

Article

Critical Cavities Design for High-Repetition-Frequency Self-Starting Kerr-Lens Mode-Locked Lasers

Xinxin Wang ¹, Xing Chen ^{2,*}, Zhigang Zhang ², Bin Luo ¹ and Song Yu ¹

¹ State Key Laboratory of Information Photonics and Optical Communications, Beijing University of Posts and Telecommunications, 10 Xitucheng Road, Beijing 100876, China; wx2001@bupt.edu.cn (X.W.)

² State Key Laboratory of Advanced Optical Communication System and Networks, School of Electronics Engineering and Computer Science, Peking University, Beijing 100871, China

* Correspondence: chenxing@bupt.edu.cn; Tel.: +86-15981802676

Abstract

The mode-locking mechanism of Kerr-lens mode-locked lasers is analyzed using the nonlinear ABCD matrix formalism. Our findings demonstrate that positioning the Kerr medium at the beam waist and operating the resonator near the stability boundary significantly enhances the nonlinear effect, thereby facilitating self-starting mode-locking without requiring any external initiation mechanisms.

Keywords: Kerr-lens mode-locked; ABCD matrix; nonlinear effect; self-starting

1. Introduction

Kerr-lens mode-locking (KLM) is one of the most effective techniques for generating ultrashort pulses in solid-state lasers [1]. A critical challenge, however, lies in its startup characteristics, which significantly impact operational stability and have hindered broader practical adoption. In 1990, Fujimoto et al. [2] investigated the self-starting dynamics of additive pulse mode-locked lasers and developed a simplified model to describe the self-starting behavior and pulse evolution. In 1992, Tamura et al. [3] demonstrated that suppressing mode competition and intracavity scattering could enhance self-starting performance; they achieved this in a ring-cavity Ti:sapphire laser by introducing an optical isolator. By 1994, the self-starting behavior of passively mode-locked lasers employing fast saturable absorbers (SESAMs) had been analyzed, establishing specific conditions for self-initiation [4]. A common limitation across these studies was their reliance on auxiliary cavities, external saturable absorbers, or mechanical modulation of cavity components to initiate mode-locking. Achieving self-starting in pure KLM lasers remains particularly challenging due to the weak intrinsic nonlinearity and constraints such as the saturation intensity of the gain medium. Theoretically, optimizing the cavity length is essential for developing a self-starting mode-locked laser with high repetition rates and output power. However, a fundamental trade-off exists, as achieving high output power is often difficult in solid-state lasers with short cavities. This study addresses this challenge by systematically simulating the parameter space of a laser resonator to identify configurations that enable stable, high-performance operation in short-cavity designs. Recent developments in cavity design have further expanded the parameter space for achieving self-starting operation in compact resonators. Yang et al. [5] reported a high-repetition-rate KLM laser operating at 1.7 GHz with self-starting behavior, emphasizing the importance of precise cavity length optimization. In a related study, Yang et al. [6] systematically examined the influence of



Received: 26 January 2026

Revised: 5 March 2026

Accepted: 17 March 2026

Published: 18 March 2026

Copyright: © 2026 by the authors.

Licensee MDPI, Basel, Switzerland.

This article is an open access article distributed under the terms and conditions of the [Creative Commons Attribution \(CC BY\)](https://creativecommons.org/licenses/by/4.0/) license.

the Kerr medium position on the self-starting threshold, demonstrating that placing the gain medium at the cavity beam waist enhances the nonlinear effect and reduces the pump power required for self-initiation. These findings provide valuable guidance for the cavity design presented in this work. This study systematically investigates the relationship between resonator structural parameters and the self-starting characteristics of KLM lasers. Our findings reveal that operating near the edge of the cavity stability region increases the sensitivity of the intracavity beam spot size to variations in pump power, thereby facilitating the onset of stable mode-locking. To suppress spurious sub-cavity effects induced by internal reflections, a fixed hard aperture was incorporated into the linear cavity design. Numerical simulations confirmed that this configuration enhances the output pulse power. Furthermore, using ABCD matrix analysis, we explored the influence of beam propagation characteristics on the self-starting behavior of KLM lasers. By applying mode-locking theory analogous to that of saturable absorber lasers—without any external triggering mechanism—we derived the fundamental conditions for self-starting. The simulation results further demonstrated that positioning the Kerr medium at the intracavity beam waist enhances the nonlinear effect and significantly improves self-starting performance.

2. Materials and Methods

2.1. Theoretical Analysis of Self-Starting of Kerr Lens Mode Locking

The phenomenon of Kerr-lens mode-locking (KLM) was first observed in 1990 by D. E. Spence [7], who demonstrated its occurrence without any external mode-locking elements. Initially, the underlying mechanism was not fully understood, and the effect was termed “self-mode-locking.” Subsequent studies revealed that titanium-doped sapphire (Ti:sapphire) crystals possess a strong Kerr nonlinearity, which causes self-focusing and produces an intensity-dependent lensing effect—leading to the established term, Kerr-lens mode-locking.

