

Article

Rapid Dynamic Separation of Radial and Azimuthal Polarization Components in Circular Airy Vortex Beams via Linear Electro-Optic Effect in Uniaxial Crystals

Guoliang Zheng ^{1,*} , Tiefeng He ¹ , Zikun Xu ¹, Jiawen Li ¹, Xuhui Zhang ^{1,*} , Lili Wan ¹ and Qingyang Wu ²

¹ Sino-German College of Intelligent Manufacturing, Shenzhen Technology University, Shenzhen 518118, China

² College of Big Data and Internet, Shenzhen Technology University, Shenzhen 518118, China

* Correspondence: zhengguoliang@sztu.edu.cn (G.Z.); zhangxuhui@sztu.edu.cn (X.Z.)

Abstract

This paper presents a rapid approach for the dynamic separation of radial polarization (R-pol) and azimuthal polarization (A-pol) components in circular Airy vortex beams (CAVBs) by utilizing the linear electro-optic (EO) effect in uniaxial crystals. By applying an external electric field along the z-axis of a strontium barium niobate (SBN) crystal, tunable spatial separation of the R-pol and A-pol components is achieved. Under positive electric fields, the crystal maintains negative uniaxial properties with increased birefringence, extending the focal separation distance. Conversely, negative electric fields initially reduce the birefringence of the crystal; further increases in negative field strength will transition the crystal to a positive uniaxial state, subsequently enhancing birefringence and restoring focal separation. Experimental simulations demonstrate a focal separation of 1.4 mm at ± 15 kV/mm, with R-pol focusing first at +15 kV/mm and A-pol preceding at -15 kV/mm. The polarization distributions at the foci confirm the successful separation of the two components. This approach overcomes the static limitation of conventional polarization splitters in separating R-pol and A-pol components, showing significant potential for optical manipulation, high-resolution imaging, and quantum information processing.

Keywords: linear electro-optic effect; uniaxial crystal; vortex circular Airy beam; radial polarization; azimuthal polarization



Received: 5 August 2025

Revised: 31 August 2025

Accepted: 2 September 2025

Published: 5 September 2025

Citation: Zheng, G.; He, T.; Xu, Z.; Li, J.; Zhang, X.; Wan, L.; Wu, Q. Rapid Dynamic Separation of Radial and Azimuthal Polarization Components in Circular Airy Vortex Beams via Linear Electro-Optic Effect in Uniaxial Crystals. *Photonics* **2025**, *12*, 894. <https://doi.org/10.3390/photonics12090894>

Copyright: © 2025 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (<https://creativecommons.org/licenses/by/4.0/>).

1. Introduction

Circular Airy beams (CABs)—radially symmetric optical fields characterized by Airy intensity profiles—have garnered substantial research attention due to their “abrupt auto-focusing property”, which enables intensity concentration by several orders of magnitude immediately before reaching the focal point [1–6]. This unique characteristic makes CABs valuable for biomedical interventions [7] and nonlinear optical processes [5]. CABs further facilitate microparticle trapping or guidance [8,9] and generate specialized beams including optical “bottles”, “needles”, “cages”, “dark channels” [10], and spatiotemporal “bullets” [5]. CABs carrying optical vortices (OVs), termed circular Airy vortex beams (CAVBs), combine the “abrupt autofocusing” property with the characteristic of vortex beams carrying orbital angular momentum (OAM). They have also attracted considerable attention in recent years. Following Davis’s [11] pioneering theoretical and experimental CAVB investigation, Lu et al. [12] demonstrated that OVs significantly enhance the abrupt autofocusing intensity while creating hollow-core distributions. Deng et al. [13] established tunability of focal

length and intensity through distribution factors, topological charge, and beam dimensions. Our recent work confirms that left-circularly polarized CABs with on-axis OV's generate both left- and right-handed circularly polarized vortex components in crystals [14]. In 2022, Liu Qiang and colleagues proposed and generated a type of structured light that is based on CAVBs and satisfies the mathematical form of SU(2) coherent states. This structured light exhibits strong auto-focusing ability, along with radial and angular self-acceleration, and 3D inhomogeneous rotation characteristics [15]. Recently, by applying a cubic phase to traditional skyrmion lattices, Yijie Shen et al. experimentally demonstrated, for the first time, accelerating skyrmion lattices conveyed by Airy structured light, which are characterized by topologically stable skyrmion textures that self-accelerate along parabolic trajectories [16].

Uniaxial crystals, as anisotropic optical media, play a pivotal role in theoretical investigations and light-field modulation. Their capacity to regulate light intensity distributions and polarization states enables diverse novel optical phenomena. Ciattoni et al. established a paraxial propagation model for uniaxial crystals using Fourier transforms (FT), demonstrating that specific circularly polarized beams propagating along the crystal's optical axis generate optical vortices with reversed circular polarization and a topological charge shift of +2 [17–19]. Subsequent studies extended this framework to explore propagation characteristics of Airy beams [20], Airy vortex beams [21], Airy-Gaussian beams [22], Airy-Gaussian vortex beams [23], and CABs [24]. Zhu et al. [25] and Liu et al. [26] demonstrated pronounced photonic spin Hall effects in uniaxial crystals, while Liu et al. [27] further showed their ability to tailor the OAM spectrum of quadratic-power-exponent-phase vortex beams, achieving flat-superimposed OAM states. Earlier research primarily focused on interactions between left- and right-handed circularly polarized components (LHCP/RHCP), analyzing their coupling and interconversion during propagation. In a paradigm shift, Liu et al. in 2023 investigated the evolution of R-pol and A-pol components [28]. They observed that CAVBs propagating along the optical axis undergo polarization separation, forming distinct focal points; however, this mechanism lacked dynamic adjustability.

On the other hand, the linear EO effect represents a powerful mechanism for optical field modulation, with significant applications spanning traditional optical communication, optical micromanipulation, and quantum information processing. In 2012, Zhu et al. demonstrated a method for generating and controlling multi-ring vector modes with LHCP and RHCP using the linear EO effect in a single SBN crystal [29]. Our prior research theoretically and numerically confirmed that EO coupling effectively manipulates the abrupt autofocusing in CABs, and generates OAM states with topological charge $l = 2$ [30]. In 2024, Zhu et al. [31] reported a novel optical Stern–Gerlach effect in EO crystals, enabling high-speed, EO-tunable polarizing beam splitters (PBSs) compatible with arbitrary orthogonal polarization states. Subsequently, Zhu et al. (2025) revealed a photonic spin Hall effect mediated by spatially engineered Pancharatnam–Berry (PB) phases in X-cut LiTaO₃ crystals, where voltage-reconfigurable PB phases induce dynamically tunable spin-dependent momentum–space separations [32]. In this work, we utilize the linear EO effect in SBN crystals to achieve rapid and precise control over R-pol and A-pol components of CAVBs.

2. Theory

To initiate our analysis, consider a monochromatic paraxial field $E(\mathbf{r}, t) = \text{Re}[E(\mathbf{r})\exp(-i\omega t)]$ propagating along the optical axis of a uniaxial crystal. Starting from Maxwell's equations and incorporating the second-order nonlinear polarization intensity induced by the linear EO effect, we derive the wave equation for the complex amplitude as follows:

$$\nabla^2 \mathbf{E}(\mathbf{r}) - \nabla[\nabla \cdot \mathbf{E}(\mathbf{r})] + k_0^2 \boldsymbol{\varepsilon} \cdot \mathbf{E}(\mathbf{r}) + \mu_0 \omega^2 \mathbf{P}^{\text{EO}}(\mathbf{r}) = 0 \quad (1)$$

where $k_0 = \omega/c$ is the wavenumber in vacuum, ω is the angular frequency of light, $\boldsymbol{\varepsilon}$ is the relative dielectric tensor, μ_0 is the magnetic susceptibility, $\mathbf{P}^{\text{EO}} = \varepsilon_0 \chi^{(2)}(-\omega, \omega, 0) : \mathbf{E} \mathbf{E}_0$ denotes the nonlinear polarization associated with the LEO effect, $\mathbf{E}_0$ is the external electric field, and $\chi^{(2)}(-\omega, \omega, 0)$ is the second-order susceptibility tensor of the linear EO effect. For uniaxial electro-optic SBN crystals, the non-vanishing electro-optic coefficients are $\gamma_{13} = \gamma_{23} = 67$, $\gamma_{33} = 1340$, and $\gamma_{42} = \gamma_{51} = 42$ (in pm/V), and the refractive indices are $n_o = 2.3117$ and $n_e = 2.2987$ at $\lambda = 632.8$ nm [33]. To utilize the largest electro-optic coefficient, we apply the external electric field along the z-axis of the crystal.

