



Article Silicon-Based TM₀-to-TM₃ Mode-Order Converter Using On-Chip Shallowly Etched Slot Metasurface

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Abstract: Mode-order converters drive the on-chip applications of multimode silicon photonics. Here, we propose a TM_0 -to- TM_3 mode-order converter by leveraging a shallowly etched slot metasurface pattern atop the silicon waveguide, rather than as some previously reported TE-polarized ones. With a shallowly etched pattern on the silicon waveguide, the whole waveguide refractive index distribution and the corresponding field evolution will be changed. Through further analyses, we have found the required slot metasurface pattern for generating the TM_3 mode with high conversion efficiency of 92.9% and low modal crosstalk <-19 dB in a length of 17.73 μ m. Moreover, the device's working bandwidth and the fabrication tolerance of the key structural parameters are analyzed in detail. With these features, such devices would be beneficial for the on-chip multimode applications such as mode-division multiplexing transmission.

Keywords: silicon photonics; photonic integrated components; mode-order converters

1. Introduction

Silicon photonics has attracted intensive research interest as an excellent platform for developing some ultradense on-chip photonic integrated circuits, due to its high refractive index contrast, low cost and compatibility with mature complementary metal-oxide-semiconductor (CMOS) technologies [1–4]. However, to meet the exponential growth demand of the optical interconnection capacity, various multiplexing technologies have been explored, such as wavelength-division multiplexing (WDM), polarization-division multiplexing (PDM), and mode-division multiplexing (MDM) [5–9]. Among them, MDM technology becomes more and more important for the on-chip optical interconnects, since the total transmission capacity can be easily extended by using more eigenmodes of optical waveguides [5–9]. Under this condition, we require the waveguide to be able to generate more higher-order eigenmodes on-chip. In order to solve this issue, high-performance mode-order converters which can change the input fundamental mode into the higher-order mode are the pivotal components for the on-chip MDM applications [6,8–10].

Recently, various schemes and concepts have been proposed for the mode-order converters. Using subwavelength grating structures and slots on the waveguide, the device size can be greatly shortened to less than 10 μ m [11–15], contributing to the compact integration on-chip, however the transmission loss will become relatively large due to the deeply-etched components. Some prevailing optimization methods (e.g., deep learning and



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Copyright: © 2021 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (https:// creativecommons.org/licenses/by/ 4.0/). topology optimization [16-18] have also been used to find the optimal structure pattern at the cost of quite time-consuming iterative calculations and complex patterns for the device fabrication. Compared with these latest reports, some classical structures such as Mach–Zehnder interferometers (MZI) [19,20] and Bragg gratings [21,22] are still employed to realize the mode-order conversion due to their simple structures, high performance and clear working mechanisms. However, the main issue of these typical schemes are their device lengths (>50 μ m). Furthermore, some common taper designs can also be used for mode conversion, by piecewise optimization of the taper shape, the TE polarized mode conversion can be achieved [23]. By using Ge on the silicon waveguide, a compact modeorder converter from TM mode to TE mode can be achieved with an average insertion loss <1 dB and a polarization extinction ratio \geq 15 dB [24]. By analyzing these previous works, we should note that almost all these reports are about the TE-polarized mode-order conversions, and few works focus on the TM-polarized ones. Faced with more mode channel requirements of high-capacity on-chip MDM transmission [7,8], TM-polarized mode-order converters should also be developed and their structures cannot refer to the TE-polarized ones directly due to their different polarization states. Therefore, efficient TM-polarized mode-order converters that can realize the mode conversion from input TM_0 and output TM-polarized higher-order mode with low insertion loss (IL), low modal crosstalk (CT), high conversion efficiency (CE), broad bandwidth and ease of fabrication are still our desired goal.

