

Article

Lensless Formation and Turbulence Response of Airy-Type Beams Generated by Direct Fresnel Propagation

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Abstract

We investigate a lensless Airy-type optical beam generated by direct Fresnel propagation of a Gaussian apodized real space cubic phase. Starting from the scalar Fresnel diffraction integral, we derive a closed-form expression showing that the propagated field can be written in terms of an Airy function, but with an argument different from that of the conventional finite energy Airy beam. This difference leads to distinct free space dynamics: the conventional beam is already formed at the input plane and follows a parabolic trajectory, whereas the lensless beam forms during propagation and undergoes stronger transverse reshaping and broadening. The two fields are also compared under atmospheric turbulence using a phase screen approach and multiple performance metrics. The comparison shows that no single beam performs best for all detection scenarios: the lensless field gives lower scintillation at the beam maximum, the two beams provide comparable aperture collected power, and the conventional Airy beam gives higher coherent fiber coupling efficiency.

Keywords: Airy beams; lensless optical fields; Fresnel propagation; cubic phase modulation; free-space optical communication; atmospheric turbulence; phase screen method; scintillation

1. Introduction

Airy beams are a class of structured optical fields known for their transverse acceleration, resistance to diffraction over a finite propagation range, and self-healing behavior [1,2]. Since the ideal Airy solution carries infinite energy, experimentally realizable Airy beams are usually implemented in a finite-energy form by introducing an exponential or Gaussian type apodization [3,4]. Such beams have been studied in different optical contexts, including beam shaping [5,6], particle manipulation [7,8], plasma guiding [9], nonlinear optics [10,11], turbulent free-space optical propagation [12,13], and optical quasiparticles [14–16].

In the conventional optical realization, a finite-energy Airy beam is generated by imposing a cubic phase on an input beam [17] and then applying a Fourier-transforming lens system. In this configuration, the Airy profile is already formed at the output plane of the Fourier system [18] and then propagates as a pre-formed accelerating beam. This generation mechanism is well established and provides the usual reference model for finite-energy Airy beams.

Several compact or lensless Airy-beam generation schemes have also been reported, but they address different optical configurations from the one considered here. Cottrell et al. directly generated accelerating beams by encoding a $3/2$ phase-only pattern on an LCD, using a caustic-based phase design rather than a Gaussian-apodized real-space



Received: 29 May 2026

Revised: 8 July 2026

Accepted: 23 July 2026

Published: 26 July 2026

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cubic phase [19]. Latychevskaia et al. studied compact transmissive SLM implementations, including a numerical lens phase combined with a cubic phase, where the Airy beam is produced at the focal plane of the numerical lens, and a separate method based on directly loading an Airy-function transmission distribution onto the SLM [17]. Lu et al. combined the cubic Airy phase with a phase-only Fresnel holographic lens, so that the holographic lens implements the Fourier-transform operation while reducing the system length relative to the conventional Fourier-lens arrangement [20]. Thus, these studies either replace the physical Fourier lens by a numerical or Fresnel holographic lens, or use a different caustic phase mask.

Against this background, we consider a different physical situation. Instead of using a Fourier-transforming lens, the Gaussian-apodized cubic phase is placed directly in the input plane and allowed to propagate in free space. The central question is whether this direct Fresnel propagation can form an Airy-type field, and, if so, how this lensless field differs from the conventional Fourier-transform Airy beam. Although both fields contain Airy function structure, they are generated by different propagation mechanisms. Therefore, they should not be assumed to have identical dynamics. From a practical point of view, this configuration is attractive because it replaces the Fourier-transforming lens system with direct Fresnel propagation from a real-space cubic-phase element. Therefore, it may provide a simpler and more compact route for generating an Airy-type field at a prescribed propagation plane.

The first objective of the paper is to derive the analytical expression for the lensless Airy-type beam generated by direct propagation of a Gaussian-apodized real-space cubic phase. The derivation shows that the field can indeed be written in terms of an Airy function, but with an argument that differs from the conventional finite-energy Airy beam. This difference leads to a different transverse evolution: the conventional Airy beam follows the familiar parabolic displacement, whereas the lensless field undergoes a formation stage before developing an Airy-like structure.

The second objective is to compare the free-space propagation dynamics of the two fields, the conventional Airy beam and the lensless one. For this purpose, we analyze the main-lobe trajectories, transverse profiles, two-dimensional intensity distributions, and three-dimensional intensity isosurfaces. We also introduce characteristic propagation distances that help interpret the formation of the lensless field. These quantities provide a physical picture of when the caustic-forming part of the cubic phase becomes sufficiently illuminated by the finite Gaussian aperture and when the dominant lensless Airy lobe crosses the optical axis.

The final objective is to examine how the two fields behave in a turbulent propagation channel. Atmospheric turbulence is an important limitation in free-space optical systems, where random refractive index fluctuations lead to beam wander, spreading, and intensity scintillation [21,22]. To make the comparison meaningful, the turbulent propagation is evaluated at a free-space matched plane where the conventional and lensless fields have the same normalized transverse intensity distribution in the absence of turbulence. This allows differences in the scintillation behavior to be related not simply to different receiver-plane beam shapes, but to the different complex field evolution of the two beams along the propagation path.

The paper is organized as follows. Section 2 recalls the conventional finite-energy Airy beam and derives the analytical expression for the lensless Airy-type field generated by direct Fresnel propagation. Section 3 compares the free-space propagation dynamics of the conventional and lensless fields. Section 4 describes the phase-screen model used for atmospheric turbulence. Section 5 introduces the matched comparison plane and presents

the pixel-by-pixel scintillation results together with power and coherent coupling. Section 6 discusses the physical interpretation, limitations, and possible future directions of the work.

2. Airy Beam Generated by Direct Propagation

Before introducing the lensless configuration considered in this work, it is useful to briefly recall the conventional finite-energy Airy beam. In the paraxial approximation, the scalar field envelope satisfies the normalized one-dimensional diffraction equation

$$i \frac{\partial \psi}{\partial \zeta} + \frac{1}{2} \frac{\partial^2 \psi}{\partial s^2} = 0, \quad (1)$$

where $s = x/x_0$ is the normalized transverse coordinate and $\zeta = z/(kx_0^2)$ is the normalized propagation distance. The ideal Airy solution is non-spreading and accelerating, but it carries infinite energy. Therefore, experimentally realizable Airy beams are usually described by the exponentially apodized input field

$$\psi(s, 0) = \text{Ai}(s) \exp(as), \quad a > 0, \quad (2)$$

where a is the finite-energy truncation parameter. The corresponding propagated field can be written as

$$\psi(s, \zeta) = \text{Ai}\left(s - \frac{\zeta^2}{4} + ia\zeta\right) \exp\left(as - \frac{a\zeta^2}{2}\right) \exp\left[i\left(\frac{s\zeta}{2} + \frac{a^2\zeta}{2} - \frac{\zeta^3}{12}\right)\right]. \quad (3)$$

The term $s - \zeta^2/4$ in the Airy-function argument determines the characteristic parabolic transverse displacement of the beam. Thus, even after finite-energy truncation, the main intensity structure follows an accelerating trajectory during propagation.

In the usual Fourier-transform realization, an Airy beam is obtained by imposing a cubic phase on a Gaussian beam and then using a lens system to perform the Fourier transformation. In contrast, here we consider a different physical situation. The cubic phase is not Fourier-transformed by a lens. Instead, the field is allowed to propagate directly in free space. The question is then whether the Fresnel propagation of such a real-space cubic phase can form an Airy-type field, and how the resulting field differs from the conventional Airy beam.

We start from the scalar Fresnel diffraction integral

$$E(x, y, z) = \frac{e^{ikz}}{i\lambda z} \iint_{-\infty}^{\infty} E(x', y', 0) \exp\left[\frac{ik}{2z}((x - x')^2 + (y - y')^2)\right] dx' dy', \quad (4)$$

where $k = 2\pi/\lambda$. As an input field, we choose a separable Gaussian-apodized real-space cubic phase,

$$E(x, y, 0) = \exp\left(\frac{(a + ix/x_0)^3}{3}\right) \exp\left(\frac{(a + iy/y_0)^3}{3}\right), \quad a > 0. \quad (5)$$

The parameter a plays the role of a finite-energy truncation parameter: after expansion, the real part of $(a + ix/x_0)^3/3$ contains the Gaussian apodization term, while the imaginary part contains the cubic phase contribution.

