

Supplementary Information

Developing a New Biophysical Tool to Combine Magneto-Optical Tweezers with Super-Resolution Fluorescence Microscopy.
***Photonics* 2015, 2, 758-772**

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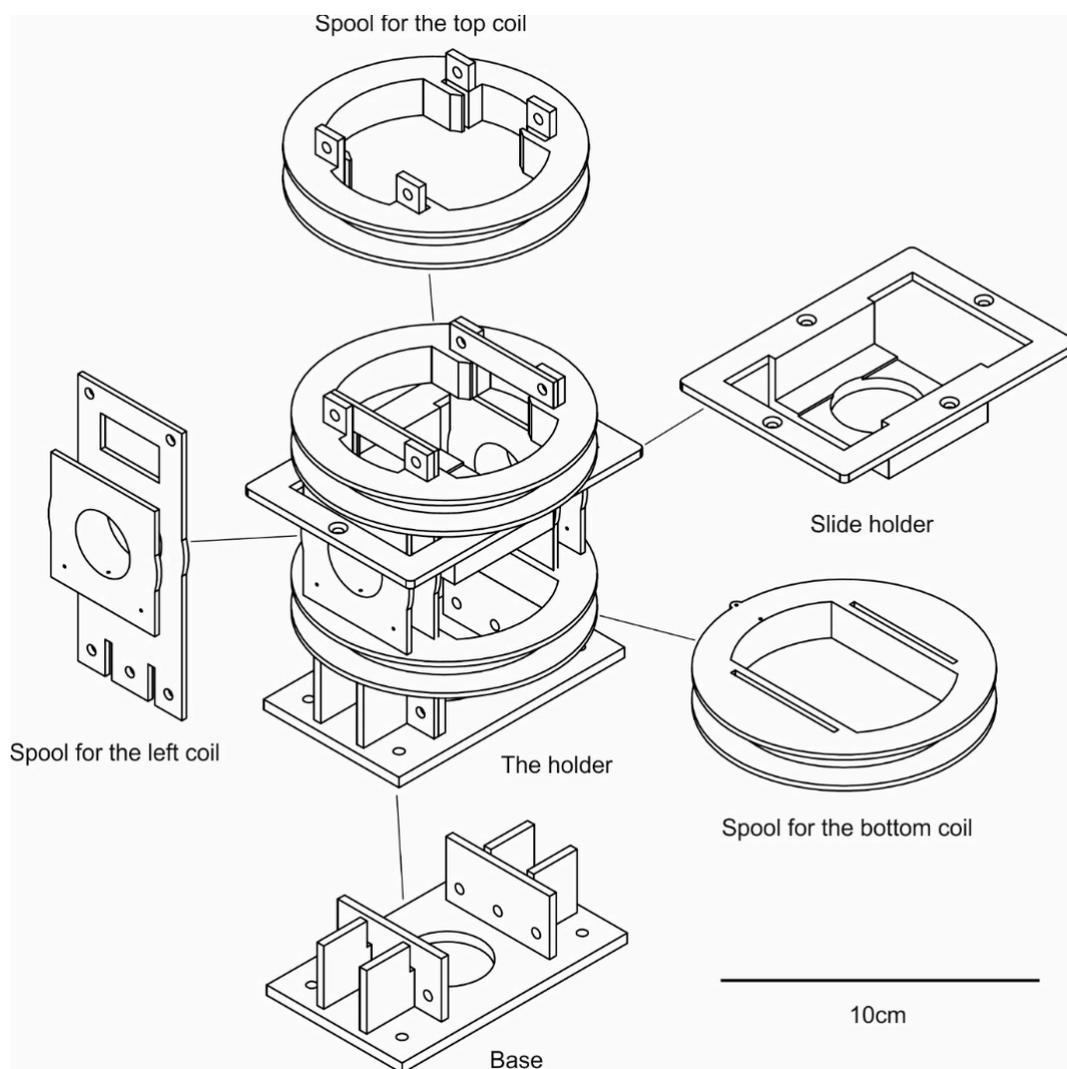
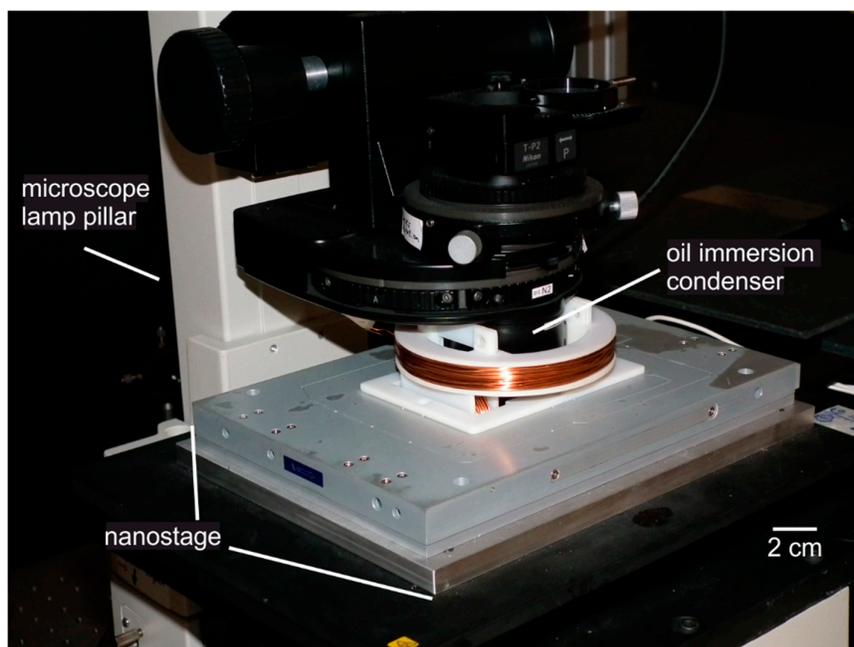
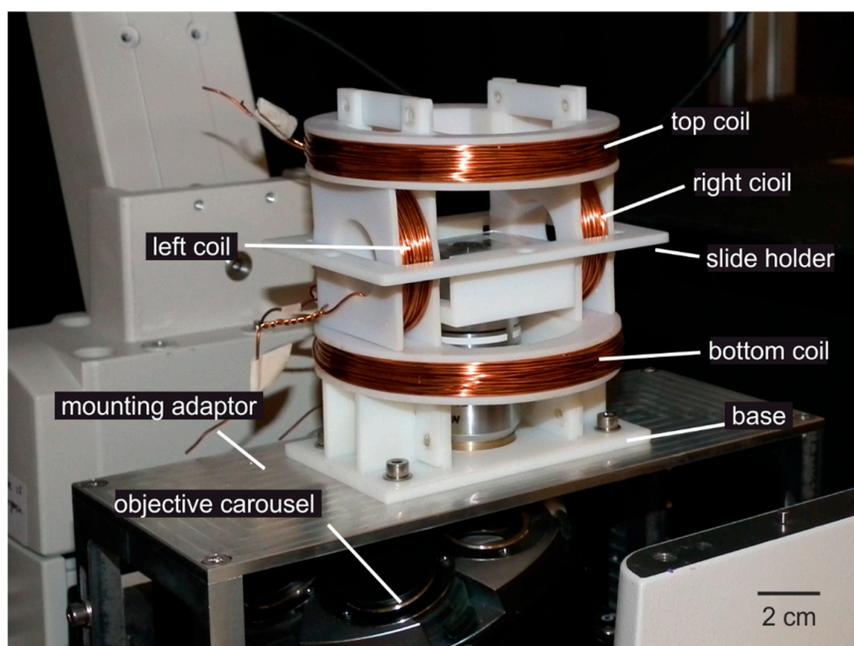