In this paper, we propose an efficient TM_0 -to- TM_3 mode-order converter based on the shallowly etched slot metasurface, which is located atop the input and output silicon waveguide. By introducing such slot metasurface patterns on the silicon waveguide, the corresponding optical refractive index distribution of the etched silicon waveguide can be changed. Then the light transmission pattern will also reveal some perturbations as it enters into the etched silicon waveguide. To realize the designated TM_0 -to- TM_3 modeorder conversion, we carefully designed and optimized the shallowly etched slot pattern on the silicon waveguide, where its function and optimization strategy were different from our previous work [25]. From results, we have obtained a silicon-based TM₀-to-TM₃ mode-order converter with high CE of 92.9%, low CT of <-19 dB, and low IL of 0.7 dB at $\lambda = 1.55 \ \mu\text{m}$, where the mode conversion length was 17.73 μm . Moreover, such a device can operate from $\lambda = 1530$ to 1600 nm without significant performance degeneration. More importantly, we found that this shallowly etched slot scheme had some scalability for the mode-order conversion, and more higher-order modes could be generated using such etched multimode waveguide structures. Based upon these unique features, the proposed scheme and related devices would well support the on-chip multimode applications [26,27], such as on-chip MDM systems.

2. Materials and Methods

Figure 1a shows the schematic of the TM₀-to-TM₃ mode converter, where the perspective and cross-sectional views along several key positions are illustrated to better present the device structure. Through analyzing the field pattern of the TM₃ mode, the four light beam spots with non-axisymmetric distribution were very clear, as can be seen in Figure 1b. So, the shallowly etched slot pattern also had to be designed to be non-axisymmetric atop the silicon waveguide and its main function was used to generate four beams (adjacent beams having a phase difference of π) at the output port [15]. To achieve this goal, we introduced six different shallowly etched rectangular slots (numbers 1 to 6) which were located atop the silicon waveguide along the length direction, as shown in Figure 1. The lengths and widths of these slots were $L_1 \sim L_6$ and $W_1 \sim W_6$ in the *x*- and *y*-direction, respectively. Slots 1 and 4, slots 2 and 5, were aligned in the *x*-direction with the same widths $W_1 = W_4$, $W_2 = W_5$, respectively. Further, one side of slots 1 and 4 in the *x*-direction was coincident with the width boundary of the silicon waveguide, benefiting the device fabrication, and the relative distance between slots 1 (4) and 2 (5) in the *y*-direction, and the relative distances

between them and the waveguide width boundary in the *y*-direction were W_s and W_m , respectively, shown in the insets of Figure 1. In addition, the internal relative distances of these shallowly etched slots in the *x*-direction were denoted as L_c , L_s , L_m , respectively, referred to the length boundary of slot 1, where all slots had a uniform etching depth H_2 . The width and thickness of the input and output silicon waveguide were $W = 2 \mu m$ and $H_1 = 300 nm$, and the upper cladding was silicon nitride, whose refractive index difference with the silicon waveguide was smaller than that of silica. Such a reduced refractive index difference can help to reduce the coupling loss in the shallowly etched waveguide thicknesses, the top silicon layer thickness of 300 nm would be recommended for the TM-polarized mode-order conversion due to its good mode confinement feature. For the TE-polarized one, standard 220 nm thick silicon layer would be the first choice.



Figure 1. (a) Schematic diagram of the proposed TM_0 -to- TM_3 mode-order converter based on the shallowly etched slot metasurface. Insets show the cross-sectional views of the device along its four key positions (marked by a, b, c, d) and the structural parameters are labeled. (b) The electric field patterns (dominant component: E_z) of TM_0 and TM_3 mode in the *y*-*z* cross section.

By etching these rectangular slots on the top surface of the silicon waveguide, the refractive index distribution of the waveguide region would not be uniform and the etched slots could be regarded as the refractive index perturbations of the whole waveguide according to the effective medium theory [28,29]. So, the light transmission pattern would not be stable along the length direction and revealed some irregular interference distributions. By analyzing these formed interference patterns, we found that we could generate the specified higher-order mode by engineering the slot metasurface pattern atop the silicon waveguide. Based upon this mechanism, we designed and optimized the shallowly etched slot structure for obtaining the mode-order conversion from input TM_0 to output TM_3 mode, as illustrated in Figure 2. First, we introduced slots 1 and 2 with relatively low effective index to generate four beams for input TM_0 mode, while obvious multimode interference could be observed and could not be easily removed, only by using these two slots [30]. Second, we added another two slots 4 and 5 with optimized dimensions and found that four beams could be observed, while there were some different phase differences between these four beams and the recorded mode CE was only 52.1% after optimization ($L_1 = 6070$ nm, $W_1 = 600$ nm, $L_2 = 5430$ nm, $W_2 = 630$ nm, $L_4 = 5340$ nm, $L_5 = 4680 \text{ nm}, W_c = 370 \text{ nm}, L_m = 4940 \text{ nm}, H_2 = 80 \text{ nm})$. Third, we added a phase-tuning slot 3 connected with slot 5 on its left side in the *x*-direction to adjust these phase differences and the result was relatively good with mode CE increasing to 85.9%, where the four light beams were clearer with a slightly uneven distribution. Finally, another phase-tuning slot 6 (connected with slot 5 on its right side in the x-direction) was also added to fine-tune these phase differences between adjacent beams. For the effective medium theory, the refractive indices of etched and unetched regions were different and the whole waveguide could be separated into several uniform regions with different refractive indices. Then we used the aforementioned way to design and optimize the structure to achieve the best performance. Therefore, through structure design and performance optimization with the

