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# Anomalous Interactions of Airy Solitons Modulated by a Fundamental Gaussian Beam and Fourth-Order Diffraction

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**Abstract:** We investigate the interactions of in-phase Airy beams modulated by a fundamental Gaussian beam and fourth-order diffraction in Kerr nonlinear media. Directly numerical simulations show that normal (anomalous) fourth-order diffraction and an in-phase (out-of-phase) Gaussian beam affect the interactions of solitons generated from Airy beams in unique ways. Different from previous results, suggesting that interactions of in-phase (out-of-phase) conventional beams are always attractive (repulsive), many anomalous interactions of Airy beams are obtained. Stable breathing Airy soliton pairs can be formed with fourth-order diffraction and a fundamental Gaussian beam.

**Keywords:** interactions; Airy beams; fourth-order diffraction; in-phase; out-of-phase



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## 1. Introduction

In past decades, the investigation of Airy beams [1–9] was a hot topic in the area of optics due to their novel propagation properties, e.g., ballistic motion [10,11], self-healing [12], etc. In a linear regime, the mechanisms of Airy beams are widely used in particle clearing [13], light bullets [14], surface plasmons [15–17], Airy vortexes [18], high-dimensional structured light [19] and electron beams [20,21]. On the contrary, the nonlinear manipulation of Airy beams is another novel problem, including nonlinear generation [22,23] and the propagation of Airy beams [24–31], solitons generated from Airy beams [32], and spatiotemporal Airy light bullets [33–35].

In nonlinear media, interactions between Airy beams have also been studied. In particular, interactions were demonstrated in photorefractive crystal [36–41], Kerr media [42–44], optical fiber [45–47], quadratic nonlinear media [48], fractional media [49,50], photonic lattices [51], and nonlocal nonlinear media [52–54]. Compared with the interactions of conventional beams [55], many anomalous interactions were reported [53,54].

In nonlinear media, linear diffraction is an important effect induced by optical beams themselves during propagation [56]. Aside from quadratic diffraction, high-order diffraction should not be ignored [57]. For instance, fourth-order diffraction has deep impacts on nonlinear optics, e.g., the self-similar propagation of optical pulses [58], modulation instability [59], pure-quartic soliton lasers [60], and soliton states [61–66].

The effects of third-order [67,68] and fourth-order [69–71] diffraction/dispersion on the propagation of Airy pulses/beams have been investigated in detail. We have also studied the interactions of Airy beams in local Kerr, saturable, and nonlocal media with fourth-order diffraction [72]. We demonstrated that stable Airy soliton pairs can be obtained in nonlocal media with fourth-order diffraction, which are always repulsive in local media [72].

As discussed above, interactions between Airy beams can be affected by different kinds of nonlinearities and diffraction. In our previous work, we also demonstrated that the interactions of Airy beams can be controlled by a fundamental Gaussian beam [73]. Anomalous interactions can be obtained, i.e., repulsions (attractions) appear when an in-phase (out-of-phase) Gaussian beam is added [73].

In this paper, based on our previous works [72,73], we demonstrate that the interactions of solitons generated from two in-phase Airy beams in Kerr nonlinear media can be controlled by a fundamental Gaussian beam and fourth-order diffraction. The numerical results reveal that the interactions of the Airy beam are sensitive to beam intervals, fourth-order diffraction, and the relative phase and amplitude of the fundamental Gaussian beam. In particular, many anomalous interactions are obtained and physical mechanisms are explained in detail. Stable breathing Airy soliton pairs can be formed with appropriate beam parameters and coupling constants of fourth-order diffraction instead of nonlocality, as discussed in Ref. [73].

## 2. Physical Model and Basic Equations

Considering a beam in nonlinear media with fourth-order diffraction, the envelope $\psi(x, z)$ is described by the following normalized nonlinear Schrödinger equation [62,63],

$$i\frac{\partial\psi}{\partial z} + \frac{1}{2}\frac{\partial^2\psi}{\partial x^2} - \beta\frac{\partial^4\psi}{\partial x^4} + \delta n(I)\psi = 0, \tag{1}$$

where $x$ and $z$ denote transverse and longitude spatial coordinates [42,43], respectively, and $\beta$ represents the strength of fourth-order diffraction [62–64]. When $\beta > 0$, the fourth-order diffraction is normal, whereas it is anomalous when $\beta < 0$ [62–64]. The nonlinear refractive index change in the Kerr media $\delta n(I)$ can be written as

