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Analysis of Error Sources in the Lissajous Scanning Trajectory Based on Two-Dimensional MEMS Mirrors

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Abstract: Deviations or distortions in the trajectoy of MEMS-based Lissajous scanning imaging platforms might be detrimental to imaging quality. These deviations often arise from differences in MEMS mirror frequency response characteristics and asymmetry in parameters within the measurement and control circuit. This study concentrated on the measurement and control circuit unit as it identified and analyzed four error sources: the MEMS mirror frequency response error, the AD acquisition synchronization error, the drive source error, and the cross-coupling error between the MEMS mirror axes. This study constructed a Lissajous trajectory test platform based on oscilloscopes and a position sensitive detector. Consequently, its experimental results guided the error processing methods to access the feasibility of the compensation methods by combining measured trajectories. Overall, regarding MEMS-based Lissajous scanning platforms for biomedical imaging, this study could provide quantitative numerical references for error analysis, image reconstruction, and aberration correction.

Keywords: two-dimensional MEMS mirrors; Lissajous scanning; trajectory distortion; error sources

1. Introduction

The MEMS mirror, employing Micro-Electro-Mechanical Systems (MEMS) and beam control technologies, offers advantages such as rapid response, compact size, and relatively wide scanning angles [1,2]. Two-dimensional MEMS mirrors have extensive applications in portable and miniature microscopic imaging instruments, including confocal microscopy [3–5], nonlinear microscopy [6–10], and photoacoustic microscopy [11–13]. They come in various types, driven by different mechanisms like electrostatic, electromagnetic, thermoelectric, and piezoelectric methods [1]. Among these, electrostatic MEMS mirrors offer the advantage of a relatively higher scanning frequency to enable faster frame rates. Researchers have explored various scanning schemes, including raster scanning, spiral scanning, and Lissajous scanning [14]. Lissajous scanning, in particular, offers light uniformity, trajectory parameter selection flexibility, and robust adaptability [15,16].

Several research groups have explored MEMS-based Lissajous scanning techniques for confocal and two-photon microscopy. Previous work primarily focused on aspects like the frequency selection principle of the Lissajous patterns, the design and optimization



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Copyright: © 2023 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (https:// creativecommons.org/licenses/by/ 4.0/). of MEMS mirrors, enhancements to optical and mechanical probe schemes, as well as hardware and software techniques to improve image quality [17–20]. However, limited research has focused on characterizing the control circuitry within Lissajous imaging systems, specifically in indentifying the error sources. The complex control circuit, coupled with several error factors, can introduce deviations or distortions into Lissajous trajectories, posing challenges for image reconstruction and imaging quality degradation. Moreover, closed-loop control strategies for integrated portable/miniature confocal and two-photon microscopy systems remain a persistent challenge when relying on feedback from sensing elements. Alternatively, open-loop feedforward control strategies are typically employed, which necessitate prior knowledge of quantitative error data. This information proves crucial for fine-tuning control circuitry drive signal parameters during feedforward compensation control and determining key parameters, such as the amplitude and phase for image reconstruction. Consequently, investigating error sources within the measurement and control circuitry holds theoretical and practical importance in advancing MEMS-based biomedical platforms.

In this work, we performed an analysis of error sources in the Lissajous scanning trajectory based on two-dimensional MEMS mirrors. The theory of Lissajous graphs and signal parameters that define the Lissajous trajectory were introduced, providing a theoretical basis for subsequent error analysis and compensation. Furthermore, we identified four error sources within the measurement and control circuit unit: the MEMS mirror frequency response error, the AD acquisition synchronization error, the drive source error, and the cross-coupling error between MEMS mirror axes. We designed and implemented a test platform using oscilloscopes and a position sensitive detector (PSD). For each type of error source, experimental methods were developed to quantitatively determine the errors. Finally, error calibration experiments were conducted, to verify the existence of error types, assess their magnitudes, and demonstrate the feasibility of error compensation. This work could provide quantitative references for the implementation of error analysis, image reconstruction, and aberration correction in MEMS-based Lissajous scanning microscopy.

2. Methods

2.1. Lissajous Theory

The Lissajous trajectory represents the compounded path followed by a point undergoing harmonic motion along both x and y axes [21]. Its mathematical description can be expressed by Equation (1).

$$\begin{cases} x = A_x \sin(2\pi f_x t + \varphi_x) \\ y = A_y \sin(2\pi f_y t + \varphi_y) \end{cases}$$
(1)

 A_x and A_y are the amplitudes of the *x* and *y* axes, f_x and f_y are the frequencies of the *x* and *y* axes, and φ_x and φ_y are the initial phases of the *x* and *y* axes.