The KLM mechanism relies on the beam self-focusing induced by the third-order nonlinear Kerr effect within the gain medium. This effect occurs when the laser peak power exceeds 1 GW, resulting in a refractive index change proportional to the incident light intensity.

$$n(I) = n_0 + n_2 I(t) \quad (1)$$

where n_0 is the inherent medium of the material, n_2 is the nonlinear refractive index coefficient. In the case of solid materials, the nonlinear refractive index coefficient n_2 is in the order of magnitude $10^{-16} \text{ cm}^2/\text{W}$, so that the medium reveals its nonlinear effect only under strong light irradiation. In KLM lasers, self-phase modulation (SPM) plays a critical role in pulse shortening. It arises from the Kerr effect in the time domain and serves as a key mechanism for temporal pulse compression. When a laser pulse traverses a medium exhibiting a nonlinear refractive index over a distance L , a phase shift is produced:

$$\Delta\varphi = \frac{2\pi n_2 L}{\lambda} I(t) \quad (2)$$

The instantaneous frequency is defined as the time derivative of the optical phase, representing the transient frequency shift within the pulse. This can be expressed mathematically as follows:

$$\Delta\omega(t) = -\frac{d\Delta\varphi(t)}{dt} = -\frac{2\pi n_2 L}{\lambda} \frac{dI(t)}{dt} \quad (3)$$

This time-dependent frequency shift, known as frequency chirping, generates new frequency components that broaden the pulse spectrum—a process essential for mode-locked laser operation and the formation of ultrashort pulses. In the spatial domain, the Kerr effect manifests as self-focusing [8]. When a Gaussian beam propagates through a

nonlinear Kerr medium, the higher optical intensity at the beam center induces a stronger nonlinear refractive index change compared to the periphery. This transverse index gradient causes the phase front to bend inward, analogous to the effect of a thin lens, resulting in beam focusing. This phenomenon is therefore termed self-focusing.

The realization of KLM relies on the combined action of the self-focusing effect and an intracavity aperture—a mechanism functionally analogous to introducing a fast saturable absorber into the cavity. The Kerr-lens effect, by reducing the beam size at high intensities, enables this saturable absorber-like behavior through two primary mechanisms: hard-aperture and soft-aperture mode-locking. In hard-aperture mode-locking, the Kerr lens reduces diffraction losses at an intracavity aperture. In soft-aperture mode-locking, it enhances the spatial overlap between the laser and pump beams, leading to a larger mode area at the pulse center and consequently higher gain in that region. This paper focuses on the hard-aperture KLM mechanism, which is analyzed using the ABCD matrix formalism to model the back-and-forth beam propagation within the optical cavity. The equivalent radius of the intracavity diaphragm spot can be expressed as follows:

$$\omega = \left(\frac{\lambda |B|}{\pi} \right)^{1/2} \left[1 - \left(\frac{A+D}{2} \right)^2 \right]^{-1/4} \quad (4)$$

where A , B , and D are matrix elements for the round-trip transmission of the beam in the cavity as a function of the structural parameters of the optical cavity. Experiments have shown [9] that mode-locked operation is favored when the cavity parameters are near the edge of the stability region for continuous-wave (CW) operation. This is precisely where the beam spot size on the aperture exhibits the highest sensitivity to changes in intracavity power, where the relative variation δ is maximized:

$$\delta = -\frac{1}{\omega} \frac{d\omega}{dp} \quad (5)$$

Transmittance function of the equivalent saturable absorber formed in the cavity is $T(p)$:

$$T(p) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dx dy S(x, y) \left(\frac{2}{\pi \omega^2} \right) \exp \left(-2 \frac{x^2 + y^2}{\omega^2} \right), \quad S(x, y) = \begin{cases} 1, & \sqrt{x^2 + y^2} \leq a \\ 0, & \sqrt{x^2 + y^2} \geq a \end{cases} \quad (6)$$

where $S(x, y)$ is the transmittance function of the diaphragm, a is diaphragm radius, therefore mode locking can be facilitated by finding a suitable radius of the diaphragm. The pulse compression mechanism, governed by the equivalent saturable absorber, is directly linked to the instantaneous Kerr effect of the gain medium, which occurs on a femtosecond timescale. Consequently, this effective saturable absorber operates at an extremely fast response rate, enabling the generation of pulses as short as a few femtoseconds. This performance significantly surpasses that of conventional saturable absorbers, which are typically limited to the picosecond scale. Thus, KLM provides a theoretical pathway to achieve laser pulses with durations on the order of a few femtoseconds.

The use of a linear resonator for KLM presents a challenge for self-starting due to the inherent sub-cavity effect. In such configurations, multiple Fabry–Perot sub-cavities formed within the main cavity introduce optical bistability [10], which further inhibits the self-starting process. To mitigate this issue, we propose the integration of a solid hard aperture into the linear cavity design. This aperture serves to reduce intracavity reflections and suppress the sub-cavity effect, thereby facilitating pulse selection.

Additionally, the resonator parameters are optimized to operate near the edge of the stability region. Within the unstable region, continuous-wave operation is strongly suppressed by the diffraction losses induced by the Kerr lens effect, while the cavity stability becomes highly sensitive to intracavity power fluctuations [8]. Near this stability boundary, the intracavity spot size exhibits a pronounced dependence on power, a condition that preferentially favors the initiation and stabilization of mode-locking.