$$\delta n(I) = |\psi|^2. \tag{2}$$

In this paper, we assume that the incident beam is composed of two in-phase shifted counter propagating Airy beams, but with an additional fundamental Gaussian beam located in the middle with a relative phase between them [73],

$$\begin{aligned} \psi(x) = {} & A\{Ai[(x - B)] \exp[a(x - B)] \\ & + Ai[-(x + B)] \exp[-a(x + B)]\} \\ & + \exp(i\rho\pi) C \exp(-x^2/2), \end{aligned} \tag{3}$$

where $A$ represents the amplitude (we set $A = 3$ to ensure solitons can be generated), $B$ is beam separations (intervals), $C$ denotes the amplitude of the fundamental Gaussian beam [73], and $a = 0.2$ is the decaying factor [1]. $\rho$ is the parameter controlling the phase shift; in particular, $\rho = 0$ ($\rho = 1$) describes how the Airy beams and the fundamental Gaussian beam are in-phase (out of phase) [73].

## 3. Case of In-Phase Gaussian Beam and Fourth-Order Diffraction

The interactions of the beams can be investigated numerically by integrating Equation (1) directly with the split-step Fourier transform method. Firstly, we consider the interactions of Airy beams and an in-phase ($\rho = 0$) Gaussian beam with fourth-order diffraction. In nonlinear media, solitons can be generated from the main lobes of the Airy beams, and the tails of the Airy beams keep self-accelerating [53]. Here, we focus on the interactions between Airy solitons. In Figure 1, we display the numerical results of the interactions with some different beam intervals $B = 0, 1, 2$ and fourth-order diffraction coupling constants $\beta$ in the case of weak Gaussian beams ($C << A$). Previous work has shown that the dynamics of interactions depend crucially on the amplitude of the Gaussian beam [73].

When $\beta = 0$ and $C = 0$, this represents the interactions of two in-phase Airy beams in Kerr media with only quadratic diffraction [42,43,53,72,73]; attractions between beams are always dominant. It is obvious that the bound states of breathing soliton pairs with different breathing periods are obtained (Figure 1(a1–c1)). The strongest attraction occurs at $B = 1$ (Figure 1(b1)) due to strong self-trapping effect [42,43,53,73]. With an in-phase ($\rho = 0$) weak fundamental Gaussian beam ($C << A$), the attraction between Airy beams is enhanced and the corresponding breathing periods are reduced significantly (Figure 1(a2–c2)).

The physical mechanisms for the above results have been explained extensively in previous works [73].