When an external electric field E_0 is applied along the z-axis, the linear EO effect in the SBN crystal modulates the refractive index ellipsoid through the EO coefficients γ_{13} and γ_{33} . The SBN crystal has a symmetry of 4 mm, and its EO coefficients are in the following form:

$$\begin{pmatrix} 0 & 0 & \gamma_{13} \\ 0 & 0 & \gamma_{13} \\ 0 & 0 & \gamma_{33} \\ 0 & \gamma_{51} & 0 \\ \gamma_{51} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \quad (2)$$

Now, considering the case when the external electric field E_0 is along the c-axis of the crystal (z-axis), the refractive index ellipsoid equation can be rewritten as follows:

$$x^2 \left(\frac{1}{n_o^2} + \gamma_{13} E_0 \right) + y^2 \left(\frac{1}{n_o^2} + \gamma_{13} E_0 \right) + z^2 \left(\frac{1}{n_e^2} + \gamma_{33} E_0 \right) = 1. \quad (3)$$

From Equation (3), one can see that the coefficients in the x and y directions change equally due to $r_{13}E_0$, while the coefficient in the z direction changes independently due to $r_{33}E_0$. The absence of mixed terms in the equation indicates that the principal axis directions of the refractive index ellipsoid remain consistent with the original coordinate axes, thus preserving the uniaxial symmetry of the crystal. Incorporating the electro-optic effect into the dielectric tensor, the modified dielectric tensor remains a symmetric matrix:

$$\boldsymbol{\varepsilon}' = \begin{bmatrix} n_o'^2 & 0 & 0 \\ 0 & n_o'^2 & 0 \\ 0 & 0 & n_e'^2 \end{bmatrix}, \quad (4)$$

where

$$n_o' = \frac{n_o}{\sqrt{1 + n_o^2 \gamma_{13} E_0}} \approx n_o - \frac{1}{2} n_o^3 \gamma_{13} E_0, \quad (5)$$

and

$$n_e' = \frac{n_e}{\sqrt{1 + n_e^2 \gamma_{33} E_0}} \approx n_e - \frac{1}{2} n_e^3 \gamma_{33} E_0. \quad (6)$$

Therefore, the birefringence of the uniaxial crystal under linear EO effect becomes the following:

$$n_o' - n_e' = n_o - n_e + \frac{1}{2} (n_e^3 \gamma_{33} - n_o^3 \gamma_{13}) E_0. \quad (7)$$

As can be seen from Equation (7), when a positive electric field is applied, $n_e' < n_o'$, the crystal remains a negative crystal, and its birefringence (anisotropy) increases with the magnitude of the electric field. When a negative electric field is applied, the birefringence of the crystal decreases. Specifically, when $E_0 = -1.67$ kV/mm, we have $n_e' = n_o'$, and

under this condition, the crystal becomes an isotropic medium. However, if the magnitude of the negative electric field continues to increase, it will cause $n'_e > n'_o$, and the crystal will transform into a positive crystal.

With the modified dielectric tensor ϵ' , Equation (1) can be rewritten accordingly:

$$\nabla^2 \mathbf{E}(\mathbf{r}) - \nabla[\nabla \cdot \mathbf{E}(\mathbf{r})] + k_0^2 \epsilon' \cdot \mathbf{E}(\mathbf{r}) = 0. \quad (8)$$

To analyze circularly polarized (CP) incident fields, we introduce two complex unit vectors, $\hat{\mathbf{e}}_+ = \frac{(\hat{\mathbf{e}}_x + i\hat{\mathbf{e}}_y)}{\sqrt{2}}$ and $\hat{\mathbf{e}}_- = \frac{(\hat{\mathbf{e}}_x - i\hat{\mathbf{e}}_y)}{\sqrt{2}}$, corresponding to LHCP and RHCP. Following the same approach as in [17], the two circular polarization components in a uniaxial crystal can be obtained as follows:

$$\begin{bmatrix} E_+(\mathbf{r}_\perp, z) \\ E_-(\mathbf{r}_\perp, z) \end{bmatrix} = \exp(ik_0 n_o z) \begin{bmatrix} F_+(\mathbf{r}_\perp, z) + G_-(\mathbf{r}_\perp, z) \\ F_-(\mathbf{r}_\perp, z) - G_+(\mathbf{r}_\perp, z) \end{bmatrix}, \quad (9)$$

where

$$F_\pm(\mathbf{r}_\perp, z) = \frac{1}{2} \int_0^\infty d^2 \mathbf{k}_\perp \exp(i\mathbf{k}_\perp \cdot \mathbf{r}_\perp) \left[\exp\left(-\frac{i(k_x^2 + k_y^2)}{2k_0 n'_o} z\right) + \exp\left(-\frac{in'_o(k_x^2 + k_y^2)}{2k_0 n'^2_e} z\right) \right] \tilde{E}_\pm(\mathbf{k}_\perp, 0), \quad (10)$$

$$G_\pm(\mathbf{r}_\perp, z) = \frac{1}{2} \int_0^\infty d^2 \mathbf{k}_\perp \exp(i\mathbf{k}_\perp \cdot \mathbf{r}_\perp) \frac{(k_x \pm ik_y)^2}{k^2} \left[\exp\left(-\frac{i(k_x^2 + k_y^2)}{2k_0 n'_o} z\right) - \exp\left(-\frac{in'_o(k_x^2 + k_y^2)}{2k_0 n'^2_e} z\right) \right] \tilde{E}_\pm(\mathbf{k}_\perp, 0). \quad (11)$$

Here, $\mathbf{r}_\perp = x\hat{\mathbf{e}}_x + y\hat{\mathbf{e}}_y$, $\mathbf{k}_\perp = k_x\hat{\mathbf{e}}_x + k_y\hat{\mathbf{e}}_y$, and $\tilde{E}_\pm(\mathbf{k}_\perp, 0) = \frac{1}{(2\pi)^2} \int d^2 \mathbf{r} \exp(-i\mathbf{k}_\perp \cdot \mathbf{r}_\perp) E_\pm(\mathbf{r}, 0)$ represents the two-dimensional (2D) FT of the initial transverse field distribution $E_\pm(\mathbf{r}, 0)$. Equations (10) and (11) can be understood from the angular spectrum theory of plane wave, where $\left[\exp\left(-\frac{i(k_x^2 + k_y^2)}{2k_0 n'_o} z\right) + \exp\left(-\frac{in'_o(k_x^2 + k_y^2)}{2k_0 n'^2_e} z\right) \right]$ and $\frac{(k_x \pm ik_y)^2}{k^2} \left[\exp\left(-\frac{i(k_x^2 + k_y^2)}{2k_0 n'_o} z\right) - \exp\left(-\frac{in'_o(k_x^2 + k_y^2)}{2k_0 n'^2_e} z\right) \right]$ are the propagation functions of $F_\pm(\mathbf{r}_\perp, z)$ and $G_\pm(\mathbf{r}_\perp, z)$, respectively. Notably, the propagation function of $G_\pm(\mathbf{r}_\perp, z)$ contains $\frac{(k_x \pm ik_y)^2}{k^2}$, indicating the presence of vortices with a topological charge number of ± 2 here. This result is consistent with previous reports [19]. From Equations (5) to (11), the transverse field distribution at any plane can be determined using the FT of the incident field $E_\pm(\mathbf{r}, 0)$, i.e., $\tilde{E}_\pm(\mathbf{k}_\perp, 0)$. For a crystal with length L , the two circularly polarized components that propagate in free space after exiting the crystal can be expressed as follows [34]:

$$\begin{bmatrix} E_+(\mathbf{r}_\perp, z) \\ E_-(\mathbf{r}_\perp, z) \end{bmatrix} = \exp[ik_0(z - L)] \begin{bmatrix} A_+(\mathbf{r}_\perp, z) \\ A_-(\mathbf{r}_\perp, z) \end{bmatrix}, \quad (12)$$

where

$$A_\pm(\mathbf{r}_\perp, z) = \int_0^\infty d^2 \mathbf{k}_\perp \exp(i\mathbf{k}_\perp \cdot \mathbf{r}_\perp) \exp\left[-i\frac{(k_x^2 + k_y^2)}{2k_0} (z - L)\right] \tilde{E}_\pm(\mathbf{k}_\perp, L), \quad (13)$$

with $\tilde{E}_\pm(\mathbf{k}_\perp, L) = \frac{1}{(2\pi)^2} \int d^2 \mathbf{r}_\perp \exp(-i\mathbf{k}_\perp \cdot \mathbf{r}_\perp) E_\pm(\mathbf{r}, L)$. Through coordinate transformation, the R-pol component $E_r(r, \varphi, z)$ and A-pol component $E_\varphi(r, \varphi, z)$ can be easily obtained [28]:

$$\begin{bmatrix} E_r(r, \varphi, z) \\ E_\varphi(r, \varphi, z) \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} \exp(i\varphi) & \exp(-i\varphi) \\ i\exp(i\varphi) & -i\exp(-i\varphi) \end{bmatrix} \begin{bmatrix} E_+(\mathbf{r}_\perp, z) \\ E_-(\mathbf{r}_\perp, z) \end{bmatrix}. \quad (14)$$

Combining Equations (9)–(14), we can obtain the R-pol and A-pol field distributions inside and outside the crystal.

3. Numerical Results

A schematic diagram of controlling separation of R-pol and A-pol components is shown in Figure 1, in which the blue areas represent uniaxial crystals of length $L = 30$ mm, a High-Voltage Direct Current (HVDC) power supply is applied along z -axis of the crystal, and the red lines and the blue dotted lines represent the propagation trajectory of the R-pol component and the A-pol component, respectively.