Applying the inverse sine function to both sides of Equations (1) and (2) is calculated as follows:

$$\begin{cases} 2\pi f_x t + \varphi_x + 2i\pi = \pm \sin^{-1}\frac{x}{A_x} \\ 2\pi f_y t + \varphi_y + 2j\pi = \pm \sin^{-1}\frac{y}{A_y} \end{cases}$$
(2)

where *i*, *j* are arbitrary integers. By performing calculations on Equation (2), Equation (3) is obtained as follows:

$$\frac{f_x}{f_y} = \frac{-\varphi_x - 2i\pi \pm \sin^{-1}\frac{x}{A_x}}{-\varphi_y - 2j\pi \pm \sin^{-1}\frac{y}{A_y}}.$$
(3)

The frequency ratio parameters $m = \frac{f_x}{gcd(f_x, f_y)}$ and $n = \frac{f_y}{gcd(f_x, f_y)}$ are defined, with $gcd(f_x, f_y)$ being the maximum common divisor of f_x and f_y . Taking $\frac{m}{n} = \frac{f_x}{f_y}$ (where m, n are mutually prime integers) into Equation (3), Equation (4) is deduced as follows:

$$m\varphi_y - n\varphi_x + 2(jm - in)\pi = \pm m\sin^{-1}\frac{y}{A_y} \mp n\sin^{-1}\frac{x}{A_x}.$$
(4)

By taking the sine of both sides of Equation (4), Equation (5) is presented as follows:

$$\sin(m\varphi_y - n\varphi_x) = \sin\left(m\sin^{-1}\frac{y}{A_y} \pm n\sin^{-1}\frac{x}{A_x}\right).$$
(5)

From the trajectory equation presented in Equation (5), the Lissajous trajectory is evidently determined by the frequency ratio parameters m, n and $m\varphi_y - n\varphi_x$ [22] when the values of amplitudes A_x and A_y are fixed. The trajectory refresh frequency (frame rate) of the Lissajous scanning trajectory corresponds to the maximum common divisor of f_x and f_y .

2.2. MEMS Scanning Lissajous Trajectory Test Platform Overview

To investigate error factors within the MEMS-based Lissajous scanning system, a test platform was constructed as illustrated in Figure 1. The platform mainly consisted of a customized ZYNQ control board, a high-voltage amplification circuit, a low-pass filtering circuit, a MEMS mirror, a laser, a PSD, an analog-to-digital module (AD), a personal computer (PC), and oscilloscopes.



Figure 1. The schematic diagram of the MEMS-based Lissajous trajectory test platform. PSD: position sensitive detector. AD: analog-to-digital module. PC: personal computer. DC power: direct current power. Red arrows indicate the optical path. Blue arrows signify the signal paths generated by the signal generator and observed by the oscilloscope. Finally, the black arrows represent the signal paths generated by the ZYNQ control board and the X/Y trajectory paths.

The ZYNQ control board was used to generate the differential signals for driving the MEMS mirror, with a phase difference of $\pm 180^{\circ}$. These signals offered programmable control over their frequency (range from 0 to 10 kHz), amplitude (range from 0 V to +10 V), and phase (range from 0° to 360°). The high-voltage amplification circuit generated a high-voltage differential driving signal with a DC bias of 80 V and a peak-to-peak voltage ranging from -150 V to +150 V. The signal contained high-frequency noise, which was filtered out by the low-pass filtering circuit with a cutoff frequency of 20 kHz before being utilized to drive the MEMS mirror.

The laser (M-16A650-10, Hengjiu LED, Guangzhou, China) operated at a wavelength of 650 nm, delivered 10 mW of power, and featured an adjustable output spot diameter. The fixtures were used to position the laser and the MEMS mirror, ensuring that the laser beam was incident onto the center of MEMS mirror surface at a 45° angle with slightly shifts in the incident point position within $\pm 4^{\circ}$ range. During scanning, the reflected

beam was incident on the PSD, which converted the light trajectory into electrical signals. After being processed by the filtering circuit, these signals could be either visualized on an oscilloscope to observe the scanning trajectory or collected by the AD module and subsequently uploaded to the PC. The digital oscilloscope (TDS2012C, Tektronix, Beaverton, OR, USA) boasted a bandwidth of 100 MHz and a sampling rate of 2 GS/s. Given that the signal frequencies did not exceed 5 kHz, the presence of the oscilloscope had a negligible impact on the measurement and control circuitry. In YT mode and XY mode, the single and combined electrical characteristics could be observed, respectively.

For the AD acquisition module, the PXI card (PXI3223) was employed, featuring eight parallel acquisition channels, 14-bit acquisition precision, adjustable sampling rates ranging from 1 to 20 MSa/s, and a measurement range of ± 10 V. The PC was a chassis-controller configuration manufactured by National Instruments. The PSD (PDP90A, Thorlabs, Newton, NJ, USA) was selected for its 15 kHz bandwidth, a typical output signal amplitude accuracy of 0.3 mV, and a detection area of up to 9 mm \times 9 mm.

