

Article

Reconstruction Algorithm of Absorption Spectral Field Distribution Based on a Priori Constrained Bivariate Polynomial Model

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Abstract: Computed Tomography–Tunable Diode Laser Absorption Spectroscopy (CT-TDLAS) is an effective diagnostic method for analyzing combustion flow fields within engines. This study proposes an adaptive reconstruction algorithm utilizing constrained polynomial fitting within the CT-TDLAS framework. Based on existing polynomial fitting models, the proposed algorithm integrates physical boundary constraints on temperature and concentration fields, optimizing integrated absorbance errors. This method significantly enhances reconstruction accuracy and computational efficiency, while also lowering computational complexity. The adaptive strategy dynamically adjusts the polynomial order, effectively mitigating issues of overfitting or underdetermination typically associated with fixed polynomial orders. Numerical simulations demonstrate reduced temperature reconstruction errors of 2%, 1.6%, and 2% for single-peak, dual-peak, and mixed distribution flow fields, respectively. Corresponding concentration errors were 2%, 1.8%, and 2.6%, which are all improvements over those achieved by the Algebraic Reconstruction Technique (ART). Experimental results using a McKenna flat-flame burner revealed an average reconstruction error of only 0.3% compared to thermocouple measurements for high-temperature regions (>1000 K), with a minimal central temperature difference of 6 K. For lower-temperature peripheral regions, the average error was 188 K, illustrating the practical applicability of the proposed algorithm.

Keywords: tunable diode laser absorption spectroscopy (TDLAS); combustion flow field diagnosis; constrained polynomial fitting; adaptive strategy



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1. Introduction

With the ongoing advancement in aerospace technology, accurately diagnosing combustion flow fields in engines, including aircraft and scramjets, has become critically important. Traditional diagnostic tools, such as pressure sensors, intrusive probes, and thermocouples, typically disrupt flow, exhibit slow responses and low sensitivity, and are unsuitable for harsh, high-temperature, and high-speed environments typical of hypersonic vehicles. TDLAS technology, employing molecules such as H₂O, CO₂, CO, and O₂ as probes [1–4], enables non-intrusive, rapid, and real-time measurements of temperature, pressure, flow velocity, and gas concentration in engine combustion flow fields. This technique robustly supports aerodynamic studies of burners, key component experiments, and ground-based performance evaluations [5–8]. However, since TDLAS measurements provide line-of-sight averages, obtaining two-dimensional flow field parameter distributions

necessitates multi-beam and multi-angle measurement approaches. By integrating absorption spectral data obtained from multiple laser paths with Computed Tomography (CT) algorithms, two-dimensional reconstructions of flow field parameters can be effectively achieved, as exemplified by the CT-TDLAS method.

Conventional CT-TDLAS algorithms typically use linear tomographic methods, such as the Algebraic Reconstruction Technique (ART), to reconstruct integrated absorbance data across multiple wavelengths. Subsequently, they employ two-line ratio or Boltzmann plot methods to determine temperature and concentration distributions. For instance, Bao et al. [9] validated an enhanced two-line ratio method using numerical simulations and conceptual experiments. Xu et al. [10] applied various Radial Basis Function (RBF) models to fit integrated absorbance data across multiple wavelengths. They subsequently utilized a two-line ratio method for temperature and concentration reconstruction, obtaining results that closely matched the original distributions. However, linear tomographic algorithms, which ignore correlations among different wavelengths, often suffer from inaccuracies due to insufficient projections [11] and noise interference [9]. This frequently results in edge distortions within complex flow fields. Nonlinear CT-TDLAS methods, such as simulated annealing (SA), jointly process multi-wavelength absorption information to find optimal solutions [12,13]. However, these methods typically require excessive computational time. Recent studies have investigated deep neural networks to improve reconstruction capabilities [14–16]. Nevertheless, the accuracy of these models heavily depends on the quality of training data, and their generalizability to different combustion configurations remains uncertain. Additionally, Wang et al. [17] employed polynomial models to fit temperature and concentration distributions, resolving these based on residuals from multi-wavelength absorbance curves. This method uses multi-objective optimization of absorbance calculations and achieves high reconstruction accuracy. Currently, reconstruction algorithms based on absorbance calculations exhibit high computational complexity and introduce lineshape errors during processing. Furthermore, determining the optimal polynomial order to balance descriptive accuracy and computational efficiency remains unaddressed. Thus, developing an efficient, robust, and scalable reconstruction algorithm remains a critical challenge in this field.

In this study, we introduce a CT-TDLAS reconstruction algorithm utilizing constrained polynomial fitting. The algorithm optimizes integrated absorbance errors by incorporating physical boundary constraints for temperature and concentration fields. Additionally, we develop an adaptive strategy for selecting the polynomial order. This strategy dynamically adjusts polynomial orders according to the complexity of the flow field, effectively mitigating problems such as overfitting and underdetermination. Numerical simulations demonstrate that the temperature reconstruction errors remain below 3% for various flow field configurations. This represents an improvement of more than 50% compared to the traditional ART algorithm. Experiments further confirm that the reconstruction results of the algorithm are consistent with the measurement trend of the thermocouple.