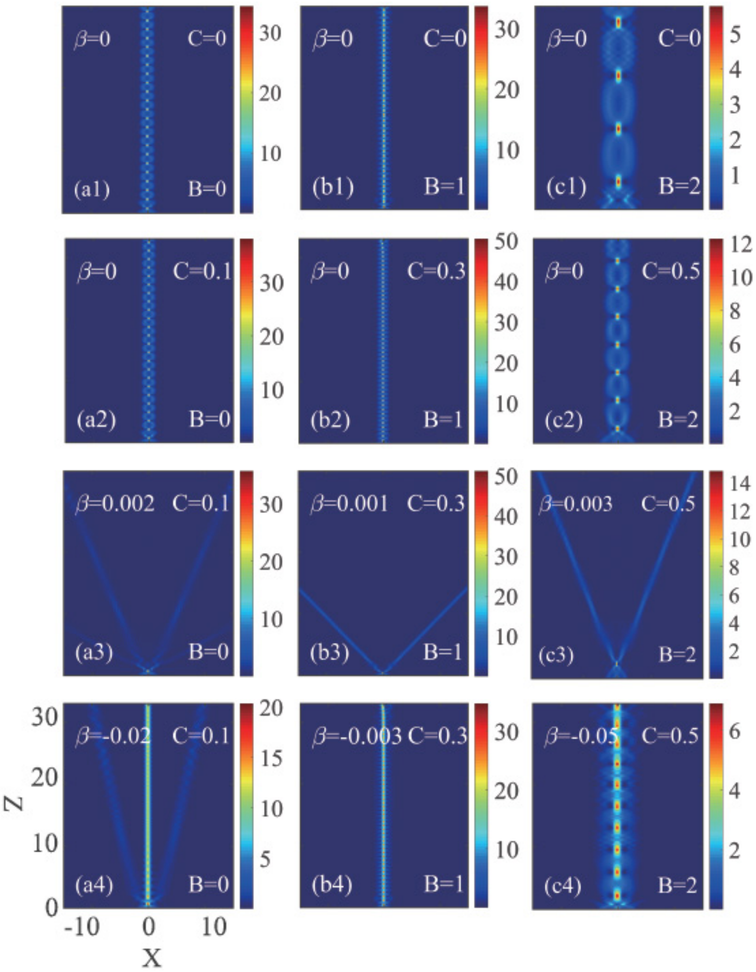


**Figure 1.** The intensity trajectories of interactions between Airy beams and in-phase ($\rho = 0$) weak light Gaussian beams. The beam intervals are $B$ = 0 (**a1**–**a4**), $B$ = 1 (**b1**–**b4**), and $B$ = 2 (**c1**–**c4**). The fourth-order diffraction coupling constants are $\beta > 0$, $\beta = 0$, and $\beta < 0$ for (**a3**–**c3**), (**a1**–**c2**), and (**a4**–**c4**), respectively. The amplitude is $A = 3$.

The dynamics of bound states can be modulated by fourth-order diffraction. In particular, without the fundamental Gaussian beam ($C$ = 0), the interactions between both in-phase and out-of-phase Airy beams under fourth-order diffraction have been studied previously [72]. Thus, in this paper we focus on the condition of $C \neq 0$ when fourth-order diffraction is taken into account ($\beta \neq 0$). For normal fourth-order diffraction ($\beta > 0$), with an in-phase Gaussian beam, as shown in Figure 1(a3–c3), the interactions of Airy beams exhibit repulsions for all beam intervals due to the fact that the diffraction effect of the beams is enhanced. The nonlinearity is unable to overcome the strong diffraction induced by normal fourth-order diffraction, and therefore no stable bound state can be formed [72]. On the other hand, when fourth-order diffraction is anomalous ($\beta < 0$), situations become different. Compared with the case without fourth-order diffraction $\beta = 0$ (Figure 1(a2–c2)), the attractions between Airy beams are significantly strengthened when the breathing periods of all the bound states become smaller (Figure 1(a4–c4)). The reason is that the diffraction effect of the beams is weakened with anomalous fourth-order diffraction, so nonlinearity will reinforce the interactions between Airy beams [72].

In general, the beam interval $B$ is an important parameter in controlling the interactions of Airy beams. When $B$ is larger, e.g., $B = 4$, only two parallel solitons can be generated from two well-separated Airy beams [42,43]. Apart from the results obtained above, anomalous interactions happen when $B = 3$. As illustrated in Figure 1, an in-phase fundamental Gaussian beam always enhances the attractions of two in-phase Airy beams when $B = 0, 1, 2$. When increasing the amplitude $C$ gradually, attraction between the Airy beams with $B = 3$ decreases firstly and subsequently increases [73], as shown in Figure 2(a2–c2). This interesting phenomena can be explained as follows: the nonlinearity induced by the intensity of the incident beam (Equation (3)) decreases first and then increases, leading to the weakness and then enhancement of the attractions [73]. Similar to Figure 1, normal ($\beta > 0$) and anomalous ($\beta < 0$) fourth-order diffraction will weaken and strengthen the attractions of the Airy beams, resulting in the repulsions (Figure 2(a1–c1)) and bound states (Figure 2(a3–c3)) with smaller breathing periods, respectively.