help of three-dimensional finite-difference time-domain methods (3D-FDTD) [31,32], we realized the mode-order conversion from input TM_0 to output TM_3 mode only by etching six well-designed rectangular slots atop the silicon waveguide. It should be noted that our proposed mode-order converter was an asymmetric structure and the input TM_0 mode could only be injected at the left side.



Figure 2. Electric field evolution (E_z component) of input TM₀ mode transmitted along the silicon waveguide which was etched by (**a**) slots 1 and 2, (**b**) slots 1, 2, 4, 5, (**c**) slots 1~5, and (**d**) slots 1~6.

3. Results

For the optimization process, we introduced slots 1 and 2 with a relatively low effective index to generate four beams for input TM₀ mode, and then we added another two slots 4 and 5 to adjust the phase difference between these four beams. In the beginning, all these four slots were with the same size and then we optimized and adjusted the dimensions of each rectangular slot and their relative positions one by one. By considering the interplay between those four slots, we made an overall optimization. Meanwhile, the most key point of the proposed device was its phase-tuning structure (slots 3 and 6) which could greatly affect the device performance [25]. Figure 3 shows the calculated mode CE and CT of the proposed TM_0 -to- TM_3 mode-order converter as functions of the length $L_3(L_6)$, width $W_3(W_6)$, and relative distance $W_s(W_m)$ of slot 3(6). Note that for the mode-order converter, high mode CE was a paramount evaluation index which reflected well its pivotal mode conversion performance [11-15,19-22]. Meanwhile, we also employed CT between the required mode (TM₃ mode) and other modes at the output port to characterize the mode purity for the mode conversion process. From the results, their optimum values were determined by both considering the mode CE and CT simultaneously, that is, $L_3 = 3470$ nm, $W_3 = 600 \text{ nm}, W_s = 790 \text{ nm}, L_6 = 1660 \text{ nm}, W_6 = 280 \text{ nm}, \text{ and } W_m = 800 \text{ nm}, \text{ respectively},$ with mode CE of 92.9% and highest CT<-19 dB. For the mode CT, we considered the CT derived from five TE modes ($TE_0 \sim TE_4$) and three TM modes ($TM_0 \sim TM_2$) with the required TM_3 mode, and chose the highest value as the mode CT of the suggested device. Moreover, by studying the variation curves in Figure 3, the tolerances of widths (W_3, W_6) and relative distances (W_s , W_m) were clearly smaller than those of lengths (L_3 , L_6), where $L_c = 380$ nm and $L_s = 1850$ nm. For slot 3, L_3 , W_3 and W_s should be kept within ± 300 nm, ± 60 nm and ± 40 nm for keeping CE > 87% and CT < -17 dB. By contrast, L_6 , W_6 and W_m should be kept within ± 600 nm, ± 120 nm and ± 60 nm for keeping CE > 91% and CT < -18 dB. As a consequence, slot 3 had more influence on the device performance with relatively tight



tolerance and slot 6 was an auxiliary component to further enhance the device performance with relatively large tolerance, which was matched with our theoretical analysis.

Figure 3. Mode CE and CT of the proposed TM_0 -to- TM_3 mode-order converter as functions of (**a**) length L_3 , (**b**) width W_3 , (**c**) relative distance W_s of slot 3, and (**d**) length L_6 , (**e**) width W_6 , (**f**) relative distance W_m of slot 6, respectively.

