows that the ratio of loss over gain is more important than their absolute values. In other words, if entanglement is erased by a particular rate of gain, one can offset it by increasing the loss rate so that there is still a preserved entanglement. This fact reflects a tricky relation between the noises respectively associated with the optical gain and loss in the entanglement dynamics.

4.3. Entanglement Generation with Coherent Drive

A more interesting scenario is the one of coupled microresonators, which is driven by a coherent driving field (a pump laser). The light fields evolve from a vacuum before pumping the system. If the system dynamics is described by the non-Hermitian Hamiltonian in Equation (1), the coherent drive term in Equation (3) makes no contribution to the covariant matrix elements, leading to no entanglement. However, with the full quantum version of Hamiltonian including the noise drive terms in Equation (2), we find that considerable amounts of CV entanglement can be created as in Figure 4. In this setup the entanglement generation is due to the noise terms in Equation (2). The noise term associated with the optical gain takes a form of two-mode squeezing between the field mode and noise mode [37], so that the commutation relation in quantum mechanics should be preserved under optical gain. Through a coupling between two field modes, these two field modes can be correlated indirectly by such a squeezing-type action, thus having a certain amount of CV entanglement.

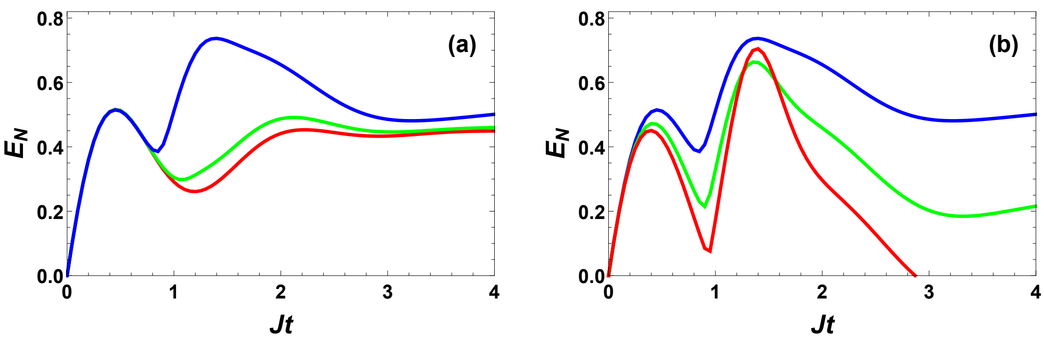

Figure 4. Time evolution of the logarithmic negativity for the coupled microresonators driven by a coherent field. In (**a**), the system parameters are $\gamma/J = 0.8$, $G_0 = 0.8$, $I_{sat} = 0.05$, and $E_0/J = 0.9$ (blue), 0.7 (green), and 0.1 (red). In (**b**), the saturation intensity is changed to $I_{sat} = 0.05$ (blue), $I_{sat} = 0.3$ (green), $I_{sat} = 0.7$ (red), while the ratio $E_0/J = 0.9$ is kept.

By the illustrated numerical calculations in Figure 4a, we observe a transient increase in E_N for stronger drive fields, but the finally stabilized degrees of entanglement have insignificant difference. It indicates that the interplay between the coherent drive and noise drive through the evolving optical gain has an almost identical tendency. In Figure 4b, we choose different saturation intensities and observe the similar behavior of the evolving entanglement to those in the coupled waveguide systems. A higher saturation intensity will nonetheless result in entanglement sudden death, implying that a prolonged noise action is harmful to the concerned entanglement. Entanglement sudden death exists when there is a high gain rate or a slower gain saturation process. However, if the optical gain quickly drops to very low level due to a small saturation intensity I_s, a considerable CV entanglement can be generated out of the noise action. The overall dynamics of the entanglement under the noise effect is rather complicated, and the entanglement generation under coherent drive should be within a proper range of optical gain g_0 and, especially, the gain saturation rate I_{sat}. This interesting property of quantum noise was not discussed before.

5. Discussion

The concerned CV entanglement start from the input states of single-mode squeezed vacuum or from driving of coherent field. On the one hand, stronger quantum noise kills the nonclassicality and results in entanglement sudden death when the gain saturation is less significant. Then, with a faster gain saturation, this tendency can be reversed if the gain rate quickly decreases and approaches a steady value smaller than the loss rate (while the intensity of the propagating light also stabilizes). As the ratio of loss over gain increases, one can obtain a steady entanglement in the end. On the other hand, more obviously in the coupled microresonators driven by a coherent field, the quantum noise can realize an amount of entanglement under the condition of a sufficiently quick gain saturation. This second role of amplification noise in complicated entanglement dynamics is rather against the intuition.

A crucial element for the CV entanglement generation is the saturated optical gain. The choice of the gain materials can be flexible, but the gain saturation rate I_{sat} should be properly designed so that the amplification of the light field can be well controlled. If the waveguide setup is realized with an erbium doped fiber amplifier (EDFA), for example, the theoretical working range of the coupled system will be between 1300 nm and 1560 nm, corresponding to a bandwidth about 394 THz. For the semiconductor materials the gain recovery time in operation [38–40] should be considered. Moreover, the system coupling can lead to more complicated dynamics involving optical gain (see, e.g., [41,42]). All these factors should be taken into account in a setup to perform the CV entanglement preparation.

6. Conclusions

We have investigated the effect of gain saturation on the entanglement dynamics in non-Hermitian systems of coupled waveguides and coupled microresonators. By analyzing with various gain saturation intensities, we conclude that a quick gain saturation is beneficial to entanglement generation in any kind of such coupled systems. If the saturation intensity of the used material is sufficiently high, the CV entanglement vanishes immediately and the evolving entanglement displays no difference from the unsaturated situation of constant gain rate. However, once the gain saturation intensity is lower, the entanglement dynamics in these coupled systems will become rather richer to realize considerable amount of stable entanglement. Furthermore, we find that the entanglement in coupled-microresonator setups driven by a coherent field is due to the noise of optical gain. A quickly saturated optical gain rate $g(t)$ in such coupled microresonators can lead to an entanglement induced by the noise with a small magnitude $\sqrt{g(t)}$, which acts in a way such that its squeezing effect can properly balance its decoherence effect. These understandings could be useful to quantum engineering with various systems of coupled components.

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