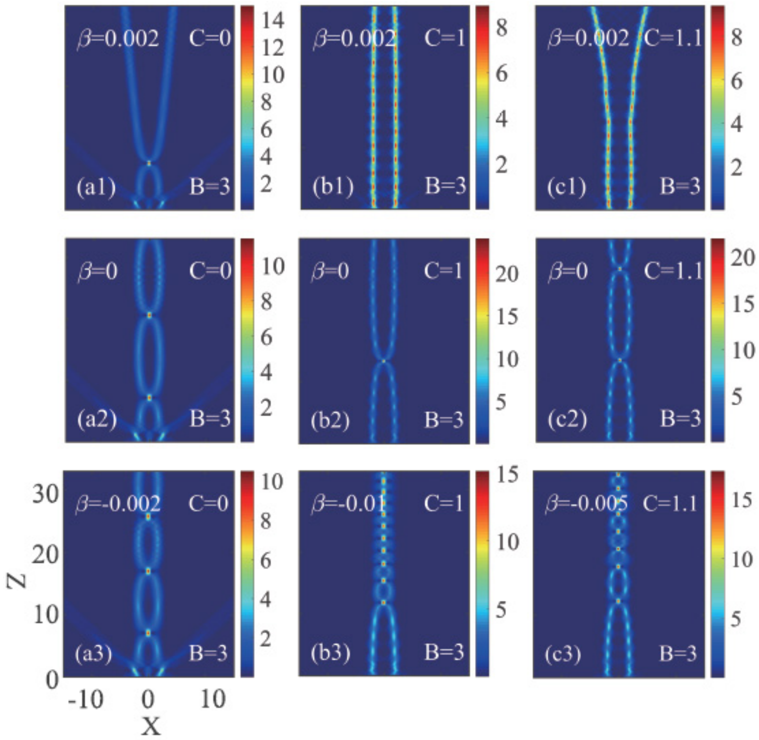


**Figure 2.** The intensity of interactions between Airy beams and in-phase ($\rho = 0$) weak light Gaussian beams. The beam interval is $B = 3$. The fourth-order diffraction coupling constants are $\beta > 0$, $\beta = 0$, and $\beta < 0$ for (**a1**–**c1**), (**a2**–**c2**), and (**a3**–**c3**), respectively. The amplitude is $A = 3$.

When the Gaussian beam is strong light, the amplitude of the fundamental Gaussian beam $C$ is large enough. Similarly, we show in Figure 3 the interactions of airy beams with different beam intervals $B = 0, 1, 2, 3$ and fourth-order diffraction coupling constants $\beta$. When $\beta = 0$, a single soliton generated from a fundamental Gaussian beam is located in the cental of two solitons shedding from Airy beams (Figure 3(a2–d2)). Different from the case of the enhancement of the attractions with an in-phase weak light Gaussian beam (Figure 1(a2–c2)), repulsions appear for all beam intervals (Figure 3(a2–d2)). For smaller beam intervals, $B = 0$ (Figure 3(a2)) and $B = 1$ (Figure 3(b2)), repulsions appear after a very short propagation distance. Although bound states are formed initially, for larger beam intervals, $B = 2$ (Figure 3(c2)) and $B = 3$ (Figure 3(d2)), repulsions overcome attractions eventually. When an in-phase strong light Gaussian beam is taken into account, the width of the breathing soliton pairs will decrease because of strong self-trapping induced by strong intensity, and attractions become stronger initially [43,73]. However, diffractions are also enhanced, induced by the decrease in the width, leading to final repulsions of the Airy beams [43,73].

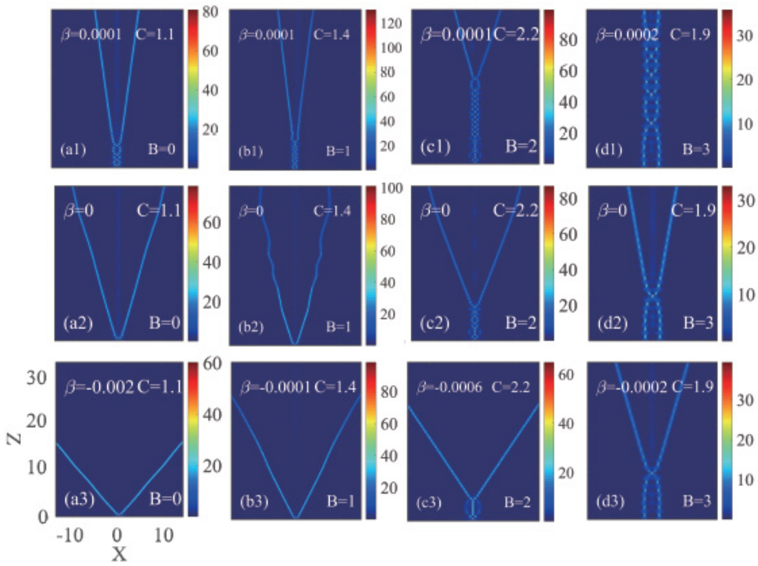

**Figure 3.** The trajectories of interactions between Airy beams and in-phase ($\rho = 0$) strong light Gaussian beams. The beam intervals are $B$ = 0 (**a1**–**a3**), $B$ = 1 (**b1**–**b3**), $B$ = 2 (**c1**–**c3**) and $B$ = 3 (**d1**–**d3**). The fourth-order diffraction coupling constants are $\beta > 0$, $\beta = 0$, and $\beta < 0$ for (**a1**–**d1**), (**a2**–**d2**), and (**a3**–**d3**), respectively. The amplitude is $A = 3$.

