

Article

Generation of Long-Term Stable Squeezed Vacuum States Using Dither-Locking Technique

Daohua Wu ^{1,2,*} , Yashuai Han ³, Xuehua Zhu ¹ and Zhuoliang Cao ⁴

¹ Anhui Key Laboratory of Detection Technology and Energy Saving Devices, School of Electrical Engineering, Anhui Polytechnic University, Wuhu 241000, China; xhzhu@ahpu.edu.cn

² Department of Engineering Science, The University of Electro-Communications, 1-5-1 Chofugaoka, Chofu-shi, Tokyo 182-8585, Japan

³ School of Physics and Electronic Information, Anhui Normal University, Wuhu 241000, China; hanyashuai@ahnu.edu.cn

⁴ School of Physics and Materials Engineering, Hefei Normal University, Hefei 230061, China; zlcao@ahu.edu.cn

* Correspondence: daohuawu@ahpu.edu.cn

Abstract: We report the generation of long-term stable squeezed vacuum states at 1064 nm using a degenerate optical parametric amplifier (DOPA) with a periodically poled KTiOPO₄ crystal (PPKTP). The OPA is pumped by a 532 nm light produced by frequency doubling the fundamental light with a bow-tie enhancement second harmonic generator (SHG). When the DOPA and relative phases are locked using a dither-locking method, the squeezed vacuum states are stably measured over 2 h at 11 MHz. The highly compact and simple squeezed light source is suitable for applications in quantum optics experiments.

Keywords: squeezed vacuum state; long-term stability; dither-locking technique



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1. Introduction

Squeezed states of light, in which the quantum noise of one quadrature is reduced below that of coherent state, have become a key resource in quantum optics [1–3]. They also have attracted great attention in various practical applications in quantum communications [4], quantum information processing [5,6], ultrahigh-precision measurement [7,8]. For these applications, one of the important issues is to obtain a high-level squeezed state. Since the squeezed light was firstly experimentally produced with 0.7 dB using a four-wave mixing by Slusher et al. in 1985 [9], various processes and techniques have been used to generate a squeezed state [10], including: OPA [11–15], SHG [16] and Kerr mediums [17]. Up to now, however, the OPA has been proved to be the most successful and the most widely used way to produce squeezed states due to the advantages of low noise, high stability and high purity of generated squeezed state. Recently, the highest level of squeezing to date with 15 dB at analysis frequencies from 3 to 8 MHz was measured using a semi-monolithic OPA cavity with PPKTP crystal [18]. These works ensure that the practicality and industrialization of squeezed state can be implemented in the future.

Another crucial issue about squeezed light is long-term stability. To increase the sensitivity of gravitational-wave detectors, a high-level squeezed light should be integrated into the detectors continuously for hours or even days [8,19]. In addition, to fulfill long-term data acquisitions and information processing, squeezed states have to be kept stable for a long time in many quantum optics experiments and practical quantum information protocols [20–22]. Up to now, there are only a few reports on producing a stable squeezed state light [23–26]. To obtain a stable squeezed vacuum state, it is challenged to control the mechanical disturbances of the OPA cavity and the influence from the external environment. Thus, the OPA cavity and the relative phases need to be locked. The Pound–Drever–Hall

(PDH) technique [27] is an option in which an external electro-optic modulator (EOM) is used. However, the PDH technique will induce residual amplitude modulation which distorts the locking error signal and yields a frequency offset. Recently, a mutual compensation method was used in frequency locking [24]. To decrease the phase fluctuations from the residual amplitude modulation in the phase modulation process, the authors controlled the direction of the phase fluctuation by choosing the relative position between the photodetector and the OPA to mutually compensate the phase fluctuations using one EOM. As a result, a -12.6 dB bright squeezed light was stably observed for 3 h. Another alternative method for frequency locking is a dither-locking technique [28], in which the fundamental light leaks from the cavity as a feedback signal to lock frequency of the cavity. To this end, the piezoelectric transducer (PZT) of the cavity is dithered by a modulated signal and the light leaking from the output mirror of the cavity is detected by a photodetector. The output of the detector is then demodulated by a lock-in amplifier to provide an error signal and feedback to the PZT. Thus, the length of cavity can be actively controlled for tuning onto the resonance with the laser frequency. The dither-locking technique, different from a PDH technique using the EOM, is characterized with a simple and low cost system and has already been successfully used in quantum optics experiments [21,22].

