

Article

Tailoring Spectral Response of Grating-Assisted Co-Directional Couplers with Weighting Techniques and Rational Transfer Functions: Theory and Experiment

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Abstract: This work addresses the tailoring spectral response of grating-assisted co-directional couplers (GADCs) in the context of wavelength filtering for fiber-to-the-home (FTTH) applications. Design methods for spectral response engineering by means of coupling profile apodization-type weighting techniques and also more advanced rational transfer functions fitting a predefined spectral window template are presented. Modeling results based on coupled mode theory are then applied for the design and experimental fabrication of InGaAsP/InP GADCs targeting 1.3+ / 1.3− μm diplexer application in FTTH access networks. The experimental results are found to be in good agreement with the modeling predictions. The design tools presented are quite general and can be easily adapted to other technology platforms, such as silicon photonics for the use of GADCs as add-drop wavelength division multiplexers. The field of parity–time symmetry is another avenue where these types of gain–loss-assisted GADCs as active components are of interest for switching applications, and the design methods presented here may find utility.

Keywords: integrated optics; wavelength division multiplexing; fiber-to-the-home applications; grating-assisted co-directional couplers; apodization techniques; inverse-scattering methods; variational optimization methods; parity–time symmetry; optical switching applications



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1. Introduction

Wavelength division multiplexers (WDMs) are among the key components of optical bidirectional communications. In order to meet the growing demand of optical communications traffic, several types of WDMs, such as Mach–Zehnder interferometers (MZIs), arrayed waveguide gratings (AWG), ring resonators (RRs), waveguide Bragg gratings (WBGs), and others, have been proposed and developed from the early 1970s of the last century [1–3]. The field of use of the different types of WDMs depends on the target applications. In the frequent case when it is necessary to handle a single-wavelength channel in a dense WDM system rather than demultiplexing all channels simultaneously, grating-assisted directional couplers (GADCs) are generally preferred among various wavelength filter solutions [4–11]. An important advantage with regard to RRs [12–14] or MZIs [15–17] is that their operation is not limited by the free spectral range. On the other hand, unlike WBGs, no circulator is necessary to separate the drop port from the input port. Another important advantage of GADCs as compared to WBGs is the higher sidelobe suppression ratio (SLSR) that can be engineered by means of apodization techniques based on different methods of weighting the coupling strength profile [4,18–26]. This topic has recently attracted attention again due to interest in the development of WDMs for silicon

photonics [27–30], although no experimental demonstrations of GADCs on this technology platform have been reported so far. It should be noted, however, that the experimental realization of high-SLSR GADCs meeting the requirements of practical applications is more complex than it appears at first glance. Additional factors, generally not considered in simplified analytical models, can adversely modify the spectral response of the GADCs. Furthermore, engineering the spectral response of the GADCs is not limited to simple sidelobe suppression but also allows for the implementation of a more advanced spectral response control through an appropriate design of the grating coupling profile. Providing insight into advanced GADCs' spectral response engineering methods is the objective of the present work.

The article is organized as follows. The use of a GADC filter based on the InGaAsP/InP platform designed for fiber-to-the-home (FTTH) access with a 1.3+/1.3– μm diplexer is presented in Section 2. Section 3 discusses advanced methods of GADCs' spectral response apodization using various coupling strength weighting techniques and provides associated experimental results. The theoretical basis of GADC spectral response engineering by means of rational transfer filter functions is presented in Section 4. Experimental results on the flat-top GADC spectral response are presented in Section 5. Section 6 concludes the article with a summary and outlook on future prospects.

2. GADC Filter for FTTH Access 1.3+/1.3– μm Diplexer Application

The GADCs considered in this work are designed for a 1.3+/1.3– μm diplexer application to be used either in an optical network unit (ONU) or in an optical line terminal (OLT) in an FTTH access network. As shown in Figure 1a, a downstream light signal at a 1.28 μm wavelength emitted from the OLT is received by an ONU photodetector, while the light signal at the user side toward the OLT is emitted at a 1.32 μm wavelength. The filtering function performing the separation of 1.28 μm and 1.32 μm wavelengths is achieved by a GADC integrated with a laser and a photodiode on the same monolithic InGaAsP/InP platform by means of selective epitaxy growth [31]. A schematic view of the III-V semiconductor meander-type grating GADC is shown in Figure 1b. A cross-sectional view of the InGaAsP/InP heterostructure with a composition corresponding to a quaternary semiconductor alloy bandgap at $\lambda_g 1.22 \mu\text{m}$ [32] is shown in Figure 1c. Additional details on the design, fabrication, and experimental characterization of the GADC are presented in the Supplementary Materials.

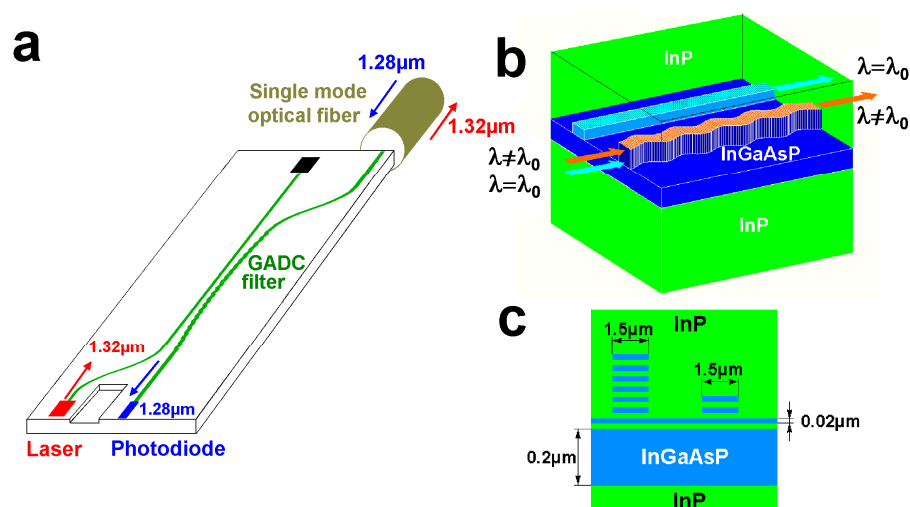


Figure 1. (a) Schematic of the integrated ONU 1.3+/1.3– μm diplexer. (b) Prospect view of the III–V semiconductor meander grating directional coupler. (c) Cross-sectional view of the InGaAsP/InP heterostructure used for the experimental realization of the GADC.

Note that the two waveguides have the same width of $1.5\ \mu\text{m}$ but different heights, resulting in the asymmetry of the waveguide effective index n_{eff} that corresponds to a grating period Λ of approximately $125\ \mu\text{m}$.

$$\lambda_0 = \Lambda(n_{\text{eff}1} - n_{\text{eff}2}) \quad (1)$$

The reliable operation of such a diplexer critically depends of the GADC filter performances. For example, in the case of the $1.32\ \mu\text{m}$ ONU shown in Figure 1a, the downstream light signal at a wavelength of $1.28\ \mu\text{m}$, cross-coupled to the laser with a $1.32\ \mu\text{m}$ emission wavelength, can affect its performance and can even lead to a coherence collapse of the laser radiation [33–38]. In turn, parasitic back-reflection of the upstream $1.32\ \mu\text{m}$ laser signal towards the photodetector channel can cause interference with the downstream $1.28\ \mu\text{m}$ light signal and lead to the corruption of the received data.

An ideal filtering function, such as the one shown in Figure 2a, should display a 100% cross-channel transmission over a certain bandwidth determined by target application specifications and suppress as much cross-channel transmission as possible (at least by 20 dB) outside this band. In practice, the typical transmission of an apodized spectral response GADC, such as the one shown in Figure 2b, is quite different from that of an ideal filter.

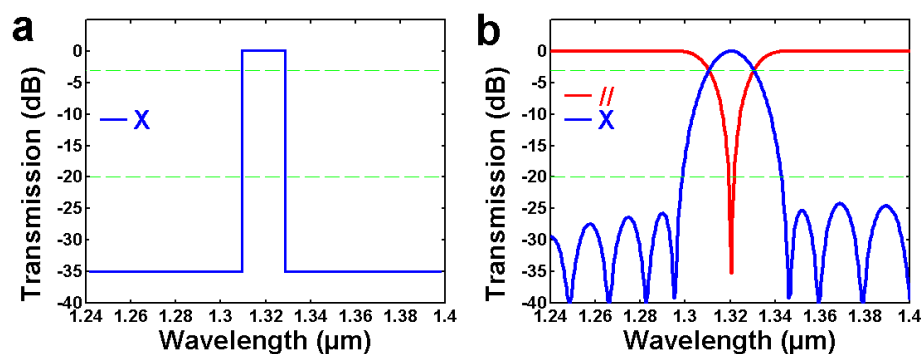


Figure 2. (a) Ideal filter cross-channel spectral response. (b) GADC-apodized spectral response (cross-channel—blue curve; bar-channel—red curve). The green dashed lines at $-3\ \text{dB}$ and $-20\ \text{dB}$ delineate the pass band and attenuation band levels, respectively.

