photonics

MDPI

Article

Generation of a Flat Optical Frequency Comb via a Cascaded Dual-Parallel Mach–Zehnder Modulator and Phase Modulator without Using the Fundamental Tone

Shiyu Zhang [1], Zixiong Wang [1], Xunhe Zuo [1], Chuang Ma [1,*], Yang Jiang [2] and Jinlong Yu [1]

[1] School of Electrical and Information Engineering, Tianjin University, Tianjin 300072, China; zhangsy_1613@tju.edu.cn (S.Z.); wangzixiong@tju.edu.cn (Z.W.); ZuoXunhe@tju.edu.cn (X.Z.); yujinlong@tju.edu.cn (J.Y.)
[2] College of Physics, Guizhou University, Guiyang 550025, China; jiangy@gzu.edu.cn
* Correspondence: machuang@tju.edu.cn

Abstract: Under the conventional scheme to generate an optical frequency comb (OFC) using an electro-optic modulator (EOM), the frequency interval of the OFC is determined via the frequency of the fundamental tone of the radio frequency (RF) driving signals. In this work, we use two harmonics without the fundamental tone to drive two EOMs, where the frequency interval of the generated flat OFC is the frequency of the fundamental tone. The orders of the two harmonics are coprime. Specifically, one harmonic drives the first branch of the dual-parallel Mach–Zehnder modulator (DPMZM) only, and the other harmonic drives the phase modulator (PM). The flatness of the OFC is achieved by adjusting the amplitude and phase of the RF driving harmonics as well as the bias of the EOM. Both a simulation and an experiment were carried out to verify the effectiveness of the proposed scheme. When the second harmonic drives the DPMZM and the third harmonic drives the PM, an 11-comb line OFC is generated, where the flatness of the OFC was 0.63 dB and 0.65 dB under the simulation and experiment, respectively. When the third harmonic drives the DPMZM and the second harmonic drives the PM, a 13-comb line OFC is generated, where the flatness of the OFC was 0.62 dB and 0.95 dB under the simulation and experiment, respectively. We also investigate the performance of the generated OFC when one harmonic drives two branches of the DPMZM and the other harmonic drives the PM. The comparison of the OFCs' performance demonstrates the effectiveness of the proposed scheme.

Keywords: optical frequency comb; DPMZM; PM; no fundamental tone

check for updates

Citation: Zhang, S.; Wang, Z.; Zuo, X.; Ma, C.; Jiang, Y.; Yu, J. Generation of a Flat Optical Frequency Comb via a Cascaded Dual-Parallel Mach–Zehnder Modulator and Phase Modulator without Using the Fundamental Tone. *Photonics* **2023**, *10*, 1340. https://doi.org/10.3390/photonics10121340

Received: 17 October 2023
Revised: 20 November 2023
Accepted: 29 November 2023
Published: 4 December 2023

Copyright: © 2023 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (https://creativecommons.org/licenses/by/4.0/).

1. Introduction

An optical frequency comb (OFC) is widely used in communication, signal processing, short pulse generation, and microwave photonics [1–3]. A flat OFC can be generated by using an electro-optic modulator (EOM), where the number of comb lines and flatness are the criteria used to evaluate the performance of the OFC [4–12]. The number of comb lines is determined by the modulation indices of the EOM. The larger the modulation indices the EOM provides, the higher the number of comb lines the OFC achieves. The flatness of the OFC is affected by the amplitude and phase of the radio frequency (RF) driving signal as well as the bias voltage of the EOM. Therefore, in order to generate a flat OFC with a large number of comb lines, the above parameters should be adjusted precisely and higher-order harmonics should also be applied to drive the EOMs.

A flat OFC can be generated by using one RF signal to drive one branch of a dual-parallel Mach–Zehnder modulator (DPMZM), where the number of comb lines was seven and the flatness was 1 dB in an experiment [13]. A flat OFC can also be generated when one RF signal drives both branches of the DPMZM. When the amplitude of the RF driving signal and the bias voltages are adjusted precisely, a seven-comb line OFC can be generated

with a flatness of 0.6 dB [14]. In [15], one RF, one signal-drove intensity modulator (IM) and two phase modulators (PMs) in a cascade, where the phase of the RF signal was adjusted. Consequently, a 29-comb line OFC was generated, where the flatness was 1.5 dB.

Besides the approach of using one RF driving signal, multiple RF driving signals are used to generate a flat OFC with more comb lines. The fundamental tone and third harmonics were combined to drive one PM to generate a nine-comb line OFC with a flatness of 0.8 dB, where the phase of the third harmonic was adjusted [16]. A flat OFC was achieved when the fundamental tone and second harmonic drove two branches of the DPMZM. Consequently, a nine-comb line OFC was generated, where the flatness was within 3 dB [17]. The fundamental tone and second harmonic were superposed to drive an IM, where the phases of the two RF signals were adjusted precisely. The generated OFC had 11 comb lines and the flatness was within 1 dB [18]. A flat OFC with 15 comb lines and 0.65 dB flatness was generated in an experiment, when the fundamental tone, third harmonic, and fifth harmonic were superposed to drive two cascaded PMs [19]. When the superposed fundamental tone, second harmonic, and third harmonic drove DPMZM, a 13-comb line OFC was generated in an experiment, where the flatness was 0.58 dB [20].

In the literature so far, although multiple harmonics were used simultaneously to generate a flat OFC, the fundamental tone remained indispensable. This is because phase modulation is the fundamental process in electro-optic modulation, and the frequency interval of the OFC is determined by the lowest frequency component of all of the RF driving signals when the phase modulation is used under PM, IM, and even DPMZM.