In this paper, we report the generation of squeezed vacuum states with a squeezing level of -2.3 dB and anti-squeezing of 2.6 dB in the range of 5 MHz to 25 MHz using a linear degenerate optical parametric amplifier (OPA) with a PPKTP crystal. Furthermore, using a dither-locking technique, a long-term stability of the output squeezed light is continuously measured over 2 h. The present method is different from Ref. [24], and the differences can be divided into two aspects. Firstly, in Ref. [24], the authors generated the squeezed state from a semi-monolithic cavity with a PPKTP crystal and a curved mirror. Since the cavity has only one crystal/air interface, it has good operational stability, and the optical locking is usually robust. In this experiment, a linear cavity with a PPKTP crystal and two curved mirrors was used to generate a squeezed vacuum state. The cavity has two crystal/air interfaces and the cavity length is usually larger than the semi-monolithic cavity; this leads to higher losses and operational instability, and the frequency is usually difficult. Secondly, in Ref. [24], the authors locked the cavity using a complex mutual compensation method in bright squeezing generation. The method was not used in squeezed vacuum generation. However, the dither-locking technique is a simple and compact system, which is suitable to be a squeezed light source for quantum optics.

2. Experimental Setup

The experimental setup is illustrated schematically in Figure 1. The fundamental laser source is a commercially available continuous-wave (cw) Nd:YAG laser which emits 2.0 W of coherent light at wavelength 1064 nm with a linewidth of 1 kHz. The fundamental light firstly went through a half-wave plate (HWP) and a polarizing beam splitter (PBS) to control the power of the pump beam and the seed beam. The PBS separated the fundamental light into two parts: the majority of the fundamental light was efficiently frequency doubled in a bow-tie enhancement second harmonic generation (SHG) cavity, which was described in detail in Ref. [29], to produce a pump light at 532 nm for an subthreshold OPA. The remaining light was spatially filtered by a high finesse mode-cleaner cavity (MCC) and split-off to provide a seed beam for the OPA and a local oscillator (LO) beam for the homodyne detection system. The MCC in our experiment is a ring cavity that consists of two identical flat input–output mirrors with reflectivity of $R = 99.0 \pm 0.5\%$ and a concave mirror with reflectivity of $R = 99.99\%$, and it has a total resonator length of 56 cm with 1 m radius of curvature. We observed a finesse up to 1500 which provided the linewidth of the MCC about 300 kHz. We measured the intensity noise of the transmitted beam, and the results showed that the intensity noise was closing the quantum noise limit (QNL) at 3 MHz.