Experimentally, the lensless Airy-type beam can be approximated by illuminating a phase-only optical element with a Gaussian beam. The Gaussian beam supplies the finite-energy apodization, while a spatial light modulator, transmissive diffractive optical element, or geometrical phase element imposes the required real-space cubic phase. Unlike conventional Airy beam generation, this configuration does not require a Fourier-transforming

lens after the cubic phase element. The Airy-type structure is instead formed by direct Fresnel propagation, which may provide a simpler and more compact arrangement when the desired intensity distribution is needed at a prescribed propagation plane.

We introduce dimensionless variables.

$$s_x = \frac{x}{x_0}, \quad \sigma_x = \frac{x'}{x_0}, \quad \zeta_x = \frac{z}{kx_0^2}, \quad s_y = \frac{y}{y_0}, \quad \sigma_y = \frac{y'}{y_0}, \quad \zeta_y = \frac{z}{ky_0^2}. \quad (6)$$

Because both the Fresnel kernel and the input field are separable in x and y , the propagated field factorizes into two one-dimensional Fresnel integrals,

$$E(x, y, z) = e^{ikz} I_x(s_x, \zeta_x) I_y(s_y, \zeta_y). \quad (7)$$

For compactness, the following derivation is written for one generic transverse coordinate using s , σ , and ζ :

$$I(s, \zeta) = \frac{1}{\sqrt{i2\pi\zeta}} \int_{-\infty}^{\infty} \exp\left[i\frac{(s-\sigma)^2}{2\zeta}\right] \exp\left[\frac{(a+i\sigma)^3}{3}\right] d\sigma. \quad (8)$$

The x -dependent factor is $s \rightarrow s_x$, $\sigma \rightarrow \sigma_x$, and $\zeta \rightarrow \zeta_x$ and the y -dependent factor has the same form after the substitutions $s \rightarrow s_y$, $\sigma \rightarrow \sigma_y$, and $\zeta \rightarrow \zeta_y$.

The integral in Equation (8) can be reduced to the standard Airy form by expanding the exponent and collecting powers of σ :

$$\frac{(a+i\sigma)^3}{3} + i\frac{(s-\sigma)^2}{2\zeta} = \frac{a^3}{3} + i\frac{s^2}{2\zeta} + i\left(-\frac{\sigma^3}{3} + p\sigma^2 + q\sigma\right), \quad (9)$$

where $p = 1/(2\zeta) + ia$ and $q = a^2 - s/\zeta$. The quadratic term is removed by the shift $\sigma = u + p$, which transforms the cubic polynomial as

$$-\frac{(u+p)^3}{3} + p(u+p)^2 + q(u+p) = -\frac{u^3}{3} + Su + D,$$

with $S = p^2 + q$ and $D = 2p^3/3 + pq$. Substituting this result into Equation (8), the field becomes

$$I(s, \zeta) = \frac{e^{a^3/3} e^{is^2/(2\zeta)} e^{iD}}{\sqrt{i2\pi\zeta}} \int_{-\infty}^{\infty} \exp\left[i\left(-\frac{u^3}{3} + Su\right)\right] du. \quad (10)$$

The remaining integral can be identified using the standard representation of the Airy function,

$$\text{Ai}(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \exp\left[i\left(\frac{u^3}{3} + xu\right)\right] du. \quad (11)$$

In Equation (10), the cubic term has the opposite sign. This is removed by the change of variable $u \mapsto -u$, which leaves the integration limits unchanged and changes the linear term from Su to $-Su$. Therefore, the remaining integral is equal to $2\pi \text{Ai}(-S)$, and Equation (10) becomes

$$I(s, \zeta) = \frac{\sqrt{2\pi}}{\sqrt{i\zeta}} \exp\left[\frac{a^3}{3} + i\frac{s^2}{2\zeta} + iD\right] \text{Ai}(-S). \quad (12)$$

Introducing $\alpha = 1/(2\zeta) + ia$ and $\beta = s/\zeta - a^2$, one has $S = \alpha^2 - \beta$ and $D = 2\alpha^3/3 - \alpha\beta$. Therefore, the one-dimensional field generated by direct Fresnel propagation of the real-space cubic phase can be written as

$$I(s, \zeta) = \frac{\sqrt{2\pi}}{\sqrt{i\zeta}} \exp\left[\frac{a^3}{3} + i\frac{s^2}{2\zeta} + i\left(\frac{2\alpha^3}{3} - \alpha\beta\right)\right] \text{Ai}(\beta - \alpha^2) \quad (13)$$

where the Airy-function argument is

$$\beta - \alpha^2 = \frac{s}{\zeta} - \frac{1}{4\zeta^2} - \frac{ia}{\zeta}. \quad (14)$$

Equation (13) shows that a direct propagating cubic-phase input indeed produces an Airy-type field. However, the Airy argument differs from that of the conventional finite-energy Airy beam. In the conventional case, the transverse coordinate appears as $s - \zeta^2/4$, whereas in the present lensless case it appears as $s/\zeta - 1/(4\zeta^2)$. This difference is important: the field is not launched as an Airy beam at $z = 0$, but rather forms an Airy-type structure through Fresnel propagation.

Finally, combining the two transverse directions gives the two-dimensional field given by Equation (7). This result provides the analytical basis for comparing the lensless Fresnel-zone Airy beam with the conventional Fourier-transform Airy beam.

3. Propagation Dynamics and Comparison with the Conventional Airy Beam

The analytical expressions derived above allow the propagation of the lensless field to be compared directly with the conventional finite-energy Airy beam. Although both fields contain an Airy function, their propagation dynamics are fundamentally different. In the conventional case, the Airy profile is already formed at the input plane. The field then mainly translates along its accelerating trajectory while retaining its transverse structure over a finite propagation range. In the lensless case, however, the input field is not an Airy beam. It is a Gaussian-apodized cubic phase distribution, and the Airy-type structure appears only after Fresnel propagation.

This difference can be seen directly from the Airy function arguments. For the conventional finite-energy Airy beam, Equation (3) contains the argument $s - \zeta^2/4 + ia\zeta$. Therefore, the real part of the Airy argument is shifted by $\zeta^2/4$, which gives the familiar parabolic displacement of the beam. If $\eta_0 \simeq -1.01879$ denotes the position of the first maximum of the Airy function, defined by $\text{Ai}'(\eta_0) = 0$, then the approximate trajectory of the dominant conventional Airy lobe is

$$\begin{aligned} A_{\text{conv}}(s, \zeta) &= s - \frac{\zeta^2}{4} + ia\zeta, \\ s_{\text{peak,conv}}(\zeta) &= \frac{\zeta^2}{4} + \eta_0. \end{aligned} \quad (15)$$

For the lensless field, Equation (14) shows that the transverse coordinate enters the Airy argument as s/ζ , rather than simply as s . This gives a different propagation law. Using the same Airy maximum η_0 , the corresponding dominant-lobe trajectory is

$$\begin{aligned} A_{\text{lensless}}(s, \zeta) &= \frac{s}{\zeta} - \frac{1}{4\zeta^2} - i\frac{a}{\zeta}, \\ s_{\text{peak,lensless}}(\zeta) &= \frac{1}{4\zeta} + \eta_0\zeta. \end{aligned} \quad (16)$$

Equations (15) and (16) summarize the main analytical difference between the two beams. The conventional Airy beam follows a parabolic trajectory proportional to ζ^2 . The lensless Airy structure, in contrast, is governed by two competing terms: $1/(4\zeta)$, which is large at short propagation distances, and $\eta_0\zeta$, which shifts the dominant lobe toward negative s at larger distances. Thus, the lensless field does not behave as a conventional Airy beam generated without a lens; it forms through Fresnel propagation and follows a different transverse evolution.

It is useful to introduce two characteristic propagation distances for the lensless field. The first one is an approximate formation distance. To see where it comes from, we can look at the phase of the one-dimensional Fresnel integral in Equation (8). Neglecting the slowly varying Gaussian amplitude, the relevant phase is

$$\Phi(\sigma; s, \zeta) = -\frac{\sigma^3}{3} + \frac{(s - \sigma)^2}{2\zeta}. \quad (17)$$

The stationary-phase condition $\partial\Phi/\partial\sigma = 0$ gives the ray relation $s = \sigma - \zeta\sigma^2$. The caustic is formed where neighbouring stationary-phase contributions merge, i.e., where $ds/d\sigma = 0$, or equivalently $\partial^2\Phi/\partial\sigma^2 = 0$. This gives the input-plane coordinate of the caustic-forming contribution,

$$\sigma_c = \frac{1}{2\zeta}. \quad (18)$$

Thus, σ_c is not a propagation coordinate, but the transverse input coordinate from which the caustic-forming ray originates at a given propagation distance ζ .