One major issue impairing GADC filter performance is that usually the cross-channel transmission pass band around the maximum at the central wavelength λ_0 is not flat but rather exhibits a parabolic shape on a logarithmic intensity scale. Therefore, a high extinction ratio for the bar-channel can only be achieved in a narrow spectral range around λ_0 . For the example shown in Figure 2b, the bandwidth of the spectral response with a bar-channel extinction ratio greater than 20 dB is only 2.6 nm.

Considering the sensitivity of the GADC central filter wavelength to manufacturing tolerances [39] or its variation with temperature [40], this may cause some inconvenience for the diplexer configuration where the photodetector is connected to the bar-channel (//) and the laser to the cross-channel (X). This is particularly important for the case of the $1.28\ \mu\text{m}$ OLT (Figure 3c) but less critical for the $1.32\ \mu\text{m}$ ONU (Figure 3b), since the level of laser emission power is lower on the user side due to considerations of cost.

On the other hand, the advantage for positioning the photodetector on the bar-channel is to relax the constraints related to the GADC TE/TM polarization sensitivity. In this case, the GADC can tolerate a fairly significant difference between λ_0^{TE} and λ_0^{TM} provided that this does not affect the operation of the cross-channel in the rejection band where a high SLSR is required. Note that the difference between λ_0^{TE} and λ_0^{TM} in this case is not at all critical for laser operation since its emission essentially occurs in one polarization (usually the TE).

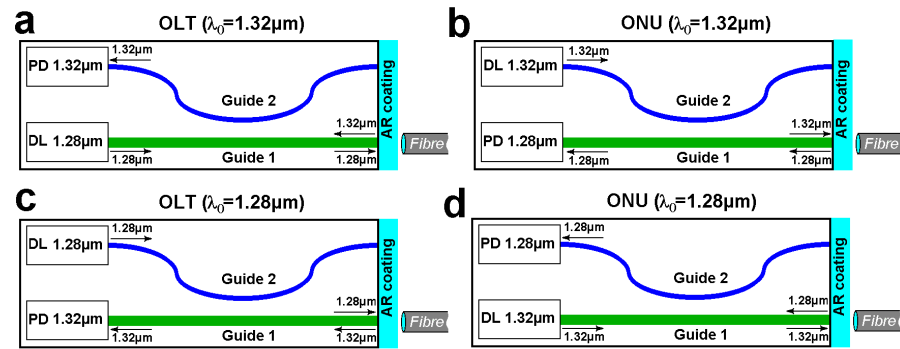


Figure 3. Schematic of different types of optical network terminal configurations for 1.3+ / 1.3− μm diplexer application: (a) 1.32 μm OLT; (b) 1.32 μm ONU; (c) 1.28 μm OLT; (d) 1.32 μm ONU.

Another type of diplexer configuration that can be used in the 1.28 μm ONU (Figure 3d) or 1.32 μm OLT (Figure 3a) involves placing the laser on the bar-channel and the photodetector on the cross-channel. In this case, the condition of polarization independence $\lambda_0^{\text{TE}} = \lambda_0^{\text{TM}}$ is mandatory. The solution considered in the present work to solve the polarization sensitivity problem of the GADC is that of effective index difference compensation [41]. The idea is that, although the effective index for InGaAsP/InP waveguides in the TE polarization is unavoidably different from that in TM, through an appropriate design, the effective index difference between GADC waveguides can be the same:

$$n_{\text{eff}1}^{\text{TE}} - n_{\text{eff}2}^{\text{TE}} = n_{\text{eff}1}^{\text{TM}} - n_{\text{eff}2}^{\text{TM}} \quad (2)$$

The GADC design shown in Figure 1c is explicitly calculated to meet the polarization insensitivity condition given by Equation (2). As will be shown below, the advantage of this diplexer configuration, once the polarization sensitivity problem is solved, is that it is easier to obtain a high SLSR around 1.28 μm for 1.32 μm OLT and thus filter out interference with the 1.28 μm laser emission than to obtain a high broadband extinction ratio required in the case of 1.28 μm OLT configuration.

Another argument for which it is preferable to set λ_0 at 1.32 μm rather than at 1.28 μm is that the coupling coefficient between waveguides increases with an increase in the wavelength [42–44]. This is why the SLSR is higher at the wavelength below the cross-channel transmission band and lower above it.

For these reasons, the optimal diplexer configurations are those corresponding to 1.32 μm ONU and 1.32 μm OLT. In the case of 1.32 μm ONU, the photodiode is connected to the bar-channel and the laser to the cross-channel. The polarization independence condition is strongly relaxed. Only a moderate broadband extinction ratio (~4 dB) [45] is required to avoid interference of the 1.28 μm wavelength downstream light signal received by the photodiode with an upstream back-reflected stray light signal at a wavelength of 1.32 μm. The required SLSR at a 1.28 μm wavelength is around 10 dB.

Meanwhile, for the case of 1.32 μm OLT configuration, this time, the photodiode is connected to the cross-channel and the laser to the bar-channel. The condition of polarization independence for the cross-channel is, in this case, mandatory. A moderately high broadband extinction ratio (~13 dB) is required to avoid perturbation of the upstream 1.28 μm wavelength emission laser caused by the 1.32 μm wavelength downstream light signal. The required SLSR at a wavelength of 1.28 μm must be at least 20 dB to avoid interference of the downstream light signal arriving at the photodetector with upstream stray light at a wavelength of 1.32 μm.

It thus appears that the most critical requirements are those for 1.32 μm OLT. For this reason, the objective of the present work is to demonstrate the high-performance

polarization-independent GADC filter with cross-channel transmission band centered around $\lambda_0 = 1.32 \mu\text{m}$ with a bandwidth $\Delta\lambda_{3\text{dB}}$ of at least 20 nm at a transmission level of -3 dB and an SLSR greater than 20 dB at a wavelength around $1.28 \mu\text{m}$.

Note that modeling results based on the coupled mode theory (CMT) [42–44,46–51] for the configuration shown in Figure 2b are obtained with realistic GADC parameters corresponding to the design shown in Figure 1c. To quantify the efficiency of such filters using spectral response engineering, it is convenient to introduce a figure of merit. In our case, it is the filter quality factor Q defined as the ratio of the transmission bandwidth $\Delta\lambda_{-3\text{dB}}$ at the -3 dB level to that at -20 dB :

$$Q = \frac{\Delta\lambda_{-3\text{dB}}}{\Delta\lambda_{-20\text{dB}}} \quad (3)$$

The implementation of the techniques used for spectral response engineering is presented in the next few sections.

3. Apodization of GADC Spectral Response Using Weighting Techniques

The achievement of a high SLSR of the GADC filtering function is usually based on an adiabatic variation in the coupling coefficient $\kappa(z)$, along the direction of light propagation. Note that the bandwidth of the GADC spectral response with variable coupling $\kappa(z)$ is different from that with a constant coupling coefficient. A higher SLSR is accompanied by a broadening of the bandwidth. To have the same bandwidth as a GADC with a constant coupling coefficient, an increase in the coupler length L_c is necessary. The spectral response of the directional coupler is thus determined by the coupling profile $\kappa(z)$ and the coupling length L_c .

The conventional method of increasing the SLSR [4] consists in imposing a variation in $\kappa(z)$, according to one of the many apodization functions reported in the literature (Hamming, Kai-ser, and Blakman). A very high rejection ratio (30–40 dB) can theoretically be obtained with this method. This solution is not necessarily optimal when it is necessary to find a compromise between the rejection ratio, the bandwidth, and the length of the filter. Our goal is to make a filter with a wide transmission band $\Delta\lambda_{3\text{dB}} \geq 20 \text{ nm}$, a rejection ratio $T_x > 20 \text{ dB}$, and a coupling length that is as short as possible.

For this purpose, it is possible to obtain an SLSR of 20–30 dB with a very simple coupling coefficient profile $\kappa(z)$ of the “generalized cosine” type:

$$\kappa(z) = c_1(1 + c_2(\cos(2\pi z/L_c))) \quad (4)$$

To find a coupling coefficient profile (i.e., the values of the variables c_1 and c_2), we use the variational optimization method proposed by Little [24]. The general idea of this method is to impose a certain number of constraints depending on the objective that must be achieved (rejection ratio, bandwidth, etc.) and to find the necessary profile that satisfies this objective function (variables c_1 and c_2) using mathematical optimization algorithms.

It should be noted that the efficiency of mathematical optimization methods depends a lot on the type of problem considered, the number of variables to be optimized, as well as the starting point of the search for solutions. The greater the number of variables ($c_1, c_2, \dots, c_n$) that we seek to optimize, the greater the risk of the search for solutions stopping at a local minimum. It is extremely important to seek to minimize the number of variables to be optimized when we want to describe a coupling profile.

Several solutions can be considered for the physical realization of a variation in the coupling coefficient $\kappa(z)$:

1. Variation in the grating amplitude;

2. Intergride distance variation;
3. Variation in the duty cycle.

The pro and contra of these methods are briefly discussed below.