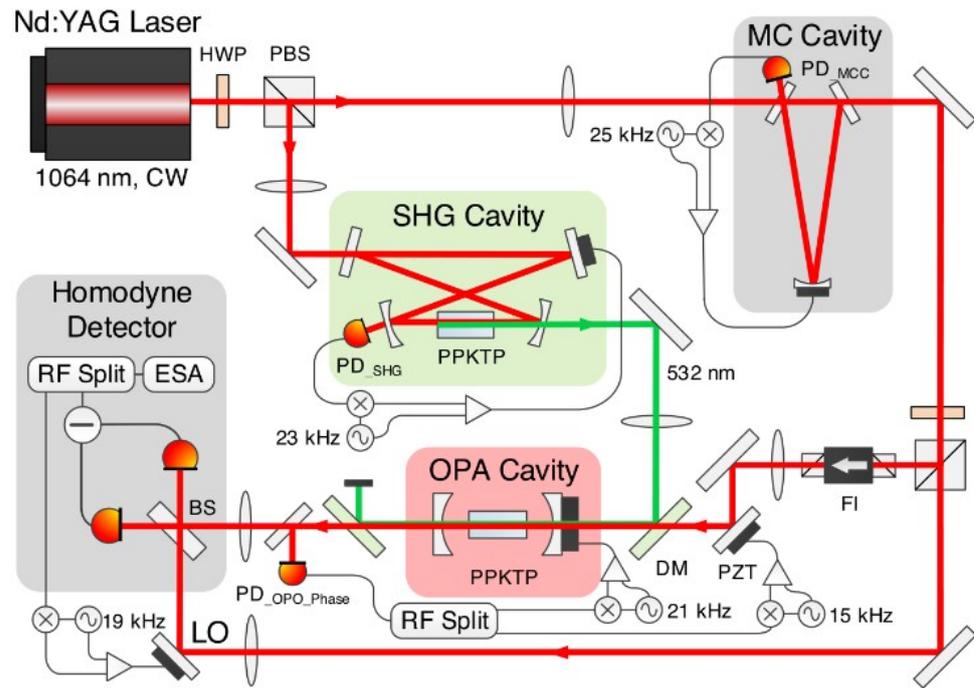


Figure 1. Sketch of experimental setup. HWP: half-wave plate, PBS: polarizing beam splitter, FI: Faraday isolator, BS: beam splitter, PD: photodetector, LO: local oscillator.

The OPA is a linear cavity which has a standing-wave configuration, using a $1 \times 1 \times 15$ mm PPKTP crystal and two curved mirrors with a 50 mm radius of curvature. The beam waist inside the crystal is about $92 \mu\text{m}$. The input mirror is high reflective at 1064 nm and high transmissive at 532 nm, and the output mirror has a reflective of 95% for 1064 nm and high transmissive of 532 nm. One of the mirrors was mounted on a PZT so that the length of the OPA cavity can be actively controlled for tuning onto the resonance with the laser frequency. The temperature of PPKTP crystal was controlled to the optimal phase-matching temperature at $36.0 \text{ }^\circ\text{C}$ within an accuracy of $0.1 \text{ }^\circ\text{C}$.

The squeezed light, produced by the OPA, was separated from the pump light by a dichroic beam splitter and submitted to a 98% transmissivity mirror. The transmitted light is directly collected by a homodyne detection (HD) system to detect the noise level, and the 2% reflected light is sent to a highly sensitive photodetector. The direct current (DC) signal from the photodetector is divided into two parts by a RF splitter: one of the signals is used to lock the OPA cavity to control the length of the cavity, and another signal is used to lock the relative phase between the pump light from the SHG and the seed light from the MCC. By choosing the relative phase between the pump light and the seed light, the OPA cavity can be controlled to parametric deamplification or amplification which ensures producing an amplitude squeezed state or a phase squeezed state.

3. Experimental Results and Discussion

To produce a squeezed light, the initial investigation is the classical parametric gain of the OPA. The classical gain is defined as

$$g = \frac{P_{out}}{P_{in}} \tag{1}$$

where P_{in} and P_{out} are the powers of output 1064 nm light from the OPA cavity with the absence and presence of the pump light, respectively. Once the pump light is transmitted to the OPA, the classical gain varies depending on the pump power and the relative phase between the seed light and the pump light. The relative phase can be controlled by scanning a triangle wave on a external PZT actuated mirror which is inserted in the path of the seed

light. Figure 2 presents the fluctuation of the output powers as a function of the changes of the relative phase. The red curve presents $P_{out} = 10 \mu\text{W}$ in the absence of the pump light, and the power of seed light was measured at $P_s = 1 \text{ mW}$. The blue curve describes the increase and decrease of the power of the output of the OPA as the relative phase is scanned with a $P_p = 292 \text{ mW}$. Pump light was transmitted to OPA. To produce a stable amplitude or phase squeezed state, the relative phase was locked at phase $\theta = \pi$ or $\theta = 0$, which corresponds to the deamplification and amplification, respectively.