In this work, we propose a new scheme to generate a flat OFC by using harmonics to drive cascaded EOMs without using the fundamental tone. More specifically, we use one harmonic to drive only one branch of the DPMZM and use the other harmonic to drive PM, where the frequencies of the two harmonics are not that of the fundamental tone. The frequency interval of the OFC is the frequency of the fundamental tone. By adjusting the amplitude and phase of the two harmonics as well as the bias voltages of the DPMZM, a flat OFC is generated. Both a simulation and an experiment were carried out. An 11-comb line OFC is generated, where the second harmonic drove the DPMZM and the third harmonic drove the PM. The flatness of the OFC was 0.63 dB and 0.65 dB under the simulation and experiment, respectively. A 13-comb line OFC was generated when the third harmonic drove the DPMZM and the second harmonic drove the PM. The flatness of the OFC was 0.62 dB and 0.95 dB under the simulation and experiment, respectively. To the best of our knowledge, this is the first work that demonstrates the feasibility of EOM-based OFC generation without using the fundamental tone.

The remainder of the paper is organized as follows. Section 2 describes the principle of the OFC generation without using the fundamental tone under cascaded DPMZM and PM. The simulation and experiment are presented in Section 3. In Section 4, we discuss the performance of the OFC by using other combined harmonics and under the scheme when one harmonic drives two branches of the DPMZM. The conclusion is summarized in Section 5.

2. Principle

Conventionally, the fundamental tone and harmonics are used to generate a flat OFC. The fundamental tone determines the frequency interval of the OFC, while the harmonics that drive the EOM are used to generate more comb lines and improve the flatness of the OFC. In this work, we would like to generate a flat OFC via cascaded DPMZM and PM without using the fundamental tone. As shown in Figure 1, the laser diode (LD) emits continuous-wave (CW) light $E_{\text{in}}(t)$, which is written as follows

$$E_{\text{in}}(t) = E_{\text{C}} \exp(j2\pi f_{\text{C}} t + j\phi_{\text{C}}), \tag{1}$$

where E_{C}, f_{C}, and ϕ_{C} are the amplitude, frequency, and phase of the electrical field of the CW light, respectively. ϕ_{C} is assumed to be zero, without loss of generality. The CW light is then sent to the DPMZM, which consists of two Mach–Zehnder modulators (MZMs).

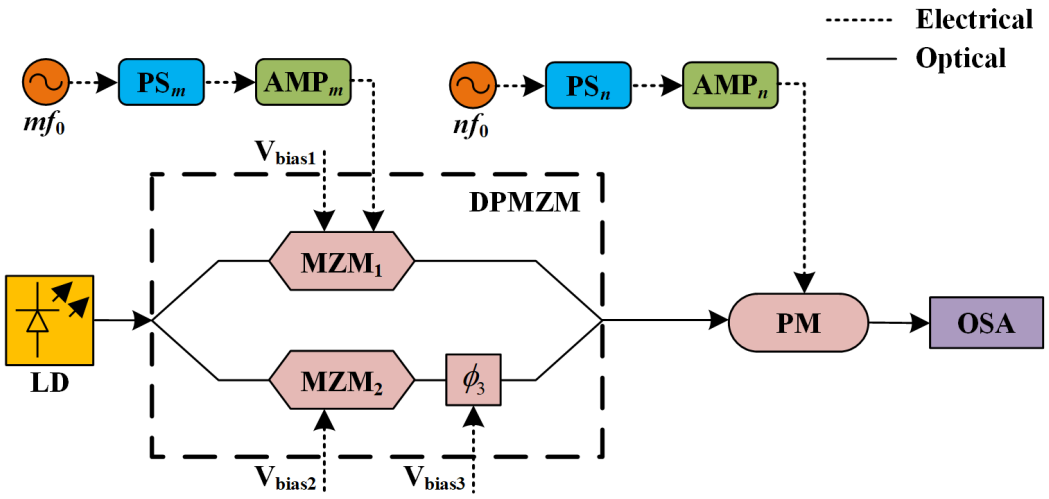


Figure 1. The principle used to generate a flat OFC via cascaded DPMZM and PM without using the fundamental tone. m and n are the order of two harmonics that drive one branch of the DPMZM and PM, respectively, where m and n are coprime. LD: laser diode, MZM: Mach–Zehnder modulator, DPMZM: dual-parallel Mach–Zehnder modulator, PS: phase shifter, AMP: amplifier, PM: phase modulator, OSA: optical spectrum analyzer.

The MZM in the first branch of the DPMZM is driven by a sinusoid whose frequency is m times that of the fundamental tone. The driving sinusoid is denoted by

$$s_m(t) = E_m \sin(2\pi m f_0 t + \phi_m), \tag{2}$$

where E_m and ϕ_m are the amplitude and phase of the mth driving sinusoid, respectively. f_0 is the frequency of the fundamental tone. Consequently, the electrical field at the output of the MZM in the first branch of the DPMZM is

$$E_1(t) = \frac{1}{2}E_{\text{in}}(t)\left\{\exp\left[j\beta_m\frac{s_m(t)}{E_m} + j\phi_1\right] + \exp\left[-j\beta_m\frac{s_m(t)}{E_m}\right]\right\}. \tag{3}$$

$\beta_m = \frac{E_m}{V_\pi}\pi$ is the modulation index of the phase modulation for the MZM in the first branch of the DPMZM, where V_π is the half-wave voltage. $\phi_1 = \frac{V_{\text{bias1}}}{V_\pi}\pi$ is the phase shift for the MZM in the first branch due to the bias voltage V_{bias1}. The expression in Equation (3) can be expanded by the Jacobi–Anger expression, which is given by [21]

$$\exp(jz\sin\theta) = \sum_{k=-\infty}^{\infty} J_k(z)\exp(jk\theta), \tag{4}$$

where $J_k(\cdot)$ is the Bessel function of the first kind for order k. Therefore, Equation (3) can be written as

$$E_1(t) = \frac{1}{2}E_{\text{in}}(t)\sum_{k=-\infty}^{\infty} J_k(\beta_m)\exp[jk(2\pi m f_0 t + \phi_m)]\cdot[\exp(j\phi_1) + \exp(jk\pi)]. \tag{5}$$