Surprisingly, fourth-order diffraction plays an unique role in the interactions of Airy beam with an in-phase strong light Gaussian beam. As shown in Figure 3, repulsions are weakened for $\beta > 0$ (Figure 3(a1–d1)) and strengthened for $\beta < 0$ (Figure 3(a3–d3)). This interesting result is quite different to the case of weak light Gaussian beams (Figures 1 and 2), where normal (anomalous) fourth-order diffraction always strengthens (weakens) the repulsions. The anomalous phenomena is very similar to the interactions of two in-phase Airy beams in saturable nonlinear media with fourth-order diffraction [72].

## 4. Case of Out-of-Phase Gaussian Beam and Fourth-Order Diffraction

Next, we study the interactions of Airy beams and out-of-phase ($\rho = 1$) fundamental Gaussian beams in nonlinear Kerr media with fourth-order diffraction. Comparing Figure 1(a1–c1), when beam intervals are smaller ($B = 0, 1, 2$) and without fourth-order diffraction ($\beta = 0$), a weak light out-of-phase fundamental Gaussian beam weakens the attractions of Airy beams with an increase in breathing period [73], as shown in Figure 4(a2–c2). This is the general case, that out-of-phase Gaussian beams provide a repulsive force between Airy beams [73].

Similar to the results of Figures 1 and 3, the dynamics of interactions of Airy beams are shown in Figure 4(a1–c1,a3–c3) when normal and anomalous fourth-order diffraction is used, respectively. Specifically, with normal fourth-order diffraction ($\beta > 0$), attractions become weaker and repulsions appear for all the beam intervals. On the contrary, attractions become stronger and the breathing periods of the bound states decrease for anomalous fourth-order diffraction ($\beta < 0$).

In the case of the beam interval $B = 3$, without fourth-order diffraction ($\beta = 0$) and a fundamental Gaussian beam ($C = 0$), the stationary bound state of two in-phase Airy beams is formed (Figure 2(a2)). When an out-of-phase Gaussian beam with amplitude $C = 0.7$ is added, repulsion appears between the two Airy beams and one fundamental soliton generated from Gaussian beam is located in the center, as shown in Figure 5(a2). However, repulsion will be weakened (Figure 5(b2)) when the amplitude of Gaussian beam $C$ increases, and the bound state of breathing mode can be formed (Figure 5(c2)) with the appropriate intensity of Gaussian beam. We emphasize that the anomalous interaction demonstrated in Figure 5(c2) is different from conventional interactions between out-of-phase beams, where repulsions are always dominant [55].