3.1. Variation in the Grating Amplitude

This is a method where a variation in the coupling coefficient is achieved by varying the amplitude of the grating geometrical amplitude $h(z)$ or its effective index profile $n_{\text{eff}}(z)$. In practice, for GADCs, it is difficult to control the modulation of the coupling coefficient with this technique because the value of this amplitude is generally less than $1\ \mu\text{m}$.

However, this method is used for fiber gratings. In this case, the grating is obtained using the photo-induced index variation in the core of the fiber, but the realization of the apodization requires fairly complex insolation techniques [23,24,52,53].

3.2. Intergride Distance Variation

This is the most commonly used method for designing co- and contra-directional couplers (Figure 4a). It allows an SLSR greater than 20 dB to be obtained [24,26]. This method requires knowledge of the exponential dependence of the coupling coefficient on the intergride distance [4]. This is a rather delicate point because typical variations in the intergride distance for this method are between $2\ \mu\text{m}$ and $5\ \mu\text{m}$.

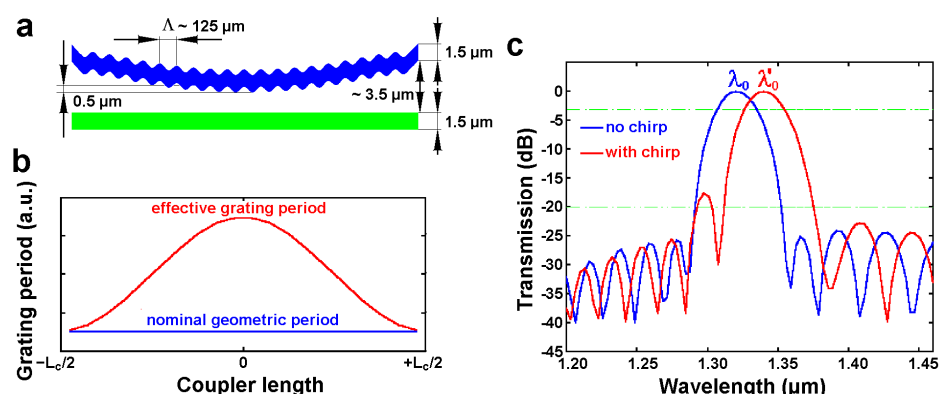


Figure 4. (a) Asymmetrical GADCs with variable intergride distance. (b) Variation in the effective grating period with respect to the nominal value caused by the variation in the coupling coefficient. (c) Impact of the “chirp” effect on the GADC spectral response.

One drawback of this method that should be taken into account is that the variation in the intergride distance is accompanied by a variation in the effective grating period, as shown in Figure 4b. This effect follows directly from the fundamentals of CMT for GADCs [48–51]. From now on, this phenomenon will be called the “chirp” of the effective grating period. This effect is all the more important when the coupling is strong and can produce a significant asymmetry of the spectral response, as represented in Figure 3c [54,55]. We see that the level of the secondary lobes increases on the short wavelength side. The asymmetry of the spectral response is also accompanied by a red-shift in the central wavelength. The physical mechanism explaining the asymmetry of the spectral response with a first marked resonance to the left of the main lobe in apodization, like that shown in Figure 4, is explained below [54]:

“Asymmetry follows from an interference picture: if Δn decreases symmetrically away from the center of the device, then for $\lambda < \lambda_0$, there exist two regions located symmetrically about the device center, which are exactly phase-matched, for which efficient polarization conversion occurs. The electric field components from both sides of the device interfere

at the output, either constructively or destructively, depending on their phase relation. Therefore the filter characteristic shows oscillations on the short wavelength side”.

3.3. Variation in the Duty Cycle

The method of varying the duty cycle consists in keeping a constant grating amplitude and interguide distance. In this case, the variation in the coupling strength $\kappa(z)$ along the light propagation direction is obtained by changing the ratio $\rho = \Lambda_+ / \Lambda$ between the section lengths where the coupling coefficient is positive and those where it is negative (Figure 5) [56]. The average value κ_i of the coupling coefficient over an i -th period of the grating is proportional to the intensity of the main harmonics when the grating coupling profile is decomposed into Fourier series. In the case of such a basic duty cycle (BDC) modulation method, the grating period is constant and the first term of the Fourier series for the i -th grating period is given using a well-known expression:

$$\kappa_i \sim \frac{1}{\pi} \sin\left(\frac{\pi(\Lambda + \Delta_i)}{2\Lambda}\right) = \frac{1}{\pi} \cos\left(\frac{\pi\epsilon_i}{2}\right) \quad (5)$$

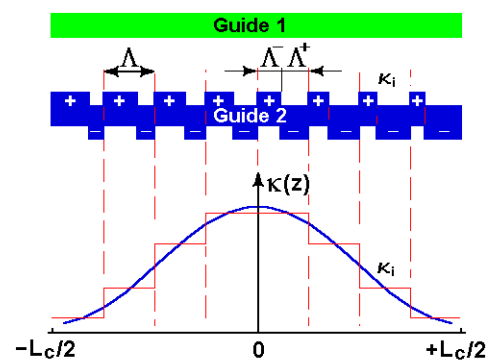


Figure 5. Schematic of GADCs with the modulation of the grating geometry profile using the BDC method. The resulting variation in the coupling coefficient $\kappa(z)$ is necessarily discrete. The finest variation that can be obtained is determined by the number of periods of the grating.

Here, $\epsilon_i = \Delta_i / \Lambda$ is the relative change in the length of the grating section. By convention, we associate ϵ with a grating section where the coupling coefficient is positive. The modulation ϵ is maximal at the ends of the grating, where the coupling strength is reduced, and it gradually decreases towards the middle of the grating, where the coupling strength is maximal. The advantage of this method is that the variation in the coupling strength is performed by controlling the lengths of the sections, which is relatively easy to achieve for co-directional couplers where the grating period is typically several tens of microns.

Inspired by this approach, several other methods differing in the way they modulate the grating duty cycle have been proposed [57–61]. An analysis of these methods, discussing their pro and contra, is reported in [62]. Based on this analysis, a supersymmetric duty cycle modulation (SDC) schematically shown in Figure 6a performs better than the others. An essential aspect of the SDC method is that the duty cycle remains constant, but the length of the two adjacent grating periods is modulated in such a way that one period is longer and the second is shorter, but their total length is equal to that of two average grating periods. This principle is illustrated in Figure 6b, although several variants of SDC are possible, as detailed in [62].

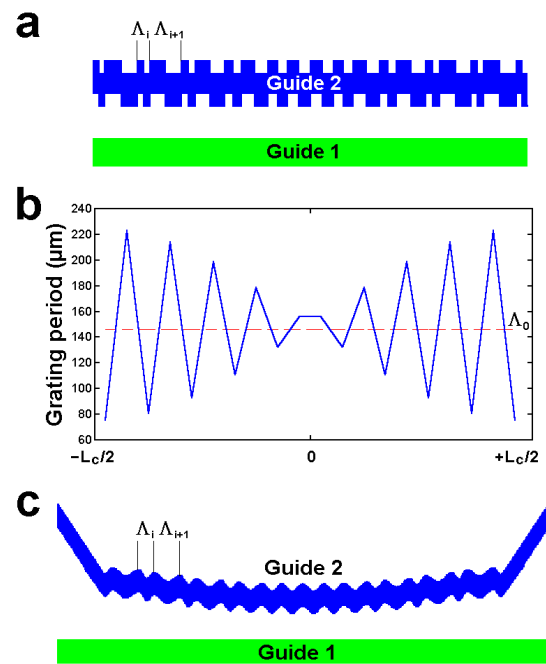


Figure 6. (a) Schematic of GADCs with the modulation of the grating geometry profile using the SDC method. (b) Variation in the local grating period along the grating length. (c) Asymmetrical GADCs with variable interguide distance combined with the modulation of the local grating period using the SDC method.

One drawback of this method that should be taken into account is that radiation losses can be significant [26,46]. To avoid these radiation losses, the grating period modulation ratio is limited to about 10%. However, such a modulation is not enough to achieve 20 dB of the SLSR. To further improve the SLSR, it is possible to combine the SDC method with that of the interguide distance variation, as illustrated in Figure 6c. Furthermore, it is also possible to additionally take into account the chirp induced by the interguide distance variation and introduce the necessary corrections in the design of the GADC in order to compensate for this effect by means of a variation in the length of the local grating period.

Other methods, which are similar to the duty cycle variation methods, have also been proposed to perform apodization [63,64]. The filter is composed of grating and non-grating sections. Apodization is performed by varying the ratio between the grating and non-grating sections along the filter. A disadvantage of these methods is that the use of the non-grating sections increases the total length of the filter. The level of apodization of the side lobes, reported by the authors, does not exceed 20 dB.

For these reasons, the following apodization techniques have been retained for the experimental implementation of an apodized GADC:

- A. Interguide distance variation;
- B. Interguide distance variation combined with chirp compensation;
- C. Interguide distance variation combined with SDC modulation;
- D. Interguide distance variation combined with SDC modulation and chirp compensation.

The obtained experimental results are presented below.