Next, we studied the spectrum properties of mode CE, CT, and IL of the proposed device shown in Figure 4, where the device parameters were based upon the above analysis and the material dispersions of silicon and silica were also considered. It was noted that the mode CE presented a clear asymmetrical response with respect to the central wavelength (1.55 µm) and the long wavelength (>1.55 µm) nearly had a higher CE than the short wavelength (<1.55 μ m). For the mode CT, its limitation mainly came from the excited TE₁ mode with highest mode CT (CT < -17 dB within the calculated wavelength range). Owing to the shallowly etched slot metasurface pattern (compared with the deeply etched slot structure with large structural changes) atop the silicon waveguide, no large transmission loss was introduced and the obtained IL curve was relatively stable within the entire wavelength range (IL < 1.4 dB, IL = 0.7 dB @1.55 μ m). Another advantage of the proposed device was its low reflection loss (<-30 dB) within the wavelength range due to relatively small structural changes. Moreover, we also plotted the field evolution through the designed device at three typical wavelengths ($\lambda = 1500, 1550, 1600$ nm) and the total conversion length was 17.73 µm. By leveraging the proposed mode-order converter, the input TM₀ mode could be well converted to the output TM₃ mode within the length <20 μ m and its optimum working wavelength range was from 1530 to 1600 nm.



Figure 4. (a) Wavelength spectra of mode CE and CT of the proposed TM₀-to-TM₃ mode-order converter ($\lambda = 1500 \sim 1600$ nm). (b) Wavelength spectra of mode IL of the proposed TM₀-to-TM₃ mode-order converter ($\lambda = 1500 \sim 1600$ nm). (c) Electric field evolution through the designed device working at different wavelengths ($\lambda = 1500$, 1550, 1600 nm). Total mode conversion length was 17.73 µm.

4. Discussion

Considering the device fabrication, we only required an extra lithography and etching step on the base of the silicon waveguide to form the designed slot metasurface pattern, which will not pose huge challenges for the current fabrication facilities [33,34]. We started from the silicon-on-insulator wafer with a 300 nm-thick top silicon layer. First, a rectangular silicon waveguide with a width of 2 μ m and an etching depth of 300 nm was fabricated using E-beam lithography (EBL) and reactive ion etching (RIE) [33,34]. Second, we employed EBL again to form the designed slot metasurface pattern atop the silicon waveguide and then used RIE to transfer the lithography pattern from photoresist to the silicon layer, where the etching depth was 80 nm. Note that the structural alignment between above steps 2 and 3 should be guaranteed during the fabrication process. Third, we grew an upper cladding of silicon nitride to further protect the waveguide through the plasma-enhanced chemical-vapor-deposition process [33,34]. Referring to these fabrication steps, we took some parameters into account for the tolerance analyses (whole pattern size deviation, round corner, etching depth deviation), as illustrated in Figure 5. It was noted that the whole pattern size and etching depth deviating from their optimum values revealed a strong influence on the device performance, leading to tight fabrication tolerances. So, the whole pattern size deviation should be controlled within ± 20 nm by keeping mode CE > 89% and CT < -16 dB, and the allowable etching depth deviation was much smaller, only within ± 5 nm by keeping mode CE > 91% and CT < -16 dB, respectively. To achieve this key etching depth, we should pay attention to the etching process, especially for the etching time and etching gas. Furthermore, owing to the proximity effect of EBL [35], sharp corners (right-angles) of the designed slot metasurface pattern will normally become round corners. Fortunately, no obvious performance deterioration could be observed from our calculations as the curvature radius *R* of round corners increased from 0 to 140 nm. Therefore, these results offer positive effect and guidance for practical device fabrication. Meanwhile, in order to find the transmission features of TE mode in the proposed device, we input the TE_0 mode through the device length direction. The result revealed that such a device could not work at TE_0 mode and we could not obtain the stable mode at the output port due to high mode CT and high IL.



Figure 5. Fabrication tolerance analysis: (a) whole pattern size deviation, (b) curvature radius *R* of round corners of the shallowly etched rectangular slots, and (c) etching depth deviation H_2 , where the influences of such variations are reflected on the mode CE and CT of the proposed device.