The fundamental equation of a mode-locked laser can be written as [11]:

$$T_R \frac{\partial}{\partial t} a(t, T) = \left\{ \left[jD \frac{\partial^2}{\partial t^2} + j\phi |a(t, T)|^2 - j\Psi \right] + \left[g - l + \frac{g}{\Omega^2} \frac{\partial^2}{\partial t^2} - q(t, T) \right] \right\} \quad (7)$$

where $a(t, T)$ is amplitude of the cavity clamping optical pulse, $|a(t, T)|^2$ is immediate power of light pulses, T_R is the round trip time of light in the cavity, D is group velocity dispersion, $\phi |a(t, T)|^2$ is self-phase modulation, Ψ is one-way phase shift, g is one-way gain provided by the gain medium, l is one-way fixed linear loss of the cavity, the parameter Ω represents the angular frequency associated with the gain bandwidth, which introduces the gain spectral window effect, $g\partial^2/\Omega^2\partial t^2$ is gained spectral window effect, $q(t, T)$ is the nonlinear loss of light by equivalent fast saturable absorbers.

The mechanism of action is known due to the formation of saturable absorbers in the lumen:

$$\frac{dq}{dP} = \left[\left(4 \frac{a^2}{\omega^2} \right) \exp \left(-2 \frac{a^2}{\omega^2} \right) \right] \left(\frac{1}{\omega} \frac{d\omega}{dP} \right) \quad (8)$$

$$\delta = - \left(\frac{1}{\omega} \frac{d\omega}{dP} \right) \quad (9)$$

δ denotes the rate of change in the intracavity spot with power, which is related to the nonlinearity and is also known as the initiation factor [9]. The expression for the equivalent saturable absorber action resulting from the self-focusing effect introduced by the Kerr effect can be written as:

$$q(\omega) = \delta s(\omega) \quad (10)$$

$s(\omega)$ is the absorption function of an equivalent saturable absorber.

Using the conditions for self-starting of mode-locked in saturable absorbers proposed by Krausz F. [12]:

$$k' P_i > \frac{1}{\ln(m_i)} \frac{T_R}{\tau_c} \quad (11)$$

where k' is the nonlinear characteristic of saturable absorber, P_i is the intracavity circulating power, m_i is the number of longitudinal modes that beat the frequency in the cavity, T_R is the time of one week's round-trip in the laser cavity, τ_c is the time width of the intracavity energy ups and downs caused by the longitudinal mode beating frequency.

In hard-aperture KLM, the effective nonlinear loss originates from the intensity-dependent transmission of the aperture. The transmission function of a circular hard aperture can be written as:

$$T = \exp \left(- \frac{2r^2}{\omega^2} \right) \quad (12)$$

where r is the aperture radius and ω is the beam radius at the aperture plane. Since the Kerr effect induces a power-dependent variation in ω , the transmission becomes indirectly dependent on the intracavity power P . Applying the chain rule, the nonlinear loss coefficient can be expressed as:

$$\frac{dT}{dP} = \frac{dT}{d\omega} \frac{d\omega}{dP} \quad (13)$$

Normalizing with respect to transmission, the effective nonlinear coefficient becomes:

$$k' = \frac{1}{T} \frac{dT}{dP} \quad (14)$$

It follows that k' is directly proportional to $(1/\omega)(d\omega/dP)$, which corresponds to the δ parameter defined earlier. Therefore, the nonlinear coefficient k' in Equation (11) can be rigorously related to δ through the derivative of the aperture transmission function, rather than being introduced as a purely phenomenological substitution. Since in the Kerr lens mode-locked laser, the change in intracavity spot size is related to the nonlinear effect, we can replace k' in Equation (11) to δ , and the self-starting condition of the Kerr lens mode-locked laser can be expressed as:

$$\delta P = -\left(\frac{1}{\omega} \frac{d\omega}{dP}\right) P > \frac{1}{\ln(m_i)} \frac{T_R}{\tau_c} \quad (15)$$

The parameters of a 100 MHz titanium gemstone laser are used as an example: where $P = 5$ W, $T_R = 10$ ns, $m_i = 1500$, $\tau_c = 0.5$ ms, and the mode-locked self-starting condition of the titanium gemstone laser can be obtained as:

$$\left(\frac{1}{\omega} \frac{d\omega}{dP}\right) < -5.4 \times 10^{-7} \quad (16)$$

let $p = P/P_c$ is the normalized power in the cavity, P_c is the self-trapping power of the Kerr medium, because in the early stage of clamping startup, the power in the cavity is much smaller than the self-trapping power of the Kerr medium, take $P_c = 2$ MW, so the above equation can be written as:

$$\left(\frac{1}{\omega} \frac{d\omega}{dP}\right) \Big|_{P=0} < 5.4 \times 10^{-7} \times P_c = -1.08 \quad (17)$$

Therefore, it can be obtained that the condition of self-starting of the Kerr lens mode-locked is $|\delta| > 1$, theoretically, which is consistent with the conclusions obtained from the paper [9], but there is a deviation between the experiment and the theory, so it needs to be revised according to the specific experiments.