Because the input field is Gaussian-apodized by $\exp(-a\sigma^2)$, the illumination weight of this caustic-forming point is approximately

$$W_c(\zeta) = \exp\left[-a\sigma_c^2\right] = \exp\left[-\frac{a}{4\zeta^2}\right]. \quad (19)$$

At small ζ , the coordinate σ_c lies far in the weakly illuminated Gaussian tail, so the Airy caustic is not yet strongly visible. As ζ increases, σ_c moves closer to the illuminated part of the input field and the Airy-like structure becomes more pronounced. We define the normalized formation distance ζ_{norm} by the condition $W_c = \exp(-1)$, which gives

$$\zeta_{\text{norm}} = \frac{\sqrt{a}}{2}. \quad (20)$$

This is not a sharp physical boundary, but rather a convenient estimate of the propagation distance at which the caustic-forming part of the cubic phase becomes appreciably illuminated by the finite Gaussian aperture.

The second characteristic distance is the point at which the dominant Airy lobe of the lensless field crosses the optical axis. Setting the lensless main-lobe trajectory $s_{\text{peak, lensless}}(\zeta) = 0$, gives

$$\zeta_{\text{axis}} = \frac{1}{2\sqrt{-\eta_0}}. \quad (21)$$

This distance should not be interpreted as a geometrical focus. It is instead the axis-crossing point of the dominant Airy lobe formed by Fresnel propagation of the Gaussian-apodized cubic phase.

Figure 1 compares the normalized propagation maps of the conventional and lensless beams. In the conventional case, the Airy structure is already present at the input plane and follows the expected parabolic trajectory. In the lensless case, the field starts from a Gaussian-apodized cubic phase and the Airy-like structure appears only after propagation.

The analytical caustic and main-lobe trajectories are overlaid on the numerical intensity maps. The agreement shows that the caustic-like reference path of the lensless Airy structure is determined by the real part of the Airy argument in Equation (16). At small propagation distances, however, the lensless field is still in a formation region, and the strongest intensity maximum does not necessarily coincide perfectly with the asymptotic Airy-lobe estimate.

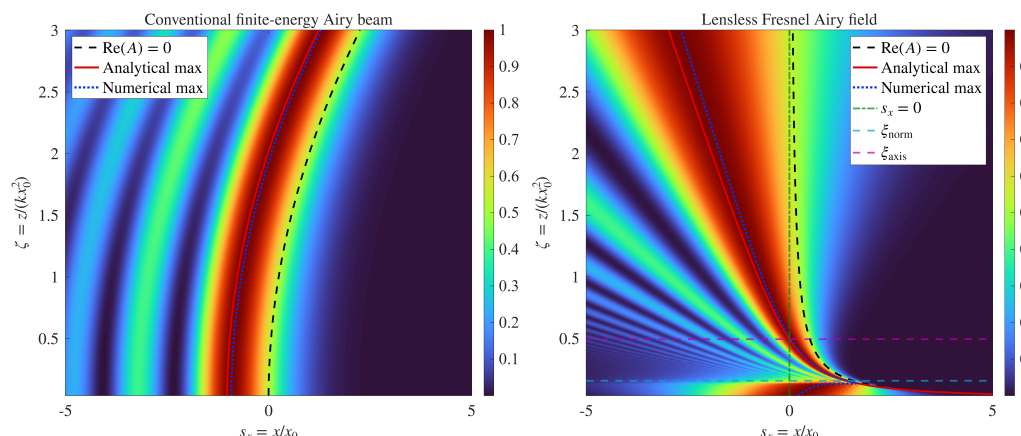


Figure 1. Normalized propagation maps of the conventional finite-energy Airy beam and the lensless Fresnel Airy beam. The conventional beam is already formed at the input plane and follows the familiar parabolic trajectory. The lensless field forms during propagation from a Gaussian-apodized cubic phase. The dashed and solid curves indicate the analytical caustic-like and main-lobe trajectories, respectively, while the dotted curve shows the numerically extracted intensity maximum.

The trajectory comparison is shown more explicitly in Figure 2. The conventional beam follows the expected parabolic law $s_{\text{peak,conv}} \sim \zeta^2/4 + \eta_0$. The lensless beam follows a different law $s_{\text{peak,lensless}} \sim 1/(4\zeta) + \eta_0\zeta$. This means that the lensless Airy lobe is not simply an ordinary Airy beam generated without optical elements. Instead, it is a Fresnel-formed Airy structure whose apparent acceleration is governed by a different scaling of the Airy argument. The markers ζ_{norm} and ζ_{axis} separate two useful parts of the lensless propagation: the approximate onset of Airy structure formation and the later crossing of the dominant lobe through the optical axis.

The difference between the two propagation mechanisms becomes clearer in the transverse cross-sections shown in Figure 3. The profiles are plotted in the coordinate $s_x = x/x_0$. Therefore, the transverse displacement, formation, and reshaping of the beams can be observed directly. For the conventional beam, the Airy profile is present from the beginning and mainly shifts along its propagation trajectory. In contrast, the lensless field initially corresponds to the intensity of the Gaussian-apodized input field. As propagation increases, an Airy-like asymmetric profile emerges, reaches the axis-crossing region near ζ_{axis} , and then broadens further. This behavior reflects the fact that the coordinate in the lensless Airy argument appears as s/ζ , so the transverse scale of the structure changes during propagation.

The same behavior can be observed in two transverse dimensions. Figure 4 shows intensity snapshots in the (s_x, s_y) plane. The conventional field preserves the familiar two-dimensional Airy structure, which is displaced along the diagonal direction as propagation increases. The lensless field, on the other hand, undergoes a visible formation process. At shorter distances, the field is still strongly affected by the finite input aperture and the Fresnel scaling. At later distances, an Airy-like structure is visible, but it is broader and less propagation-invariant than the conventional Airy beam. This supports the interpretation that the lensless field combines Airy-type caustic formation with ordinary finite-aperture diffraction.

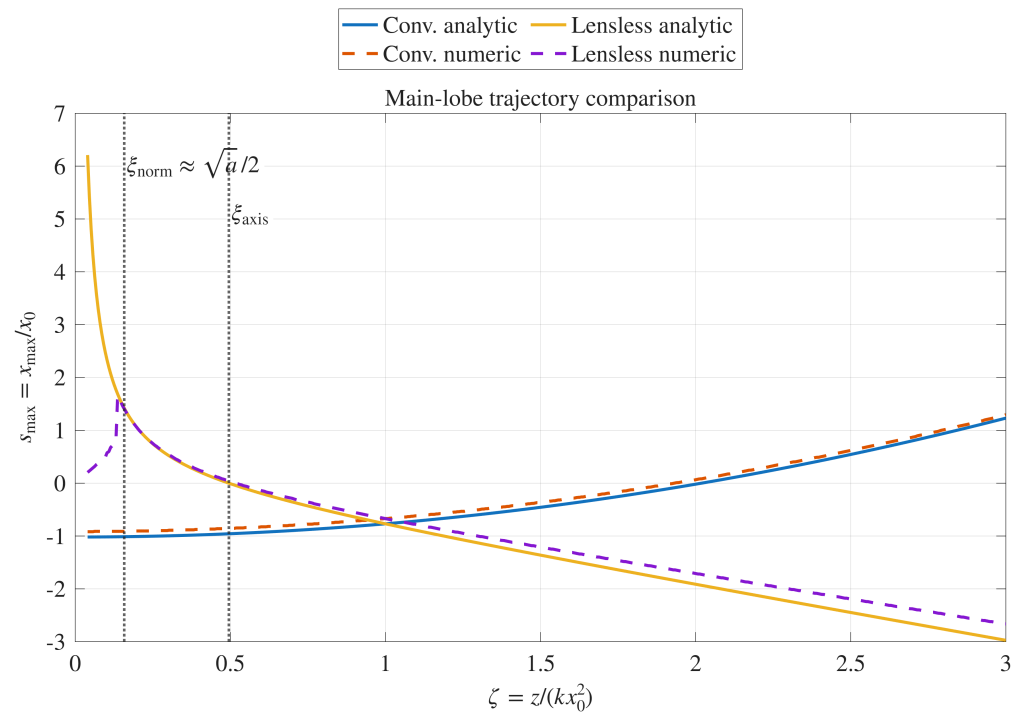


Figure 2. Analytical and numerically extracted main-lobe trajectories. The conventional Airy beam follows a parabolic trajectory, whereas the lensless Airy field follows the trajectory $s_{\text{peak,lensless}} = 1/(4\zeta) + \eta_0\zeta$. The characteristic distances ζ_{norm} and ζ_{axis} indicate the approximate formation distance and the axis-crossing distance of the lensless main lobe.