Since the RF signal drives the MZM in the first branch of the DPMZM only, the electrical field at the output of the MZM in the second branch is

$$\begin{aligned} E_2(t) =& \frac{1}{2}E_{\text{in}}(t)[\exp(j\cdot 0 + j\phi_2) + \exp(-j\cdot 0)] \\ =& \frac{1}{2}E_{\text{in}}(t)[\exp(j\phi_2) + 1], \end{aligned} \tag{6}$$

where $\phi_2 = \frac{V_{\text{bias2}}}{V_\pi}\pi$ is the phase shift for MZM in the second branch due to the bias voltage V_{bias2}. An additional phase shift $\phi_3 = \frac{V_{\text{bias3}}}{V_\pi}\pi$ is applied to the electrical field in the second branch, due to the bias voltage V_{bias3}. The overall electrical field at the output of the DPMZM is

$$\begin{aligned} E_{\text{DPMZM}}(t) =& E_1(t) + E_2(t)\exp(j\phi_3) \\ =& \frac{1}{2}E_{\text{in}}(t)\Bigg\{\sum_{k=-\infty}^{\infty} J_k(\beta_m)\exp[jk(2\pi m f_0 t + \phi_m)] \\ & \cdot[\exp(j\phi_1) + \exp(jk\pi)] + [\exp(j\phi_2 + j\phi_3) + \exp(j\phi_3)]\Bigg\}. \end{aligned} \tag{7}$$

The modulated light at the output of the DPMZM is then sent to the PM, which is driven by another sinusoid. This driving sinusoid has the same form as that shown in Equation (2), where the order of the harmonic is changed from m to n. Note that m and n are coprime. By applying the Jacobi–Anger expression, the electrical field at the output of PM is

$$\begin{aligned} E_{\text{DPMZM-PM}}(t) =& E_{\text{DPMZM}}(t)\exp\left(j\beta_n\frac{s_n(t)}{E_n}\right) \\ =& E_{\text{DPMZM}}(t)\sum_{\ell=-\infty}^{\infty} J_\ell(\beta_n)\exp[j\ell(2\pi n f_0 t + \phi_n)], \end{aligned} \tag{8}$$

where E_n is the amplitude of the nth driving sinusoid and $\beta_n = \frac{E_n}{V_\pi}\pi$ is the corresponding modulation index. Substituting Equation (7) into Equation (8), we have

$$\begin{aligned} E_{\text{DPMZM-PM}}(t) =& \frac{1}{2}E_{\text{C}}\exp(j2\pi f_{\text{C}} t + j\phi_{\text{C}}) \\ & \cdot\Bigg\{\sum_{k=-\infty}^{\infty} J_k(\beta_m)\exp[jk(2\pi m f_0 t + \phi_m)] \\ & \cdot[\exp(j\phi_1) + \exp(jk\pi)] + [\exp(j\phi_2 + j\phi_3) + \exp(j\phi_3)]\Bigg\} \\ & \cdot\sum_{\ell=-\infty}^{\infty} J_\ell(\beta_n)\exp[j\ell(2\pi n f_0 t + \phi_n)]. \end{aligned} \tag{9}$$

Taking the Fourier transform (FT) of Equation (9), the overall spectrum of the OFC under the scheme of Figure 1 is

$$\begin{aligned} \widetilde{E}_{\text{DPMZM-PM}}(f) =& \frac{1}{2}E_{\text{C}}\exp(j\phi_{\text{C}})\delta(f - f_{\text{C}}) \\ & \circledast\Bigg\{\sum_{k=-\infty}^{\infty} J_k(\beta_m)\exp(jk\phi_m)[\exp(j\phi_1) + \exp(jk\pi)]\cdot \\ & \delta(f - km f_0) + [\exp(j\phi_2 + j\phi_3) + \exp(j\phi_3)]\delta(f)\Bigg\} \\ & \circledast\sum_{\ell=-\infty}^{\infty} J_\ell(\beta_n)\exp(j\ell\phi_n)\delta(f - \ell n f_0) \\ =& \frac{1}{2}E_{\text{C}}\exp(j\phi_{\text{C}})\Bigg\{\sum_{k=-\infty}^{\infty}\sum_{\ell=-\infty}^{\infty} J_k(\beta_m)J_\ell(\beta_n)\exp(jk\phi_m)\cdot \\ & \exp(j\ell\phi_n)[\exp(j\phi_1) + \exp(jk\pi)]\cdot \\ & \delta(f - f_{\text{C}} - km f_0 - \ell n f_0) + \sum_{\ell=-\infty}^{\infty} J_\ell(\beta_n)\exp(j\ell\phi_n)\cdot \\ & [\exp(j\phi_2 + j\phi_3) + \exp(j\phi_3)]\delta(f - f_{\text{C}} - \ell n f_0)\Bigg\}, \end{aligned} \tag{10}$$

where $\circledast$ represents convolution and $\delta(\cdot)$ denotes the delta function. From Equations (7) and (8), we can see that the intervals among adjacent tones of the OFC are multiples of

mf_0 under the modulation of the DPMZM and multiples of nf_0 under the modulation of PM, respectively.

The fundamental process of electro-optic modulation is phase modulation. When we use one RF signal to drive the PM, according to the Jacobi–Anger expansion, i.e., Equation (4), the OFC can be generated, where the repetition rate is the same as the frequency of the fundamental tone. The intensity modulation process of the MZM consists of the processes of two parallel phase modulations, and the modulation process of the DPMZM consists of the process of two parallel intensity modulations, i.e., four phase modulations. In this work, we use the mth harmonic to drive the DPMZM, where the repetition rate of the generated spectrum is the frequency of mth harmonic, i.e., mf_0. The frequency of each comb line of the OFC is denoted by $f_C + kmf_0$, where f_C is the carrier frequency and k is an arbitrary integer, as shown in the Jacobi–Anger expansion. Regarding the phase modulation process in the PM after the DPMZM, the nth harmonic drives the PM. Consequently, the phase modulation process occurs for each frequency component $f_C + kmf_0$ of the generated OFC at the output of the DPMZM, and the corresponding frequency component at the output of the PM is $f_C + kmf_0 + \ell nf_0$, where ℓ is also an arbitrary integer. Once $km + \ell n$ is ± 1, the repetition rate of the generated OFC is f_0, i.e., the fundamental tone, although we do not use the fundamental tone to drive either DPMZM or PM.