2. Materials and Methods

2.1. Principles of CT-TDLAS

CT-TDLAS combines tunable diode laser absorption spectroscopy (TDLAS) with computer tomography (CT) to achieve noncontact spectral measurements of two-dimensional fields. TDLAS employs a tunable laser to generate a beam over a specific wavelength range, which, upon passing through the test gas, undergoes absorption at characteristic wavelengths. The transmitted beam is then detected, and the obtained spectral information

is related to the gas species, temperature, concentration, and pressure. According to the Beer–Lambert law, the relationship is given by

$$\alpha_v = -\ln\left(\frac{I_t}{I_0}\right) = \int_0^L C(l) \cdot P(l) \cdot S[T(l)] \cdot \phi dl \quad (1)$$

where α_v is the absorbance; I_0 is the incident light intensity in the absence of absorption; I_t is the transmitted light intensity after absorption; $C(l)$ is the concentration of the target gas species; $P(l)$ [atm] represents the pressure in the absorption region; $S[T(l)]$ [cm^{−2} atm^{−1}] is the line strength dependent on temperature; ϕ [cm] is the lineshape function; and L [cm] is the absorption path length. Because the lineshape function is normalized, the integral absorption A_v can be described as

$$A_v = \int_0^L C(l) \cdot P(l) \cdot S[T(l)] \cdot \phi dl \quad (2)$$

Using this principle, TDLAS technology can retrieve spectral information for specific absorption lines, which in turn can be used to infer gas temperature and concentration [18,19]. By combining data from multiple angles and laser paths with CT reconstruction algorithms, a two-dimensional map of flow field parameters is obtained [20,21].

2.2. Constrained Polynomial Fitting Reconstruction Algorithm

The constrained polynomial fitting (CPF) algorithm for reconstructing combustion fields begins by discretizing the measurement domain into pixels, each carrying independent temperature and concentration information. Since the distributions of temperature and concentration vary continuously over the region, a bivariate polynomial in spatial coordinates can be used as the basis function to approximate these distributions [17]. Assuming a maximum polynomial order m , the temperature and concentration fields are expressed as

$$T(x, y) = \sum_{k=0}^m \sum_{l=0}^k b_{k-l,l} x^{k-l} y^l = F_T(b_{kl}) \quad (3)$$

$$C(x, y) = \sum_{k=0}^m \sum_{l=0}^k a_{k-l,l} x^{k-l} y^l = F_C(a_{kl}) \quad (4)$$

a_{kl} and b_{kl} are polynomial coefficients of concentration and temperature, respectively. $F_T(b_{kl})$ and $F_C(a_{kl})$, respectively, represent polynomials describing the temperature concentration distribution. Thus, the number of basis functions is $N_{bf} = (m + 1)(m + 2)/2$, leading to $2N_{bf}$ unknown coefficients for both temperature and concentration. The integrated absorbance signal and its corresponding error for wavelength λ can be expressed as follows when the number of projection paths is designated as P_r , the total grid number is defined as Q , and the total number of wavelengths is specified as N_λ :

$$A_{\lambda,p} = \sum_{q=1}^Q L_{p,q} \cdot C_q \cdot S_\lambda(T_q) \cdot P = \sum_{q=1}^Q L_{p,q} \cdot F_C(a_{kl})_q \cdot S_\lambda[F_T(b_{kl})_q] \cdot P \quad (5)$$

$$Error = \sum_{\lambda,p} \left[(A_{\lambda,p})_{theory} - (A_{\lambda,p})_{experiment} \right]^2 \quad (6)$$

Based on the physical a priori information, boundary constraints for the temperature and concentration fields are imposed for each grid cell, resulting in $2Q$ inequality constraints. With the projection paths fixed, the calculation of temperature and concentration can be reformulated in a matrix form as

$$T_q(x_q, y_q) = \sum_{k=0}^m \sum_{l=0}^k b_{k-l,l} x_q^{k-l} y_q^l = \sum_{h=1}^{N_{base}} b_h \Psi_{h,q} = (\Psi b)_q \quad (7)$$

where the basis function matrix Ψ is precomputed according to the grid partitioning. Thus, the constraints on temperature and concentration can be transformed into the form

$$\begin{aligned} T_{min} &\leq \Psi b \leq T_{max} \\ C_{min} &\leq \Psi a \leq C_{max} \end{aligned} \quad (8)$$

The overall algorithm is illustrated in Figure 1. It integrates multi-spectral measurements via TDLAS with physical a priori information, imposing upper and lower bounds on temperature and concentration for each grid cell. By solving the minimization of the error equations using a Sequential Quadratic Programming (SQP) algorithm, the bivariate polynomial coefficient matrices a and b in Equations (3) and (4) are determined, yielding the reconstructed temperature and concentration fields.

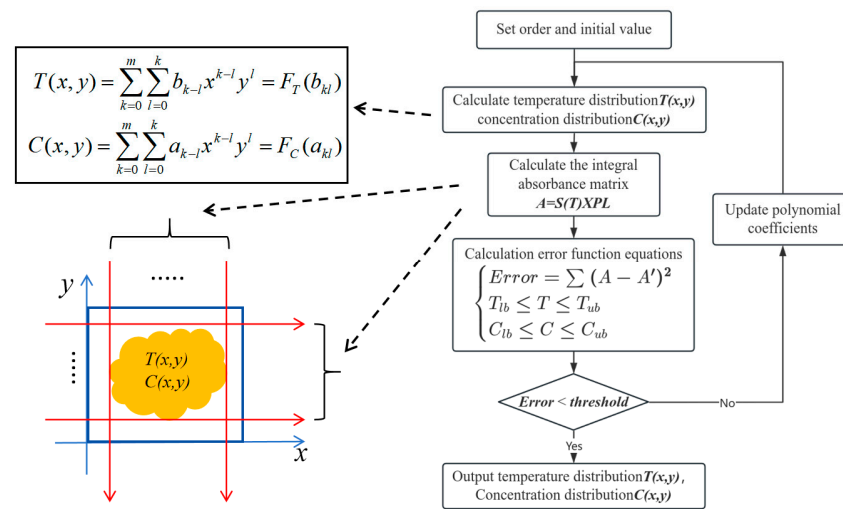


Figure 1. Process of the constrained polynomial model algorithm.