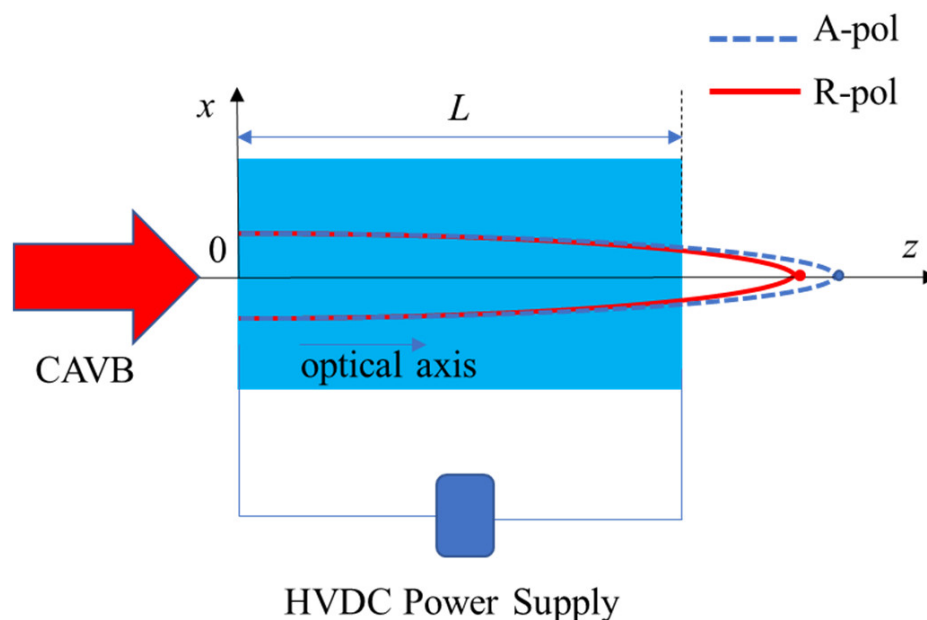


Figure 1. The schematic diagram of controlling separation of R-pol and A-pol components.

The CAVB used in this study is an LHCP vortex beam with topological charge $l = -1$, described by the following:

$$E(r, \varphi, 0) = C \cdot \text{Ai}\left(\frac{r_0 - r}{w}\right) \exp\left(a \frac{r_0 - r}{w}\right) \exp(-i\varphi) e_+ \quad (15)$$

where C is a constant, while Ai , r , w , and a stand for the Airy function, radial distance, radially scaled coefficient, and decay parameter, respectively. Since the incident beam has only the LHCP component, its Fourier spectrum distribution only includes $\tilde{E}_+(\mathbf{k}_\perp, 0)$; i.e., $\tilde{E}_-(\mathbf{k}_\perp, 0) = 0$. Equation (9) then becomes

$$\begin{bmatrix} E_+(\mathbf{r}_\perp, z) \\ E_-(\mathbf{r}_\perp, z) \end{bmatrix} = \exp(ik_0 n_o z) \begin{bmatrix} F_+(\mathbf{r}_\perp, z) \\ -G_+(\mathbf{r}_\perp, z) \end{bmatrix}. \quad (16)$$

It can be seen from Equations (10), (11), and (16) that the LHCP component in the crystal carries the same vortex as the incident light beam, while the RHCP component has a vortex with a topological charge number of 2 superimposed on it. In our numerical study, we choose $r_0 = 0.25$ mm, $w = 15$ μm , $a = 0.01$, $\lambda = 632.8$ nm, and the crystal length is $L = 30$ mm.

Unlike Reference 28, numerical calculation methods are employed herein to conduct simulations. The following is a description of the simulation methodology: First, the Fourier spectral distribution of the incident light, $\tilde{E}_\pm(\mathbf{k}_\perp, 0)$, is obtained by directly performing a 2D FT on Equation (15). Subsequently, according to Equations (10) and (11), the functions $F_+(\mathbf{r}_\perp, z)$ and $G_+(\mathbf{r}_\perp, z)$ can be obtained by multiplying $\tilde{E}_+(\mathbf{k}_\perp, 0)$ by the corresponding propagation function and then performing an 2D inverse FT (IFT). Finally, the complex

amplitude distributions of the LHCP and RHCP components within the crystal are obtained from Equation (16). For the free space propagation stage, simulations are carried out in line with Equations (12) and (13), using a methodology analogous to that applied to the crystal. Specifically, we first perform a 2D FT on the optical field $E_{\pm}(\mathbf{r}, L)$ at the exit face of the crystal to obtain $\tilde{E}_{\pm}(\mathbf{k}_{\perp}, L)$. Then, by multiplying $\tilde{E}_{\pm}(\mathbf{k}_{\perp}, L)$ by the propagation function in free space and performing an inverse 2D FT, we can obtain $A_{\pm}(\mathbf{r}_{\perp}, z)$. Finally, the complex amplitude distributions of the LHCP and RHCP components in free space are derived from Equation (12). Furthermore, whether in the crystal or in free space, we can obtain the complex amplitude distributions of the R-pol component $E_r(r, \varphi, z)$ and A-pol component $E_{\varphi}(r, \varphi, z)$ through Equation (14). MATLAB R2023a is used for the simulations, where 2D FT and 2D IFT are implemented utilizing the fast Fourier algorithms built into the software.

According to the quantum theory of solids, the optical dielectric impermeability tensor depends on the charge distribution within a crystal [33]. The application of an electric field causes the redistribution of bond charges and may lead to slight deformation of the ionic lattice. The net effect of these changes is a variation in the optical dielectric impermeability tensor, which is known as the electro-optic effect [33]. Jiang et al. calculated the electro-optic coefficients of two ferroelectric oxides, $\text{Pb}(\text{Zr,Ti})\text{O}_3$ and BaTiO_3 , at the atomic scale using first-principles methods [35]. They investigated the electro-optic effects of these two materials by varying the applied electric field (ranging from 0 to 10^8 V/m), fitted the linear and quadratic electro-optic coefficients, and found that these electro-optic coefficients do not change with the magnitude of the electric field. Therefore, in our simulations, the EO coefficients γ_{13} and γ_{33} of the SBN crystal are set as constant values. The introduction of additives can significantly enhance the breakdown strength of SBN. For example, Liu et al. studied MgO-modified SBN, whose breakdown strength can reach 22.6 kV/mm [36]. Therefore, in our simulations, the applied electric field is set within the safe range of -15 kV/mm to 15 kV/mm.

First, we investigate the evolution of the light beam in the absence of an applied electric field; the result is shown in Figure 2. To observe the changes in intensity during propagation, we use the normalized intensity I_n , which is defined as $I_n = I/I_{0\max}$, where $I_{0\max}$ is the maximum intensity of the incident light field distribution, and I is the intensity at any position. Since the light field before the focus is extremely weak, we plot the square root of the normalized intensity distribution in Figure 2 so that the light field can be seen clearly during the whole propagation. Similarly, we also adopt this plotting method in the following same cases. The figure shows that light in both polarization states focuses around $z = 35.6$ mm without spatial separation, owing to SBN's relatively low birefringence ($\Delta n = |n_o - n_e|$). To observe the proportion of the R-pol and A-pol components, we plotted the normalized intensity distributions of these two components at the exit face of the crystal ($z = 30$ mm) and the focal point ($z = 35.6$ mm), as shown in Figure 3. As can be seen from Figure 3, the two components account for a comparable proportion, whether at the crystal's exit face or the focal point. Further, we plotted the intensity distributions and polarization distributions of the light at the exit face of the crystal and near the focal point, as shown in Figure 4. As can be seen from Figure 3b, the light field at the focal point exhibits non-uniform elliptical polarization due to the overlap of the two components.