Considering the structural alignment between step 2 and step 3, we analyzed the device performance dependent on the relative position shift *D* (in *y*-direction) between them as shown in Figure 6a. From the results, if the relative position shift could be controlled within the range from -20 to 20 nm, the CE could still be kept higher than 86% with CT lower than -15 dB. By contrast, if such a relative position shift was larger than ± 20 nm, both CE and CT would degenerate quickly, corresponding to bad device performance. Figure 6b shows the device performance dependent on the length variation of slot 5 (L_j) and width variation of slot 3 and slot 6 (W_j), where the total lengths of slots 3, 5, and 6 were kept unchanged. To make it more clear, we only drew the curve of the highest CT. From Figure 6b, we can see that the length variation and width variation of these slots should be kept within the range from -100 to 100 nm and from -20 to 20 nm, respectively. Therefore, we should pay attention to these key device size requirements in the practical device fabrication.



Figure 6. Device performance dependent on (**a**) the relative position shift *D* between the shallowly etched pattern and the silicon waveguide, (**b**) the length variation of slot 5 (L_j) and width variation of slot 3 and slot 6 (W_j) where the total lengths of slots 3, 5, and 6 are kept unchanged.

Finally, how to apply the proposed mode-order converter to the on-chip MDM system? The following procedures can be used for reference. First, the input single-wavelength laser was coupled to the input waveguide and then separated into N waveguide channels through splitter, where the input and output waveguide modes were kept as fundamental modes. Second, the N waveguide channels were connected with different mode-order converters to generate higher-order modes, where only the TM₀-to-TM₃ mode-order converter was analyzed in here and other TM_0 -to- $TM_1/-TM_2$ ones were also studied in our previous work [25,36] and they could also be added in such an MDM system. Other more higher-order converters could also be realized using the shallowly etched scheme due to its scalability in controlling the mode-order conversion. Third, these generated higher-order modes should be further combined to the bus waveguide through mode multiplexer or using densely packed waveguide array (DPWA) [37]. So, these higher-order modes can be transmitted in the bus waveguide or DPWA through the mode multiplexing way. At the output port, these multiplexing channels should be demultiplexed into different single mode channels based on the reverse process of mode multiplexing. In addition, some modulators and detectors should also be added if high-speed data transmission features should be studied. Therefore, if we want to construct such on-chip MDM systems, various components will be required not only the mode-order converters.

5. Conclusions

In conclusion, we have proposed a TM_0 -to- TM_3 mode-order converter using a shallowly etched slot metasurface atop the silicon waveguide. The introduced slot structure was employed to change the refractive index distribution of the silicon waveguide through the extra etching process, and then the light transmission pattern would show some perturbations, leading to the generation of multimode interference in the waveguide. Hereby, we designed and optimized the shallowly etched slot pattern atop the silicon waveguide to realize the TM_0 -to- TM_3 mode-order conversion, where six different rectangular slots were used. Compared with some reported TE-polarized mode-order converters, the TM-polarized ones are limited. According to our results, the silicon-based TM_0 -to- TM_3 mode-order converter was achieved with high CE (92.9%), low CT (<-19 dB), and low IL (~0.7 dB) in a conversion length of 17.73 µm. Furthermore, such shallowly etched slot schemes can be employed to generate other higher-order modes, such as TM_0 -to- $TM_1/-TM_2$, which would be very beneficial for the development of on-chip MDM transmission and other multimode applications.