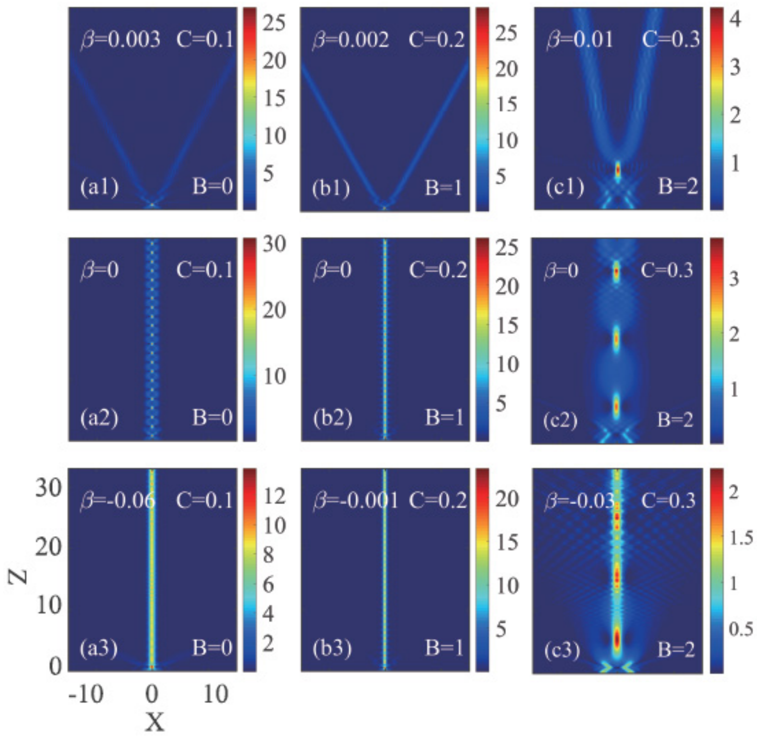


**Figure 4.** Evolution of interactions between Airy beams and out-of-phase ($\rho = 1$) weak light Gaussian beams. The beam intervals are $B$ = 0 (**a1**–**a3**), $B$ = 1 (**b1**–**b3**), and $B$ = 2 (**c1**–**c3**). The fourth-order diffraction coupling constants are $\beta > 0$, $\beta = 0$, and $\beta < 0$ for (**a1**–**c1**), (**a2**–**c2**), and (**a3**–**c3**), respectively. The amplitude is $A = 3$.

Similar to the interactions of Airy beams with an in-phase strong light Gaussian beam, as shown in Figure 3, or interactions of two in-phase Airy beams in saturable nonlinear media [72], fourth-order diffraction plays the same role in the interactions between Airy beams and out-of-phase weak light Gaussian beams when the beam interval is $B = 3$. As shown in Figure 5(a1,b1), repulsions are also weakened (or attractions are enhanced) and the breathing period decreases for $C = 1.2$ (Figure 5(c1)) when fourth-order diffraction is normal. On the other hand, repulsions are enhanced for all the interactions with anomalous fourth-order diffraction (Figure 5(a3–c3)).

In the case of out-of-phase strong light Gaussian beams, for beam intervals $B = 0$ and $B = 1$, similar to Figure 4(a2,b2), we numerically check that the breathing periods of the bound states become larger (not shown). When $B = 2$, as shown in Figure 6(a2), repulsion appears instead of attraction (Figure 4(c2)) when $C = 2.4$. Similar results happen in the case of $B = 3$, where repulsion emerges again (Figure 6(b2)) despite the bound state being formed with an appropriate out-of-phase fundamental Gaussian beam (Figure 5(c2)).

Similar to Figures 3 and 5, anomalous fourth-order diffraction ($\beta < 0$) strengthens the repulsions of the interactions, as shown in Figure 6(a3,b3). However, normal fourth-order diffraction ($\beta > 0$) weakens the repulsions for $B = 3$ (Figure 6(b1)), and the stationary bound state can even be formed for $B = 2$ (Figure 6(a1)). These impacts of fourth-order diffraction are different to the outcomes shown in Figures 1, 2 and 4.