2.3. Adaptive Polynomial Order Selection Algorithm

A critical issue in the bivariate polynomial fitting of temperature and concentration fields is the selection of the polynomial order m . In practice, the precise distribution of the temperature and concentration fields is unknown. A polynomial with too low an order may be insufficient to describe a complex field, resulting in high reconstruction errors; conversely, a polynomial with too high an order increases computational time and the risk of converging to local minima due to an increased number of unknowns. In this study, an adaptive order selection algorithm based on error correction is developed. The algorithm flow is illustrated in Figure 2.

Initially, parameters are set, including the initial polynomial order (Initial_Order, typically set to 8), the maximum allowable order (Max_Order, typically 13), an initial residual value (Initial_Error, set to a large value), and a maximum number of iterations (N , set to 10). The Initial_Error serves to mark the current minimum residual, an Error_flag indicates the number of iterations X during which the error does not improve (typically $X = 3$), and an Order_flag tracks the consecutive increases in the polynomial order with no change in error (threshold typically set to 2). The algorithm begins with the lowest-order polynomial and iteratively computes the solution; if the residual does not improve for several consecutive iterations, the polynomial order is increased, and the parameters from the previous order are used as the initial values for further iterations. When the residual reaches a threshold, the maximum order is reached, or when the number of successive order increases exceeds the preset limit without error improvement, the algorithm terminates. This indicates that within the current order range, further increases do not significantly

reduce the reconstruction error and only decrease computational efficiency. This method not only ensures sufficient reconstruction accuracy but also enhances efficiency and avoids errors associated with inappropriate order selection in unknown fields.

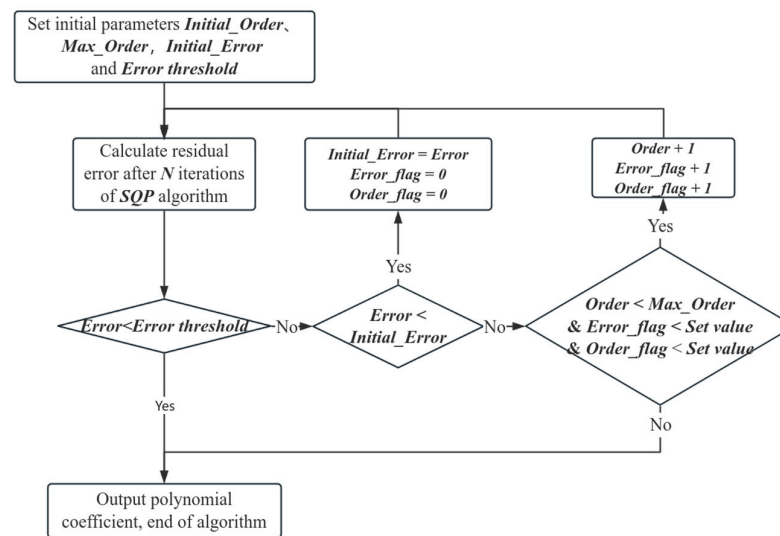


Figure 2. Process of adaptive polynomial order selection algorithm.

3. Results

3.1. Numerical Simulations

Numerical simulations are conducted to verify the performance of the proposed bivariate polynomial algorithm. Three different combustion field configurations are simulated. Figure 3a represents a single Gaussian peak distribution, Figure 3b represents a dual Gaussian peak distribution, and Figure 3c represents a mixed dual Gaussian distribution. For the single-peak and double-peak cases, the temperature ranges from 500 to 1500 K with a H₂O concentration between 0.05 and 0.12, while the mixed peak spans 500–1800 K with a H₂O concentration from 0.05 to 0.15. The measurement domain is discretized into a 30 × 30 grid. The side length of the simulated distribution is set at 6 cm with a spacing of 0.2 cm between beams.

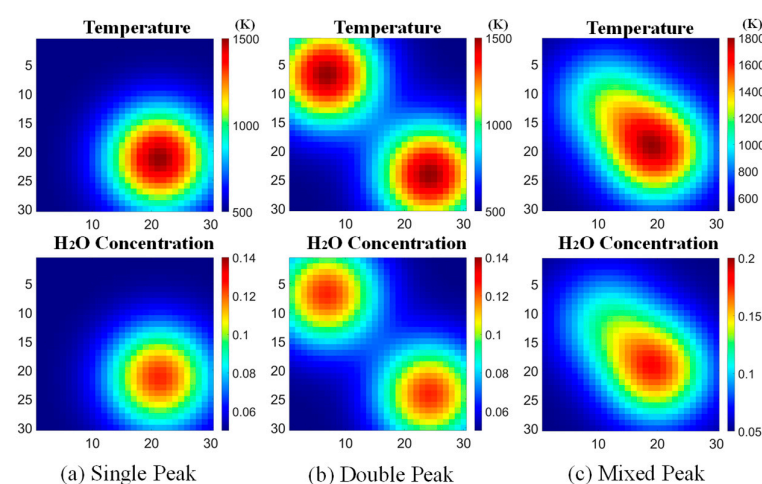


Figure 3. Three simulated fields: (a) single peak; (b) double peak; (c) mixed peak.

The simulation employs four projection angles (0°, 45°, 90°, and 135°) with 30 beams per angle, yielding a total of 120 laser paths as depicted in Figure 4 (red lines represent laser paths, and the black grid lines indicate the discretization). Four absorption

wavelengths (1339 nm, 1343 nm, 1392 nm, and 1469 nm) are selected, and the integrated absorbance at these wavelengths is computed using absorption line parameters from the HITRAN database.