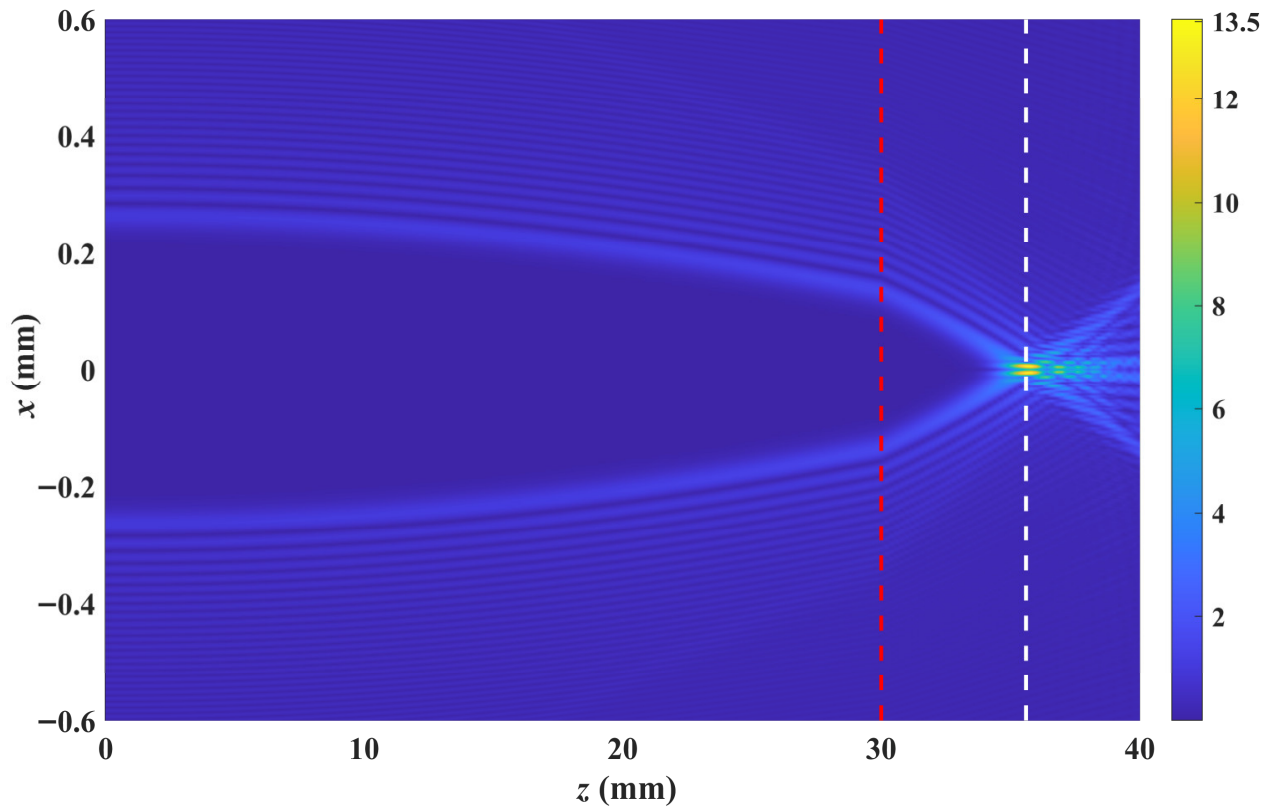


Figure 2. The evolution of the square root of the normalized intensity of the light beam in the absence of an applied electric field. The red and white dashed lines represent the exit surface of the crystal and the focal plane, respectively.

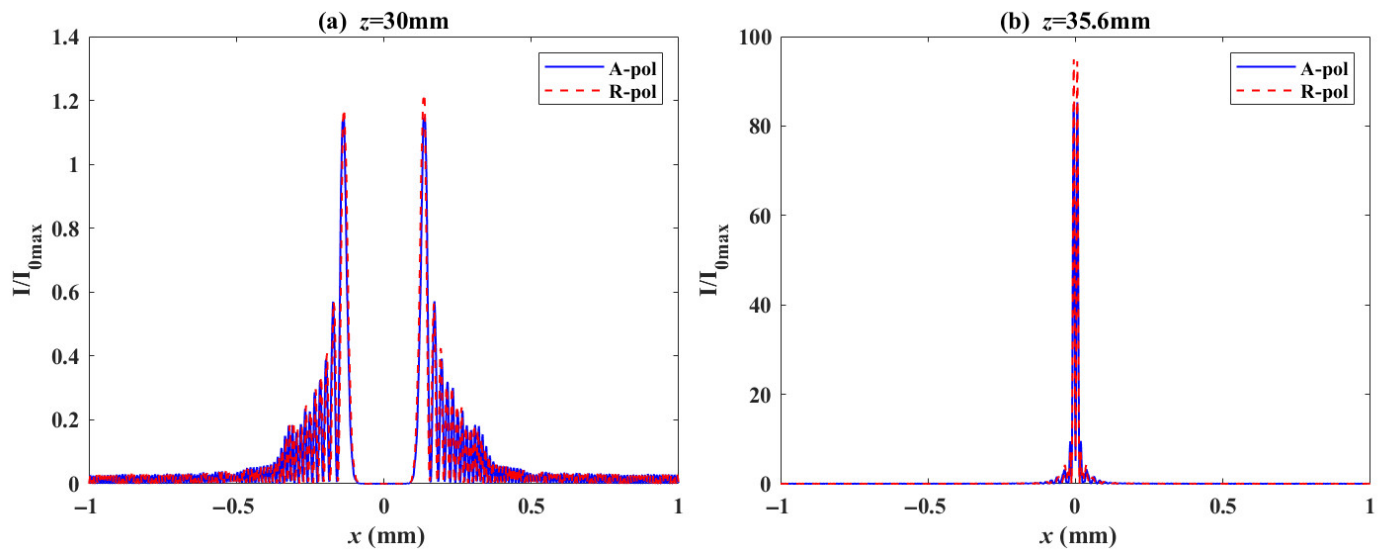


Figure 3. The normalized intensity distribution of the R-pol and A-pol components on x-axis at (a) $z = 30$ mm, (b) $z = 35.6$ mm in the absence of an applied electric field.

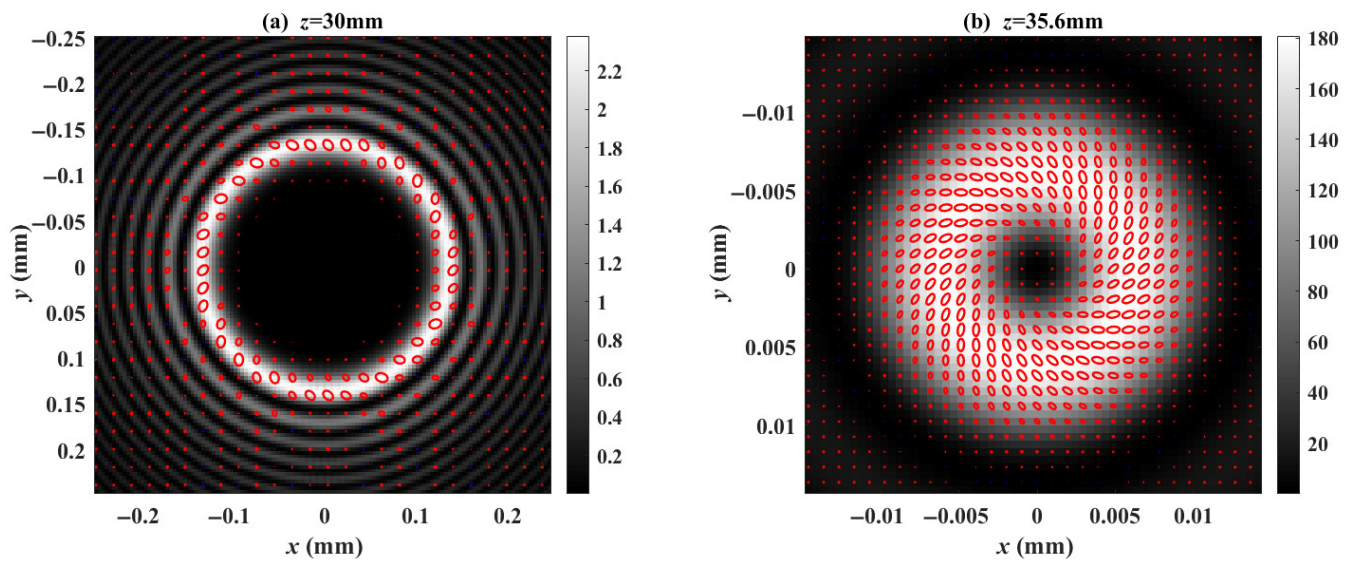


Figure 4. The normalized intensity distribution and polarization distribution at (a) $z = 30 \text{ mm}$ and (b) $z = 35.6 \text{ mm}$ in the absence of an applied electric field. The red and blue ellipses represent the left-handed polarized ellipse and the right-handed polarized ellipse, respectively.

Next, we apply an external electric field of $E_0 = 15 \text{ kV/mm}$ along the z -axis of the crystal. The evolution of the light beam is shown in Figure 5. As clearly shown in Figure 5, after the light beam exits the crystal, it focuses at two positions of 34.0 mm and 35.6 mm , achieving a focal separation of 1.4 mm . These two focusing actions are completed by the R-pol and A-pol components of the beam, respectively. We plotted the normalized intensity distributions of the two components at the exit face of the crystal ($z = 30 \text{ mm}$), and the first focal point $z = 34.0 \text{ mm}$ and the second focal point $z = 35.6 \text{ mm}$, as shown in Figure 6. As can be seen from Figure 6, no separation for the two components is achieved at the exit surface. Near the first focal point, the intensity of the R-pol is significantly stronger than that of the A-pol. More importantly, since the A-pol has not yet focused, a high-purity R-pol light beam can be obtained within a finite transverse region. We plotted the intensity distribution and polarization distribution at $z = 30 \text{ mm}$, $z = 34.0 \text{ mm}$, and $z = 35.6 \text{ mm}$, which are shown in Figure 7. From Figure 7a, it can be observed that the beam distribution exhibits a ring-like pattern. As the R-pol component focuses earlier, the inner ring corresponds to an R-pol component, while the outer ring exhibits an elliptical polarization distribution. The outer ring exhibits elliptical polarization because it has both R-pol and A-pol components simultaneously, and there is a phase difference between these two components. At the focal point, we confined the observation area to a 30-micron window. Figure 7b clearly demonstrates that the focal spot corresponds to the R-pol component. However, as seen in Figure 7c, at the second focal point, the focal spot is not entirely formed by the A-pol component due to the slight influence of the R-pol component.

As mentioned in Section 2, when the applied electric field is $E_0 = -1.67 \text{ kV/mm}$, the crystal becomes an isotropic medium. Figures 8 and 9 illustrate beam evolution behavior and the polarization distributions at the crystal exit surface and focal point under this condition. As shown in Figure 8, the beam exhibits only one focal point. Figure 9 reveals that both at the crystal exit surface and the focal point, the polarization state of the beam remains identical to the incident light, maintaining a uniform left-handed circularly polarized light.