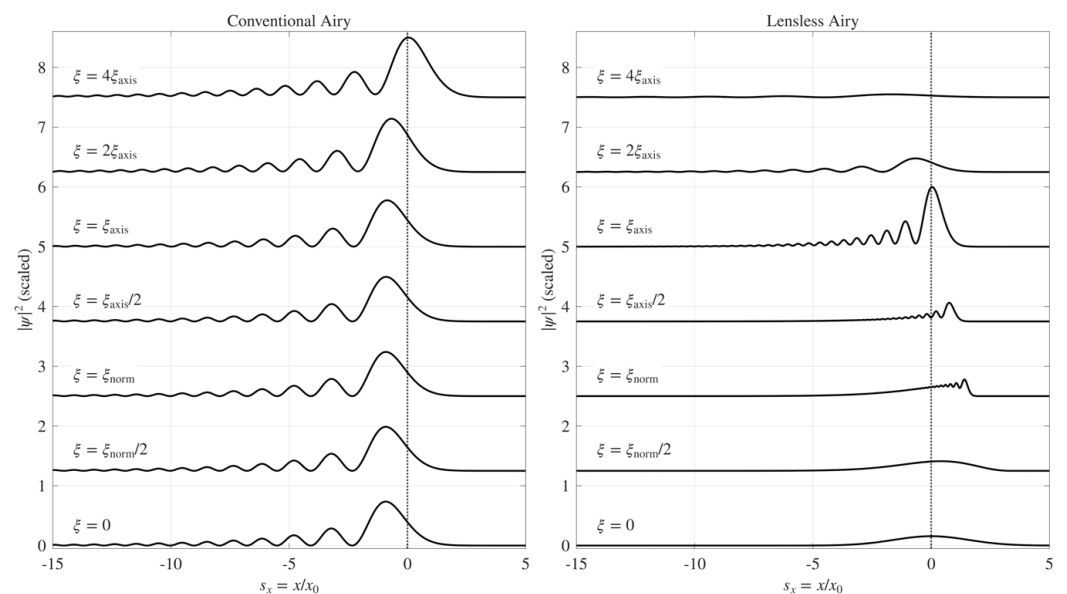


Figure 3. Transverse intensity profiles at selected normalized propagation distances: 0, $\zeta_{\text{norm}}/2$, ζ_{norm} , $\zeta_{\text{axis}}/2$, ζ_{axis} , $2\zeta_{\text{axis}}$, and $4\zeta_{\text{axis}}$. The profiles are shown in the $s_x = x/x_0$ and vertically shifted for clarity. The conventional Airy beam is already formed at the input plane, while the lensless beam evolves from a Gaussian-apodized cubic-phase input into an Airy-like structure during propagation.

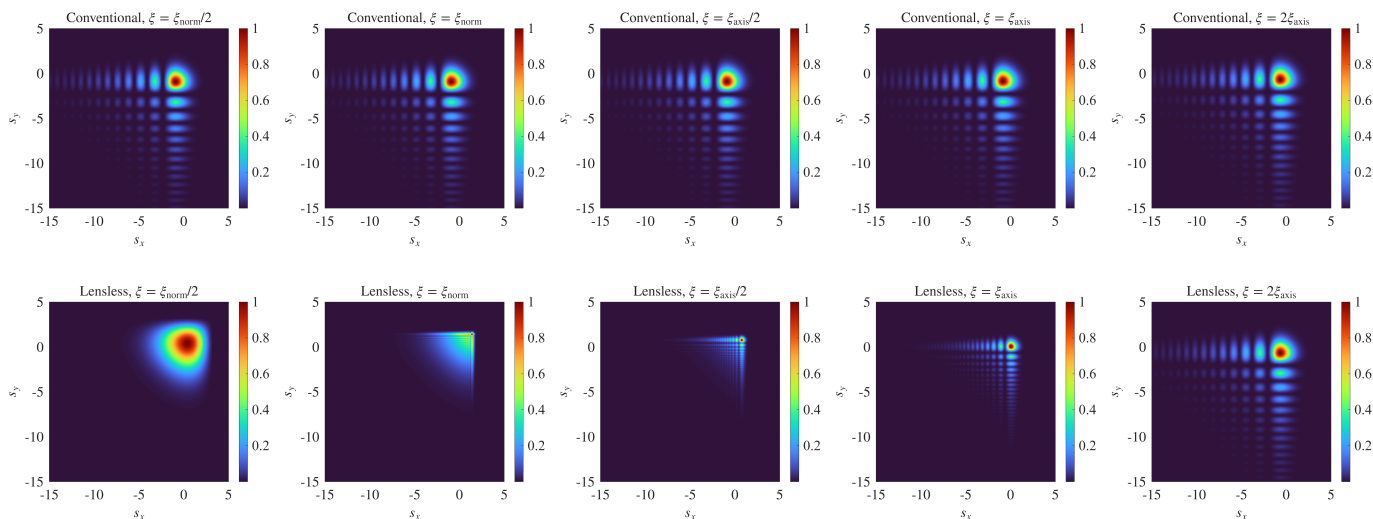


Figure 4. Two-dimensional transverse intensity snapshots of the conventional and lensless fields in laboratory coordinates. The conventional Airy beam is already formed and mainly translates during propagation. The lensless field forms through Fresnel propagation of the Gaussian-apodized cubic phase and exhibits stronger transverse reshaping and broadening.

A complementary view of the same propagation dynamics is given by the three-dimensional intensity isosurfaces in Figure 5. While the transverse snapshots in Figure 4 show selected propagation planes, the isosurface representation shows how the high- and low-intensity regions are connected along the propagation coordinate. In the conventional case, the Airy structure forms a continuous accelerating volume, indicating that the beam is already established and mainly follows its trajectory during propagation. In the lensless case, the intensity volume is more localized near the formation region and then expands, showing that the field first develops an Airy-like structure and subsequently undergoes stronger transverse broadening. Thus, the three-dimensional representation supports the same interpretation as the transverse snapshots, but makes the propagation-induced formation of the lensless field more visually explicit.

Finally, Figure 6 summarizes the two analytical quantities that help interpret the formation of the lensless Airy field. The first one is the caustic illumination weight $W_c(\zeta) = \exp[-a/(4\zeta^2)]$. At very small propagation distances, the caustic-forming point lies far out in the Gaussian-apodized tail of the input beam and is therefore weakly illuminated. As ζ increases, this point moves into the effective aperture and the Airy-like structure becomes more visible. The second quantity is the magnitude of the imaginary part of the lensless Airy argument, a/ζ . This term is large near the input plane and decreases during propagation, meaning that the Airy argument becomes progressively closer to a real Airy argument. Together, these two effects explain why the lensless Airy field is not fully formed at $z = 0$, but appears after a finite propagation distance.

Overall, the comparison shows that the lensless Airy field should not be viewed as a conventional Airy beam with the lens removed. The conventional beam is produced by a Fourier-transform operation and is already an Airy beam at the beginning of propagation. The lensless beam is produced by direct Fresnel propagation of a Gaussian-apodized cubic phase. It therefore contains an Airy-type caustic, but this caustic forms during propagation and is accompanied by transverse broadening due to the finite input aperture. This distinction is important for free-space transmission, because turbulence interacts with a field that may still be undergoing Airy-structure formation rather than with a fully formed Airy beam.