3.4. Experimental Results on GADC Spectral Response Apodization

Figure 7 shows the experimental results of the GADC spectral response obtained with the apodization techniques listed above. As can be seen in Figure 7a, a simple variation in the distance between the waveguides is not sufficient to reliably achieve $T_x = 20$ dB of the SLSR around $1.28 \mu\text{m}$ simultaneously for both TE and TM polarizations. The sidelobe level

is notably higher at wavelengths above the 1.32 μm transmission band. However, this last point is unimportant since, as detailed above for the 1.3+/1.3– μm diplexer application, only the SLRS around 1.28 μm matters. For this reason, calculation of the filter quality factor is performed by only taking into account the short-wavelength side of the cross-channel spectral response below $\lambda_0 \approx 1.32 \mu\text{m}$. Note that even though no local grating period variation is used to compensate for the chirp effect, a grating period correction is still introduced to avoid a red-shift towards higher wavelengths. The high extinction ratio $> 20 \text{ dB}$ observed at λ_0 provides an almost ideal total light transmission to the cross-channel. It is also important to note the remarkable independence of polarization. The difference between λ_0^{TE} and λ_0^{TM} is approximately 1 nm. The results, $\Delta\lambda_{3\text{dB}} \approx 23 \text{ nm}$ and $Q \approx 0.45$, demonstrate the precision of the design and manufacturing of the GADC with a weighted coupling strength profile.

It should be noted that the obtained performances are mostly similar to those reported in [26] for a GADC with essentially the same type of design, but with about 50% higher coupling length.

The introduction of chirp compensation allows us to further improve the GADC filter performances. As can be seen in Figure 7b, the SLRS at 1.28 μm now occurs at about 25 dB, $\Delta\lambda_{3\text{dB}} \approx 26 \text{ nm}$, and $Q \approx 0.39$. The rest of the parameters are similar to those in the previous case.

The effect of introducing the additional modulation of the grating period length using the SDC method, the results of which are shown in Figure 7c, is largely similar to that of chirp compensation. However, it is important to note a more steep transition from the pass band to the rejection one, leading to an improvement of $Q \approx 0.45$, despite the fact that $\Delta\lambda_{3\text{dB}} \approx 22 \text{ nm}$ is slightly narrower with respect to the previous case.

Introducing additional chirp compensation does not provide further improvement to the filter performances. As shown in Figure 7d, the transition slope from the pass band to the rejection one is less steep, resulting in a slightly lower $Q \approx 0.4$.

It thus appears that the combination of the weighting of the coupling profile using the interguide distance separation and the SDC modulation of the grating period length is optimal in order to obtain a spectral response with a filter quality factor up to $Q \approx 0.45$. The further improvement of the filter quality factor using apodization techniques is however not possible and requires a radically different approach which is presented in the following section.

The main figure-of-merit parameters related to the apodized GADCs considered above are summarized in Table 1.

Table 1. Main figure-of-merit parameters of GADCs apodized using different weighting methods.

Apodization Method	Polarization	λ_0 (nm)	$\Delta\lambda_{3\text{dB}}$ (nm)	$Q = \Delta\lambda_{3\text{dB}}/\Delta\lambda_{20\text{dB}}$	SLSR (dB)
(a) Interguide distance variation.	TE	1323.0	23.0	0.45	>20
	TM	1323.7	21.0	0.45	>19
(b) Interguide distance variation + chirp compensation	TE	1319.5	25.7	0.39	>25
	TM	1321.7	23.7	0.39	>25
(c) Interguide distance variation + SDC modulation.	TE	1315.3	22.5	0.46	>22
	TM	1316.9	18.5	0.44	>24
(d) Interguide distance variation + SDC modulation + chirp compensation.	TE	1328.2	25.7	0.40	>26
	TM	1329.8	22.8	0.39	>25

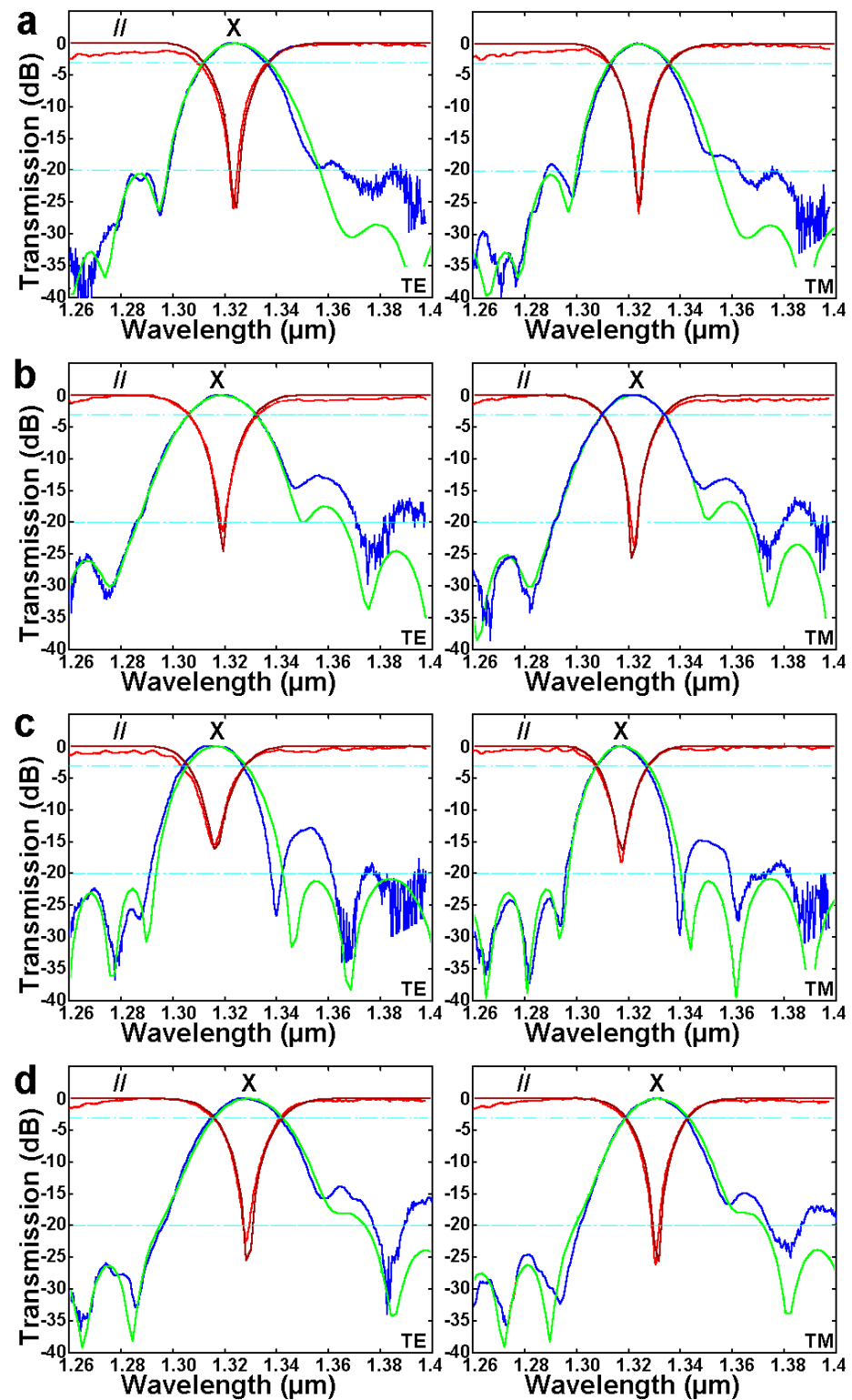


Figure 7. Experimental (red and blue curves) and CMT (brown and green curves) spectral response of GADCs using different apodization techniques. (a) Interguide distance variation. (b) Interguide distance variation combined with chirp compensation. (c) Interguide distance variation combined with SDC modulation. (d) Interguide distance variation combined with SDC modulation and chirp compensation.

4. Design and Spectral Response of GADCs with Rational Transfer Functions

As discussed in the previous section, effective sidelobe level reduction can be achieved using conventional apodization techniques with cosine-type variation in the coupling profile. However, achieving a square spectral shape, otherwise called a flat-top band-pass filter, closer to that of an ideal filter, requires the use of more advanced techniques. These techniques can be broadly classified into three main categories:

1. The Gel'fand–Levitan–Marchenko (GLM) inverse-scattering method;
2. The discrete inverse-scattering or layer-peeling method;
3. Genetic or variational optimization algorithms.

A brief overview of these methods is presented below.

4.1. Gel'fand–Levitan–Marchenko Inverse-Scattering Method

The origin of the GLM approach is related to the inverse problem of recovering the differential operator via the spectral data [65–70]. If the spectral response of a differential operator, like the Schrodinger equation in quantum mechanics, is approximated by rational functions, then analytical solutions for the wave-scattering potential can be obtained [71,72]. In the case of GADCs, by exploiting the analogies between quantum mechanics and optics, the objective is to find the coupling profile $\kappa(z)$ as a function of the spectral response of the coupler. The GLM integral equation can be solved numerically, provided that the bar-channel and cross-channel spectral responses are represented as a rational fraction. Based on this approach, several GADC designs with a flat-top spectral response approximating a square shape band-pass filter function have been proposed [22,73–77].