Figure S1. Schematic of the bespoke holder of the electromagnetic coils and each individual component. The slide holder is mounted on the xyz nanostage and does not touch the other parts of the holder so it can move independently. The base provides the mechanism for the holder to be mounted on the microscope. The spool for the left coil slots into the top spool, the bottom spool and the base, holding the entire structure together. Note that the spool for the right coil is identical (but facing the opposite side) to this part so the right spool is not shown in the sketch. Readers can modify the design to suit their own experimental setups.



(a)



(b)

Figure S2. Photos of the magnetic tweezers featuring the mounting mechanism to mount the MT onto the microscope so it is stationary relative to the objective lens, the 3D printed spools on which the enamel sheathed copper wires are wound and a bespoke sample holder that features a narrow tray to make room for the coils on the left and right. (a) Shows how the condenser, the magnetic tweezers and the xyz nanostage all fit together; (b) Exposes the magnetic tweezers structure for visualisation purposes here.

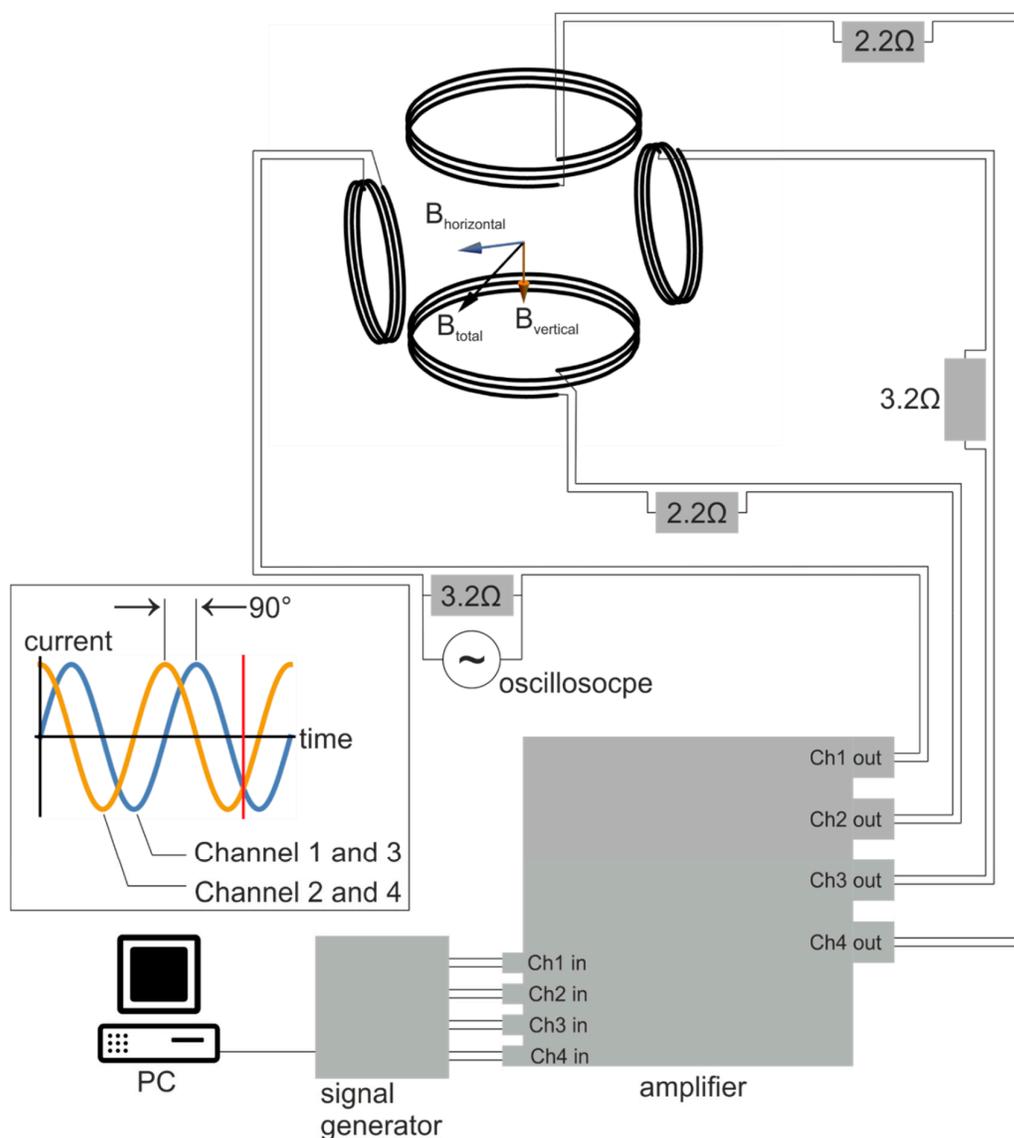


Figure S3. Magnetic tweezers circuit diagram. Sinusoidal signal variations are synthesized with our home-made LabVIEW code. The PC sends the signal to a 4-channel signal generator (NI 9263, National Instruments), which is capable of creating AC voltages between ± 10 V and at 100 kHz sample rate. This is then sent to a 4-channel car audio amplifier for high current output. Each channel has a resistor connected in series that adds to the impedance of the corresponding coil to bring the total load up to the rated output impedance of the amplifier. Also the voltage on each resistor is monitored with an oscilloscope (drawn only on Ch1 resistor) as a means to monitor the current in each coil. The inset on the left shows a typical current-time plot. Currents that are phased 90° apart are applied to each pair of coils. But the currents in both coils in either pair are the same. The red line in the inset corresponds to the B-field drawn in the centre of the coils. The orange arrow represents the combined field due to the vertical coil pair at the centre point, the blue arrow represents that due to the horizontal pair and the black arrow represents the total resultant field due to all four coils.