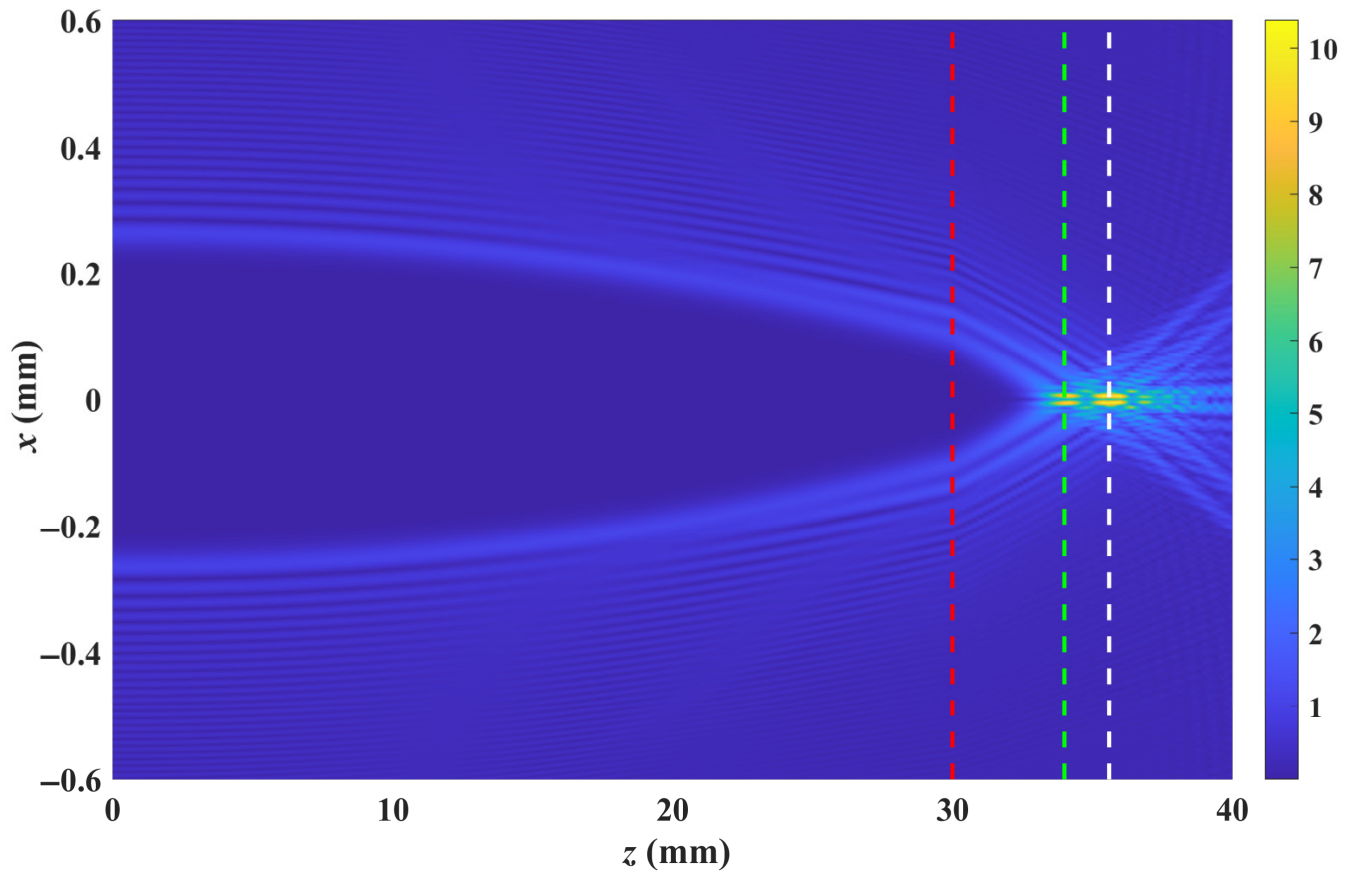


Figure 5. The evolution of the square root of the normalized intensity of the light beam with the electric field $E_0 = 15$ kV/mm. The red, green, and white dashed lines represent the exit surface of the crystal, and the front and rear focal planes, respectively.

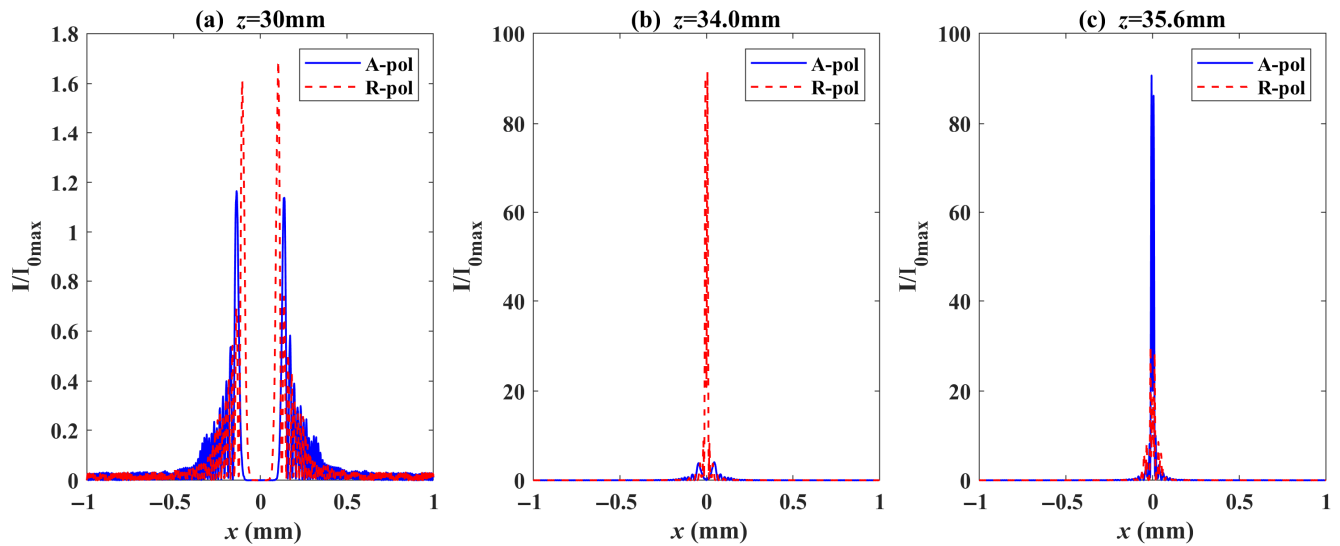


Figure 6. The normalized intensity distribution of the R-pol and A-pol components the on x-axis at (a) $z = 30$ mm, (b) $z = 34.0$ mm, and (c) $z = 35.6$ mm with the electric field $E_0 = 15$ kV/mm.

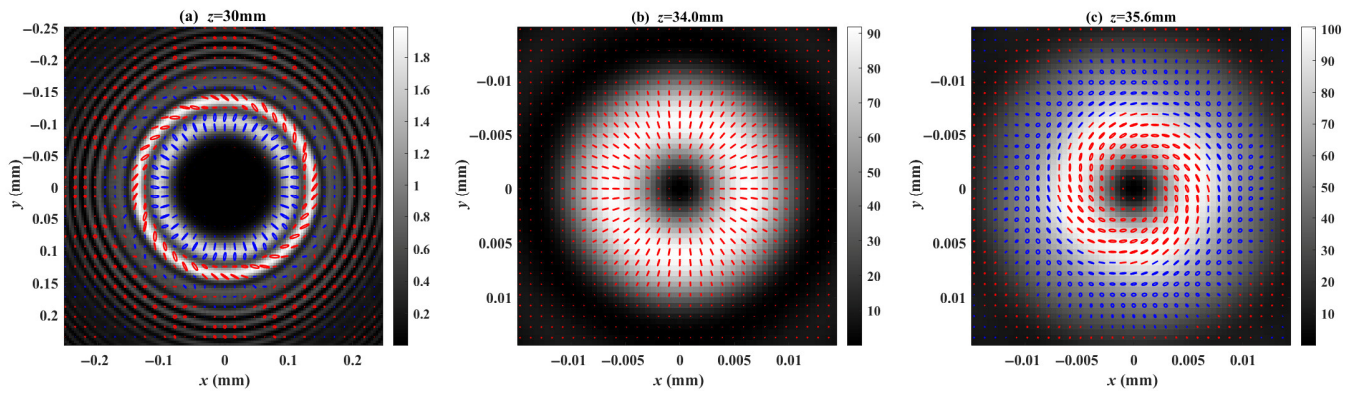


Figure 7. The normalized intensity distribution and polarization distribution at (a) $z = 30$ mm, (b) $z = 34.2$ mm, and (c) $z = 35.6$ mm with the electric field $E_0 = 15$ kV/mm. The red and blue ellipses represent the left-handed polarized ellipse and the right-handed polarized ellipse, respectively.