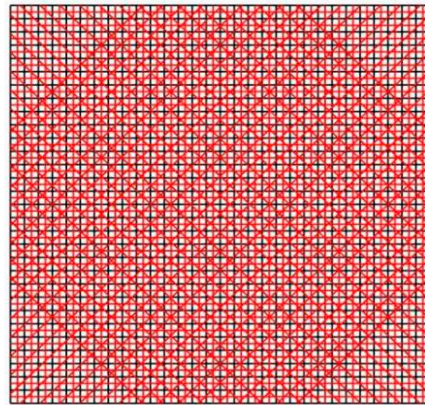


Figure 4. Laser path and grid.

The reconstruction errors for the temperature and concentration fields are defined as follows:

$$Error_T = \frac{\sum_i^Q |T_i^{true} - T_i^{cal}|}{\sum_i^Q T_i^{true}}$$

$$Error_C = \frac{\sum_j^Q |C_j^{true} - C_j^{cal}|}{\sum_j^Q C_j^{true}}$$

where Q is the total number of grid cells, T_i^{true} and C_j^{true} represent the simulated temperature and concentration values, and T_i^{cal} and C_j^{cal} are the corresponding reconstructed values.

3.1.1. Analysis of Polynomial Order Effects

The order of the bivariate polynomial affects the model's ability to accurately represent the surface; higher orders offer greater descriptive power but lead to increased computational time and risk of underdetermination due to a surge in unknown coefficients. In contrast, lower orders may fail to capture the complexity of the field, resulting in significant reconstruction errors.

For the single-peak, dual-peak, and mixed fields, the reconstruction error between the bivariate polynomial model and the simulated temperature field is computed for various polynomial orders, with the results shown in Figure 5. The results indicate that for lower orders (below 9), the error differences among different field types are significant, potentially reducing the model's generalization capability. When the order exceeds 12, the reconstruction errors across the three configurations become comparable, suggesting that a higher-order polynomial can universally describe diverse combustion fields. However, as shown by the increase in the number of polynomial coefficients with order, an excessively high order leads to a rapid increase in unknowns, which may cause underdetermined problems and significantly raise computational costs.

The proposed adaptive order adjustment algorithm based on error variation addresses the difficulty of pre-determining the polynomial order in unknown field reconstructions. Based on the performance and computational cost analysis, the polynomial order range is set between 8 and 13, ensuring applicability to most combustion fields. All subsequent simulations employ this adaptive strategy.

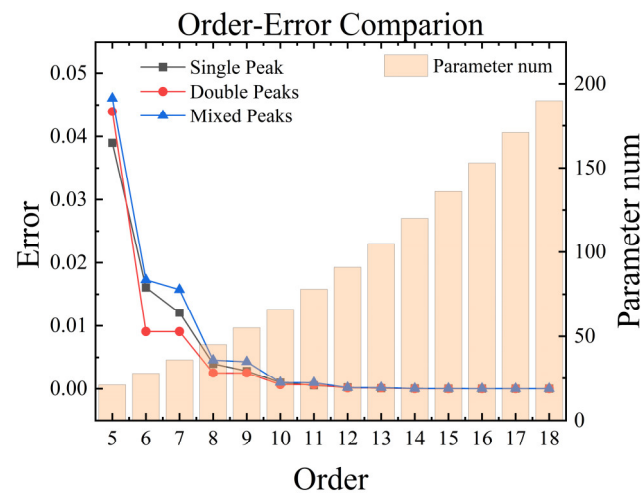


Figure 5. Error of different orders.

3.1.2. Simulation Results

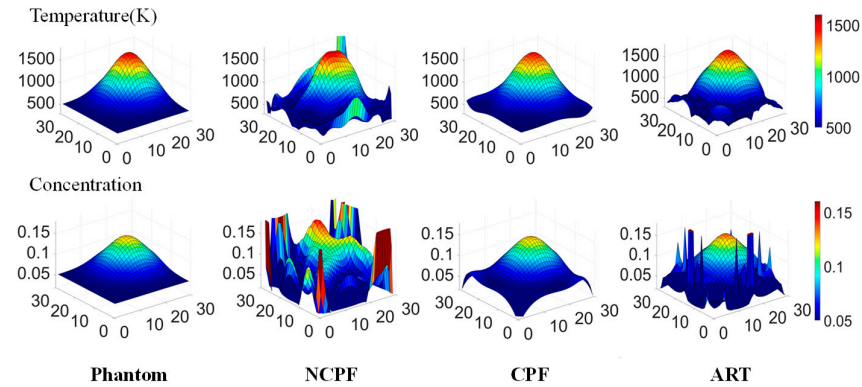
Currently, two main types of algorithms are used for reconstructing two-dimensional combustion fields: linear tomography methods (e.g., ART) and nonlinear tomography methods. In the linear approach, ART is compared with both the non-constraint polynomial fitting (NCPF) and the proposed constrained polynomial fitting (CPF) algorithms. Derived from the CPF model, the NCPF algorithm implementation removes constraint conditions while retaining core polynomial fitting principles in its computational model. In these comparisons, the temperature constraint is set to [296 K, 2500 K] and the concentration constraint to [0.0001, 0.3]. The ART algorithm uses a four-wavelength Boltzmann plot method for temperature and concentration reconstruction. The reconstruction results for the three simulated configurations are presented in Figure 6 and Table 1.

Table 1. Reconstruction error of temperature and concentration distribution.