2.2. Cavity Design

In the simulation of pulse propagation in a cavity, we conceptualize the process of transmission into two parts, linear and nonlinear. The linear part consists mainly of the saturation gain spectrum, dispersion and linear loss; the nonlinear part includes the Kerr effect and self-phase modulation. The linear part can be always expressed as a total spectral function:

$$E(\omega) \rightarrow g(\omega)T(\omega)E(\omega) \quad (18)$$

where $g(\omega)$ is the spectral gain of the active medium, $T(\omega)$ is the transfer function of the five-element element in the cavity. The nonlinear part is simulated by the total ABCD matrix. Our proposed cavity structure is shown in Figure 1:

The relationship between the spot size ω and the radius of curvature R is for a material with a refraction of n_1 :

$$\frac{1}{q(t)} = \frac{1}{R} - \frac{i\lambda}{\pi n_1 \omega^2(t)} \quad (19)$$

$q(t)$ is the beam parameter, and if q_1 is the parameter at the input of the linear optical system, the output is:

$$q_2 = \frac{(Aq_1 + B)}{(Cq_1 + D)} \quad (20)$$

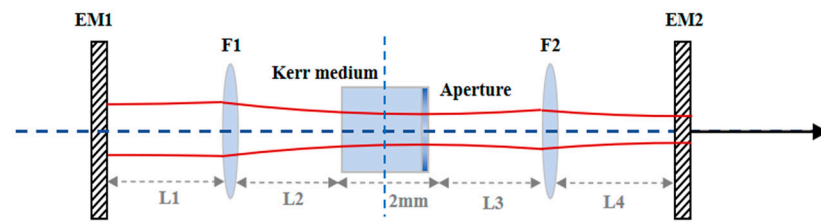


Figure 1. The linear structure of a Kerr lens mode-locked laser resonator cavity: two spherical mirrors/lenses (focus of F1 and F2 are 30 mm, 60 mm), 2 mm Kerr (gain) medium (Er:Yb:YAB), a solid hard aperture, two planar mirrors (EM1, EM2); L1: the distance from EM1 to F1; L2: the distance from F1 to Kerr medium; L3: the distance from Kerr medium to F2; L4: the distance from F2 to EM2.

The light transmission matrix of the Kerr medium [13] is:

$$M = \sqrt{1-\gamma} \begin{pmatrix} 1 & d_e \\ -\frac{\gamma}{[(1-\gamma)/d_e]} & 1 \end{pmatrix} \quad (21)$$

$$\gamma = \left[1 + \frac{1}{4} \left(\frac{2\pi\omega_c^2}{\lambda d_e} - \frac{\lambda d_e}{2\pi\omega_0^2} \right)^2 \right]^{-1} p \quad (22)$$

where $d_e = d/n_0$ is the effective medium length at $p = 0$, d is the Kerr medium length, ω_c is the radius of the spot at the center of the medium, ω_0 is the radius of the beam waist of the spot. Decompose the propagation matrix into linear and nonlinear parts:

$$M = \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & \frac{1}{2}d_e \\ 0 & 1 \end{pmatrix} M_k \begin{pmatrix} 1 & \frac{1}{2}d_e \\ 0 & 1 \end{pmatrix} \quad (23)$$

where the matrix at the right end of the equation is the propagation matrix for linear propagation through $1/2d_e$, and M_k is the nonlinear propagation, where the Kerr nonlinear effects are concentrated in the matrix M_k :

$$M_k = \frac{1}{\sqrt{1-\gamma}} \begin{pmatrix} 1 - \frac{1}{2}\gamma & -\frac{\gamma}{4}d_e \\ -\frac{\gamma}{d_e} & 1 - \frac{1}{2}\gamma \end{pmatrix} \quad (24)$$

when γ is comparatively small, M_k can be abbreviated as:

$$M_k \approx \begin{pmatrix} 1 & -\frac{\gamma}{4}d_e \\ -\frac{\gamma}{d_e} & 1 - \frac{1}{2}\gamma \end{pmatrix} \quad (25)$$

For thicker Kerr media, it can be approximated by splitting the operator [14] into multiple thin Kerr media and replacing it with a thin lens matrix with focal length f [15], whose transmission matrix is:

$$\begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix} \quad (26)$$

The propagation z -distance matrix is:

$$\begin{pmatrix} 1 & z \\ 0 & 1 \end{pmatrix} \quad (27)$$

For a first-order matrix M_k , it can be viewed as a superposition of matrices after a distance z_k for lenses with focal lengths of f_k :

$$M_k = \begin{pmatrix} 1 & z_k \\ -\frac{1}{f_k} & 1 \end{pmatrix} \quad (28)$$

where $f_k = \gamma/d_e$ denotes the self-focusing effect, $z_k = -\gamma d_e/4$ denotes the self-shortening effect of the medium. The total transmission matrix of the Kerr medium can be written as multiple superpositions of Equation (28).