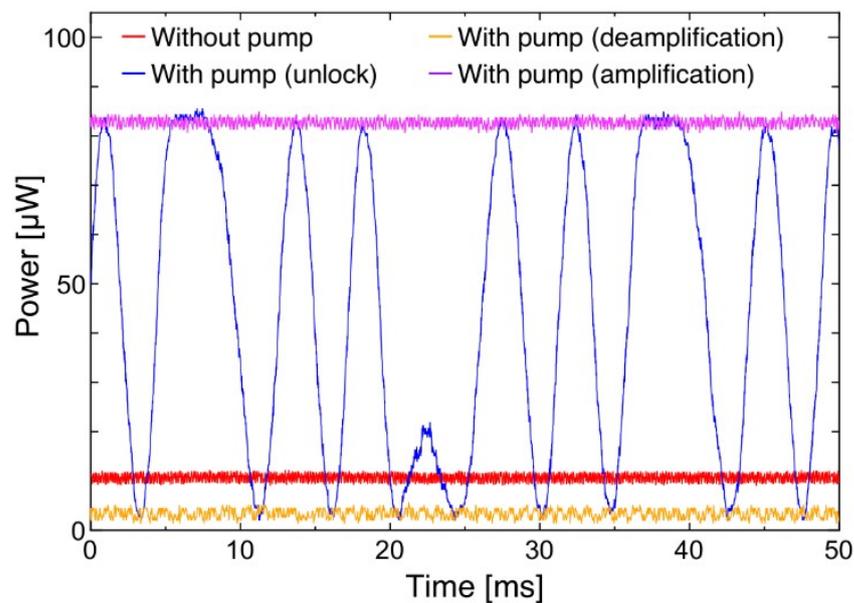


Figure 2. Locking the relative phase between the pump light and the seed light at 0 and π , which corresponds to the amplification and deamplification, respectively.

To observe squeezing, the generated squeezed beam from the OPA is combined with an LO beam on a 50:50 beam splitter (BS) and detected by a homodyne detection system. The homodyne detector is built around a pair of matched ETX 500 photodiodes with quantum efficiency estimated to be 90%. The homodyne fringe visibility between the LO beam and the measured squeezed beam is 96% and the power ratio of signal/LO is 1:1000. The total detection efficiency is estimated with 83%. Figure 3a shows the measured results of squeezed vacuum light with the power of seed and pump light at 1 mW and 450 mW, respectively. Both curves were measured at the analysis frequency $\Omega = 11 \text{ MHz}$ by a spectrum analyzer with a resolution bandwidth (RBW) of 100 kHz and a video bandwidth (VBW) of 100 Hz. The black trace corresponds to the QNL, which was measured by blocking the squeezed light and only injecting LO light at 19 mW. The blue curve shows the noise level with squeezing level of -2.3 dB and anti-squeezing level of 2.6 dB when the measured squeezed light was injected and the LO phase was scanned by driving the PZT mirror. To further understand the statistical properties of generated squeezed state, the photon number distribution is obtained from the density matrix which is reconstructed using the maximum likelihood method. Theoretically, the squeezed light generated by the parametric down conversion process is an even-number state. However, due to the system loss and the imperfect quantum efficiency of the detection system, the odd-number state (such as single-photon state and three-photon state) will emerge, as shown in Figure 3b.