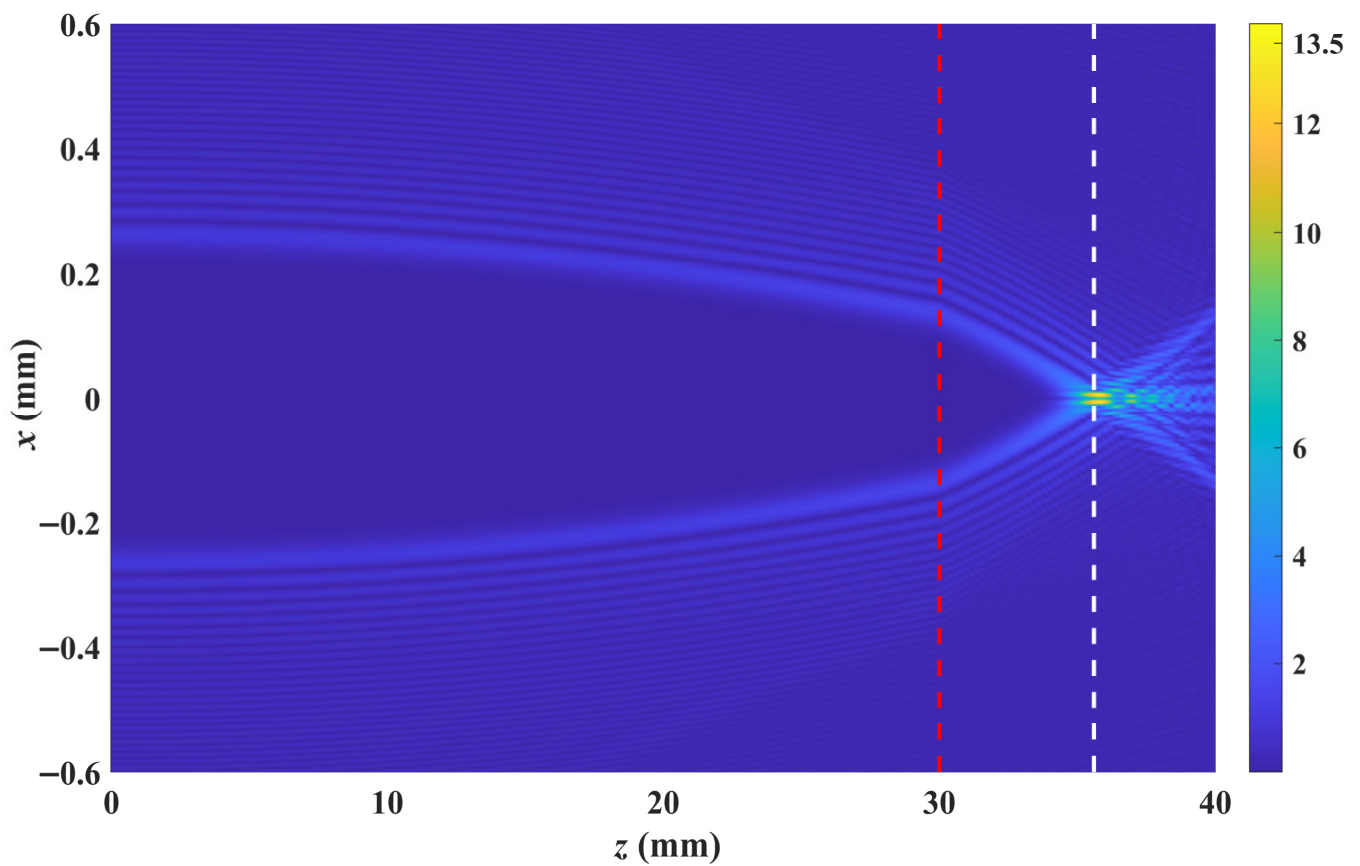


Figure 8. The evolution of the square root of the normalized intensity of the light beam with the electric field $E_0 = -1.67$ kV/mm. The red and white dashed lines represent the exit surface of the crystal and the focal plane, respectively.

Finally, we increase the magnitude of the applied electric field to $E_0 = -15$ kV/mm, where the principal refractive index of the extraordinary light (e-light) exceeds that of the ordinary light (o-light), i.e., $n'_e > n'_o$, transforming the crystal into a positive uniaxial medium. Similarly, we investigate the beam evolution, as well as the intensity and polarization distributions at the crystal exit surface and the two focal points. The results are depicted in Figures 10–12. As shown in Figure 10, two focal points emerge during beam propagation at 35.6 mm and 37.0 mm, respectively. Similarly to the case of $E_0 = 15$ kV/mm, the two focal points are also separated by 1.4 mm. Figure 11 indicates that the first focal

point is formed by the A-pol component, while the second focal point originates from the R-pol component—this is the exact inverse of the results in Figure 6. Notably, Figure 12b clearly demonstrates that the polarization distribution at the first focal point is dominated by the A-pol component.

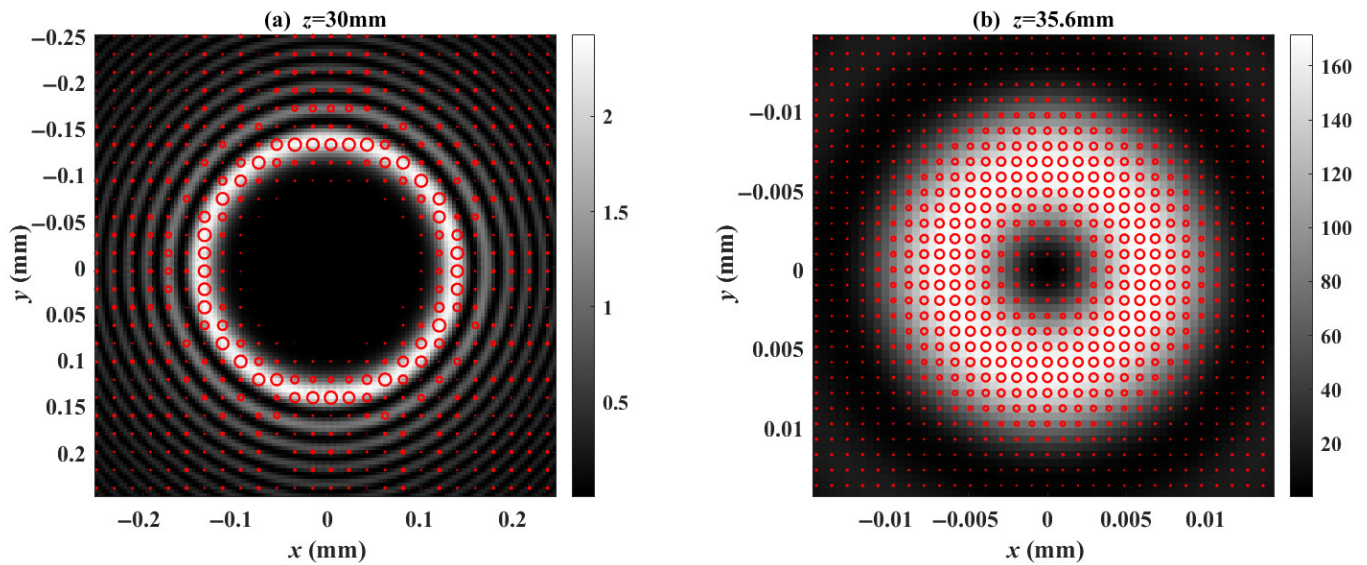


Figure 9. The normalized intensity distribution and polarization distribution at (a) $z = 30 \text{ mm}$ and (b) $z = 35.6 \text{ mm}$ with the electric field $E_0 = -1.67 \text{ kV/mm}$. The red ellipse represents the left-handed polarized ellipse.