For a given beam power, when the value of γ reaches its maximum value p , shown in Equation (22), and $1/f_k$ and z_k reaches its maximum value, the optimal focusing condition can be obtained as:

$$\omega_c \omega_0 = \frac{\lambda d_e}{2\pi} \quad (29)$$

The Kerr medium was modeled using the thin-lens approximation under the assumption of a small nonlinear phase shift ($\gamma \ll 1$). This approximation is valid when the nonlinear phase accumulation per round trip remains well below π and the medium length is short compared to the Rayleigh range. Under stronger nonlinear conditions or with a longer gain medium, higher-order effects such as self-focusing collapse, spatial distortions, and non-parabolic phase profiles may occur. In these cases, the thin-lens approximation may underestimate the effective nonlinear focusing. For the present cavity parameters and circulating power levels, the calculated nonlinear phase shift remains within the perturbative regime, ensuring that the simplified ABCD formalism provides an accurate estimation of both the stability boundary and the initiation factor δ . For extremely high peak powers, a full numerical solution of the spatially dependent nonlinear Schrödinger equation would be necessary.

3. Results

From Equation (29), we can see when the center of the Kerr medium is placed at the 1beam's beam waist position, as shown in Figure 2, below at the position of the dotted line, there is the strongest Kerr effect, which is conducive to the locking self-starting.

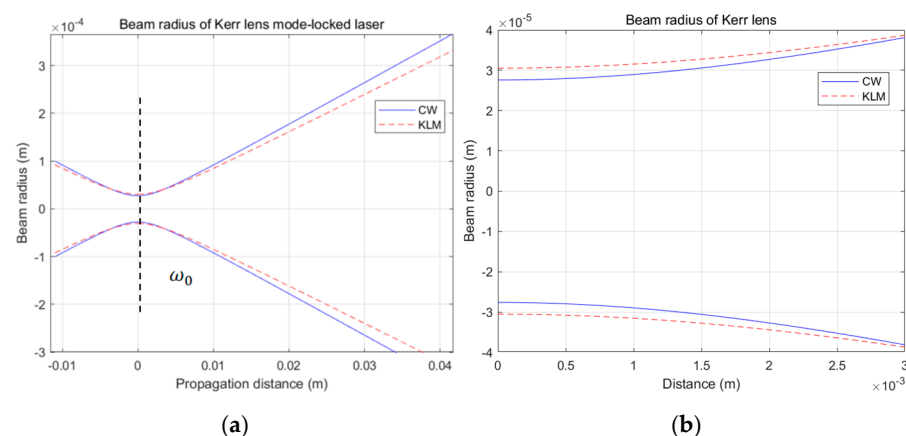


Figure 2. (a) Transmission of the beam in the cavity; (b) beam radius variation in a beam in a Kerr lens.

In this case, placing a hard diaphragm at the position where the spot size varies the most will further enhance the Kerr effect by introducing a certain amount of nonlinear loss. As illustrated in Figure 2b, the focal length of the Kerr lens is not fixed but depends on the intracavity power, the length of the nonlinear medium, and the instantaneous beam waist. Consequently, a comprehensive consideration of these factors is essential when calculating the transmission matrix for the Kerr medium. In our cavity design,

the hard aperture was placed at the location where the beam radius exhibits the highest sensitivity to intracavity power variations. This optimal position was identified numerically by evaluating $(d\omega/dP)$ along the propagation axis and corresponds to the region where the nonlinear Kerr lens induces maximal spot-size modulation—specifically, near the output coupler end where the mode size varies most rapidly near the stability boundary. Placing the aperture at this location maximizes the differential loss between the continuous-wave and mode-locked regimes by introducing a controlled amount of nonlinear loss. Regarding aperture size, the radius a must satisfy a trade-off condition. If the aperture is too small, excessive linear diffraction loss suppresses CW operation and reduces efficiency. If it is too large, the nonlinear modulation of transmission becomes too weak to sustain self-starting. To determine the optimal aperture radius, we evaluated the differential transmission as a function of normalized aperture radius. Numerical simulations indicate that the maximum differential loss occurs when the aperture radius is approximately 0.8–1.0 times of the CW beam radius at the aperture plane. For our cavity, with the $\omega_c \approx 0.85 \mu\text{m}$ at the aperture position, the optimal beam radius lies in the range of 68–85 μm . Alignment tolerance is a critical practical consideration for hard-aperture KLM lasers [16]. Longitudinal displacements of the aperture by several millimeters generally have a minor impact on mode-locking performance, as the beam size varies gradually along the cavity axis near the stability boundary. In contrast, transverse misalignment is more sensitive: lateral offsets exceeding approximately 5–10% of the beam radius can substantially reduce nonlinear loss modulation and may inhibit self-starting. Nevertheless, this level of alignment precision—on the order of a few micrometers—is readily achievable using standard optomechanical components, rendering the design experimentally feasible.

To simulate the stability region of the resonator, we employed the nonlinear ABCD matrix formalism [3], in conjunction with the transmission equation and the resonator stability condition, to derive the relationship between lengths L_2 and L_3 , as shown in Figure 3a. The resulting stability boundaries are plotted in Figure 3c,d. The critical value for L_2 is 43 mm, with the corresponding stable values for L_3 being 15 mm and 76 mm. The resonator operates in a stable state when L_3 is between 0 and 15 mm or 76 and 135 mm.