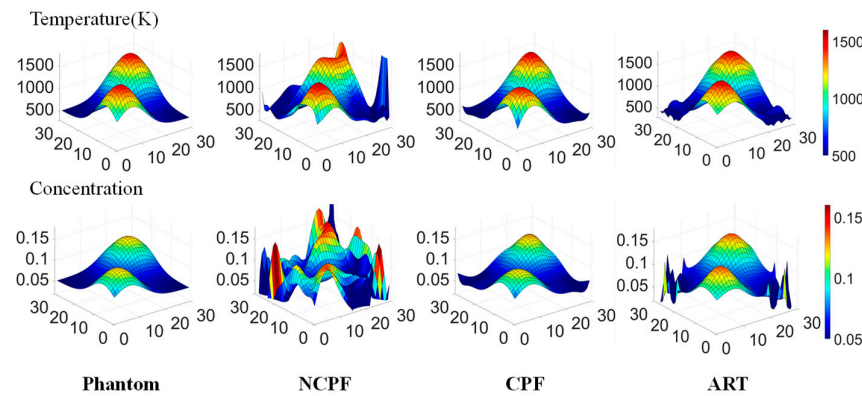
(a) Single Peak			
	NCRF	CPF	ART
Temperature	13.3%	2%	7.4%
Concentration	47.1%	2%	14.6%
(b) Double Peak			
	NCRF	CPF	ART
Temperature	6.2%	1.6%	3.1%
Concentration	29.5%	1.8%	7.9%
(c) Mixed Peak			
	NCRF	CPF	ART
Temperature	2.9%	2%	3%
Concentration	11.5%	2.6%	7.9%

Figure 6a–c show that while the NCPF algorithm yields satisfactory results for a simple single-peak field, its performance degrades significantly for more complex dual-peak and mixed fields, resulting in configuration distortions. Moreover, the unconstrained method exhibits pronounced anomalies in the peripheral regions, which are attributable to the insufficient number of laser paths at the edges. These results indicate that, without physical a priori constraints, the NCPF algorithm cannot produce reliable reconstructions. In contrast, the CPF algorithm yields accurate reconstructions for all three configurations, with overall consistency between the reconstructed and simulated fields. The temperature

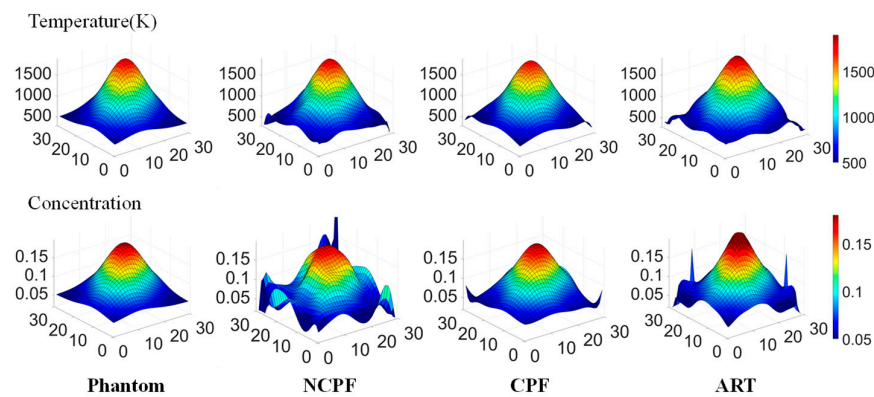
reconstruction errors for the three configurations are 2.03%, 1.58%, and 1.96%, respectively, demonstrating the method's versatility. According to the error values in Table 1, compared with the NCPF algorithm, the improved CPF method achieves increases in temperature accuracy of 84.9%, 74.4%, and 31.9% and concentration accuracy improvements of 95.8%, 94%, and 77.1%, respectively.



(a) Reconstruction results of single peak.



(b) Reconstruction results of double peak.



(c) Reconstruction results of mixed peak.

Figure 6. Reconstruction results of temperature and concentration distribution: (a) single peak; (b) double peak; (c) mixed peak.

Figure 7 presents the one-dimensional temperature profiles along a line parallel to the x-axis through the peak region. The CPF algorithm is shown to accurately capture the overall temperature trend: the high-temperature region (>1000 K) exhibits one-dimensional absolute mean differences of 55 K, 10.3 K, and 31.7 K, while the low-temperature region (≤ 1000 K) shows mean differences of 19.5 K, 6.9 K, and 16.4 K. Compared to the traditional ART algorithm, the CPF algorithm consistently reduces reconstruction errors across all

configurations, particularly in concentration, and maintains overall errors below 3%. Furthermore, unlike the NCPF and ART methods, which display marked peripheral anomalies, the CPF method effectively suppresses these anomalies, yielding a smoother and more accurate reconstruction.