The practical implementation of this method, however, has some significant drawbacks:

1. There is no guarantee that the profile found for $\kappa(z)$ is physically realizable.
2. An exact solution for the coupling profile $\kappa(z)$ is found for an infinite interaction length, and truncations of $\kappa(z)$ are then necessary. This can considerably change the spectral response.
3. The requirement to express the spectral response as a rational function is inconvenient because the response of filters with finite coupling lengths is not rational in general.
4. It is difficult or impossible to incorporate physical variations in the parameters into the model, such as variations in the coupling coefficient with changes in the wavelength.

Therefore, this method is not retained for the design of a flat-top pass band GADC. It is nevertheless important to emphasize a point arising from this methodological analysis, especially for the rest of this study. The profile of the coupling coefficient $\kappa(z)$ of a filter represented in the form of a rational fraction is similar to the impulse response of this type of filter [74]. This shows the relation between the Fourier transform used in the apodization method and the inverse-scattering method. The latter becomes the Fourier transform method when the coupling $\kappa(z)$ is weak.

4.2. Discrete Inverse-Scattering or Layer-Peeling Method

Another type of inverse-scattering technique applied for the synthesis of gratings is the discrete inverse-scattering or layer-peeling method [78–84]. This method is based on the principle of causality and reconstructs the grating recursively layer by layer. The advantage of this method is its lower complexity compared to the GLM inverse-scattering method and better convergence. However, implementing this method first requires determining a valid spectral response that corresponds to a feasible structure [78] and thus reduces the room of the GADC design. In this context, an optimal procedure would rather be to start with an initial $\kappa(z)$ profile and then, using iterative algorithms, to minimize the error between a

target spectral response and the designed spectral response. This can be achieved by means of variational optimization methods.

4.3. Genetic or Variational Optimization Algorithms

A synthesis method for GADCs based on variational optimization, partially presented in Section 3, has been developed by Little et al. [24,25,85]. The design of GADCs using genetic algorithms has also been described in the literature [86,87]. Based on a variational optimization approach, the CMT modeling results reported in [25] showed an SLSR of about 40 dB and an extinction ratio of 20 dB over a fairly wide spectral range. The profile is approximately described by a sum of three exponential functions:

$$\kappa(z) = \sum_{i=1}^3 A_i \exp[-\alpha_i (z - z_i)^2] \quad (6)$$

where $0 < z < 1$, $A_1 = 8.774$, $A_2 = -3.45$, $A_3 = 1.183$, $\alpha_1 = 51.87$, $\alpha_2 = 34.49$, $\alpha_3 = 20.63$, $z_1 = 0.28$, $z_2 = 0.60$, and $z_3 = 0.75$.

It is worth noting that these techniques may suffer more significantly from slow convergence and low algorithmic efficiency when applied to the design of sophisticated coupling profiles. This is a natural consequence of the large number of degrees of freedom involved in the grating design optimization problem. The efficiency of the optimization procedure also strongly depends on the initial insight of the coupling profile. However, no method that can be used to gain such an initial insight appears to be reported in the literature. One of the main objectives of this publication is to introduce such an approach based on the variational optimization method of GADCs whose spectral response can be described by rational filtering functions.

4.4. Optimization and Spectral Response of GADCs with Rational Transfer Functions

4.4.1. Design of a Frequency Filter Fitting a Specified Window Template

To introduce this method, let us consider the GADC as a system composed of a finite number of linear elements, with localized constants and invariant in time. An input signal, $e(t)$, is related to the output, $s(t)$, as revealed by a constant-coefficient differential equation:

$$a_n \frac{d^n s(t)}{dt^n} + \dots + a_0 s(t) = b_m \frac{d^m e(t)}{dt^m} + \dots + b_0 e(t) \quad (7)$$

With all the initial conditions being zero, if we take the Laplace transform of Equation (7), we can define the transfer function:

$$T(p) = \frac{S(p)}{E(p)} = \frac{b_m p^m + \dots + b_0}{a_n p^n + \dots + a_0} \quad (8)$$

A frequency filter with localized constants is therefore characterized by a transfer function, which is a rational fraction with real coefficients of the complex variable $p = \sigma + j\omega$.

If we limit ourselves to systems that have a numerator degree less than or equal to the degree of their denominator, the order of the system is equal to the degree (n) of the denominator. After factorization, we obtain the following:

$$T(p) = \frac{N(p)}{D(p)} = \frac{b_m (p - z_1)(p - z_2) \dots (p - z_m)}{a_n (p - p_1)(p - p_2) \dots (p - p_n)} \quad (9)$$

The roots of the numerator z_i that cancel the transfer function are the zeros of $T(p)$. The roots of the denominator p_i are the poles of $T(p)$ (or natural modes or eigenmodes). If $n > m$, then the transfer function has $(n-m)$ poles at infinity.

The realization of an ideal filter, which would transmit without attenuation the useful components of a signal and eliminate all the others, is impossible using a finite number of components. Real filters have non-zero pass band ripples and attenuations less than or equal to a value $A_{\max} = 1 - \varepsilon$, and their attenuation in the attenuated band is greater than or equal to a minimum value $A_{\min}$ instead of being infinite (Figure 8). The transition between the pass band and the attenuated band is made in a progressive manner, resulting in a transition band being limited by a pass band frequency F_p and an attenuated frequency F_a . The design of a filter shall therefore be based on a window or template, within which the spectral response curve shall be inscribed. In frequency filtering, there are essentially four types of windows: low-pass, high-pass, band-pass, and band-cut.

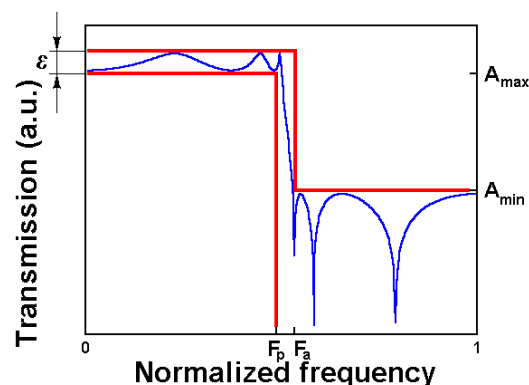


Figure 8. Low-pass frequency filter spectral response (blue curve) and target spectral window template (red lines indicate the delimited area).

A small remark is necessary to avoid any confusion. Although a GADC wavelength band-pass filter is considered in the present work, the frequency filter that corresponds to our application is the low-pass filter (Figure 8). To understand this, it is necessary to consider the spectral response of the GADC in terms of positive and negative spectral detuning $\delta(\lambda)$ with the central wavelength of the GADC as zero.

In principle, it is possible to solve the problem of filter synthesis directly by mathematically adjusting the value of the elements of a structure using optimization techniques. But such a synthesis method is quite difficult to implement because of the generally high number of variables in this case and the risk of stopping the search for solutions in a local minimum.

Filter synthesis therefore requires a preliminary approximation phase. Once the spectral window template is determined, an approximation function that can lead to a transfer function (realizable with a physical structure) is used. To solve the approximation problem, it is “sufficient” to find a set of transmission zeros, to find a set of reflection zeros, and to find a value ε such that the transmission function $T(\omega)$ respects the constraints of the window template.

Usually, the spectral response constraints imposed by window template specifications are sought to be met with the lowest possible order of transfer function. Most of the time, the window templates encountered in practice can be approximated by a function chosen among a set of polynomial or rational functions, called standard approximation functions. Although not quite optimal from the point of view of the filter order, this practice has significant advantages, as it usually leads to simple physical filter structures. There are currently commercial programs available that perform a frequency filter synthesis based on standard filtering functions.

What is the procedure for moving to the realization of a GADC with a standard filter function? As mentioned above, the profile of the coupling coefficient $\kappa(z)$ of a filter, represented as a rational fraction, is similar to the impulse response of this type of filter [74].

Under these conditions, for the design of the coupling profile, the following procedure can be adopted:

- Synthesis of a frequency filter using the specified window template. Commercial software allows the synthesis of a filter with spectral responses to be carried out according to one of many standard functions. In fact, the minimum order of the filter is determined at this stage, as well as the coefficients b_i and a_j in Equation (7), which make it possible to fit the spectral response in the specified window template.
- Calculation of the filter impulse response. This can also be carried out with filter synthesis software or ab initio calculations.
- Association of the filter impulse response to a coupling profile. This procedure is carried out in two steps. First, the filter impulse response is truncated (Figure 9a). This step is quite delicate because, to avoid an excessive length of the coupler, the truncation is completed after two or three oscillations. On the other hand, the smaller the number of oscillations before truncation, the more the spectral response is likely to be different from the initial predictions. To reduce the effect of truncation, it is preferable to perform it at a point where the amplitude of the pulse is zero. The second step is to associate the truncated impulse response with a coupling profile of a coupler composed of N sections. The scaling of the coupling coefficient value can be performed using mathematical optimization methods (Figure 9b).
- Comparison of the GADC spectral response and the specified window template. If the spectral response is too different from the predictions and does not satisfy the template, either the truncation point or the number of sections of the coupler should be revised or some parameters of the approximation function should be redefined and the procedure should be repeated. In practice, in order to meet the specified window template, it is easier to fix the number of coupler sections and to find the values of the approximation function parameters and the truncation point using mathematical optimization methods.