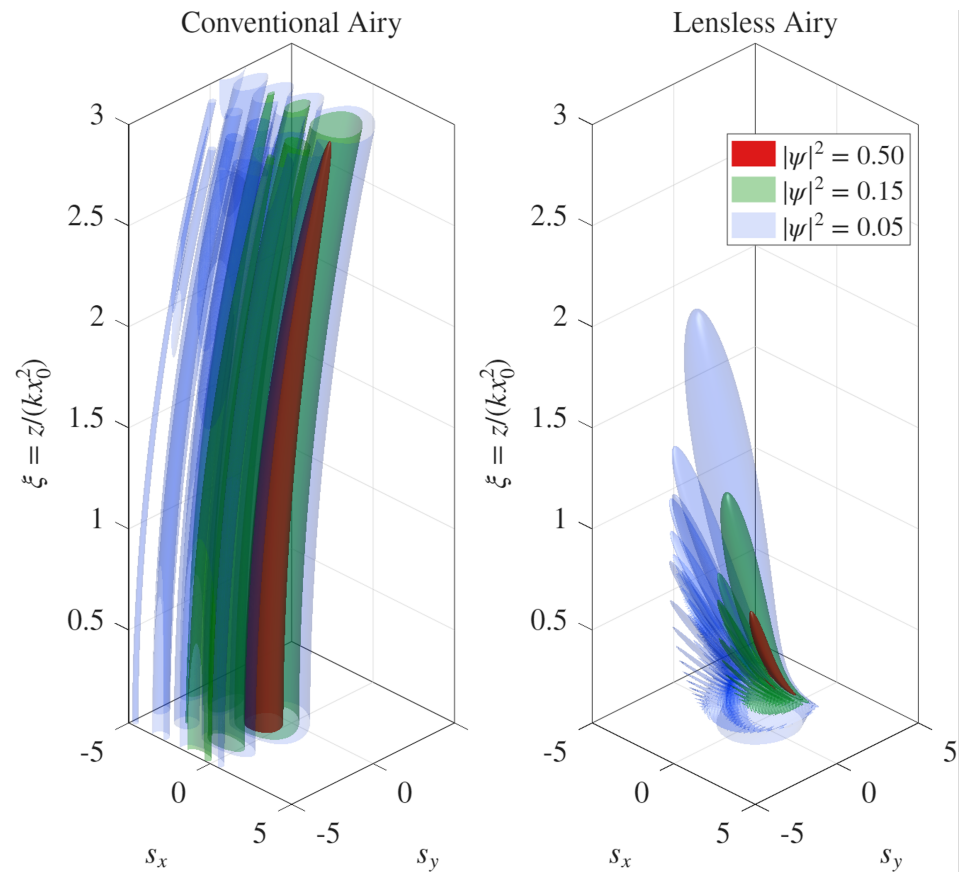


Figure 5. Three -dimensional normalized intensity isosurfaces of the conventional finite-energy Airy beam and the lensless Fresnel Airy beam. The isosurfaces correspond to normalized intensities $|\psi|^2 = 0.50, 0.15$, and 0.05 . The conventional Airy beam preserves a continuous accelerating structure over the plotted propagation interval, whereas the lensless field is concentrated near the formation region and broadens more rapidly after propagation. The coordinates are shown in normalized units $s_x = x/x_0$, $s_y = y/y_0$, and $\xi = z/(kx_0^2)$.

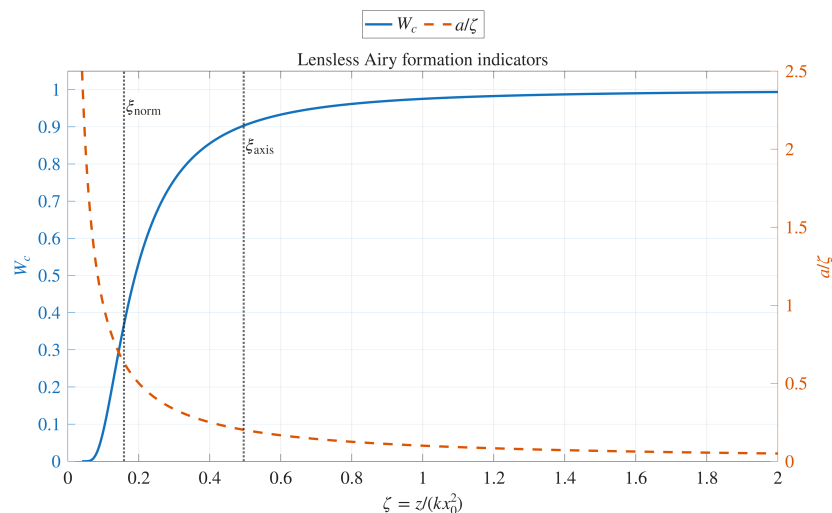


Figure 6. Analytical indicators of lensless Airy-field formation. The caustic illumination weight $W_c = \exp[-a/(4\zeta^2)]$ increases as the caustic-forming region moves into the effective Gaussian aperture. The imaginary part of the lensless Airy argument, a/ζ , decreases with propagation. The distance ξ_{norm} gives an approximate formation onset, while ξ_{axis} marks the axis crossing of the dominant Airy lobe.

4. Turbulence Phase Screen Modeling

Atmospheric turbulence is commonly modeled using random phase screens that reproduce the statistical properties of refractive index fluctuations. In this work, phase screens are generated using the inverse spectral (Fourier) method, supplemented with subharmonic components to improve low-frequency content [23,24].

The Fourier-based phase component $\phi_{\text{FFT}}(x, y)$ is obtained by filtering complex Gaussian white noise in the spatial frequency domain according to the Kolmogorov phase power spectral density (PSD), and transforming back to the spatial domain:

$$\phi_{\text{FFT}}(x, y) = \mathcal{F}^{-1} \left\{ \sqrt{\Phi_{\phi}(\mathbf{f})} W(\mathbf{f}) \right\}, \quad (22)$$

where $\mathcal{F}^{-1}\{\cdot\}$ denotes the inverse Fourier transform, $W(\mathbf{f})$ is a complex Gaussian random field, and $\Phi_{\phi}(\mathbf{f})$ is the Kolmogorov phase PSD given by

$$\Phi_{\phi}(\mathbf{f}) = 0.0229 r_0^{-5/3} |\mathbf{f}|^{-11/3}. \quad (23)$$

The Fried parameter r_0 is used to set the strength of the phase fluctuations in the generated phase screens. In the calculations, it is defined from the refractive-index structure parameter C_n^2 , propagation distance z , and optical wavenumber $k = 2\pi/\lambda$ as

$$r_0 = \left(0.423 k^2 C_n^2 z \right)^{-3/5}. \quad (24)$$

The corresponding Rytov variance is used as a measure of turbulence strength and is calculated as

$$\sigma_R^2 = 1.23 C_n^2 \left(\frac{2\pi}{\lambda} \right)^{7/6} z^{11/6}. \quad (25)$$

Thus, both r_0 and σ_R^2 are determined directly from the selected values of C_n^2 , wavelength λ , and propagation distance z .

To compensate for the lack of low-frequency components inherent to finite grids, additional subharmonic terms are introduced as a sum of plane waves:

$$\phi_{\text{SH}}(x, y) = \sum_p \sum_{m,n} a_{m,n}^{(p)} \exp \left[i 2\pi \left(f_x^{(p)} x + f_y^{(p)} y \right) \right], \quad (26)$$

where the coefficients $a_{m,n}^{(p)}$ are random variables scaled according to the turbulence spectrum, and $(f_x^{(p)}, f_y^{(p)})$ represent progressively lower spatial frequencies.

The final phase screen used in simulations is obtained as a superposition of the Fourier component and the subharmonic contribution:

$$\phi(x, y) = \phi_{\text{FFT}}(x, y) + \phi_{\text{SH}}(x, y). \quad (27)$$

The statistical consistency of the generated phase screens is verified using the phase structure function, defined as

$$D_{\phi}(r) = \left\langle [\phi(\mathbf{r}_0 + \mathbf{r}) - \phi(\mathbf{r}_0)]^2 \right\rangle, \quad (28)$$

which, for Kolmogorov turbulence, follows the analytical form

$$D_{\phi}(r) = 6.88 \left(\frac{r}{r_0} \right)^{5/3}. \quad (29)$$

Figure 7 shows an example phase screen realization and the corresponding structure function comparison. The inclusion of low-frequency subharmonic components improves the representation of large-scale phase distortions, which are responsible for effects such as beam wander and large-angle wavefront tilt. These components are important because beam displacement and pointing fluctuations can strongly affect the received intensity, especially in finite-aperture optical links. Physically, the lowest spatial-frequency phase terms can be interpreted as slowly varying phase ramps across the aperture, similar to a weak blazed grating, which locally deflects the beam. It should be noted, however, that the finite-grid spectral method is affected by spectral leakage and periodic boundary assumptions; therefore, the regions close to the edges of the generated screen are less reliable and are typically avoided in propagation calculations.

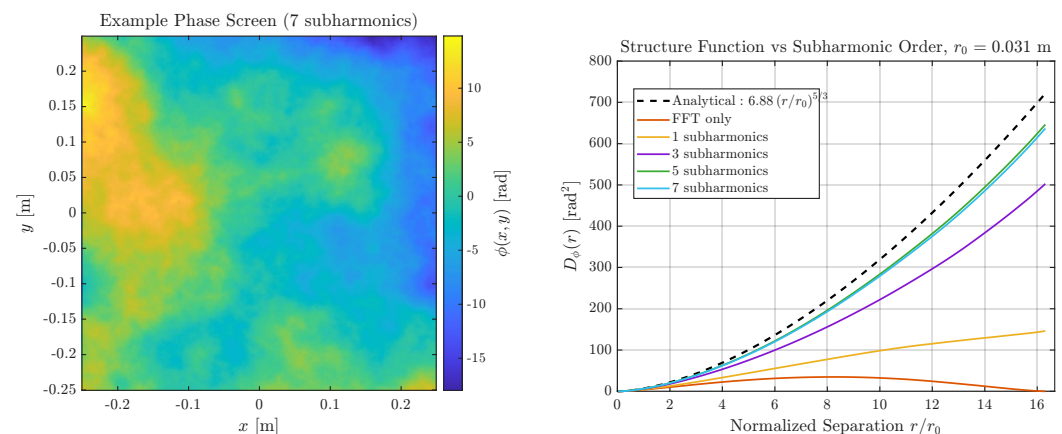


Figure 7. Turbulence phase screen modeling. Left: example phase screen $\phi(x, y)$ generated using the inverse spectral method with subharmonics. Right: phase structure function for different subharmonic orders compared to the analytical Kolmogorov model.