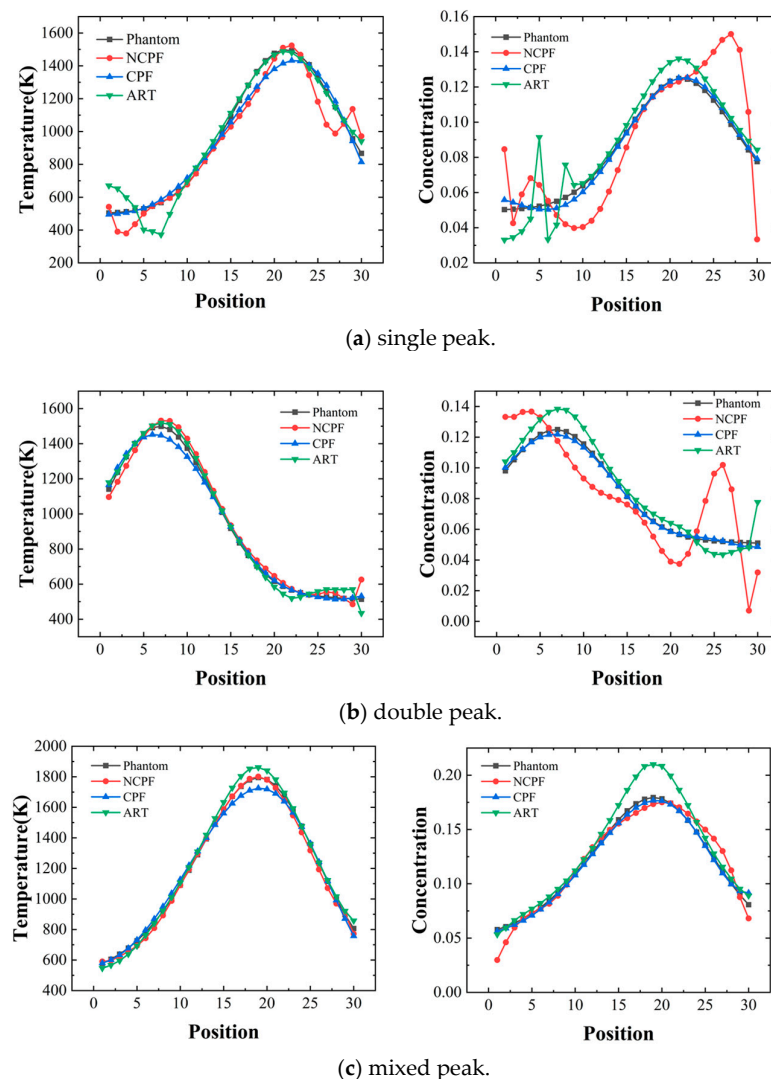


Figure 7. Temperature and concentration distribution curve with peak point parallel to X axis: (a) comparison results of single peak; (b) comparison results of double peak; (c) comparison results of mixed peak.

3.2. Effect of Noise on Reconstruction Accuracy

The superior performance of the CPF algorithm is further evidenced by its contrast with the NCPF method, which exhibits significant errors and distorted concentration reconstructions. To assess robustness, Gaussian white noise levels ranging from 0.5% to 5% are added to the integrated absorbance matrix of the three simulated distributions. For each noise level, 30 simulation trials are performed to mitigate random fluctuations, and both the CPF and ART algorithms are applied. The average reconstruction errors for the three configurations are shown in Figure 8, with error bars representing the standard deviation over 30 trials. Although the CPF algorithm's error increases more rapidly with noise compared to ART, the overall error remains low. Under 5% Gaussian white noise, the temperature reconstruction errors for the three configurations are 8.48%, 7.02%, and 6.01%. Notably, ART exhibits smaller variations with noise, indicating higher stability, while the CPF algorithm's stability requires further improvement.

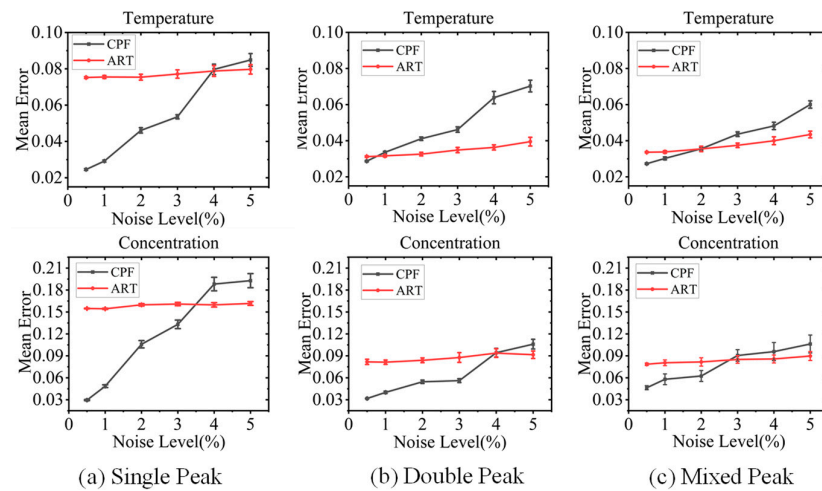


Figure 8. Reconstruction error under 1–5% Gaussian noise: (a) reconstruction error of single peak; (b) reconstruction error of double peak; (c) reconstruction error of mixed peak.

Table 2 details the average error variations for both algorithms under different noise levels. In terms of concentration reconstruction, for the single Gaussian peak model, the CPF algorithm achieves a concentration error of 2.94% under 0.5% noise—an 82.1% reduction compared with ART’s error of 15.47%. For the dual-peak and mixed distributions, the CPF concentration errors remain lower than those of ART when the noise is below 4%; for the mixed distribution, however, under 5% noise, the CPF error (10.62%) is 18.4% higher than ART (8.97%), although under low noise (0.5%), CPF still outperforms ART. In temperature reconstruction, CPF shows superior accuracy under low-noise conditions (noise level $\leq 2\%$); for instance, in the single-peak model under 0.5% noise, the error is 2.45%, only 32.6% of ART’s 7.51%. Similarly, in the mixed distribution under 0.5% noise, CPF achieves an error of 2.73%, 18.8% lower than ART’s 3.36%. Despite the more rapid error growth under high noise (noise level $\geq 4\%$), the overall performance and global advantage in concentration reconstruction highlight the CPF algorithm’s adaptability to complex combustion fields, providing a reliable theoretical basis for high-precision diagnostics.

Table 2. Reconstruction error of different configurations.