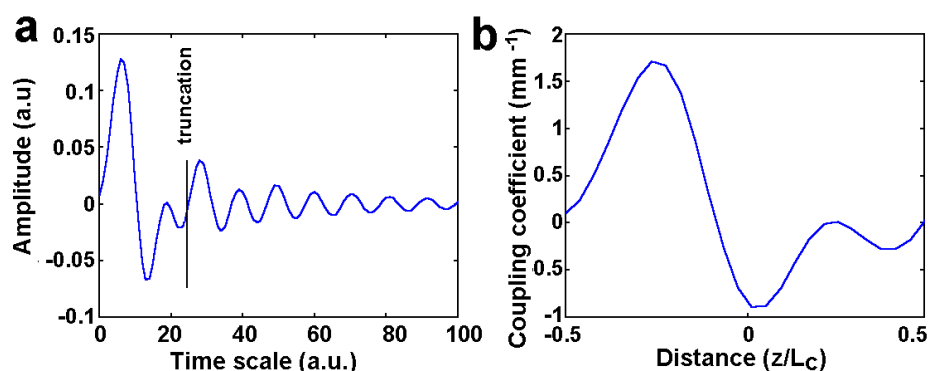


Figure 9. (a) Example of the impulse response of a filter; (b) coupling profile obtained from the truncated impulse response.

It is clear that this procedure may require several iterations before the result is satisfactory. It may be necessary to choose another approximation function better suited to the specified window template. The results of such a coupling profile design procedure are shown in the following examples.

4.4.2. Chebyshev Polynomials of Type I Approximation

This is a polynomial-type approximation. The reflection zeros are distributed in the pass band and the characteristic function has n transmission zeros at infinity. Since the ripple is uniformly distributed over the pass band, Chebycheff filters are sometimes called “ER” (equiripple) filters.

Using the procedure described in the previous subsection for a polynomial of order $n = 3$, we obtain a coupling profile (Figure 10a), which gives the spectral response shown in Figure 10b. The pass band spectral width is 46 nm and $Q = 0.69$. The coupler for which this simulation is carried out is composed of 40 sections. The ripple amplitude in the pass band is ≈ 1 dB and the SLSR at $1.28 \mu\text{m}$ is 20 dB. The price to pay for a more square spectral response is the increase in the length of the component.

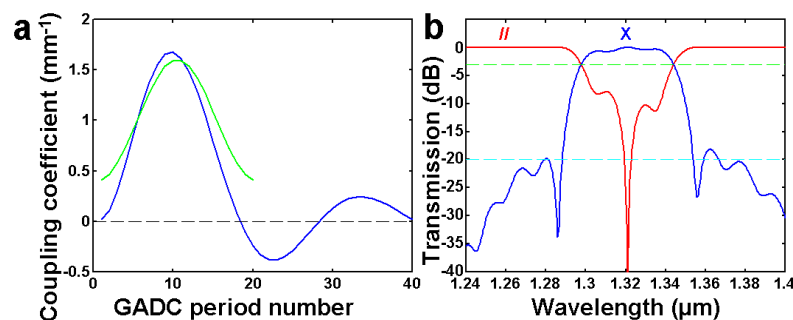


Figure 10. Chebyshev polynomials of type I approximation. (a) Coupling profile (Chebyshev type I—blue curve; generalized cosine—green curve). The dashed black line corresponds to the zero value of the coupling coefficient. (b) GADC spectral response (cross-channel—blue curve; bar-channel—red curve). The lowest levels of the pass band and the highest levels of the attenuation band are delineated by green and cyan dash-dotted lines at the -3 dB and -20 dB levels, respectively.

It may seem paradoxical that for a greater coupler length, the bandwidth is wider. But when we compare the current coupling profile with the “generalized cosine” profile ($c_1 = 1.0 \text{ mm}^{-1}$, $c_2 = 0.6 \text{ mm}^{-1}$), we understand that the interaction length with a strong coupling is lower in this case.

4.4.3. Chebyshev Type II Approximation (Inverse)

This is a non-polynomial type of approximation. The characteristic function has n reflection zeros at the origin and $n/2$ or $(n - 1)/2$ transmission zeros at finite frequencies, depending on the parity of n .

The profile obtained for $n = 3$ is shown in Figure 11a. The spectral response of a 40-section-long coupler is shown in Figure 11b. The ripple in the pass band is absent. However, the pass band spectral width is only 18 nm and therefore $Q = 0.46$. We can therefore see that for our application, this type of spectral response has no advantage compared to that of the “generalized cosine” profile.

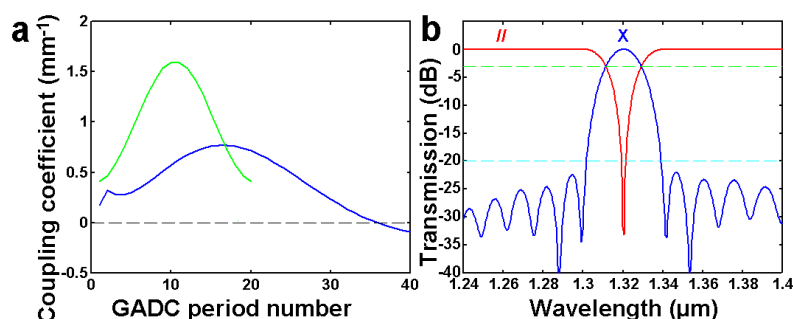


Figure 11. Chebyshev type II approximation. (a) Coupling profile (Chebyshev type II—blue curve; generalized cosine—green curve). The dashed black line corresponds to the zero value of the coupling coefficient. (b) GADC spectral response (cross-channel—blue curve; bar-channel—red curve). The lowest levels of the pass band and the highest levels of the attenuation band are delineated by green and cyan dash-dotted lines at the -3 dB and -20 dB levels, respectively.

4.4.4. Elliptic Approximation

The elliptic approximation (also known as Cauer, Zolotareff, or Darlington) is of the non-polynomial type. The characteristic function has $n/2$ or $(n - 1)/2$ reflection zeros distributed in the pass band and $n/2$ or $(n - 1)/2$ transmission zeros at finite frequencies, depending on the parity of n . It can be shown that for the same window template, the elliptic approximation is the most optimal one from the filter order point of view for all standard or non-standard approximation functions. As for the previous cases, the coupling profile is calculated for $n = 3$ (Figure 12a). The spectral response of a 40-section-long coupler is shown in Figure 12b. The spectral response is very close to the Chebyshev type I. The pass band spectral width is 45 nm and $Q = 0.68$. This result is not surprising because the coupling profiles are very close in these two cases.

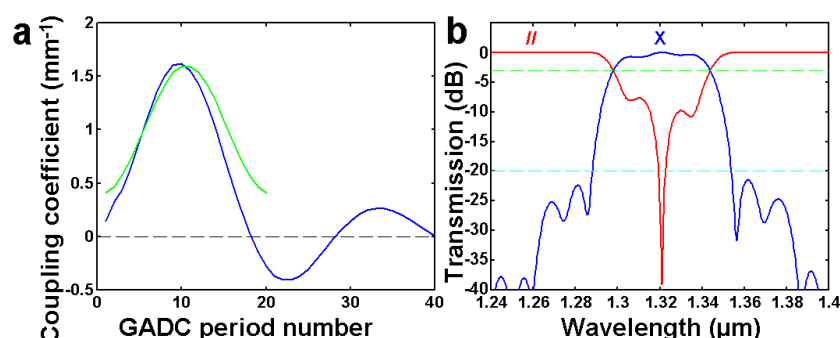


Figure 12. Elliptic approximation. (a) Coupling profile (elliptic—blue curve; generalized cosine—green curve). The dashed black line corresponds to the zero value of the coupling coefficient. (b) GADC spectral response (cross-channel—blue curve; bar-channel—red curve). The lowest levels of the pass band and the highest levels of the attenuation band are delineated by green and cyan dash-dotted lines at the -3 dB and -20 dB levels, respectively.

4.4.5. Butterworth Approximation

This is a polynomial approximation. We impose n reflection zeros at the origin and n attenuation poles at infinity on the characteristic function. Having n reflection zeros at the origin implies $n - 1$ zero derivatives at the origin. Therefore, Butterworth filters are sometimes called “MF filters” (maximally flat).

The coupling profile for $n = 4$ (Figure 13a) gives a 40-section-long coupler the spectral response which is shown in Figure 13b. The ripple in the pass band is absent. The pass band spectral width is 39 nm and $Q = 0.61$.