A flat OFC can be generated by precisely adjusting the parameters in Equation (10) including β_m, β_n, ϕ_m, ϕ_n, ϕ_1, ϕ_2, and ϕ_3. The optimized parameters can be obtained by minimizing the power variance (in dBm) for all the tones of the OFC [19,20]

$$\begin{aligned} \underset{\beta_m,\beta_n,\phi_m,\phi_n,\phi_1,\phi_2,\phi_3}{\text{minimize}} \quad & \text{Var}\left\{10\log_{10}\left|\widetilde{E}_{\text{DPMZM-PM}}\right|^2\right\} \\ \text{subject to} \quad & 0 < \beta_m \leqslant 1.5,\ 0 < \beta_n \leqslant 1.5,\ 0 < \phi_m \leqslant 2\pi,\ 0 < \phi_n \leqslant 2\pi, \\ & 0 < \phi_1 \leqslant 2\pi,\ \ 0 < \phi_2 \leqslant 2\pi,\ \ 0 < \phi_3 \leqslant 2\pi \end{aligned} \tag{11}$$

where Var represents the variance. Under the constraint of practical equipment, the modulation indices are not larger than 1.5.

3. Results

3.1. Simulation Results

We used the differential evolution (DE) algorithm to find the optimized parameters [22]. In the simulation, we set the number of generations, the population size, the mutation constant, and the crossover constant as 1000, 500, 0.5, and 0.1, respectively.

The specific implementation steps of the optimization process refer to [23].

We investigated the following cases to generate a flat OFC. In the first case, the second harmonic (12 GHz) and the third harmonic (18 GHz) drive the DPMZM and PM, respectively. In the second case, the third harmonic (18 GHz) and the second harmonic (12 GHz) drive the DPMZM and PM, respectively. In the third case, the second harmonic (12 GHz) and the fifth harmonic (30 GHz) drive the DPMZM and PM, respectively. The optimized parameters under the three cases are listed in Table 1.

Table 1. The optimized parameters of the generated OFCs under three cases. Case I: 11-comb line OFC, m = 2, n= 3. Case II: 13-comb line OFC, m = 3, n = 2. Case III: 11-comb line OFC, m = 2, n = 5.

Case	β_m	β_n	ϕ_m	ϕ_n	ϕ_1	ϕ_2	ϕ_3
I	1.19	1.50	2.16	3.05	5.61	2.17	1.62
II	1.19	1.40	2.34	3.13	5.66	4.18	3.87
III	0.99	1.43	3.45	2.40	5.78	5.19	3.42

The generated flat OFCs are shown in Figure 2 for the above three cases. We used the difference between the maximum and minimum power of N consecutive comb lines to define the flatness of the generated OFCs. For the first case, the generated OFC had

11 comb lines, where the flatness was 0.63 dB, as shown in Figure 2a. The maximum power of the 11-comb line OFC was −21.70 dBm on the eighth comb line and the minimum power was −22.33 dBm on the sixth comb line. For the second case, the generated OFC had 13 comb lines, where the flatness was 0.62 dB, as shown in Figure 2b. The maximum power of the 13-comb line OFC was −13.95 dBm on the eighth comb line and the minimum power was −14.57 dBm on the twelfth comb line. For the third case, the generated OFC had 11 comb lines, where the flatness was 0.10 dB, as shown in Figure 2c. The maximum power of the 11-comb line OFC was −26.17 dBm on the sixth comb line and the minimum power was −26.27 dBm on the ninth comb line.

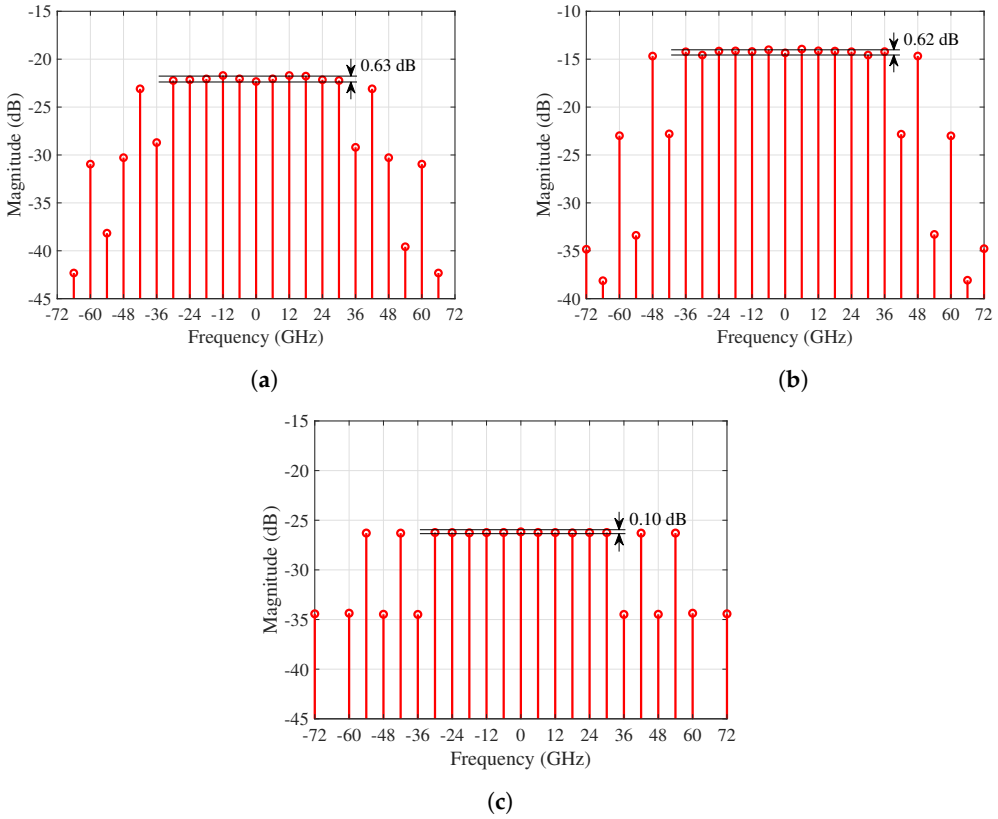


Figure 2. The generated OFC in simulation under the scheme in Figure 1, when (**a**) the second harmonic (12 GHz) and third harmonic (18 GHz) drive the DPMZM and PM, respectively; (**b**) the third harmonic (18 GHz) and second harmonic (12 GHz) drive the DPMZM and PM, respectively; (**c**) the second harmonic (12 GHz) and fifth harmonic (30 GHz) drive the DPMZM and PM, respectively.