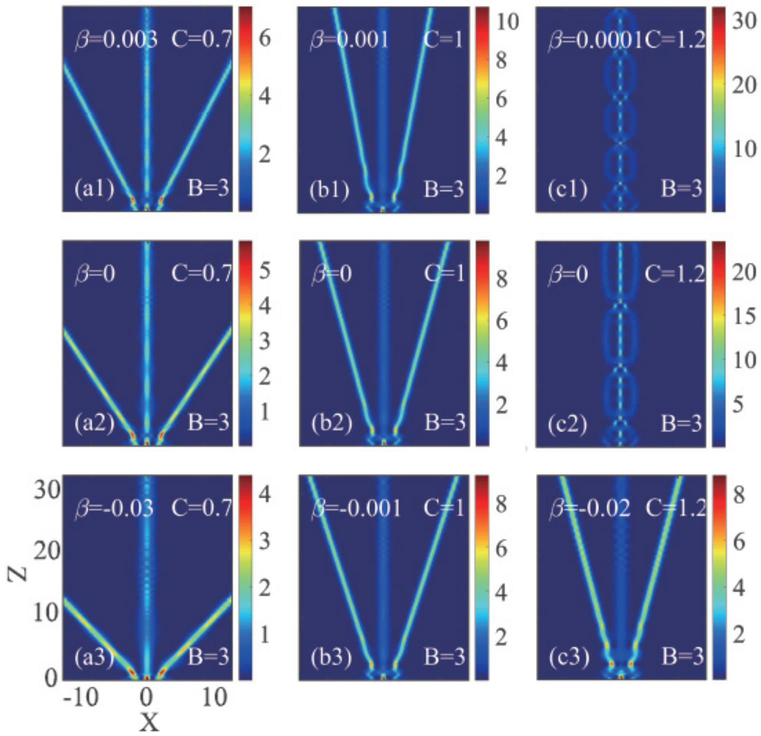


**Figure 5.** The intensity trajectories of interactions between Airy beams and out-of-phase ($\rho = 1$) weak light Gaussian beams. The beam interval is $B$ = 3. The fourth-order diffraction coupling constants are $\beta > 0$, $\beta = 0$, and $\beta < 0$ for (**a1**–**c1**), (**a2**–**c2**), and (**a3**–**c3**), respectively. The amplitude is $A = 3$.

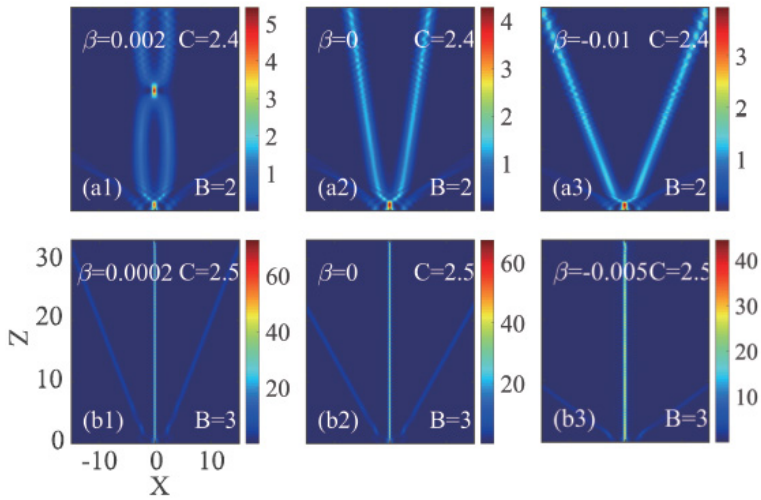


**Figure 6.** The intensity of interactions between Airy beams and out-of-phase ($\rho = 1$) strong light Gaussian beams. The beam intervals are $B$ = 2 (**a1**–**a3**) and $B$ = 3 (**b1**–**b3**). The fourth-order diffraction coupling constants are $\beta > 0$, $\beta = 0$, and $\beta < 0$ for (**a1**,**b1**), (**a2**,**b2**), and (**a3**,**b3**), respectively. The amplitude is $A = 3$.

## 5. Discussion

Particle-like interactions between solitons are one of the most interesting and important dynamics of soliton phenomena. For the coherent interactions of conventional beams, solitons exert attractive or repulsive forces on each other, depending on their relative phase [55]. In the case of in-phase interactions, bound states of soliton pairs can be formed. On the other hand, repulsions appear for out-of-phase interactions [55]. This result is also available for interactions between two in-phase or out-of-phase Airy beams [42,43,53].

With an in-phase fundamental Gaussian beam, attraction between in-phase Airy beams may cause them to become repulsive to each other, whereas the attraction can be strengthened when the Gaussian beam is out-of-phase [73]. These anomalous results are caused by the intensity changes induced by the Gaussian beam in the center region of beam overlap [73]. With the help of nonlocality, the bound states of breathing Airy soliton pairs can be obtained with the appropriate parameters of Airy beams and Gaussian beams [73]. Here, using fourth-order diffraction instead of nonlocality, similar results are demonstrated.

## 6. Conclusions

In conclusion, the interactions of in-phase Airy beams modulated by a fundamental Gaussian beam and fourth-order diffraction in Kerr nonlinear media were investigated numerically with the split-step Fourier transform method. Attractions and repulsions between the Airy beams with different beam intervals can be controlled flexibly by normal (anomalous) fourth-order diffraction and an in-phase (out-of-phase) Gaussian beam. We demonstrated the properties of both normal and anomalous interactions of Airy beams in detail. For the sake of the formation of stable bound states of breathing Airy soliton pairs, the parameters of fourth-order diffraction and the fundamental Gaussian beam should be adjusted properly. Our results may have potential applications in the areas of optical interconnection, optical switching and guided wave optics.

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