For weak to moderate turbulence regimes, a single phase screen model is often sufficient [25]. In this approximation, the entire turbulent path with constant refractive index structure parameter C_n^2 is represented by a single equivalent phase screen placed at the midpoint of the propagation path. This approach provides an efficient and widely used approximation for wave propagation through atmospheric turbulence.

5. Turbulent-Channel Comparison of Conventional and Lensless Airy Fields

Before introducing the turbulent channel, it is useful to identify a propagation distance at which the conventional and lensless Airy fields have the same transverse intensity distribution in free space. This gives a natural reference plane for comparing their behavior in turbulence. If the two beams are compared at an arbitrary distance, the observed differences may simply originate from their different free-space evolution. In contrast, comparison at a free-space matched plane allows the influence of turbulence to be isolated more clearly.

The matching condition follows directly from the Airy arguments of the two fields. For the conventional Airy beam, Equation (3) contains the argument $s - \zeta^2/4 + ia\zeta$, while for the lensless field Equation (14) contains $s/\zeta - 1/(4\zeta^2) - ia/\zeta$. At the normalized propagation distance

$$\zeta_{\text{match}} = 1, \quad (30)$$

these two arguments reduce to

$$A_{\text{conv}}(s, 1) = s - \frac{1}{4} + ia, \quad A_{\text{lensless}}(s, 1) = s - \frac{1}{4} - ia. \quad (31)$$

Thus, the two Airy arguments are complex conjugates. Since $\text{Ai}(z^*) = \text{Ai}(z)^*$, their moduli are equal. At the same distance, the exponential apodization factors also have the same transverse dependence, differing only by propagation-dependent constants. Therefore, $\zeta = 1$ represents a free-space matching plane at which the two beams have the same transverse intensity profile up to normalization. For the two-dimensional separable field, the same result holds when $x_0 = y_0$ and $a_x = a_y$, so that both transverse directions reach $\zeta = 1$ at the same physical distance.

The corresponding physical propagation distance is

$$z_{\text{match}} = kx_0^2 = \frac{2\pi x_0^2}{\lambda}. \quad (32)$$

This plane is selected as the receiver plane for the turbulent-channel comparison. In free space, the two beams have identical normalized intensity distributions at this distance. However, their complex fields and propagation histories are not identical. The lensless field is formed through Fresnel propagation of a Gaussian-apodized cubic phase, whereas the conventional Airy beam is already formed at the input plane. Therefore, turbulence may affect the two beams differently even when their free-space receiver-plane intensities are matched.

The turbulent propagation is evaluated using the phase-screen method. For each beam, an ensemble of statistically independent turbulent realizations is generated, and the received intensity at the matched plane is recorded. The local, pixel-by-pixel scintillation index is then calculated as

$$\sigma_I^2(x, y) = \frac{\langle I^2(x, y) \rangle - \langle I(x, y) \rangle^2}{\langle I(x, y) \rangle^2}, \quad (33)$$

where $\langle \cdot \rangle$ denotes ensemble averaging over turbulence realizations. This metric allows the local sensitivity of the conventional and lensless Airy fields to turbulence to be compared at a receiver plane where their free-space intensity distributions are otherwise the same.

For all turbulence simulations, the computational grid contained $N_x = N_y = 2048$ samples. The physical transverse window was $x \in [-0.2, 0.1]$ m and $y \in [-0.2, 0.1]$ m. The optical wavelength was $\lambda = 500$ nm, the beam transverse scaling parameters were $x_0 = y_0 = 3.5$ mm, and the apodization parameters were $a_x = a_y = 0.1$. The turbulent-channel comparison was performed at the matched propagation plane $\zeta = 1$, corresponding to $z = z_{\text{match}} = 1.54 \times 10^3$ m. For each Monte Carlo realization, a single statistically independent phase screen was generated and placed at the midpoint of the propagation path, $z = z_{\text{match}}/2$. The field was propagated over half of the path, multiplied by $\exp[i\phi(x, y)]$, and then propagated over the remaining half of the path to the receiver plane. The ensemble statistics were calculated from 500 independent phase-screen realizations.

Figures 8–10 show the received long-term beam distributions together with the corresponding pixel-by-pixel scintillation-index maps for three turbulence strengths. In the weakest case, $C_n^2 = 10^{-15} \text{ m}^{-2/3}$, shown in Figure 8, both beams retain a compact long-term distribution at the matched propagation plane. The scintillation-index maps show that the lowest-fluctuation region is concentrated around the main received structure, while larger values appear away from the high-intensity region.

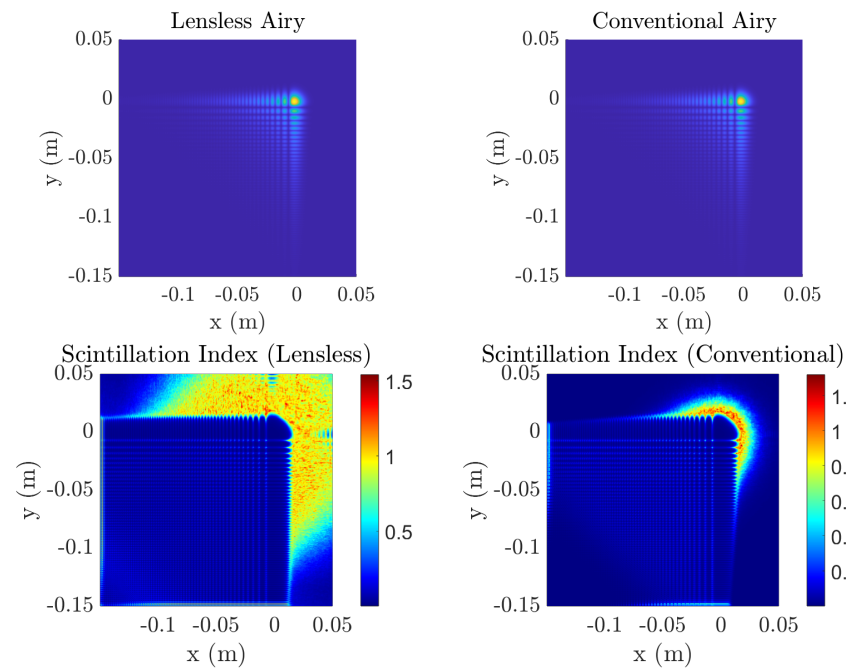


Figure 8. Turbulent propagation results for $C_n^2 = 10^{-15} \text{ m}^{-2/3}$ at the matched propagation plane $z = z_{\text{match}} = kx_0^2$, using $N = 500$ independent phase-screen realizations. The top row shows the long-term received beam distributions of the lensless and conventional Airy fields, while the bottom row shows the corresponding pixel-by-pixel scintillation index. At the position of the long-term beam maximum, the scintillation index is 3.9×10^{-4} for the lensless field and 8.3×10^{-4} for the conventional Airy beam.

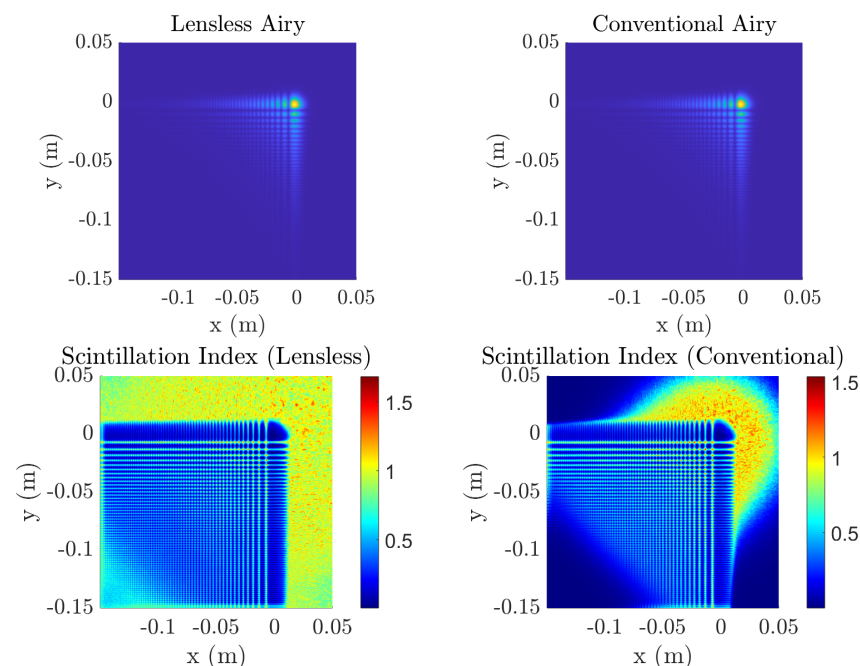


Figure 9. Turbulent propagation results for $C_n^2 = 10^{-14} \text{ m}^{-2/3}$ at the matched propagation plane $z = z_{\text{match}} = kx_0^2$, using $N = 500$ independent phase-screen realizations. The top row shows the long-term received beam distributions, and the bottom row shows the corresponding scintillation-index maps. Compared with Figure 8, the low-scintillation region becomes smaller and the long-term beam distributions become slightly more broadened. At the long-term beam maximum, the scintillation index is 4.34×10^{-3} for the lensless field and 9.17×10^{-3} for the conventional Airy beam.