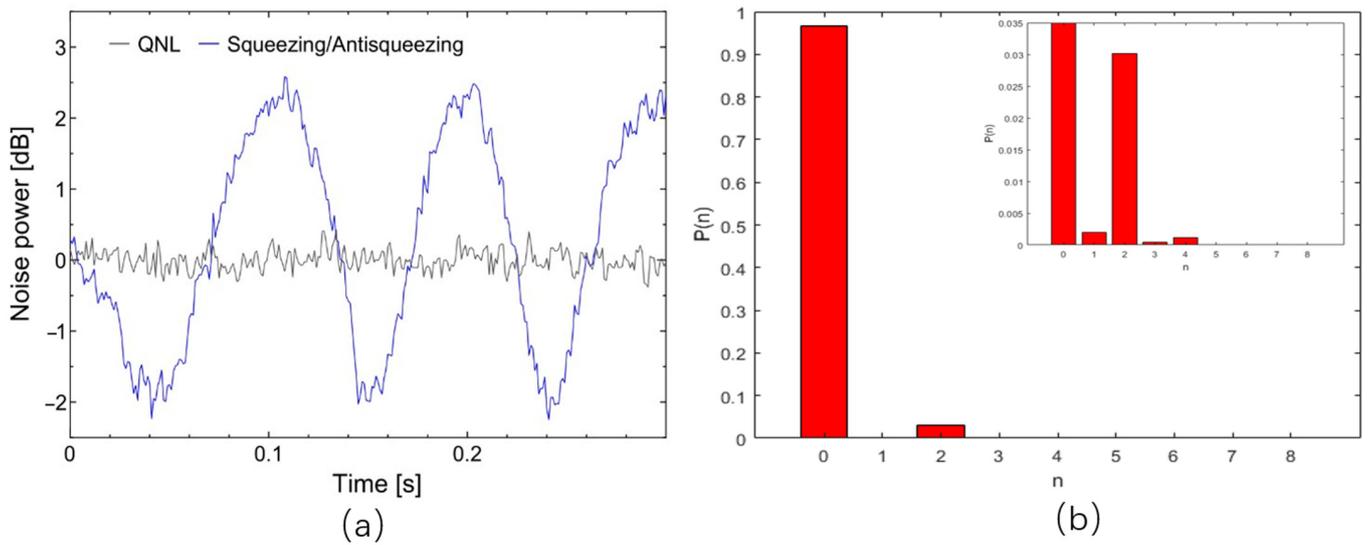


Figure 3. (a) The noise power of squeezed vacuum light at analysis frequency 11 MHz with RBW = 100 kHz and VBW = 100 Hz. The squeezing level is approximately 2.3 dB below the QNL, and the anti-squeezing level is approximately 2.6 dB above the QNL; (b) the photon number distribution of generated squeezed vacuum light.

Figure 4 shows the observed squeezing and anti-squeezing levels with different pump powers from 0 to 450 mW. The blue circles indicate the observed squeezing and anti-squeezing levels, in which all data are normalized to the QNL reference. The red curve is the theoretical prediction of the component of squeezing (V_-) and anti-squeezing (V_+), which is derived from Equation (2) using the measured values in the experiment:

$$V_{+,-} = 1 \pm \eta_{tot} \frac{4\sqrt{P/P_{thr}}}{(1 \mp \sqrt{P/P_{thr}})^2 + 4(2\pi f k^{-1})^2} \quad (2)$$

where η_{tot} is the total detection efficiency, P is the pump power, and P_{thr} is the threshold power of the OPA, f is the analysis frequency of the measurement and k is the decay rate of the OPA. From Figure 2, we can find that the measured values fit the theoretical prediction very well. In addition, with the pump power of 100 mW, -1.4 dB squeezing with 1.6 dB anti-squeezing is measured, which is close to the minimum uncertainty state. The generated pure squeezed vacuum light is extremely sensitive for the applications in quantum optics, such as multiphoton interference [23].

The long-term stability of the generated squeezed vacuum state is crucial for practical application in many fields. To obtain and measure stable squeezed vacuum states, the lengths of all cavities and the relative phases in the experiment were locked using a dither-locking technique. Taking the OPA cavity as an example, the PZT of the cavity was dithered by a sinusoidal signal, and the leaked light from the high reflective output mirror of the OPA was detected by a photodetector and then fed into a servo-control system which consists of a proportional-integral-derivation controller (PID), a locking-in amplifier and a high voltage amplifier (HVA). The DC signal from the photodetector was firstly mixed with a 21 kHz sinusoidal signal, and then further processed by the PID and the lock-in amplifier—after being filtered by a servo circuit, amplified by the HVA, then fed back to the PZT via a piezo driver. This kept the length of the OPA to the resonance with the laser frequency. In order to improve the stability of the OPA cavity, we placed the OPA device, including a PPKTP crystal with an oven, the cavity mirror mounts and the thermal electric cooler, in an aluminum box which is made from a single aluminum block by digging the inner part out.