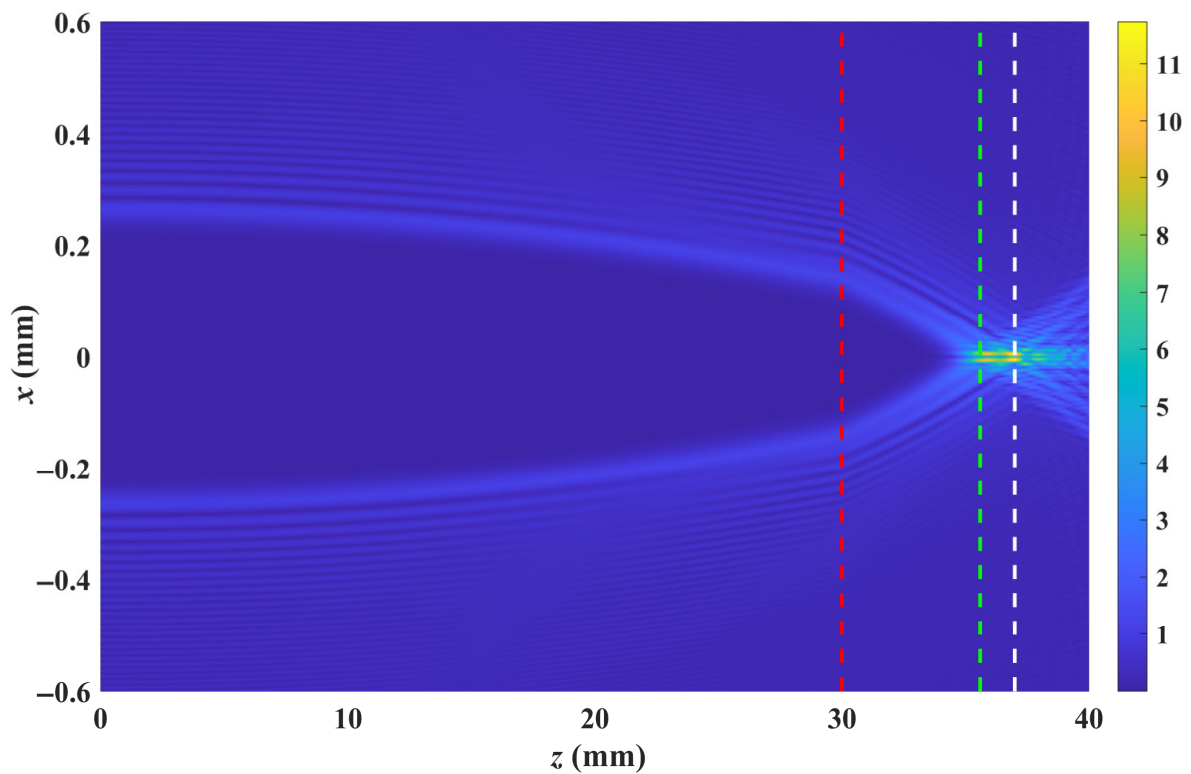


Figure 10. The evolution of the square root of the normalized intensity of the light beam with the electric field $E_0 = -15 \text{ kV/mm}$. The red, green, and white dashed lines represent the exit surface of the crystal, and the front and rear focal planes, respectively.

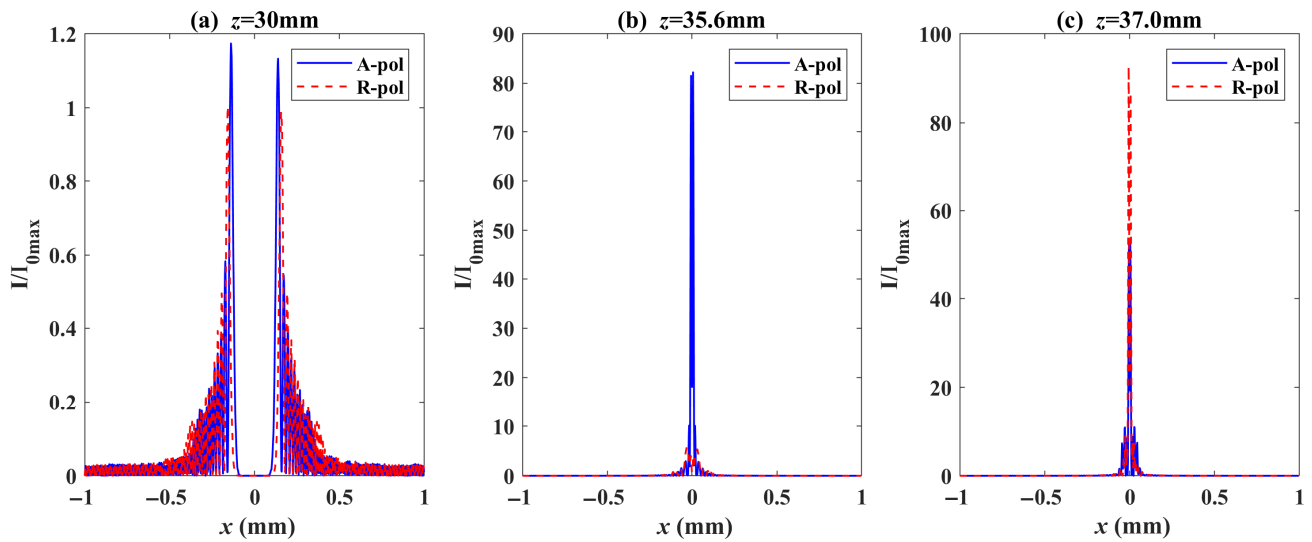


Figure 11. The normalized intensity distribution of the R-pol and A-pol components on x-axis at (a) $z = 30$ mm, (b) $z = 35.6$ mm, and (c) $z = 37.0$ mm with the electric field $E_0 = -15$ kV/mm.

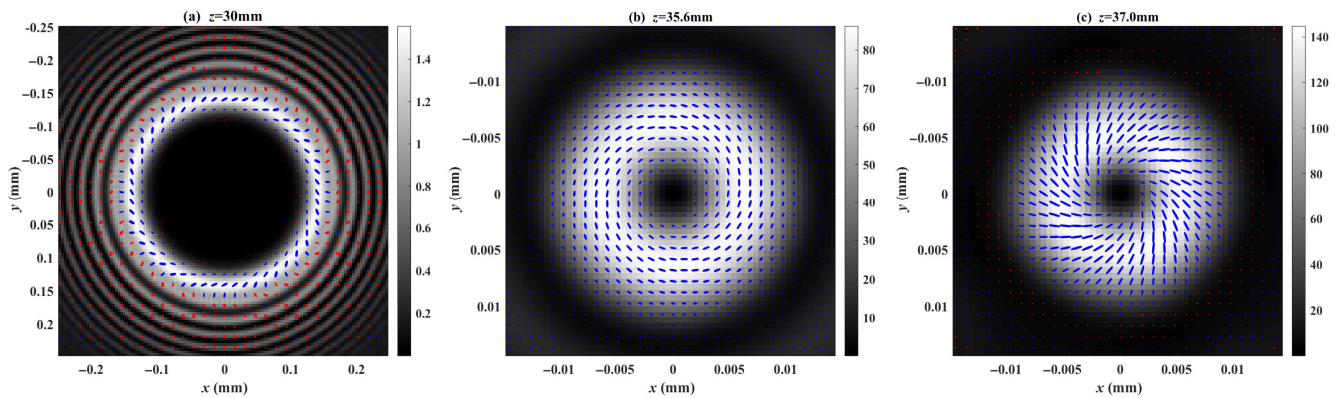


Figure 12. The normalized intensity distribution and polarization distribution at (a) $z = 30$ mm, (b) $z = 35.6$ mm, and (c) $z = 37.0$ mm with the electric field $E_0 = -15$ kV/mm. The red and blue ellipses represent the left-handed polarized ellipse and the right-handed polarized ellipse, respectively.

4. Discussion

The present study introduces a novel method for dynamically separating R-pol and A-pol components in CAVBs using the linear EO effect in uniaxial crystals. These findings can be interpreted within the context of previous studies on CAVBs and the linear EO effect in uniaxial crystals. Liu et al. (2023) [28] demonstrated the static separation of R-pol and A-pol components in CAVBs using positive and negative uniaxial crystals with large birefringence, where the separation distance between the two focal points can be adjusted by changing the crystal length. Here, we utilize the longitudinal linear EO effect of SBN crystals, which can rapidly modify the magnitude of crystal birefringence and even convert the crystal into a positive uniaxial state. More importantly, this process does not alter the uniaxial crystal property of SBN. The ability to dynamically control the polarization components of CAVBs offers versatility in optical experiments and applications. For instance, precise control over the polarization state is crucial for manipulating micro-particles.

There are several issues that can be further studied. First, the natural birefringence of SBN crystals is relatively weak, and a 1.4 mm focal separation requires an applied electric field of 15 kV/mm. Future research could focus on seeking materials with both larger birefringence and higher EO coefficients to reduce the requirement for electric fields, or optimizing the applied electric field—for example, by cascading multiple crystals to lower

the field requirement. Second, we chose CAVBs as the research subject because of their unique properties. A CAVB can remain hollow before being focused, and both the radius of the main ring and the depth of focus can be adjusted via the incident beam parameters. If we aim to extend CAVBs to other types of beams, such as Laguerre–Gaussian beams, certain adjustments must be made. For instance, the beam should be hollow, require lens focusing, and have a small depth of focus, among other requirements [26]. Finally, experimental validation is also crucial for theoretical simulations.

5. Conclusions

In conclusion, we have demonstrated a method to dynamically separate R-pol and A-pol components of CAVBs using SBN's linear EO effect. Numerical calculations show that by varying the magnitude and direction of the applied electric field, the birefringence of the crystal can be altered, enabling the rapid separation of the two components. This method overcomes the static limitation of traditional polarization splitters, enabling dynamic modulation of R-pol and A-pol polarization components. It holds significant promise for applications in optical manipulation, high-resolution imaging, and quantum information processing, providing a versatile platform for real-time beam control.

Author Contributions: Conceptualization, G.Z.; Methodology, G.Z., T.H. and X.Z.; Software, G.Z. and Q.W.; Investigation, Z.X., J.L. and L.W.; Writing—original draft, G.Z. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by the Shenzhen Science and Technology Programs, grant numbers: 20220719141014001, 20220717104706001, 20231129112637001, Guangdong Provincial Engineering Technology Research Center for Materials for Advanced MEMS Sensor Chip, grant number 2022GCZX005, and National Key Research and Development Program of China, grant numbers: 2024YFB4709503.

Data Availability Statement: The original contributions presented in this study are included in the article. Further inquiries can be directed to the corresponding author(s).

Conflicts of Interest: The authors declare no conflicts of interest.