For the subsequent output pulse simulation, we selected resonator parameters at the boundary of the stability region. This is because the Kerr effect parameter [9] reaches its maximum at the edge of the stable zone, which is optimal for initiating mode-locking. The specific parameters used were: $L_1 = 36\text{--}59$ mm, $L_2 = 43$ mm, $L_3 = 76$ mm, and $L_4 = 30$ mm, yielding a corresponding repetition rate of 1.22 GHz. To verify that the simulated 1.22 GHz resonator fulfills the theoretical self-starting requirement, the initiation factor δ was evaluated using the power-dependent nonlinear ABCD matrix framework established in Section 2. Because the Kerr lens introduces a power-dependent focal contribution, the round-trip ABCD matrix of the cavity becomes explicitly dependent on the intracavity circulating power. For each selected power value, the self-consistent Gaussian eigenmode of the resonator was obtained numerically, and the corresponding beam radius at the hard-aperture position was determined. The nonlinear sensitivity of the cavity mode was quantified by evaluating the variation in the beam radius with respect to power using a finite-difference approach around the operating point at the stability boundary. The initiation factor δ was then calculated according to its definition in Section 2.1. For the stability-boundary configuration defined by $L_2 = 43$ mm and $L_3 = 76$ mm, the resulting initiation factor is $\delta \approx -1.32$. Since the magnitude of δ exceeds unity, the cavity satisfies the theoretical criterion for spontaneous self-starting. Operating near this critical point facilitates mode-locking and self-starting. The case where $L_3 = 15$ mm, which lies within the stable region, was not simulated as the enhanced nonlinearity at the stability boundary is a key requirement for our study.

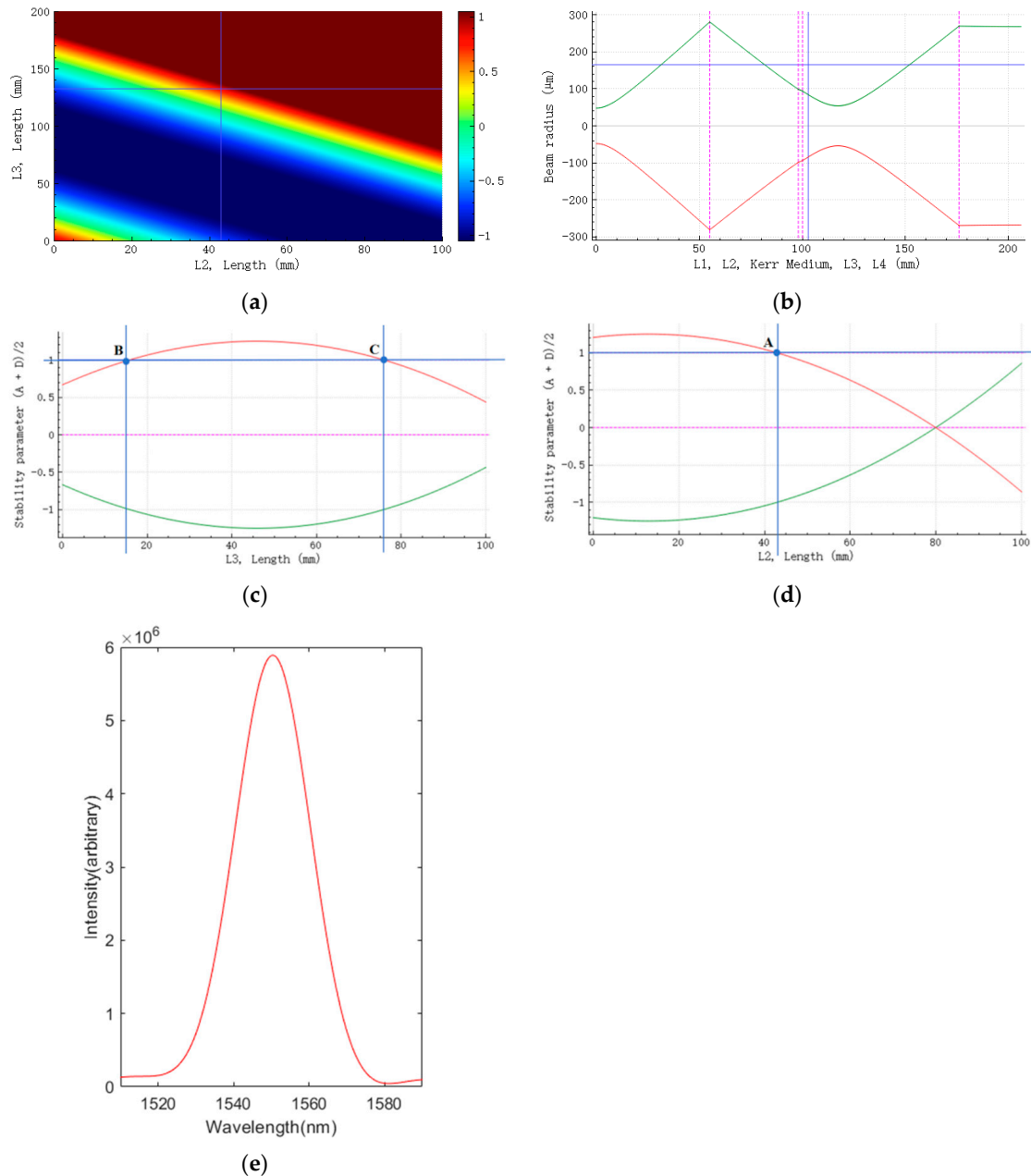


Figure 3. (a) Stable region of the resonance cavity corresponding to the values of L_2 and L_3 ; (b) beam radius inside the resonance cavity; (c,d) stability parameter profile of L_2 and L_3 (A, B, and C are the stable boundary points); (e) output power of the single pulse at the boundary: $L_1 = 59$ mm, $L_2 = 43$ mm, $L_3 = 76$ mm, $L_4 = 30$ mm.