(a) Single Peak							
Noise Level		0.5%	1%	2%	3%	4%	5%
CPF	Temperature	2.45%	2.91%	4.61%	5.35%	7.96%	8.48%
	Concentration	2.94%	4.85%	10.58%	13.3%	18.82%	19.28%
ART	Temperature	7.51%	7.55%	7.54%	7.71%	7.88%	7.97%
	Concentration	15.47%	15.43%	15.98%	16.08%	15.97%	16.16%
(b) Double Peak							
Noise Level		0.5%	1%	2%	3%	4%	5%
CPF	Temperature	2.86%	3.36%	4.11%	4.61%	6.38%	7.02%
	Concentration	3.16%	4.01%	5.45%	5.61%	9.42%	10.6%
ART	Temperature	3.12%	3.16%	3.26%	3.48%	3.63%	3.94%
	Concentration	8.18%	8.13%	8.39%	8.77%	9.38%	9.15%
(c) Mixed Peak							
Noise Level		0.5%	1%	2%	3%	4%	5%
CPF	Temperature	2.73%	3.02%	3.55%	4.36%	4.82%	6.01%
	Concentration	4.64%	5.79%	6.24%	9.05%	9.58%	10.62%
ART	Temperature	3.36%	3.38%	3.55%	3.75%	3.99%	4.34%
	Concentration	7.85%	8.05%	8.17%	8.51%	8.57%	8.97%

3.3. Experimental Verification

Following the numerical simulations, the proposed algorithm was further validated using a standard McKenna flat-flame burner in a laboratory setting to assess its feasibility in practical combustion fields. The TDLAS system and a thermocouple measurement system were used concurrently under the same operating conditions, with the measurement plane located 50 mm above the burner surface. As illustrated in Figure 9, the TDLAS scanning range extended from the burner center to its periphery, with a 4 mm shift per measurement (totaling 12 positions). The thermocouple measurement path corresponded with the TDLAS scanning path. In the figure, the red lines represent the actual laser paths, the black dashed lines indicate the reconstruction grid (4 mm grid size), and the circular region denotes the actual combustion zone (6 cm in diameter). Green dots represent thermocouple measurement points, spaced consistently with the laser paths. Due to the axisymmetric distribution of the flat-flame burner, single-angle measurements were transformed into equivalent data for four angles (0° , 45° , 90° , and 135°) using an equivalent optical path method. Background absorption interference was removed by proportionally subtracting background signals from the peripheral measurements. The final reconstruction resolution was 17×17 , with each grid measuring $4 \text{ mm} \times 4 \text{ mm}$, yielding a square reconstruction area of 68 mm per side that is centered on the burner and covers the entire combustion zone.

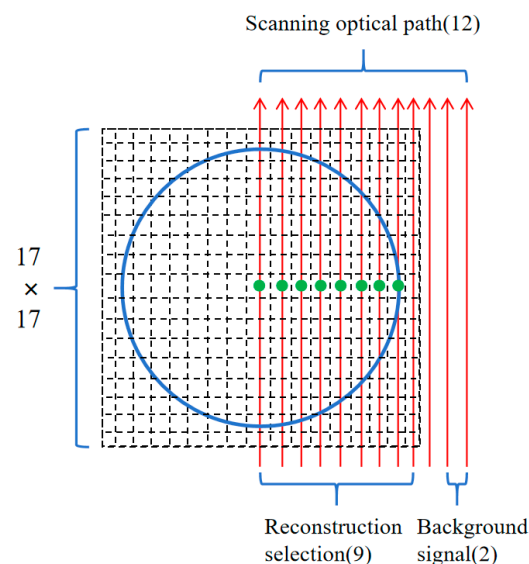


Figure 9. Laser path of the experiment.

The experimental setup of the TDLAS system is depicted in Figure 10. The TDLAS system employs five DFB lasers operating at discrete wavelengths of 1339 nm, 1343 nm, 1392 nm, 1398 nm, and 1469 nm, with time-division multiplexing synchronization. Four characteristic absorption lines (1339 nm, 1343 nm, 1392 nm, and 1469 nm) were algorithmically selected for spectral analysis. A beam splitter configuration established dual measurement paths: a probing beam traversing the combustion field and a reference beam propagating through a temperature-stabilized etalon. Both optical signals were acquired by matched photodetectors, with subsequent signal conditioning comprising low-noise amplification prior to digitization via a data acquisition system. The acquired spectral datasets were stored for subsequent multivariate analysis. A translation stage controlled the laser scanning with a step size of 4 mm. To mitigate the effects of flame pulsation during measurement, 4000 cycles of data were acquired at each position (approximately 1 min of acquisition per position) and averaged. The equivalence ratio of the flat-flame

burner was maintained at 0.8 in continuous combustion until all data were collected. The reconstruction results obtained from the algorithm are shown in Figure 11.

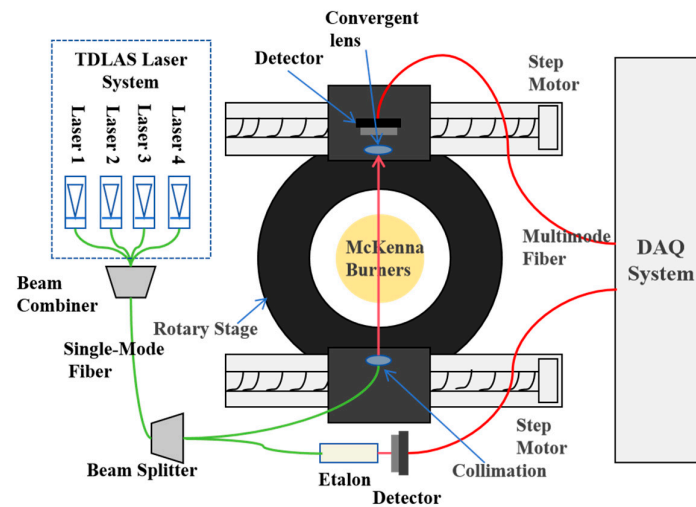


Figure 10. TDLAS measurement system.