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References

- 1. Bogaerts, W.; Chrostowski, L. Silicon photonics circuit design: Methods, tools and challenges. *Laser Photonics Rev.* **2018**, *12*, 1700237. [CrossRef]
- Helkey, R.; Saleh, A.A.M.; Buckwalter, J.; Bowers, J.E. High-performance photonic integrated circuits on silicon. *IEEE J. Sel. Top. Quantum Electron.* 2019, 25, 8300215. [CrossRef]
- 3. Chen, X.; Milosevic, M.; Stankovic, S.; Reynolds, S.; Bucio, T.D.; Li, K.; Thomson, D.J.; Gardes, F.; Reed, G. The emergence of silicon photonics as a flexible technology platform. *Proc. IEEE* **2018**, *106*, 2101. [CrossRef]
- 4. Saber, M.G.; Xu, L.; Sagor, R.H.; Wang, Y.; Plant, D.V. Integrated polarisation handling devices. *IET Optoelectron.* 2019, 14, 109–119. [CrossRef]
- Dong, P. Silicon photonic integrated circuits for wavelength-division multiplexing applications. *IEEE J. Sel. Top. Quantum Electron.* 2016, 22, 6100609. [CrossRef]
- Luo, L.-W.; Ophir, N.; Chen, C.P.; Gabrielli, L.H.; Poitras, C.B.; Bergmen, K.; Lipson, M. WDM-compatible mode-division multiplexing on a silicon chip. *Nat. Commun.* 2014, *5*, 3069. [CrossRef]
- Dai, D.; Li, C.; Wang, S.; Wu, H.; Shi, Y.; Wu, Z.; Gao, S.; Dai, T.; Yu, H.; Tsang, H.-K. 10-channel mode (de)multiplexer with dual polarizations. *Laser Photonics Rev.* 2018, 12, 1700109. [CrossRef]
- 8. Tan, Y.; Wu, H.; Dai, D. Silicon-based hybrid (de)multiplexer for wavelength-/polarization-division-multiplexing. *J. Lightwave Technol.* 2018, *36*, 2051–2058. [CrossRef]
- 9. Han, X.; Xiao, H.; Liu, Z.; Zhao, T.; Jia, H.; Yang, J.; Eggleton, B.J.; Tian, Y. Reconfigurable on-chip mode exchange for modedivision multiplexing optical networks. *J. Lightwave Technol.* **2019**, *37*, 1008–1013. [CrossRef]
- 10. Jia, H.; Chen, H.; Wang, T.; Xiao, H.; Ren, G.; Mitchell, A.; Yang, J.; Tian, Y. Multi-channel parallel silicon mode-order converter for multimode on-chip optical switching. *IEEE J. Sel. Top. Quantum Electron.* **2020**, *26*, 8302106. [CrossRef]
- Wang, H.; Zhang, Y.; He, Y.; Zhu, Q.; Sun, L.; Su, Y. Compact silicon waveguide mode converter employing dielectric metasurface structure. *Adv. Opt. Mater.* 2018, 7, 1801191. [CrossRef]
- 12. Cheng, Z.; Wang, J.; Yang, Z.; Zhu, L.; Yang, Y.; Huang, Y.; Ren, X. Sub-wavelength grating assisted mode order converter on the SOI substrate. *Opt. Express* **2019**, *27*, 34434–34441. [CrossRef]
- 13. Greenberg, Y.; Karabchevsky, A. Spatial eigenmodes conversion with metasurfaces engraved in silicon ridge waveguides. *Appl. Opt.* **2019**, *58*, F21–F25. [CrossRef]
- 14. Zhao, Y.; Guo, X.; Zhang, Y.; Xiang, J.; Wang, K.; Wang, H.; Su, Y. Ultra-compact silicon mode-order converters based on dielectric slots. *Opt. Lett.* **2020**, *45*, 3797–3800. [CrossRef]
- 15. Abu-elmaaty, B.E.; Sayed, M.S.; Pokharel, R.K.; Shalaby, H.M.H. General silicon-on-insulator higher-order mode converter based on substrip dielectric waveguides. *Appl. Opt.* **2019**, *58*, 1763–1771. [CrossRef]
- 16. Liu, D.; Tan, Y.; Khoram, E.; Yu, Z. Training deep neural networks for the inverse design of nanophotonic structures. *ACS Photonics* **2018**, *5*, 1365. [CrossRef]
- 17. Ma, W.; Liu, Z.; Kudyshev, Z.A.; Boltasseva, A.; Cai, W.; Liu, Y. Deep learning for the design of photonic structures. *Nat. Photonics* **2021**, *15*, 77–90. [CrossRef]
- Frellsen, L.F.; Ding, Y.; Sigmund, O.; Frandsen, L.H. Topology optimized mode multiplexing in silicon-on-insulator photonic wire waveguides. *Opt. Express* 2016, 24, 16866–16873. [CrossRef] [PubMed]
- 19. Ahmmed, K.T.; Chan, H.P.; Li, B. Broadband high-order mode pass filter based on mode conversion. *Opt. Lett.* **2017**, *42*, 3686–3689. [CrossRef] [PubMed]
- 20. Sun, C.; Yu, Y.; Chen, G.; Zhang, X. Integrated switchable mode exchange for reconfigurable mode-multiplexing optical networks. *Opt. Lett.* **2016**, *41*, 3257–3260. [CrossRef] [PubMed]
- 21. Xiao, R.; Shi, Y.; Li, J.; Dai, P.; Zhao, Y.; Li, L.; Lu, J.; Chen, X. On-chip mode converter based on two cascaded Bragg gratings. *Opt. Express* **2019**, 27, 1941–1957. [CrossRef]
- Okayama, H.; Onawa, Y.; Takahashi, H.; Shimura, D.; Yaegashi, H.; Sasaki, H. Polarization insensitive silicon waveguide wavelength filter using polarization rotator and mode conversion Bragg grating with resonator cavity. *Jpn. J. Appl. Phys.* 2020, 59, 128002. [CrossRef]
- Chen, D.; Xiao, X.; Wang, L.; Yu, Y.; Liu, W.; Yang, Q. Low-loss and fabrication tolerant silicon mode-order converters based on novel compact tapers. *Opt. Express* 2015, 23, 11152–11159. [CrossRef] [PubMed]
- 24. Gallacher, K.; Millar, R.W.; Griskeviciute, U.; Sinclair, M.; Sorel, M.; Baldassarre, L.; Ortolani, M.; Soref, R.; Paul, D.J. Ultrabroadband mid-infrared Ge-on-Si waveguide polarization rotator. *APL Photonics* **2020**, *5*, 026102. [CrossRef]
- Xu, Y.; Zhu, C.; Hu, X.; Dong, Y.; Zhang, B.; Ni, Y. On-chip silicon shallowly etched TM₀-to-TM₁ mode-order converter with high conversion efficiency and low modal crosstalk. *J. Opt. Soc. Am. B* 2020, *37*, 1290–1297. [CrossRef]
- 26. Li, C.; Liu, D.; Dai, D. Multimode silicon photonics. Nanophotonics 2019, 8, 227. [CrossRef]
- 27. Zhang, G.; Mojaver, H.R.; Das, A. Liboiron-Ladouceur, Mode insensitive switch for on-chip interconnect mode division multiplexing systems. *Opt. Lett.* **2020**, *45*, 811–814. [CrossRef]
- 28. Cheben, P.; Halir, R.; Schmid, J.H.; Atwater, H.A.; Smith, D.R. Subwavelength integrated photonics. *Nature* **2018**, *560*, 565–572. [CrossRef]