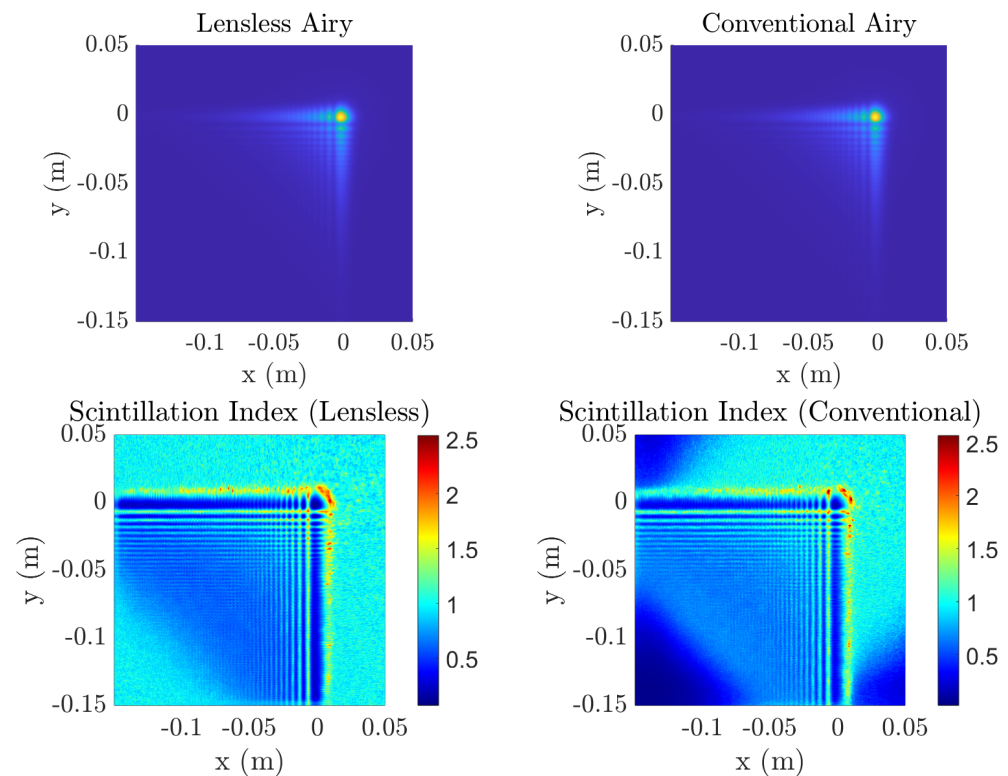


Figure 10. Turbulent propagation results for $C_n^2 = 10^{-13} \text{ m}^{-2/3}$ at the matched propagation plane $z = z_{\text{match}} = kx_0^2$, using $N = 500$ independent phase-screen realizations. The stronger turbulence further reduces the low-scintillation region and produces more visible broadening of the long-term beam distributions. At the long-term beam maximum, the scintillation index is 7.218×10^{-2} for the lensless field and 1.1757×10^{-1} for the conventional Airy beam.

When the turbulence strength is increased to $C_n^2 = 10^{-14} \text{ m}^{-2/3}$, as shown in Figure 9, the long-term beam distributions become slightly more spread, and the region of low scintillation becomes smaller. This trend becomes more pronounced for $C_n^2 = 10^{-13} \text{ m}^{-2/3}$ in Figure 10, where the received distributions are broader and the scintillation-index maps show higher values over a larger part of the observation plane.

At the position of the long-term beam maximum, the scintillation index increases monotonically with C_n^2 for both beams. For the lensless field, the values are 3.9×10^{-4} , 4.34×10^{-3} , and 7.218×10^{-2} for $C_n^2 = 10^{-15}$, 10^{-14} , and $10^{-13} \text{ m}^{-2/3}$, respectively. For the conventional Airy beam, the corresponding values are 8.3×10^{-4} , 9.17×10^{-3} , and 1.1757×10^{-1} . Thus, for the parameters considered here, the two beams show different scintillation levels even though they are compared at the free-space matched plane $z = z_{\text{match}}$. This confirms that matching the free-space receiver-plane intensity does not fully determine the turbulent propagation response, since the conventional and lensless fields have different complex-field evolutions before reaching the receiver plane.

Receiver-Aperture Power and Single-Mode Fiber Coupling

The scintillation index discussed above is a local intensity fluctuation metric. To obtain a receiver-level comparison, we additionally calculated two quantities: the detector power-in-bucket (PIB) and the single-mode-fiber (SMF) coupling efficiency. These metrics represent different detection scenarios. The PIB describes the optical power collected by a finite-area direct detector, whereas the SMF coupling efficiency describes coherent coupling into a fundamental fiber mode.

The detector power-in-bucket was calculated as the fraction of the received optical power falling inside a circular receiver aperture,

$$\text{PIB} = \frac{\int_{A_R} |U_r(x, y)|^2 dx dy}{\int_{\mathbb{R}^2} |U_r(x, y)|^2 dx dy}, \quad (34)$$

where $U_r(x, y)$ is the complex optical field at the receiver plane and A_R denotes the circular receiver aperture. This quantity depends only on the intensity distribution and therefore does not account for the phase structure of the received field.

The SMF coupling efficiency was calculated using a coherent overlap integral between the received optical field and the backpropagated fundamental mode of the fiber. Following the usual free-space-to-fiber coupling approach [26], the fundamental fiber mode was approximated by a Gaussian mode and propagated backward from the fiber facet to the receiver aperture plane. The coupling efficiency was then evaluated as

$$\eta_{\text{SMF}} = \frac{\left| \int_{A_R} U_r(x, y) U_m^*(x, y) dx dy \right|^2}{\left(\int_{A_R} |U_r(x, y)|^2 dx dy \right) \left(\int_{A_R} |U_m(x, y)|^2 dx dy \right)}, \quad (35)$$

where $U_m(x, y)$ is the backpropagated fiber mode at the receiver aperture plane. In contrast to the PIB, this metric is sensitive to both amplitude matching and phase matching. Therefore, phase distortions, wavefront curvature, and spatial mismatch across the receiver aperture can reduce the SMF coupling efficiency even when the aperture-collected power remains high.

The backpropagated fiber mode was represented by the Gaussian profile $U_m(x, y) = C \exp[-(x^2 + y^2)/w_R^2]$, where C is a normalization constant and w_R is the radius of the backpropagated Gaussian mode at the receiver plane. The mode was truncated by the circular receiver aperture before evaluating the overlap integral, so that only the collected portion of the received optical field contributed to the fiber-coupling calculation.

The coupling geometry was specified by the parameter $a_c = \pi W_m D_R / (2\lambda f)$, where D_R is the receiver aperture diameter, W_m is the fiber mode radius at the fiber facet, λ is the optical wavelength, and f is the focal length of the coupling lens. The value $a_c = 1.12$ was used because it corresponds to the optimum coupling condition for a coherent incident field [26]. The fiber parameters were chosen according to the Corning RGB 400 specialty single-mode fiber, which is designed for visible wavelength operation. At $\lambda = 500$ nm, the mode-field diameter was taken as $\text{MFD} = 3.2$ μm , corresponding to $W_m = 1.6$ μm . With $D_R = 5$ cm and $a_c = 1.12$, the resulting coupling-lens focal length is $f = \pi W_m D_R / (2a_c \lambda) \approx 0.224$ m. This choice ensures that the backpropagated Gaussian fiber mode has the appropriate size at the receiver aperture for the selected coupling geometry.