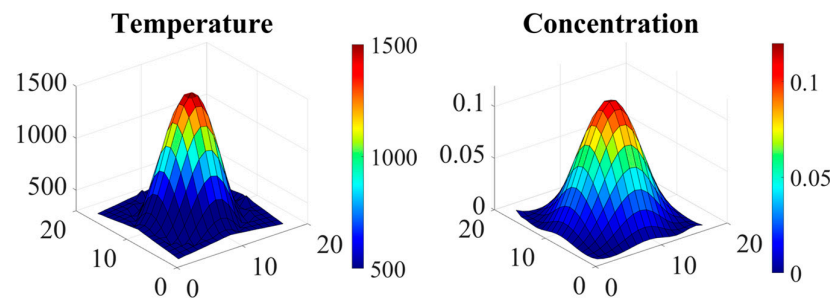


Figure 11. Reconstruction results of experimental temperature and concentration distribution.

Figure 12 compares the reconstructed temperatures with thermocouple measurements. The experimental results demonstrate excellent agreement in the high-temperature central region, with a temperature difference of only 6 K at the center (position = 10), indicating high accuracy in the central region. The overall temperature trend in the reconstructed results closely follows that of the thermocouple measurements. In the high-temperature region (>1000 K), the average temperature difference was only 33 K, whereas in the low-temperature region (≤ 1000 K), the average difference reached 188 K. The two-dimensional reconstruction also reveals anomalously high temperatures at the periphery, indicating partial distortion in low-temperature areas.

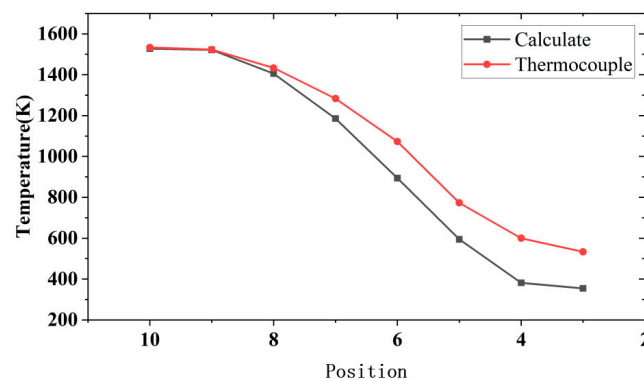


Figure 12. Algorithm reconstruction results and thermocouple measurement results.

Figure 13 presents the raw measurement signals, with the black line representing the central region and the red line the periphery. It can be observed that the absorption in the peripheral region is significantly weaker compared to the center. Notably, the absorption peaks for the 1343 nm and 1339 nm lines are almost indiscernible, and those for the 1392 nm and 1469 nm lines are markedly diminished. As indicated by the one-dimensional comparisons in Figure 7, although the algorithm describes the low-temperature region reasonably well, its performance degrades under high noise conditions. This suggests that the pronounced deviations in the low-temperature region are due to a reduced effective optical path and lower signal-to-noise ratio (SNR) as the temperature decreases toward the burner edge, resulting in diminished measurement sensitivity.

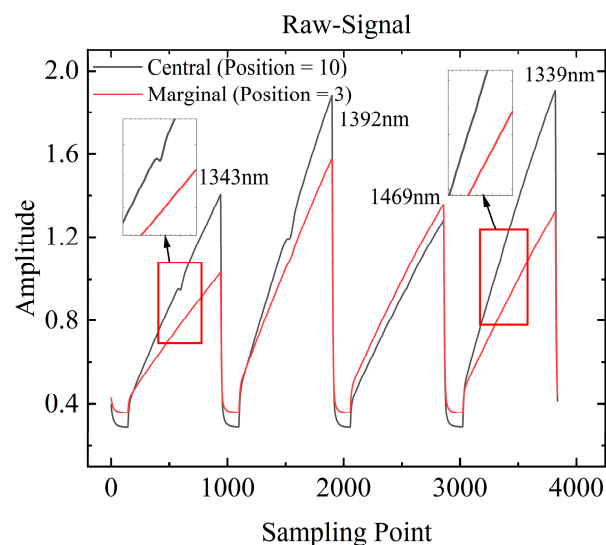


Figure 13. Original signals in the center and the edge of the experiment.

4. Conclusions

Based on the polynomial CT-TDLAS algorithm, this study proposes a nonlinear constrained polynomial fitting algorithm. By employing a single-objective optimization method for the integrated absorbance under constraint conditions—rather than the traditional multi-objective approach—and incorporating an adaptive order selection strategy, the proposed method addresses the challenge of determining the appropriate polynomial order for unknown fields. Comparisons with the traditional ART algorithm demonstrate that, for three distinct distribution configurations, the proposed algorithm achieves reconstruction errors below 3%, outperforming ART. Moreover, when Gaussian white noise levels of 0–5% are introduced, the overall error remains low, underscoring the method's robustness. However, compared with ART, the CPF algorithm exhibits greater error variability, indicating that its stability may still be improved.

Experimental validation on a flat-flame burner shows that in high-temperature regions—especially at the center—the reconstruction error is extremely low, with a central difference of only 6 K (approximately 0.3%). The overall trend of the reconstructed temperature field is consistent with that of the thermocouple measurements. Nevertheless, discrepancies remain in the low-temperature regions, which are attributed to reduced SNRs resulting from weaker absorbance signals. Future work will further investigate the impact of absorption line selection on the algorithm to enhance its performance in low-temperature regions and improve reconstruction stability.

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Abbreviations

The following abbreviations are used in this manuscript:

CT-TDLAS	Computed Tomography–Tunable Diode Laser Absorption Spectroscopy
ART	Algebraic Reconstruction Technique
RBF	Radial Basis Function
SA	Simulated Annealing
CPF	Constrained Polynomial Fitting
SQP	Sequential Quadratic Programming
NCPF	Non-Constraint Polynomial Fitting

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