3.2. Experimental Results

We carried out the experiment as follows to verify the effectiveness of the proposed scheme in generating a flat OFC; the experimental setup is shown in Figure 3. The LD (Han's Raypro Sensing, China) emitted CW light with a 1550 nm wavelength and 16.2 dBm power, which was sent consecutively to the DPMZM (Fujitsu FTM7961EX, USA) and PM (iXblue MPZ-LN-40, France). The frequencies of the second and third harmonics that drove the EOMs were 12 GHz and 18 GHz, respectively, and were generated by HP 83732A, USA and Agilent E8257D, USA, respectively. The phase and amplitude of the driving harmonic were adjusted precisely using a phase shifter (PS) and an electrical amplifier. The performance of the generated OFC was observed using an optical spectrum analyzer (OSA) (Yokogawa AQ6370D, Japan) that has a resolution of 0.02 nm.

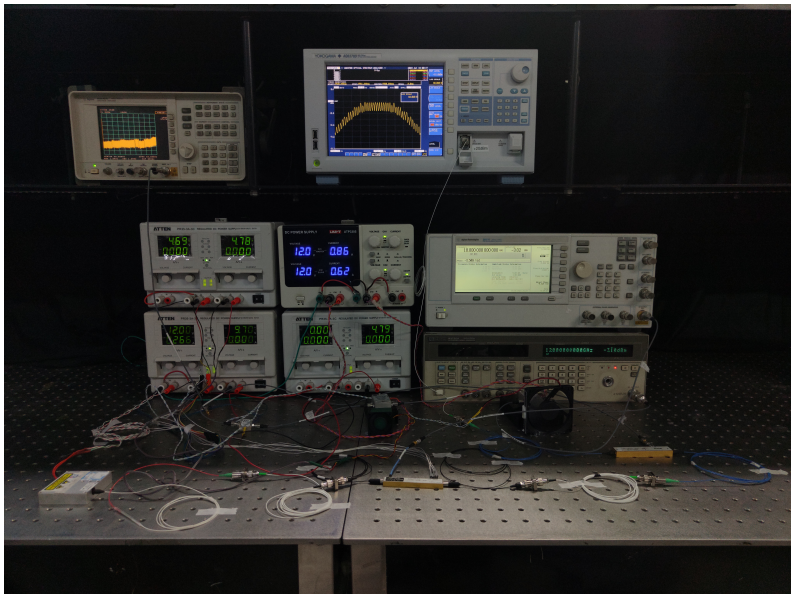
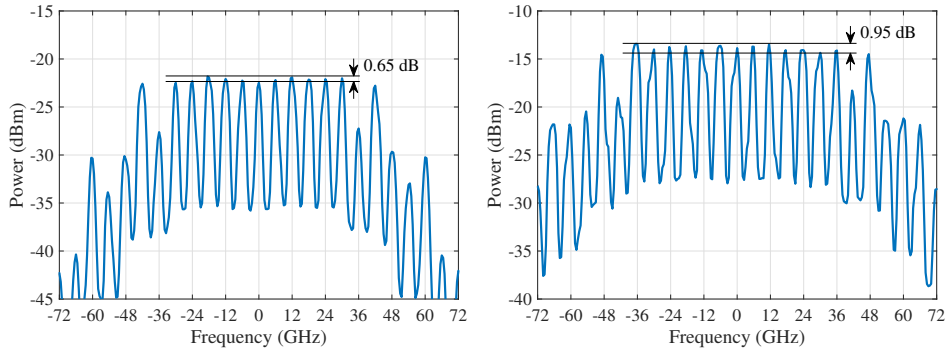


Figure 3. The experimental setup for OFC generation under the scheme in Figure 1.

For the first case, 12 GHz and 18 GHz sinusoids drove the DPMZM and PM, respectively. The powers of the two sinusoids at the inputs of the two EOMs were both 20.2 dBm. The generated 11-comb line OFC is depicted in Figure 4a, where the flatness is 0.65 dB, the maximum power of the comb lines is −21.82 dBm on the eighth comb line, and the minimum power of the comb lines is −22.47 dBm on the sixth comb line. For the second case, 18 GHz and 12 GHz sinusoids drove the DPMZM and PM, respectively. The powers of the two sinusoids at the inputs of the two EOMs were 21.2 dBm and 21.0 dBm, respectively. The generated 13-comb line OFC is depicted in Figure 4b, where the flatness is 0.95 dB, the maximum power of the comb lines is −13.41 dBm on the sixth comb line, and the minimum power of comb lines is −14.36 dBm on the twelfth comb line. The well-matched profiles of the OFCs under the simulation and experiment verify the effectiveness of the proposed scheme shown in Figure 1. Note that due to the constraint of the electrical amplifier, the experiment the for third case was not carried out.



(**a**) (**b**)

Figure 4. The generated OFCs in the experiment under the scheme in Figure 1, when (**a**) the second harmonic and third harmonic drove the DPMZM and PM, respectively; (**b**) the third harmonic and the second harmonic drove the DPMZM and PM, respectively.

4. Discussion

In this section, we investigate the performance of the generated OFCs under the scheme in which one harmonic drives both branches rather than one branch of the DPMZM and the other harmonic drives the PM for the cases of the harmonic combinations shown in Figure 2.