Figure 11 compares the PIB and SMF coupling efficiency as functions of the turbulence strength C_n^2 . The PIB values of the conventional and lensless Airy beams are very similar over the investigated turbulence range. For weak and moderate turbulence, the collected power remains nearly constant, and a noticeable decrease appears only for stronger turbulence. This indicates that, for the chosen receiver aperture, both beams deliver a comparable amount of optical power to a finite-area direct detector.

The SMF coupling efficiency shows a different trend. The conventional Airy beam gives higher coupling efficiency than the lensless Airy beam over the investigated range of C_n^2 . This result demonstrates that similar collected power does not necessarily imply similar coherent coupling into a single-mode fiber. The difference arises because the fiber-coupling metric is sensitive to the complex field, not only to the intensity.

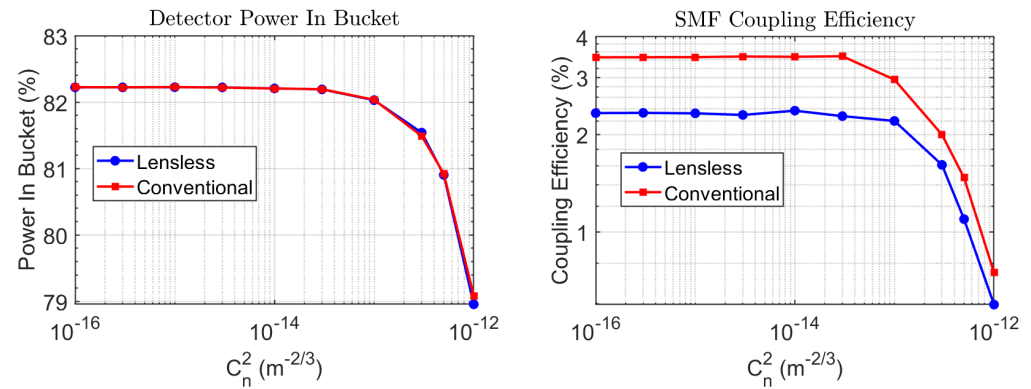


Figure 11. Comparison of detector power-in-bucket (PIB) and single-mode-fiber (SMF) coupling efficiency as a function of turbulence strength C_n^2 . The left plot shows that the aperture-collected PIB values of the conventional and lensless Airy fields remain similar for weak and moderate turbulence and decrease mainly under stronger turbulence. The right plot shows the SMF coupling efficiency, which depends not only on collected power but also on the phase coherence and mode matching of the received field with the fundamental fiber mode. Although the two beams have comparable PIB, the conventional Airy beam provides higher SMF coupling efficiency than the lensless Airy beam over the investigated turbulence range. Fiber used: Corning RGB 400 specialty optical fiber.

This behavior is illustrated in Figure 12. The main intensity lobes of both beams lie inside the receiver aperture, which explains the similar PIB values. However, the phase distributions across the aperture are different. The conventional Airy beam retains a more regular phase structure, whereas the lensless Airy field exhibits stronger and more rapidly varying phase modulation. This additional phase variation is associated with the Fresnel formation of the lensless field from the Gaussian-apodized cubic phase. As a result, the coherent overlap with the backpropagated Gaussian fiber mode is reduced, leading to lower SMF coupling efficiency.

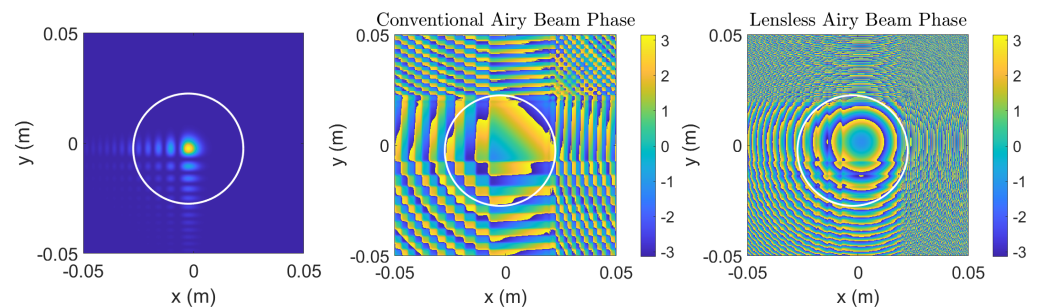


Figure 12. Intensity and phase distributions of the Airy beams at the receiver plane. The left figure shows the beam intensity profile together with the circular collection aperture, indicating that the main Airy lobes are located within the aperture. The middle figure shows the phase distribution of the conventional Airy beam, where the phase structure remains relatively organized across the aperture. The right figure shows the phase distribution of the lensless Airy beam, exhibiting a more distorted and rapidly varying phase pattern. These phase differences affect the spatial coherence of the received field and therefore influence the single-mode fiber coupling efficiency.

When the PIB is considered, the conventional and lensless Airy beams perform similarly for the chosen receiver aperture. When coherent SMF coupling is considered, the conventional Airy beam performs better because its receiver-plane phase is more compatible with the fundamental fiber mode. Therefore, the relative performance of the two beams depends on the receiver architecture and on whether the relevant metric is intensity collection or coherent mode matching.

6. Discussion

The results of this work show that a Gaussian-apodized cubic phase can generate an Airy-type field by direct Fresnel propagation, without using a Fourier-transforming lens. This lensless field should not be interpreted simply as a conventional Airy beam with the lens removed. In the conventional case, the Airy profile is already formed at the input plane, whereas in the lensless case the input field is a Gaussian-apodized cubic phase and the Airy-type structure appears only after propagation. This distinction is reflected analytically in the different Airy arguments and is observed numerically in the propagation maps, transverse profiles, two-dimensional snapshots, and three-dimensional isosurfaces.

The free-space analysis shows that the two beams follow different transverse evolution laws. The conventional Airy beam follows the familiar parabolic displacement, while the lensless field is governed by the scaled coordinate s/ζ and therefore undergoes a formation stage before developing a recognizable Airy-like structure. The characteristic distances ζ_{norm} and ζ_{axis} help interpret this process: the first gives an approximate formation onset related to the illumination of the caustic-forming part of the cubic phase, while the second marks the axis crossing of the dominant lensless Airy lobe. These quantities do not represent sharp boundaries, but they provide useful markers for understanding the field evolution.

For the turbulence study, the comparison was made at the matched free-space plane ($z_{\text{match}} = kx_0^2$), where the conventional and lensless fields have the same normalized transverse intensity distribution in the absence of turbulence. This choice reduces the possibility that differences at the receiver are caused only by different free-space beam shapes. The phase-screen simulations show that, as C_n^2 increases, the low-scintillation region becomes smaller and the long-term beam distributions become broader. At the position of the long-term beam maximum, the lensless field gives lower pixel-by-pixel scintillation-index values than the conventional Airy beam for the three turbulence strengths considered here.

This result, however, should be interpreted with care because the pixel-by-pixel scintillation index is a local metric. It does not directly determine the performance of a finite-aperture or fiber-coupled receiver. For this reason, the comparison was extended to the detector power-in-bucket and the single-mode-fiber coupling efficiency. The power-in-bucket results show that the two beams provide comparable aperture-collected power for the chosen receiver aperture. In contrast, the conventional Airy beam gives higher single-mode-fiber coupling efficiency than the lensless field. This difference is attributed to the more regular receiver-plane phase structure of the conventional Airy beam, whereas the lensless field contains stronger phase variations associated with its Fresnel formation from the Gaussian-apodized cubic phase.

Therefore, the relative performance of the two beams in turbulence is not universal, but depends on the chosen evaluation metric and receiver configuration. The lensless field performs better according to the local scintillation index at the beam maximum, the two fields are similar in terms of aperture-collected power, and the conventional Airy beam is favoured for coherent single-mode-fiber coupling. This shows that matching the free-space intensity profile at the receiver plane is not sufficient to predict the practical turbulence response; the phase structure and detection method must also be considered.

The present analysis is limited to the selected beam parameters, matched propagation plane, receiver aperture, fiber-coupling geometry, and single-phase-screen turbulence model. Future work should examine how these conclusions change for other beam parameters, propagation distances, receiver apertures, and distributed turbulence models.

Author Contributions: Conceptualization, J.B. and S.O.; derivation, J.B.; software, J.B. and K.L.; validation, J.B. and K.L.; formal analysis, J.B. and K.L.; investigation, J.B. and K.L.; writing—original draft preparation, J.B.; writing—review and editing, K.L. and S.O.; visualization, J.B. and K.L.; supervision, S.O. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Data Availability Statement: The raw data supporting the conclusions of this article will be made available by the authors on request.

Conflicts of Interest: The authors declare no conflicts of interest.

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