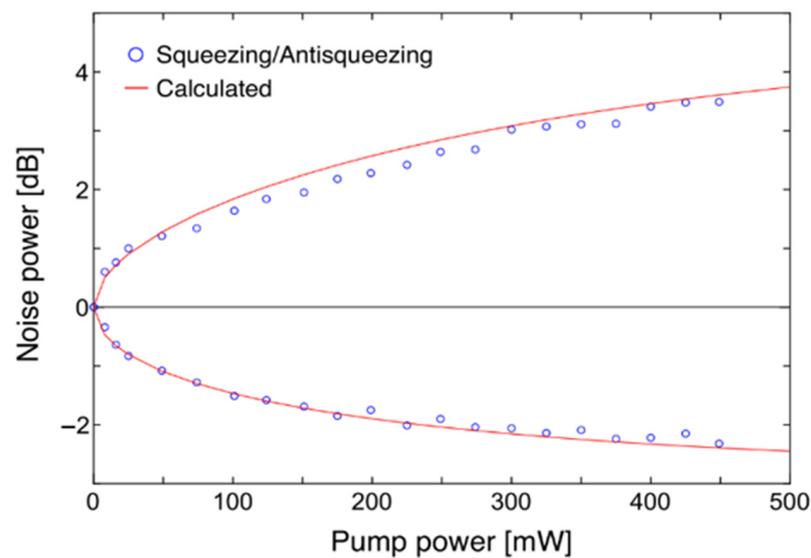


Figure 4. Measurement of squeezing and anti-squeezing as a function of pump power. Curves are theoretical predictions.

When the length of the OPA was controlled, the pump light was injected into the OPA, and the relative phase between the pump light and seed light was locked at π ; it is possible to observe stable amplitude squeezed vacuum states (Figure 3). After locking the relative phase between the squeezed vacuum light and LO for the HD at $\pi/2$, the generated amplitude squeezed state can be measured continuously. Figure 5 shows the noise power of a generated squeezed vacuum state at analysis frequency 11 MHz by a spectrum analyzer with an RBW of 100 kHz and a VBW of 100 Hz. As shown in Figure 5, we find that the noise of the generated squeezed state can be measured and recorded continuously for 2 h.

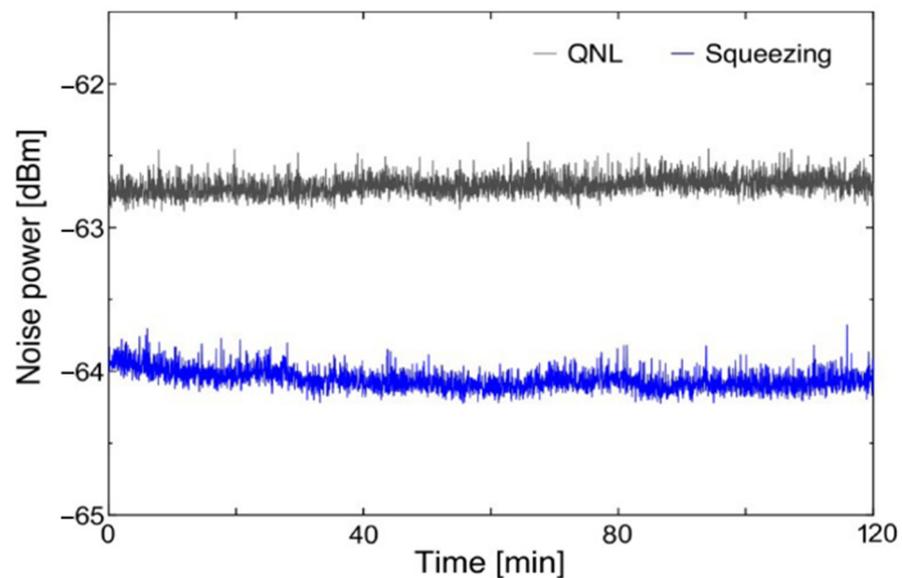


Figure 5. Long-term stability of the noise suppression recorded continuously for 2 h at 11 MHz, with an RBW = 100 kHz and VBW = 100 Hz.

4. Conclusions

In conclusion, we demonstrated an experimental realization of long-term stable squeezed vacuum states using a periodically poled KTiOPO4 crystal (PPKTP) placed in a resonant subthreshold resonant OPA cavity. When the OPA cavity was locked using the dither-locking technique, and the relative phase between the pump light and seed light was

locked at π , an amplitude squeezed vacuum state with a squeezing-level of -2.3 dB and anti-squeezing-level of 2.6 dB could be generated. By locking the relative phase between the generated squeezed vacuum state and LO at $\pi/2$, the squeezed vacuum state could be continuously observed at a squeezing level over 2 h. The long-term noise reduction of the squeezed vacuum state could be used as a squeezed light source for many quantum optics experiments, such as multi-photon interference [22,30].

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