3. Experimental Validation

In the experimental validation, we focused on four error sources in MEMS-based Lissajous scanning platforms: the MEMS mirror frequency response error, the AD acquisition synchronization error, driver source errors, and the cross-coupling error between MEMS mirror axes. These error sources were analyzed and tested, as described in Sections 3.1–3.4.

3.1. Frequency Response Error of a Two-Dimensional MEMS Mirror

A two-dimensional quasi-static MEMS mirror (A3I12.2, Mirrorcle Technologies, Richmond, CA, USA) was selected for the Lissajous scanning platform [23], as depicted in Figure 2. This aluminum-coated scanning mirror operated with a differential drive on both of its axes, featuring a mirror diameter of 1.2 mm, a maximum differential voltage of 150 V, and a single-ended signal DC bias of 80 V. The mechanical deflection angle of the mirror had a range of up to $\pm 5.4^{\circ}$, with resonant frequencies of 2937 Hz for the X axis and 2913 Hz for the Y axis. This mirror exhibited operational capability across the frequency range from zero to its resonant frequencies, thereby offering flexible frequency selectivity for Lissajous trajectory [24].



Figure 2. A photograph of a two-dimensional MEMS mirror.

To investigate the influence of frequency characteristics on the trajectory of the MEMS mirror, we designed a model for its frequency response test, as depicted in Figure 3 [25]. The control circuit generated two-channel high-voltage differential signals with a DC bias of 80 V and a phase difference of π , which drove both the X and Y axes of the MEMS mirror. The signals induced mechanical angular vibrations of the mirror, resulting in the generation of a Lissajous trajectory [26]. A PSD was employed to convert the light signal into electrical signals along both axes [27]. Oscilloscopes were used to measure the amplitude and phase information of the Lissajous trajectory. Given the sufficiently high bandwidth of the PSD



and the oscilloscopes compared to the MEMS mirror, the obtained results reflected the operational characteristics of the MEMS mirror.

Figure 3. The MEMS mirror frequency response test model.

To account for the MEMS's mirror lifespan and practicality, this study limited the operating frequency to 2600 Hz. The output amplitudes of the measurement and control circuit (also the input amplitude of the MEMS mirror), denoted as V_x , and V_y , were restricted to 40% of the full-scale range V_{max} . Employing the model presented in Figure 3, we conducted frequency response tests for both axes of the MEMS mirror. The measured output amplitudes of the PSD signals were recorded as V'_x , and V'_y . The measured phase differences between the input MEMS drive signals and PSD output signals were measured as $\Delta \varphi_x = \Theta_x - \Phi_x$, and $\Delta \varphi_y = \Theta_y - \Phi_y$. At a specific frequency of 50 Hz, the measured output amplitudes of the PSD signals were recorded as V'_{x0} and V'_{y0} . The measured amplitude of the PSD signals were recorded as V'_{x0} and V'_{y0} . The measured amplitude of the PSD signals were recorded as V'_{x0} and V'_{y0} . The measured amplitude of the PSD signals versus the input amplitude ratio curves is depicted in Figure 4a. Based on this figure, when the input amplitude frequency response curves obtained in this situation are depicted in Figure 4b. In this figure, the horizontal axis represents the frequency, and the vertical axis represents the output amplitude ratios of two axes, $\frac{V'_x}{V''_{x0}}$ and $\frac{V'_y}{V''_{y0}}$.

the phase frequency response curves obtained in this situation are depicted in Figure 4c. The horizontal axis represents the frequency, and the vertical axis represents the phase differences of two axes, $\Delta \varphi_x$ and $\Delta \varphi_y$. The variations in the MEMS mirrors' X and Y axes, in conjunction with their associated DA circuits, high-voltage amplification circuits, and low-pass filtering circuits, led to differences in the frequency response characteristics. The experimental results indicated that when the frequency was below 800 Hz, the frequency response characteristics of both axes closely matched. However, within the frequency range of 800 Hz to 2600 Hz, the amplitude ratio of the X axis exceeded that of the Y axis by less than 0.1. The phase difference of the X axis exceeded that of the Y axis, with a difference within a range of 10° .

Furthermore, we selected specific drive signal parameters as follows: $f_x = 2300$ Hz, $f_x = 2300$ Hz, $V_x = V_y = 0.2$ V_{max} , and $\Phi_x = \Phi_y = 0^\circ$. By referring to Figure 4, we derived the actual scanning trajectory for the X axis, which had an amplitude of $V'_x = 0.51$ V'_{x0} = 1.91 V and a phase of $\Theta_x = -107^\circ$, and for the Y axis, which had an amplitude of $V'_y = 0.45$ $V'_{y0} = 1.43$ V and a phase of $\Theta_x = -112^\circ$. Note that the amplitudes and phases of the two axes were not equal. This quantitative information can be used to compensate for the frequency response error caused by the MEMS mirror.





