Supplementary Materials

Image deconvolution with hybrid reweighted adaptive total variation (HRATV) for optoacoustic tomography

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Supplementary Figure S1: Deconvolution results of mouse kidney

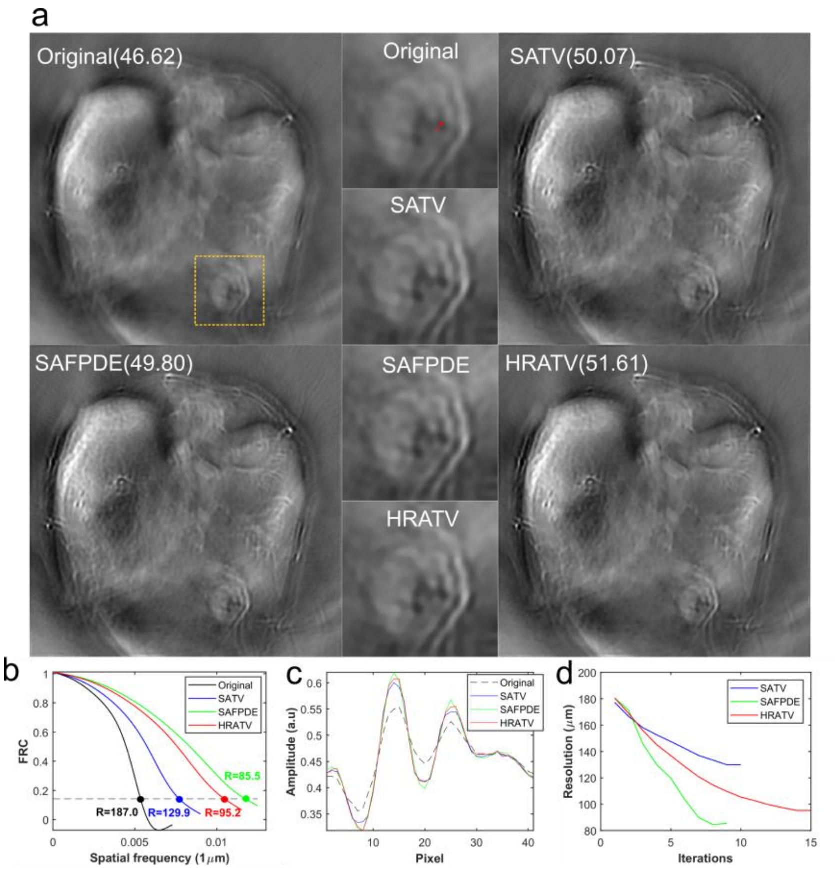

Figure S1 Deconvolution results of mouse kidney. (a) Original image and images after deconvolutions. The vessel indicated by the red arrow is selected to calculate the SNR and the results are shown beside the image titles. (b) FRC measurements of the original image and the images after deconvolutions. (c) Lateral line profiles of the structures indicated by the red arrow. (d) Estimated resolution during iterations. From the close-ups, it can be seen that the micro-structures are clearly distinguished from the background after deconvolutions. The result obtained by RLBD-SATV method appears sharper than that obtained by RLBD-SAFPDE and RLBD-HRATV methods, partly due to the "false edges" generated by the staircase effect in the noise region. The images restored by RLBD-SAFPDE and RLBD-HRATV looks more natural than that restored by RLBD-SATV. Figure (b) and (c) demonstrate that RLBD-SAFPDE achieves the best performance in regard to the resolution. However, its SNR is lower as it produces higher oscillations in the noise region. Overall, RLBD-HRATV method combines the strength of SATV and SAFPDE regularizations, and thus enables recover of both fine details and sharp edges in OAT.