We also validated the scheme that utilizes one harmonic to drive both branches of the DPMZM to generate the OFCs. The specific system block diagram is shown in Figure 5, and the overall spectrum of the OFC under the scheme of Figure 5 is

$$\begin{aligned}\widetilde{E}_{\text{DPMZM-two-branches-PM}}(f) =& \frac{1}{2}E_C\exp(j\phi_C)\delta(f-f_C)\\ &\circledast\left\{\sum_{k_1=-\infty}^{\infty}J_{k_1}(\beta_{m,1})\exp(jk_1\phi_{m,1})[\exp(j\phi_1)+\exp(jk_1\pi)]\cdot\right.\\ &\delta(f-k_1mf_0)+\sum_{k_2=-\infty}^{\infty}J_{k_2}(\beta_{m,2})\exp(jk_2\phi_{m,2})\cdot\\ &\left.[\exp(j\phi_2+j\phi_3)+\exp(jk_2\pi+j\phi_3)]\delta(f-k_2mf_0)\right\}\\ &\circledast\sum_{\ell=-\infty}^{\infty}J_\ell(\beta_n)\exp(j\ell\phi_n)\delta(f-\ell nf_0)\\ =&\frac{1}{2}E_C\exp(j\phi_C)\left\{\sum_{k_1=-\infty}^{\infty}\sum_{\ell=-\infty}^{\infty}J_{k_1}(\beta_{m,1})J_\ell(\beta_n)\exp(jk_1\phi_{m,1})\cdot\right.\\ &\exp(j\ell\phi_n)[\exp(j\phi_1)+\exp(jk_1\pi)]\cdot\\ &\delta(f-f_C-k_1mf_0-\ell nf_0)+\sum_{k_2=-\infty}^{\infty}\sum_{\ell=-\infty}^{\infty}J_{k_2}(\beta_{m,2})J_\ell(\beta_n)\cdot\\ &\exp(jk_2\phi_{m,2})\exp(j\ell\phi_n)[\exp(j\phi_2+j\phi_3)+\exp(jk_2\pi+j\phi_3)]\cdot\\ &\left.\delta(f-f_C-k_2mf_0-\ell nf_0)\right\},\end{aligned} \tag{12}$$

where the subscripts 1, 2 of k, β, and ϕ in Equation (12) correspond to the parameters of the upper and lower arms of the DPMZM, respectively. The generated OFCs when one harmonic drove both branches of the DPMZM are shown in Figure 6. Comparing Figure 6 with Figure 2, we can see that the numbers of comb lines for the OFCs under the two schemes are identical.

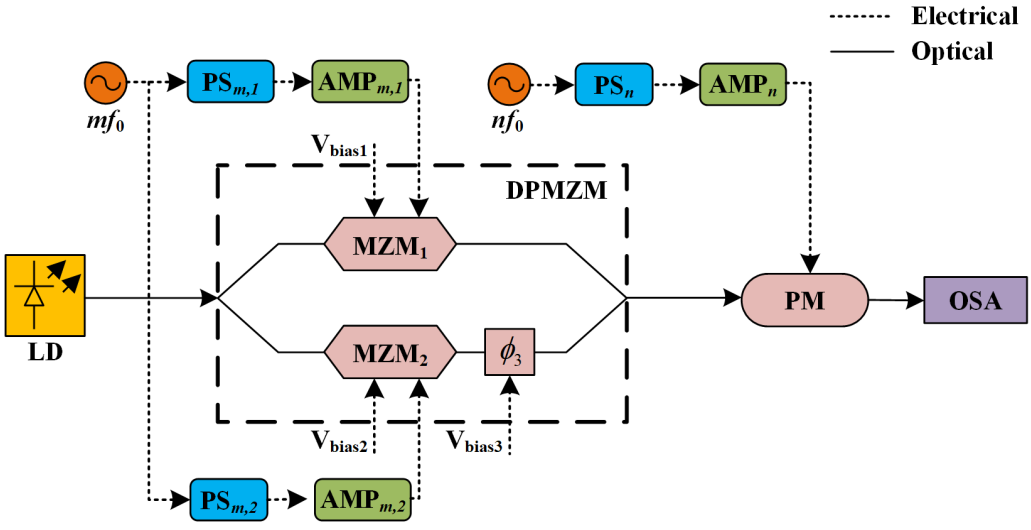


Figure 5. The principle used to generate a flat OFC via cascaded DPMZM and PM without using the fundamental tone. m and n are the orders of two harmonics that drive two branches of the DPMZM and PM, respectively, where m and n are coprime.