Heat Dissipation Calculation

Each coil is modelled as a series of concentric rings for ease of calculation. The rings stack up in 10 layers. Each layer has a unique diameter and each layer has 10 rings (except the outermost layer of the small coil, which has 5, due to the fact that small coils only have room for 95 windings). The big coil is treated as 100 rings of diameters ranging from 38.5 mm to 47.5 mm; the small coil 95 rings from 14.5 mm to 23.5 mm.

The resistance of copper $\rho = 1.68 \times 10^{-8} \Omega \text{ m}$. The SWG 20 wire has diameter 0.914 mm, giving a cross-sectional area $A = 6.56 \times 10^{-7} \text{ m}^2$. Thus the resistance of a ring is:

$$R = \rho \frac{l}{A} = 1.68 \times 10^{-8} \times \frac{2 \times 3.14 \times r}{6.56 \times 10^{-7}} = 0.161 r$$

where r is the radius of the ring and SI units are used throughout.

The total resistance of the big coils is found by summing the resistance of each ring (r takes values between 38.5 mm and 47.5 mm inclusive in steps of 1 mm):

$$10 \times \sum_r 0.161 r = 0.692 \Omega$$

And small ring (r takes values between 14.5 mm and 22.5 mm inclusive in steps of 1 mm):

$$10 \times \sum_r 0.161 r + 5 \times 0.161 \times 23.5 = 0.287 \Omega$$

So the total resistance is:

$$R = 2 \times (0.692 + 0.287) = 1.96 \Omega$$

The root-mean-square current in the coils is

$$I = \sqrt{\frac{1}{T} \int_0^T [I_0 \sin(\omega t + \phi)]^2 dt} = \frac{I_0}{\sqrt{2}}$$

where I_0 is the amplitude of the current, ω angular frequency, ϕ angular offset, t time and T period. For $I_0 = 0.1 \text{ A}$ (a typical operating current), power

$$P = I^2 R = 20 \text{ mW}$$

Next we calculate heat capacity. The total length of all the coils is 76.4 m. Again the concentric ring model is assumed to obtain this value.

$$\text{Capacity} = V \times C = 76.4 \times 6.56 \times 10^{-7} \times 3.45 \times 10^6 = 172.9 \text{ J K}^{-1}$$

where V is the volume of the coils and C is the isobaric volumetric heat capacity of copper. To raise the temperature by $0.1 \text{ }^\circ\text{C}$, it takes a minimum of

$$t = \frac{E}{P} = \frac{172.9 \times 0.1}{20 \times 10^{-3}} = 8.6 \times 10^2 \text{ s} = 15 \text{ min}$$

And this is in complete negligence of heat dissipation from the coils. In practice the temperature rise will be much slower.

Dissipation vs. Wire Thickness Calculation

Here we calculate the dependence of the rate of Joule heat generation on the thickness of copper wires that make up the magnetic tweezers. All other variables are held constant, such as the B-field generated and the space available in the spools for the wire winding. Also we neglect the thickness of the enamel wrapping of the wires and the skin effect.

The B-field is linearly proportional to the current, I , and the number of turns, n :

$$B \propto I \cdot n \quad (1)$$

which gives

$$I \propto \frac{B}{n} \quad (2)$$

The resistance of the wire, R , depends on the cross-sectional area, A , and the length, l , of the wire according to the following relationship:

$$R \propto \frac{l}{A} \quad (3)$$

Since $l \propto n$ and $A \propto \frac{1}{n}$ (spool space is fixed so the more turns there are, the thinner the wire needs to be), Equation (3) can be written in terms of n :

$$R \propto \frac{n}{\frac{1}{n}} = n^2 \quad (4)$$

The equation for power dissipation is

$$P = I^2 R \quad (5)$$

Substituting (2) and (4) into (5);

$$P \propto \left(\frac{B}{n}\right)^2 n^2 = B^2 \quad (6)$$

The cross-section of the wire cancels out so Joule heating does not depend on the thickness of the wires.