4. Discussion

4.1. Influence of Thermal Effects and Practical Considerations

In practical implementations, the self-starting capability of KLM is also influenced by thermal effects [8]. To account for this effect, an effective thin thermal lens is incorporated into the round-trip ABCD matrix of the cavity. The presence of the thermal lens modifies the geometric stability region by introducing an additional linear focusing term. Consequently, the position of the stability boundary shifts slightly, depending on the thermal focal length. Because the self-starting condition is realized near the stability edge, the initiation factor δ must be re-evaluated at the thermally shifted boundary. Numerical analysis shows that moderate thermal lensing primarily affects the geometric parameters associated with the optimal stability edge, but does not suppress the nonlinear sensitivity enhancement

responsible for self-starting. The magnitude of δ remains greater than unity in the vicinity of the adjusted stability boundary. Therefore, while thermal lensing influences the precise cavity length required for operation, it does not fundamentally alter the stability-boundary-assisted self-starting mechanism described in this work. Nevertheless, a rigorous design for practical implementation should consider the ratio f_{th}/L as a critical parameter to accurately locate the new stability boundary. They also provide a plausible explanation for the deviations that are sometimes observed between the ideal theoretical model and experimental results.

4.2. Optimization via Stability Boundary Operation

The simulation results, summarized in Table 1, demonstrate that the output power of a single pulse is maximized when the resonator operates near the stability boundary (critical point), as opposed to the center of the stable region. This finding aligns with the principle established by Cerullo et al. [9], who showed that configuring the resonator near the edge of its stability region enhances the Kerr effect and promotes self-starting. The underlying mechanism is the heightened sensitivity of the intracavity beam spot size to power fluctuations at this boundary. This enhanced sensitivity amplifies the nonlinear Kerr lensing, which effectively functions as a fast saturable absorber. Therefore, operating the resonator at this critical point not only facilitates the initiation and stabilization of mode-locked pulses but also enables the generation of high-repetition-rate, high-power pulses from a compact, short-cavity design.

Table 1. The output power of the single pulse (P) is proportional to the value of L1.

L1 (mm)	36	38	40	42	45	50
P (MW)	4.63	4.23	4.29	4.91	4.78	4.59
L1 (mm)	52	55	56	59	60	
P (MW)	4.87	4.91	5.03	5.66	4.48	

4.3. Pulse Competition and Long-Term Stability

Operation near the stability boundary enhances nonlinear sensitivity but may also increase the risk of multi-pulse formation or pulse splitting due to strong gain saturation and nonlinear phase accumulation. Numerical pulse-evolution simulations indicate that, for the selected cavity parameters ($L_2 = 43$ mm, $L_3 = 76$ mm), single-pulse operation remains stable provided that the gain saturation energy sufficiently exceeds the intracavity pulse energy. Multi-pulse instabilities are more likely when the gain recovery time is short or when the cavity is driven well above threshold. To suppress pulse competition, several strategies can be employed: slightly detuning from the exact stability edge while maintaining $|\delta| > 1$; introducing moderate spectral filtering to stabilize the pulse bandwidth; optimizing pump power to avoid excessive nonlinear phase accumulation. Implementing these measures ensures robust single-pulse self-starting operation with improved long-term stability.

5. Conclusions

This study presents a systematic analysis of the relationship between resonator structural parameters and the self-starting capability of Kerr-lens mode-locked (KLM) lasers. We establish that operating near the edge of the resonator's stability region heightens the sensitivity of the intracavity beam spot size to pump power variations. This enhanced sensitivity directly facilitates higher nonlinearity and output power, which are pivotal for the autonomous initiation and stabilization of mode-locking. To counter detrimental sub-cavity effects from internal reflections, a strategic hard aperture was integrated into the linear

cavity design—a modification that simulations quantitatively confirmed to significantly enhance output pulse power. Through ABCD transfer matrix analysis, we elucidated the critical influence of specific beam propagation characteristics on self-starting. Furthermore, by adapting the mode-locking theory of saturable absorber lasers, we derived the fundamental condition for self-starting in pure KLM lasers without external stimulus, providing a crucial theoretical criterion. Simulation results robustly demonstrate that positioning the Kerr medium at the intracavity beam waist maximizes the nonlinear effect, thereby optimally promoting self-starting. Collectively, this work provides a comprehensive theoretical framework and practical design guidelines for developing optimized, high-repetition-rate, self-starting solid-state mode-locked lasers.