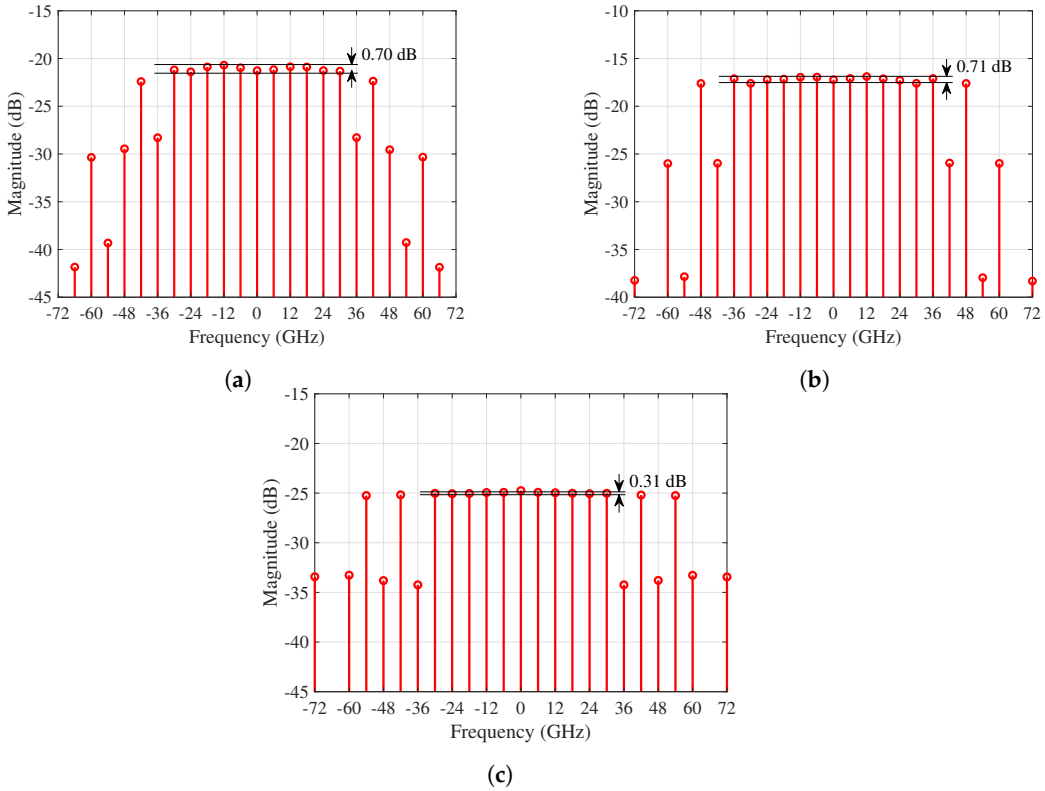


Figure 6. The generated OFCs in simulation, (**a**) when the second harmonic (12 GHz) drove two branches of the DPMZM and third harmonic (18 GHz) drove PM; (**b**) when the third (18 GHz) harmonic drove two branches of the DPMZM and the second (12 GHz) harmonic drove the PM; (**c**) when the second harmonic (12 GHz) drove two branches of the DPMZM and fifth harmonic (30 GHz) drove PM.

However, the flatness of the OFC under the scheme in Figure 2 is better than in the scheme in which one harmonic drove both branches of the DPMZM, which is depicted in Table 2. For the three cases described in Table 2, the improvement in flatness performance was 0.07 dB, 0.09 dB, and 0.21 dB, respectively, which demonstrates the effectiveness of the proposed scheme.

Table 2. Performance of the generated OFCs under the schemes in Figures 1 and 5 for three cases. Case I: second harmonic (12 GHz) and third harmonic (18 GHz) drove the DPMZM and PM, respectively. Case II: third harmonic (18 GHz) and second harmonic (12 GHz) drove the DPMZM and PM, respectively. Case III: second harmonic (12 GHz) and fifth harmonic (30 GHz) drove the DPMZM and PM, respectively. Scheme (1): Figure 5. Scheme (2): Figure 1.

Scheme	Case I		Case II		Case III	
	Number of Comb Lines	Flatness	Number of Comb Lines	Flatness	Number of Comb Lines	Flatness
(1)	11	0.70 dB	13	0.71 dB	11	0.31 dB
(2)	11	0.63 dB	13	0.62 dB	11	0.10 dB
Improvement by (2)	-	0.07 dB	-	0.09 dB	-	0.21 dB

We investigated the performance of the OFC with high modulation indices under the scheme in Figure 1. Although due to the constraint of practical equipment the modulation indices were not larger than 1.5 in Section 3, we increased the modulation indices from 1.5 to 9 in the simulation to investigate the number of comb lines and flatness of the generated OFC, which is shown in Table 3. Generally, a higher modulation index will generate more comb lines under a certain requirement of flatness. However, since the

fundamental tone is not used to generate the OFC in this study, the number of comb lines will not be increased when we increase the modulation indices of the RF driving signal, although the flatness performance can be improved.

Table 3. The effect of modulation indices on the performance of generated OFC under scheme in Figure 1. Case I: second harmonic (12 GHz) and third harmonic (18 GHz) drive the DPMZM and PM, respectively. Case II: third harmonic (18 GHz) and second harmonic (12 GHz) drive the DPMZM and PM, respectively. Case III: second harmonic (12 GHz) and fifth harmonic (30 GHz) drive the DPMZM and PM, respectively.

Case	Upper Bound of β	β_m	β_n	Flatness (dB)	Comb Lines
I	1.5	1.19	1.50	0.63	11
	3.0	2.17	1.54	0.26	11
	5.0	2.15	1.54	0.23	11
	7.0	2.21	1.55	0.17	11
	9.0	2.04	1.53	0.20	11
II	1.5	1.19	1.40	0.62	13
	3.0	2.44	1.39	0.49	13
	5.0	2.35	1.39	0.51	13
	7.0	2.35	1.39	0.47	13
	9.0	2.77	1.39	0.33	13
III	1.5	0.99	1.43	0.10	11
	3.0	0.83	1.42	0.14	11
	5.0	1.39	1.43	0.16	11
	7.0	1.09	1.42	0.22	11
	9.0	1.15	1.42	0.23	11

5. Conclusions

In this work, we proposed a novel scheme to generate a flat OFC via cascaded DPMZM and PM using harmonics. Different from previous work to generate OFCs using EOMs, the proposed scheme does not need the fundamental tone to drive the EOMs, the frequency of which is the same as the repetition rate of the OFC. In the proposed scheme, the two EOMs are driven via two harmonics, the orders of which are coprime. When the third and second harmonics drive the first branch of the DPMZM and PM, respectively, a 13-comb line OFC was generated. The corresponding flatness was 0.62 dB and 0.95 dB under the simulation and experiment, respectively. This approach is also effective when the second and third harmonics or the second and fifth harmonics drive two EOMs. We also compared the performance of the generated OFCs under the proposed scheme with those under the scheme in which one harmonic drives both branches of the DPMZM and the other harmonic drives the PM. The same performance on the number of comb lines and better performance on flatness were achieved, verifying the effectiveness of the proposed scheme.

Author Contributions: Conceptualization, S.Z. and Z.W.; software, S.Z.; validation, S.Z., Z.W., X.Z., C.M., Y.J. and J.Y.; writing—original draft preparation, S.Z.; writing—review and editing, Z.W., X.Z., C.M., Y.J. and J.Y.; supervision, Z.W., X.Z., C.M., Y.J. and J.Y. All authors have read and agreed to the published version of the manuscript.