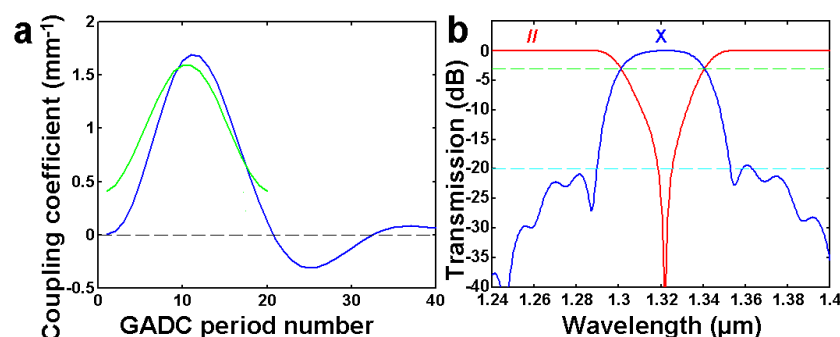


Figure 13. Butterworth approximation. (a) Coupling profile (Butterworth—blue curve; generalized cosine—green curve). The dashed black line corresponds to the zero value of the coupling coefficient. (b) GADC spectral response (cross-channel—blue curve; bar-channel—red curve). The lowest levels of the pass band and the highest levels of the attenuation band are delineated by green and cyan dash-dotted lines at the -3 dB and -20 dB levels, respectively.

Simulations of this type of spectral response have already been carried out for co- and contra-directional couplers [74–76]. The coupling profile is obtained by applying the inverse dissipation method. The spectral responses are quite similar to ours. This shows that for the case where the coupling remains moderate, our method of calculating the coupling profile is almost as efficient as the inverse dissipation method. The great advantage of our method is that, unlike the inverse dissipation method, we can introduce all kinds of variations of physical parameters into the calculations, such as, for example, the “chirp” of the grating period or the variation in the coupling coefficient with changes in the wavelength. We can also introduce different constraints on the maximum value of the coupling coefficient so that the filter remains feasible.

4.4.6. Refinement of the Coupling Profile Using Variational Optimization Methods

The conclusion that can be drawn from these few examples is that the realization of a coupler with a better-quality spectral response is quite possible.

As we have seen, the elliptic or Chebyshev type I approximation allows to considerably improve the quality factor of the spectral response, but we do not manage to obtain an extinction ratio on the bar-channel that is high enough for our application. This result can be seen as more or less predictable because we impose a certain level of the ripple in the pass band. It is possible to reduce the ripple and thus increase the extinction ratio, but simulations show that we then lose on the attenuation band rejection ratio. An alternative way is to increase the polynomial function order and/or refine the spectral window template.

The situation where ripples are completely absent in the pass band is the Butterworth filter. In this case, a high extinction ratio is obtained over a sufficiently wide spectral range, but a less good spectral response quality factor is obtained because the transition speed to the attenuated band is not steep enough.

The spectral response can be further improved by a direct variational optimization of the coupling profile. The advantage is to already have an initial guess for the target coupling profile. A comparison of the coupling profiles for the spectral responses of the Chebyshev, elliptic, or Butterworth types shows that the essential point is to have an alternation of polarity of the coupling coefficient along the propagation direction. The target profile should be similar to the Chebyshev of type I or Butterworth. It should be noted that this type of profile is globally similar to that obtained via the direct optimization of the coupling profile using variational methods proposed in [25].

The coupling profile obtained via variational optimization in [25] for a 40-section coupler, after scaling the coupling coefficient value, is shown in Figure 14a. For comparison, a Chebyshev type I coupling profile is also shown in the same figure. As can be seen, overall, the two profiles are similar. The spectral response for this optimized coupling profile is shown in Figure 14b. The bandwidth is 38 nm and $Q = 0.58$. These results are worse than those obtained for the Chebyshev type I or elliptical-type filters, but the rejection and extinction rates are improved. The rejection rate at $1.28\ \mu\text{m}$ is now greater than 27 dB and the extinction rate is greater than 13 dB over a wide spectral range of 24 nm. Finally, this coupling profile represents a good compromise for our objectives. This is the profile chosen for an experimental realization. It is worth noting that realizing such a coupling profile is more difficult due to the higher maximum coupling coefficient, which results in a smaller interguide separation distance and a more challenging technological manufacturing process. Therefore, it is a good test of the feasibility of an experimental device and for comparing experimental performance with modeling predictions.

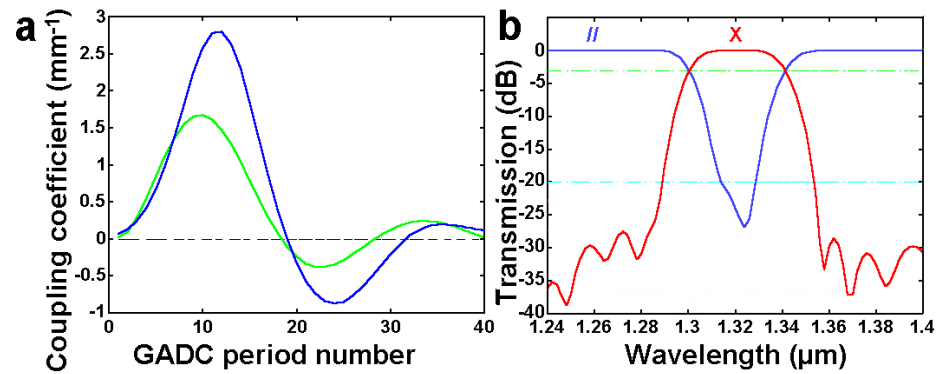


Figure 14. (a) Coupling profile (variational optimization—blue curve; Chebyshev type I—green curve). The dashed black line corresponds to the zero value of the coupling coefficient. (b) GADC spectral response (cross-channel—blue curve; bar-channel—red curve). The lowest levels of the pass band and the highest levels of the attenuation band are delineated by green and cyan dash-dotted lines at the -3 dB and -20 dB levels, respectively.

The main figure-of-merit parameters related to the GADCs considered above, optimized using different rational transfer functions, are summarized in Table 2.

Table 2. Main figure-of-merit parameters of GADCs optimized using rational transfer functions.

Transfer Function Approximation	$\Delta\lambda_{3\text{dB}}$ (nm)	$Q = \Delta\lambda_{3\text{dB}} / \Delta\lambda_{20\text{dB}}$	SLSR (dB)
Chebyshev type I	44	0.65	>20
Chebyshev type II	16	0.40	>23
Elliptic	44	0.67	>21
Butterworth	29	0.61	>21
Variational optimization	38	0.58	>27

5. Flat-Top Spectral Response GADC Experimental Results

The realization of the coupler with a coupling profile that gives a square-templated flat-top spectral response is almost identical to that of a coupler with a “generalized cosine” type of coupling profile. To translate the coupling profile into the variation in the interguide distance, the same procedure is used, but with a small difference. In the present case, there is an alternation of polarity in the coupling profile. Physically, the change in the sign of the coupling coefficient results in the introduction of 180° -phase shift sections (π rad). The geometry of the coupler guides is shown in Figure 15.

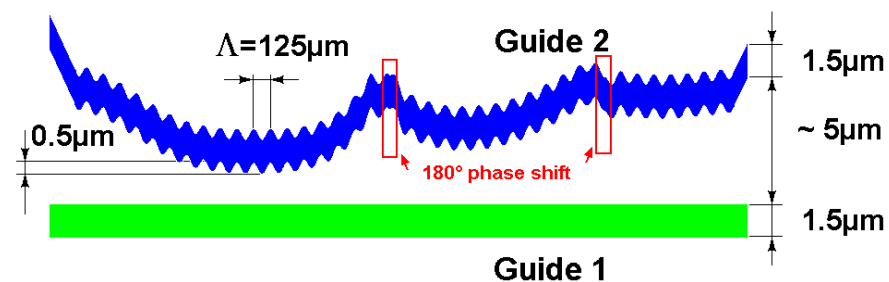


Figure 15. GADC geometry with altered polarity in the coupling profile. The red rectangles indicate the areas of 180° -phase shift sections.

The experimental spectral response for a coupler composed of 40 sections with a grating period of $125\ \mu\text{m}$ is presented in Figure 16. The bandwidth is 40 nm in the TE polarization and 38 nm in the TM. The quality factor of the spectral response is 0.61 in the TE and 0.6 in the TM. As can be seen, we obtain a very good agreement between the model predictions and the experiment for these parameters. The rejection ratio at $1.28\ \mu\text{m}$ on the cross-channel is greater than 20 dB. The extinction ratio at $1.32\ \mu\text{m}$ on the bar-channel is around 10 dB over a range of $\approx 30\ \text{nm}$. Compared to the experimental results reported in [40], we obtain much better spectral response performances and good polarization independence.

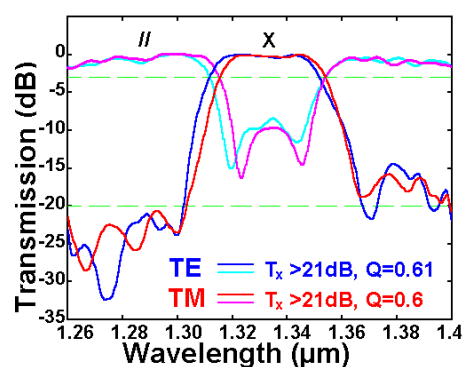


Figure 16. Square window template GADC experimental spectral response for bar (//) and cross (X) channels for the TE and TM polarizations. The lowest levels of the pass band and the highest levels of the attenuation band are delineated by green dashed lines at the $-3\ \text{dB}$ and $-20\ \text{dB}$ levels, respectively.