Author Contributions: Conceptualization, X.W.; Methodology, X.W.; Software, X.W.; Validation, X.W.; Formal analysis, X.W.; Investigation, X.W.; Resources, X.W. and X.C.; Data curation, X.W.; Writing—original draft, X.W.; Writing—review & editing, X.W., Z.Z., B.L. and S.Y.; Visualization, X.W.; Supervision, X.C.; Project administration, X.C.; Funding acquisition, X.C. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding, and the APC was funded by Beijing University of Posts and Telecommunications.

Data Availability Statement: The original contributions presented in this study are included in the article. Further inquiries can be directed to the corresponding author.

Acknowledgments: The successful completion of this work is indebted to the invaluable guidance and support of numerous individuals. Foremost, I wish to express my profound gratitude to my advisor, Chen Xing, for his unwavering encouragement, insightful critiques, and generous guidance throughout the course of this research and manuscript preparation. I am also deeply thankful to my laboratory colleagues for their collaborative spirit and technical assistance.

Conflicts of Interest: The authors declare no conflicts of interest.

References

- Herrmann, J. Starting dynamic, self-starting condition and mode-locking threshold in passive, coupled-cavity or Kerr-lens mode-locked solid-state lasers. *Opt. Commun.* **1993**, *98*, 111–114. [[CrossRef](#)]
- Goodberlet, J.; Wang, J.; Fujimoto, J.G. Starting dynamics of additive-pulse mode locking in the Ti: Al₂O₃ Laser. *Opt. Lett.* **1990**, *15*, 1300–1302. [[CrossRef](#)] [[PubMed](#)]
- Tamura, K.; Jacobson, J.; Ippen, E.P. Unidirectional ring resonators for self-starting passively mode-locked lasers. *Opt. Lett.* **1993**, *18*, 220–222. [[CrossRef](#)] [[PubMed](#)]
- Gürel, K.; Wittwer, V.J.; Hoffmann, M. Green-diode-pumped femtosecond Ti:Sapphire laser with up to 450 mW average power. *Opt. Express* **2015**, *23*, 30043–30048. [[CrossRef](#)] [[PubMed](#)]
- Yang, T.; Liu, H.; Wu, F.; Chen, H.; Wang, Q.; Wei, Z.; Zhang, J. 0.5 GHz femtosecond Yb:YAG thin-disk oscillator. *Opt. Lett.* **2025**, *50*, 2235–2238. [[CrossRef](#)] [[PubMed](#)]
- Yang, T.; Liu, H.; Wu, F.; Chen, H.; Wang, Q.; Wei, Z.; Zhang, J. Self-started Kerr-lens mode-locked thin-disk oscillator. *Opt. Lett.* **2023**, *48*, 4633–4636. [[CrossRef](#)] [[PubMed](#)]
- Spence, D.E.; Kean, P.N.; Sibbett, W. 60-fsec pulse generation from a self-mode-locked Ti:sapphire laser. *Opt. Lett.* **1991**, *16*, 42–44. [[CrossRef](#)] [[PubMed](#)]
- Yefet, S.; Pe'er, A. A Review of Cavity Design for Kerr Lens Mode-Locked Solid-State Lasers. *Appl. Sci.* **2013**, *3*, 694–724. [[CrossRef](#)]
- Cerullo, G.; De Silvestri, S.; Magni, V. Self-starting Kerr-lens mode locking of a Ti:sapphire laser. *Opt. Lett.* **1994**, *19*, 1040–1042. [[CrossRef](#)] [[PubMed](#)]
- Komarov, A.K.; Komarov, K.P.; Mitschke, F.M. Phase-modulation bistability and threshold self-start of laser passive mode locking. *Phys. Rev. A* **2002**, *65*, 053803. [[CrossRef](#)]
- Krausz, F.; Brabec, T.; Spielmann, C. Self-starting passive mode locking. *Opt. Lett.* **1991**, *16*, 235–237. [[CrossRef](#)] [[PubMed](#)]
- Gordon, A.; Gat, O.; Baruch, F.; Kärtner, F. Self-starting of passive mode locking. *Opt. Express* **2006**, *14*, 11142–11154. [[CrossRef](#)] [[PubMed](#)]
- Magni, V.; Cerullo, G.; Silvestri, S. BCD matrix analysis of propagation of gaussian beams through Kerr media. *Opt. Commun.* **1993**, *96*, 348–355. [[CrossRef](#)]

14. Parshani, I.; Bello, L.; Meller, M. Kerr-Lens Mode-Locking: Numerical Simulation of the Spatio-Temporal Dynamics on All Time Scales. *Appl. Sci.* **2022**, *12*, 10354. [[CrossRef](#)]
15. Magni, V.; Cerullo, G.; Silvestri, S.D. Closed form gaussian beam analysis of resonators containing a Kerr medium for femtosecond lasers. *Opt. Commun.* **1993**, *101*, 365–370. [[CrossRef](#)]
16. Xia, J.; Lee, M.H. Analysis of cavities for self-starting Kerr-lens mode-locked lasers. *Appl. Opt.* **2002**, *41*, 453–458. [[CrossRef](#)] [[PubMed](#)]

Disclaimer/Publisher’s Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.