- Halir, R.; Ortega-Monux, A.; Benedikovic, D.; Mashanovich, G.Z.; Wanguemert-Perez, G.J.; Schmid, J.H.; Molina-Fernandez, J.; Cheben, P. Subwavelength-grating metamaterial structures for silicon photonic devices. *Proc. IEEE* 2018, 106, 2144–2157. [CrossRef]
- 30. Soldano, L.B.; Pennings, E.C.M. Optical multi-mode interference devices based on self-imaging: Principles and applications. *J. Lightwave Technol.* **1995**, *13*, 615–627. [CrossRef]
- 31. Sullivan, D.M. Electromagnetic Simulation Using the FDTD Method; IEEE Press: Piscataway Township, NJ, USA, 2000.
- 32. Available online: https://www.lumerical.com/products/fdtd/ (accessed on 8 January 2021).
- Rahim, A.; Spuesens, T.; Baets, R.; Bogaerts, W. Open-access silicon photonics: Current status and emerging initiatives. *Proc. IEEE* 2018, 106, 2313–2330. [CrossRef]
- 34. Komljenovic, T.; Huang, D.; Pintus, P.; Tran, M.A.; Davenport, M.L.; Bowers, J. Photonic integrated circuits using heterogeneous integration on silicon. *Proc. IEEE* 2018, 106, 2246–2257. [CrossRef]
- 35. Seo, E.; Choi, B.K.; Kim, O. Determination of proximity effect parameters and the shape bias parameter in electron beam lithography. *Microelectron. Eng.* **2020**, *53*, 305–308. [CrossRef]
- Zhu, C.; Xu, Y.; Dong, Y.; Zhang, B.; Ni, Y. A compact silicon-based TM₀-to-TM₂ mode-order converter using shallowly-etched slots. J. Opt. 2020, 22, 015802. [CrossRef]
- 37. Liu, L. Densely packed waveguide array (DPWA) on a silicon chip for mode division multiplexing. *Opt. Express* **2015**, *23*, 12135. [CrossRef] [PubMed]