Funding: This research was supported by the National Natural Science Foundation of China (grant numbers: 61835003 and 62005194).

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Data are contained within the article.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Kobayashi, T.; Sueta, T.; Cho, Y.; Matsuo, Y. High-repetition-rate optical pulse generator using a Fabry-Perot electro-optic modulator. *Appl. Phys. Lett.* **1972**, *21*, 341–343. [CrossRef]
2. Torres-Company, V.; Weiner, A.M. Optical frequency comb technology for ultra-broadband radio-frequency photonics. *Laser Photon. Rev.* **2014**, *8*, 368–393. [CrossRef]
3. Fujiwara, M.; Teshima, M.; Kani, J.I.; Suzuki, H.; Takachio, N.; Iwatsuki, K. Optical carrier supply module using flattened optical multicarrier generation based on sinusoidal amplitude and phase hybrid modulation. *J. Light. Technol.* **2003**, *21*, 2705. [CrossRef]
4. Wang, J.; Cai, H.; Chen, D.; Qu, R. Generation of ultra-flat optical frequency comb using a balanced driven dual parallel Mach-Zehnder modulator. *Chin. Opt. Lett.* **2015**, *13*, 060604. [CrossRef]
5. Chen, C.; Zhang, F.; Pan, S. Generation of Seven-Line Optical Frequency Comb Based on a Single Polarization Modulator. *IEEE Photon. Technol. Lett.* **2013**, *25*, 2164–2166. [CrossRef]
6. Wu, R.; Supradeepa, V.R.; Long, C.M.; Leaird, D.E.; Weiner, A.M. Generation of very flat optical frequency combs from continuous-wave lasers using cascaded intensity and phase modulators driven by tailored radio frequency waveforms. *Opt. Lett.* **2010**, *35*, 3234–3236. [CrossRef]
7. Lin, T.; Zhao, S.; Zhu, Z.; Li, X.; Qu, K. Generation of Flat Optical Frequency Comb Based on a DP-QPSK Modulator. *IEEE Photon. Technol. Lett.* **2017**, *29*, 146–149. [CrossRef]
8. Tran, T.T.; Song, M.; Song, M.; Seo, D. Highly flat optical frequency comb generation based on pulse carving and sinusoidal phase modulation. *Opt. Eng.* **2019**, *58*, 076103. [CrossRef]
9. Gheorma, I.L.; Gopalakrishnan, G.K. Flat frequency comb generation with an integrated dual-parallel modulator. *IEEE Photon. Technol. Lett.* **2007**, *19*, 1011–1013. [CrossRef]
10. Sakamoto, T.; Kawanishi, T.; Izutsu, M. Asymptotic formalism for ultraflat optical frequency comb generation using a Mach–Zehnder modulator. *Opt. Lett.* **2007**, *32*, 1515–1517. [CrossRef]
11. Sakamoto, T.; Kawanishi, T.; Izutsu, M. Widely wavelength-tunable ultra-flat frequency comb generation using conventional dual-drive Mach–Zehnder modulator. *Electron. Lett.* **2007**, *43*, 1. [CrossRef]
12. Xu, M.; He, M.; Zhu, Y.; Yu, S.; Cai, X. Flat optical frequency comb generator based on integrated lithium niobate modulators. *J. Light. Technol.* **2022**, *40*, 339–345. [CrossRef]
13. Wang, Q.; Huo, L.; Xing, Y.; Zhou, B. Ultra-flat optical frequency comb generator using a single-driven dual-parallel Mach-Zehnder modulator. *Opt. Lett.* **2014**, *39*, 3050–3053. [CrossRef] [PubMed]
14. Shang, L.; Li, Y.; Ma, L.; Chen, J. A flexible and ultra-flat optical frequency comb generator using a parallel Mach-Zehnder modulator with a single DC bias. *Opt. Commun.* **2015**, *356*, 70–73. [CrossRef]
15. Dou, Y.; Zhang, H.; Yao, M. Generation of Flat Optical-Frequency Comb Using Cascaded Intensity and Phase Modulators. *IEEE Photon. Technol. Lett.* **2012**, *24*, 727–729. [CrossRef]
16. Ozharar, S.; Quinlan, F.; Ozdur, I.; Gee, S.; Delfyett, P.J. Ultraflat Optical Comb Generation by Phase-Only Modulation of Continuous-Wave Light. *IEEE Photon. Technol. Lett.* **2008**, *20*, 36–38. [CrossRef]
17. Yan, J.; Zhang, S.; Xia, Z.; Bai, M.; Zheng, Z. A tunable optical frequency comb generator using a single dual parallel Mach-Zehnder modulator. *Opt. Laser Technol.* **2015**, *72*, 74–78. [CrossRef]
18. Yokota, N.; Abe, K.; Mieda, S.; Yasaka, H. Harmonic superposition for tailored optical frequency comb generation by a Mach-Zehnder modulator. *Opt. Lett.* **2016**, *41*, 1026–1029. [CrossRef]
19. Cui, Y.; Wang, Z.; Xu, Y.; Jiang, Y.; Yu, J.; Huang, Z. Generation of Flat Optical Frequency Comb Using Cascaded PMs with Combined Harmonics. *IEEE Photon. Technol. Lett.* **2022**, *34*, 490–493. [CrossRef]
20. Cui, Y.; Wang, Z.; Zuo, X.; Xu, Y.; Jiang, Y.; Yu, J.; Huang, Z. Flat optical frequency comb generation by using one DPMZM and superposed harmonics. *Opt. Commun.* **2023**, *531*, 129223. [CrossRef]
21. Weber, H.J.; Arfken, G.B. *Essential Mathematical Methods for Physicists*; Elsevier Academic Press: San Diego, CA, USA, 2004.
22. Feoktistov, V. *Differential Evolution—In Search of Solutions*; Springer: New York, NY, USA, 2006.
23. Storn, R.; Price, K. Differential evolution–a simple and efficient heuristic for global optimization over continuous spaces. *J. Glob. Optim.* **1997**, *11*, 341–359. [CrossRef]

Disclaimer/Publisher's Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.