References

1. Efremidis, N.K.; Christodoulides, D.N. Abruptly autofocusing waves. *Opt. Lett.* **2010**, *35*, 4045–4047. [[CrossRef](#)]
2. Papazoglou, D.G.; Efremidis, N.K.; Christodoulides, D.N.; Tzortzakis, S. Observation of abruptly autofocusing waves. *Opt. Lett.* **2011**, *36*, 1842–1844. [[CrossRef](#)]
3. Liu, S.; Wang, M.; Li, P.; Zhang, P.; Zhao, J. Abrupt polarization transition of vector autofocusing Airy beams. *Opt. Lett.* **2013**, *38*, 2416–2418. [[CrossRef](#)] [[PubMed](#)]
4. Chremmos, I.; Zhang, P.; Prakash, J.; Efremidis, N.K.; Christodoulides, D.N.; Chen, Z. Fourier-space generation of abruptly autofocusing beams and optical bottle beams. *Opt. Lett.* **2011**, *36*, 3675–3677. [[CrossRef](#)] [[PubMed](#)]
5. Panagiotopoulos, P.; Papazoglou, D.G.; Couairon, A.; Tzortzakis, S. Sharply autofocused ring-Airy beams transforming into non-linear intense light bullets. *Nat. Commun.* **2013**, *4*, 2622. [[CrossRef](#)]
6. Siviloglou, G.A.; Broky, J.; Dogariu, A.; Christodoulides, D.N. Observation of Accelerating Airy Beams. *Phys. Rev. Lett.* **2007**, *99*, 213901. [[CrossRef](#)]
7. Dasgupta, R.; Ahlawat, S.; Verma, R.S.; Gupta, P.K. Optical orientation and rotation of trapped red blood cells with Laguerre-Gaussian mode. *Opt. Express* **2011**, *19*, 7680–7688. [[CrossRef](#)]
8. Zhang, P.; Prakash, J.; Zhang, Z.; Mills, M.S.; Efremidis, N.K.; Christodoulides, D.N.; Chen, Z. Trapping and guiding microparticles with morphing autofocusing Airy beams. *Opt. Lett.* **2011**, *36*, 2883–2885. [[CrossRef](#)]
9. Li, P.; Liu, S.; Peng, T.; Xie, G.; Gan, X.; Zhao, J. Spiral autofocusing Airy beams carrying power-exponent-phase vortices. *Opt. Express* **2014**, *22*, 7598–7606. [[CrossRef](#)]
10. Zhu, W.; She, W. Tightly focusing vector circular airy beam through a hard aperture. *Opt. Commun.* **2015**, *334*, 303–307. [[CrossRef](#)]
11. Davis, J.A.; Cottrell, D.M.; Sand, D. Abruptly autofocusing vortex beams. *Opt. Express* **2012**, *20*, 13302–13310. [[CrossRef](#)]

12. Jiang, Y.; Huang, K.; Lu, X. Propagation dynamics of abruptly autofocusing Airy beams with optical vortices. *Opt. Express* **2012**, *20*, 18579–18584. [\[CrossRef\]](#)
13. Chen, B.; Chen, C.; Peng, X.; Peng, Y.; Zhou, M.; Deng, D. Propagation of sharply autofocused ring Airy Gaussian vortex beams. *Opt. Express* **2015**, *23*, 19288–19298. [\[CrossRef\]](#)
14. Zheng, G.; Wu, Q.; He, T.; Zhang, X. Propagation Characteristics of Circular Airy Vortex Beams in a Uniaxial Crystal Along the Optical Axis. *Micromachines* **2022**, *13*, 1006. [\[CrossRef\]](#)
15. Pan, J.; Wang, H.; Shen, Y.; Fu, X.; Liu, Q. Airy coherent vortices: 3D multilayer self-accelerating structured light. *Appl. Phys. Lett.* **2022**, *121*, 141102. [\[CrossRef\]](#)
16. Wu, H.; Zhou, W.; Zhu, Z.; Shen, Y. Optical skyrmion lattices accelerating in a free-space mode. *APL Photonics* **2025**, *10*, 050804. [\[CrossRef\]](#)
17. Ciattoni, A.; Cincotti, G.; Palma, C. Propagation of cylindrically symmetric fields in uniaxial crystals. *J. Opt. Soc. Am. A* **2002**, *19*, 792–796. [\[CrossRef\]](#) [\[PubMed\]](#)
18. Ciattoni, A.; Crosignani, B.; Di Porto, P. Vectorial theory of propagation in uniaxially anisotropic media. *J. Opt. Soc. Am. A* **2001**, *18*, 1656–1661. [\[CrossRef\]](#)
19. Ciattoni, A.; Cincotti, G.; Palma, C. Circularly polarized beams and vortex generation in uniaxial media. *J. Opt. Soc. Am. A* **2003**, *20*, 163–171. [\[CrossRef\]](#)
20. Zhou, G.; Chen, R.; Chu, X. Propagation of Airy beams in uniaxial crystals orthogonal to the optical axis. *Opt. Express* **2012**, *20*, 2196–2205. [\[CrossRef\]](#)
21. Deng, D.; Chen, C.; Zhao, X.; Li, H. Propagation of an Airy vortex beam in uniaxial crystals. *Appl. Phys. B* **2013**, *110*, 433–436. [\[CrossRef\]](#)
22. Zhou, M.-L.; Chen, C.-D.; Chen, B.; Peng, X.; Peng, Y.-L.; Deng, D.-M. Propagation of an Airy–Gaussian beam in uniaxial crystals. *Chin. Phys. B* **2015**, *24*, 124102. [\[CrossRef\]](#)
23. Yu, W.; Zhao, R.; Deng, F.; Huang, J.; Chen, C.; Yang, X.; Zhao, Y.; Deng, D. Propagation of Airy Gaussian vortex beams in uniaxial crystals. *Chin. Phys. B* **2016**, *25*, 044201. [\[CrossRef\]](#)
24. Zheng, G.; Deng, X.; Xu, S.; Wu, Q. Propagation dynamics of a circular Airy beam in a uniaxial crystal. *Appl. Opt.* **2017**, *56*, 2444–2448. [\[CrossRef\]](#)
25. Zhu, W.; Zheng, H.; Zhong, Y.; Yu, J.; Chen, Z. Wave-Vector-Varying Pancharatnam-Berry Phase Photonic Spin Hall Effect. *Phys. Rev. Lett.* **2021**, *126*, 083901. [\[CrossRef\]](#)
26. Liu, H.; Yuan, L. Controlling the spin Hall effect of grafted vortex beams propagating in uniaxial crystal. *Opt. Express* **2023**, *31*, 10434–10448. [\[CrossRef\]](#)
27. Liu, H.; Deng, S.; Deng, H.; Xu, R.; Yang, H.; Teng, C.; Zhang, L.; Chen, M.; Yuan, L. Spin-orbital coupling of quadratic-power-exponent-phase vortex beam propagating in a uniaxial crystal. *Opt. Express* **2020**, *28*, 216–225. [\[CrossRef\]](#)
28. Liu, H.; Yuan, L. Separating radial and azimuthal polarizations of circular Airy vortex beam via uniaxial crystal. *Opt. Express* **2023**, *31*, 22507–22518. [\[CrossRef\]](#)
29. Zhu, W.; She, W. Electro-optically generating and controlling right- and left-handed circularly polarized multiring modes of light beams. *Opt. Lett.* **2012**, *37*, 2823–2825. [\[CrossRef\]](#)
30. Zheng, G.; Xu, S.; Wu, Q.; Wang, Q.; Ouyang, Z. Electro-optical coupling of a circular Airy beam in a uniaxial crystal. *Opt. Express* **2017**, *25*, 14654–14667. [\[CrossRef\]](#)
31. Zhu, W.; Liang, X.; Xie, W.; Zheng, H.; Zhong, Y.; Tang, J.; Yu, J.; Chen, Z. Optical Stern–Gerlach Effect in Periodically Poled Electro-Optical Crystals. *Laser Photonics Rev.* **2024**, *18*, 2301030. [\[CrossRef\]](#)
32. Xie, X.; Cheng, H.; Chen, H.; Zheng, H.; Zhong, Y.; Tang, J.; Yu, J.; Chen, Z.; Zhu, W. Photonic Spin Hall Effect by Electro-optically Induced Pancharatnam-Berry Phases. *Phys. Rev. Lett.* **2025**, *134*, 113805. [\[CrossRef\]](#)
33. Yariv, A.; Yeh, P. *Optical Waves in Crystals: Propagation and Control of Laser Radiation*; Wiley: New York, NY, USA, 1983.
34. Liu, H.; Zhang, J.; Pu, H.; Xu, J.; Xu, R.; Yuan, L. Controlling the Abrupt Autofocusing of Circular Airy Vortex Beam via Uniaxial Crystal. *Photonics* **2022**, *9*, 943. [\[CrossRef\]](#)
35. Jiang, Z.; Paillard, C.; Xiang, H.; Bellaiche, L. Linear Versus Nonlinear Electro-Optic Effects in Materials. *Phys. Rev. Lett.* **2020**, *125*, 017401. [\[CrossRef\]](#)
36. Rao, Y.; Liu, H.; Hao, H.; Yao, Z.; Zhou, X.; Cao, M.; Yu, Z. MgO-modified Sr_{0.7}Ba_{0.3}Nb₂O₆ ceramics for energy storage applications. *Ceram. Int.* **2018**, *44*, 11022–11029. [\[CrossRef\]](#)

Disclaimer/Publisher’s Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.