The performances obtained are sufficient for the correct operation of the $1.32\ \mu\text{m}$ ONU. However, for the $1.32\ \mu\text{m}$ OLT, the maximum level of non-coherent feedback ($1.32\ \mu\text{m}$ on the $1.28\ \mu\text{m}$ laser) is 4 dB too high compared to the specifications. But the critical threshold of non-coherent feedback that we set ourselves is probably overestimated here, due to the transparency of the laser for photons below the bandgap edge wavelength. It is therefore very likely that this filter would also be suitable for the $1.32\ \mu\text{m}$ OLT.

However, one could have expected a better rejection rate on the cross-channel and a better extinction on the bar-channel. There are two reasons that explain why the maximum performance of the component has not been attained:

- It is likely that the actual coupling profile is not entirely conformal to the modeling. Since the variation in the coupling coefficient is much greater than that for a “generalized cosine” type of profile, the risk of having errors in the variation in the interguide distance is therefore also greater.
- The “chirp” of the effective grating period caused by the variation in the coupling coefficient reduces the rejection rate. This “chirp” is certainly more important than that for a “generalized cosine” type of profile because of the greater range of variation in the coupling coefficient.

It should be noted that unlike the case of “generalized cosine” couplers, in the present case, this “chirp” is not compensated by a geometric variation in the grating period. This is not necessary because this type of filter is much more robust in combatting this type of problem. This robustness lies in the fact that the high rejection rate is not only determined by the adiabatic variation in the coupling coefficient, but also by the alternation of the polarity in the coupling profile. The apodization mechanism of the spectral response is therefore quite different. Nevertheless, the “chirp” compensation of the effective grating period and corrections in the coupling profile can certainly improve the performance of the component.

Another argument supporting the hypothesis that the obtained coupling profile is not entirely conformal to the modeling is the fact that for couplers where the ridge width of the waveguide carrying a grating is fixed at $1.9\ \mu\text{m}$, the rejection ratio is better (Figure 17). As can be seen, for this coupler, we actually manage to obtain an extinction ratio of 13 dB over a wide spectral range of 26 nm in TE polarization and 24 nm in TM polarization. The results for the SLSR, i.e., 19 dB, are a little worse. It should be noted, however, that for this structure, the waveguide with a ridge width of $1.9\ \mu\text{m}$ is already bi-mode. This result shows us that this type of coupler is also more robust with respect to the bimodality problem. It should be noted that for couplers with a “generalized cosine” type of coupling profile, even for a waveguide ridge width of $1.7\ \mu\text{m}$, there is a clear degradation in the filter performance.

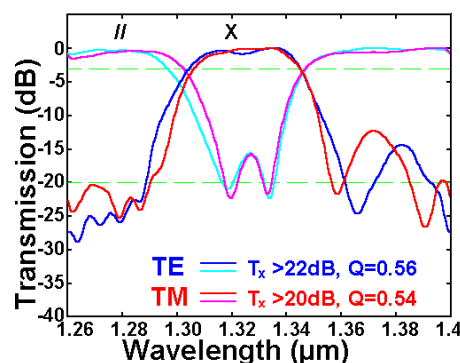


Figure 17. GADC with grating waveguide ridge width of $1.9\ \mu\text{m}$ experimental spectral response for bar (//) and cross (X) channels for the TE and TM polarizations. The lowest levels of the pass band and the highest levels of the attenuation band are delineated by green dashed lines at the $-3\ \text{dB}$ and $-20\ \text{dB}$ levels, respectively.

6. Summary and Outlook

This work was devoted to the design and experimental realization of GADCs with a high SLSR. The main challenge was to obtain a robust and reliable filter compatible with the application of the $1.28/1.32\ \mu\text{m}$ integrated diplexer. The central wavelength at $1.32\ \mu\text{m}$ was chosen as it is better suited for this realization and it allows to relax the constraints on the polarization independence at $1.28\ \mu\text{m}$ for the $1.32\ \mu\text{m}$ ONU and allows the operation of such a filter to be considered for $1.32\ \mu\text{m}$ OLT application when the polarization independence of the coupler is acquired. The objective was to obtain a filter with an SLSR that is $1.28\ \mu\text{m}$ greater than 20 dB (cross-channel), an extinction ratio at λ_0 that is greater than 13 dB (bar-channel), and a pass band width of 26 nm centered at $\lambda_0 = 1.32\ \mu\text{m}$ in a reproducible and reliable way.

The models based on the theory of coupled modes served as a tool for the design of the GADC. The method chosen to increase the SLSR is to realize along the propagation direction a coupling profile of the “generalized cosine” type by varying the interguide distance. We also decided to combine this method with that of the moderate modulation of the grating period using the SDC method to avoid radiation losses.

An SLSR of 23 dB at $1.28\ \mu\text{m}$ and a good spectral response quality factor $Q = 0.45$ were obtained for the GADC where the interguide distance variation was combined with SDC grating period modulation. The filter obtained was polarization-independent at $1.28\ \mu\text{m}$, as well as at $1.32\ \mu\text{m}$. The difference between the central wavelengths in the TE and TM polarizations was 1 nm to 3 nm.

The only problem is that for a high extinction ratio, we obtained a very narrow spectral band. The width of this spectral range for an extinction ratio of 10 dB was typically below 10 nm. We can therefore see that this value is considerably lower than the width of the pass band and that it is necessary to achieve a precise centering of the filter λ_0 .

To make the filter more robust, it is necessary to achieve a spectral response that would allow a high extinction ratio over a considerably wider spectral range (>13 dB on 26 nm) and that is compatible with an SLSR greater than 20 dB at $1.28\ \mu\text{m}$. This measure may also improve the quality factor of the spectral response. We observed that for filters with a “generalized cosine” coupling profile, the quality factor of the spectral response $Q \approx 0.4$. This value can be increased by achieving a wider pass band width and steeper transition slopes, but for this, it is necessary to consider another design of the coupling profile.

The realization of such a filter requires a coupling profile quite different from the “generalized cosine” one. We showed what kind of approach can be used to design such a filter from an imposed spectral window template. For this, we did not use the inverse dissipation method, but we adopted a pragmatic approach to synthesize the coupling profile from the impulse response of a frequency filter. It was found that this approach gives good results and provides a starting point for a possible refinement of the coupling profile by means of mathematical optimization methods.

A common aspect of the different profiles that give a square-templated flat-top spectral response was the presence of an alternation in the polarity of the coupling profile. Physically, a change in the sign of the coupling coefficient resulted in the introduction of 180° -phase shift sections (π rad, as shown in Figure 11). The realization of such a filter showed good agreement between the experiment and the simulations. The objectives targeted for $1.32\ \mu\text{m}$ ONU (OLT) applications were achieved.

It is worth noting that all the design methods developed here can be also extended for FTTH $1.3/1.5\ \mu\text{m}$ diplexer and $1.3+/1.3- /1.5\ \mu\text{m}$ triplexer applications [45,88].

The design tools presented here are quite general and can easily be adapted to other technology platforms, such as silicon photonics, for the use of GADCs as add-drop wavelength division multiplexers. A particularly promising research direction would be to consider narrowband forward coupling using the dispersion properties of Bragg grating. The theoretical premises for this type of operation are mainly detailed in [89–91]. An important point is that the design of this type of coupler involves the use of four-wave CMTs because the effects of both forward coupling and also backward coupling due to the Bragg grating reflection must be taken into account [92–95].

The field of parity-time (PT) symmetry is another avenue where this type of gain–loss-assisted active GADC is of interest for switching applications [96–99]. Here, the main idea is to modify their propagation constants by providing gain and loss in a pair of coupled waveguides, and, by extension, the beat length of the light intensity oscillations between these two waveguides. The change in the beat length can then be exploited for switching applications [96,97]. It turns out that the optimal switching operation, in terms of the minimum amount of amplification to be provided in such a system, is obtained when the gain level in one waveguide is equal to the loss level in the second waveguide and is not uniform over the entire length of the structure, or, if uniform, a variation in the coupling strength through interguide distance separation akin to that used for the apodization of the GADC spectral response is introduced [98]. Moreover, the switching operation of such a gain–loss-assisted PT-symmetric coupler critically depends on the phase matching condition between the two waveguides. In practice, it can be exactly fulfilled using a GADC with two dissimilar waveguides at the wavelength λ_0 given by Equation (1) [92]. As for the case of a basic PT-symmetric coupler, consisting of a pair of two identical waveguides, the modulation of either gain–loss or coupling strength profile brings benefits in terms of reducing the amplification level required for switching operation [98].

Of course, it would also be highly desirable to control the shape of the spectral response through the tailoring of either gain–loss and/or coupling strength profiles, as was the case for passive GADCs presented in this work. The ultimate goal would be to achieve the active

control of the GADC spectral response. So far, this problem has not really been addressed in the literature, apart from by engineering the spectral response of PT-symmetric Bragg gratings with gain–loss profiles modulated by grating duty cycle methods [61]. However, the case of GADCs seems more complex and requires further in-depth studies.

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