Supplementary Note S1: Implementation details of the RLBD-HRATV method

The proposed Richardson-Lucy blind deconvolution with hybrid reweighted adaptive total variation (RLBD-HRATV) is an iterative image deconvolution method that combines the spatially adaptive total variation (SATV) and spatially adaptive fourth-order partial differential equation (SAFPDE) regularizations in each iteration of the deconvolution progress. The core functions for updating the estimation of the object image are given as:

$$f_{k+1}(\boldsymbol{x}) = \left\{\left[\frac{i(x)}{(f_k * h_{k+1})(x)}\right] * h_{k+1}(-\boldsymbol{x})\right\} \cdot \frac{f_k(x)}{1 - \frac{\lambda_1}{1 + \beta D(x)} div\left(\frac{\nabla f_k(x)}{|\nabla f_k(x)|}\right)}, \tag{1}$$

$$|\nabla f_k(x)| = \sqrt{{(f_k)_x}^2 + {(f_k)_y}^2}, \tag{2}$$

$$g_{k+1}(\boldsymbol{x}) = \left\{\left[\frac{i(x)}{(g_k * h_{k+1})(x)}\right] * h_{k+1}(-\boldsymbol{x})\right\} \cdot \frac{g(x)}{1 - \frac{\lambda_2}{1 + \beta D(x)} F(g(x))}, \tag{3}$$

$$F(g(x)) = \left(\frac{g_{xx}}{|\nabla^2 g|}\right)_{xx} + \left(\frac{g_{xy}}{|\nabla^2 g|}\right)_{yx} + \left(\frac{g_{yx}}{|\nabla^2 g|}\right)_{xy} + \left(\frac{g_{yy}}{|\nabla^2 g|}\right)_{yy}. \tag{4}$$

To solve the Equation (1-2), we adopt the finite difference scheme:

$$div\left(\frac{\nabla f}{|\nabla f|}\right) = \frac{1}{\Delta x}\Delta_-^x\left(\frac{\Delta_+^x f_{ij}}{\sqrt{(\Delta_+^x f_{ij})^2 + m(\Delta_+^y f_{ij},\ \ \Delta_-^y f_{ij}) + \varepsilon}}\right) + \frac{1}{\Delta y}\Delta_-^y\left(\frac{\Delta_+^y f_{ij}}{\sqrt{(\Delta_+^y f_{ij})^2 + m(\Delta_+^x f_{ij},\ \ \Delta_-^x f_{ij}) + \varepsilon}}\right), \tag{5}$$

where the finite difference derivation defined as:

$$\Delta_\pm^x f_{ij} = \pm\frac{1}{\Delta x}(f_{(i\pm1)j} - f_{ij}), \tag{6}$$

$$\Delta_\pm^y f_{ij} = \pm\frac{1}{\Delta y}(f_{i(j\pm1)} - f_{ij}). \tag{7}$$

and the function $m(a, b)$ defined as:

$$m(a, b) = \frac{sign(a) + sign(b)}{2} min(|a|, |b|). \tag{8}$$

For boundary conditions, we use:

$$f_{0j} = f_{1j},\ f_{(N+1)j} = f_{Nj},\ f_{i0} = f_{i1},\ f_{i(N+1)} = f_{iN}. \tag{9}$$

To solve the Equation (3-4), we adopt the following finite difference scheme:

$$\Delta^{xx} f_{ij} = \pm\frac{1}{\Delta x}(\Delta_+^x f_{ij} - \Delta_+^x f_{(i-1)j}), \tag{10}$$

$$\Delta_\pm^{xy} f_{ij} = \pm\frac{1}{\Delta y}(\Delta_\pm^x f_{i(j+1)} - \Delta_\pm^x f_{ij}), \tag{11}$$

$$\Delta_\pm^{yx} f_{ij} = \pm\frac{1}{\Delta x}(\Delta_+^y f_{(i+1)j} - \Delta_+^y f_{ij}), \tag{10}$$

$$\Delta^{yy} f_{ij} = \pm\frac{1}{\Delta y}(\Delta_\pm^y f_{ij} - \Delta_\pm^y f_{i(j-1)}), \tag{11}$$

$$|\nabla^2 f| = \sqrt{(\Delta^{xx} f_{ij})^2 + (\Delta_+^{xy} f_{ij})^2 + (\Delta_+^{yx} f_{ij})^2 + (\Delta^{yy} f_{ij})^2 + \varepsilon}. \tag{12}$$

In the implementation, we use the spatial step $\Delta x = \Delta y = 1$. $\varepsilon = 10^{-10}$ is used to avoid dividing zero. Using Equation (5-12), the core updating functions (Equation (1) and (4)) of the RLBD-HRATV method can be solved.