Figure 4. The MEMS mirror frequency response characteristics. Red line: X axis. Blue line: Y axis. (a) Measured amplitude of the PSD output signals versus the input amplitude ratio curves. The input amplitude ratio varied from 0 to 0.4. (b) Amplitude frequency response curves. The frequency varied from 50 Hz to 2600 Hz in 50-Hz increments. The input amplitude ratio was set as 0.2. (c) Phase frequency response curves. The frequency varied from 50 Hz to 2600 Hz in 50 Hz increments. The input amplitude ratio was set as 0.2. (c) Phase frequency response curves. The frequency varied from 50 Hz to 2600 Hz in 50 Hz increments. The input amplitude ratio was set as 0.2.

3.2. AD Acquisition Synchronization Error

The AD circuit mainly comprises a conditioning circuit and an AD integrated chip. In cases involving multiple AD acquisition channels, differences in phase frequency characteristics arise due to the asymmetry of circuit parameters among the channels. When the frequency of the acquired signal varies across these channels, it exacerbates phase delays. Consequently, the initially set phase difference for the acquired signal deviates from its theoretical value, resulting in an AD acquisition synchronization error.

As per Lissajous graph theory, variations in phase between X and Y axes lead to alterations in the Lissajous trajectory [28]. To illustrate this point, we considered two sets of parameters, $V_x = V'_x = 5$ V, $V_y = V'_y = 5$ V, $f_x = f'_x = 1200$ Hz, and $f_y = f'_y = 1300$ Hz, differing only in the initial phase $\varphi_x = \varphi'_x = 0^\circ$, $\varphi_y = 0^\circ$, $\varphi'_y = 1^\circ$. Two theoretical Lissajous trajectories were plotted, as illustrated in Figure 5a,b. After zooming in Figure 5a, a deviation between the two theoretical trajectories became noticeable, as depicted in Figure 5b. This trajectory deviation could induce distortion during image reconstruction.

A signal generator was used to generate two sine signals for the X and Y channels, with the following parameters: $V_x = 5$ V, $f_x = 1200$ Hz, $\varphi_x = 0^\circ$ and $V_y = 5$ V, $f_y = 1300$ Hz, $\varphi_y = 0^\circ$. Subsequently, two independent parallel AD channels simultaneously captured the X and Y signals at a sampling rate set of 10M Sa/s, The overview and zoomed-in view of the uncompensated measured Lissajous trajectory, along with the theoretical trajectory 1, were plotted, respectively, as shown in Figure 6a,b. The synchronization between the acquisition of the two AD channels deviated, leading to a change in the initial phase difference between



the X and Y signals and causing an offset in the Lissajous trajectory. The uncompensated measured trajectory partially coincided with the theoretical trajectory 1.

Figure 5. Overall and zoomed-in diagrams of two theoretical Lissajous trajectories. (a) Overall view.(b) Zoomed-in view of the green dashed box labeled in (a).



Figure 6. Overall and zoomed-in diagrams of the measured and theoretical trajectories. (a) Overall view of the uncompensated measured and theoretical trajectory 1. (b) Zoomed-in view of the green dashed box labeled in (a). (c) Overall view of the compensated measured and theoretical trajectory 1. (d) Zoomed-in view of the green dashed box labeled in (c).

Furthermore, the initial phase of the X axis φ_x remained unaltered, and the initial phase of the Y axis φ_y was adjusted. When the compensated measured trajectory coincided with the theoretical trajectory 1, the initial phase of the Y axis, $\varphi'_y = 1.12^\circ$, corresponding to a phase adjustment, $\Delta \varphi_y = \varphi'_y - \varphi_y = 1.12^\circ$. $\Delta \varphi_y$ could be regarded as the AD acquisition

synchronization error under the specified parameters. As depicted in Figure 6c,d, we observed an improved overlap between the compensated measured Lissajous trajectory and the theoretical trajectory 1. Note that the phase adjustment values were determined for specific frequency parameters. When the frequencies of the X and Y axes signals changed, the phase adjustment values also altered accordingly.

3.3. Drive Source Error

Within the platform, the measurement and control unit generated the drive signals for the MEMS mirror, consisting of the voltage generation circuit, the bias conditioning circuit, the high-voltage amplifier circuit, and the low-pass filter circuit [29–32]. Due to asymmetry in the circuit parameters and frequency characteristics, the measured drive signals deviated from the theoretical signals, particularly in the amplitude and phase. These discrepancies in drive source resulted in a mismatch between the theoretical and measured trajectories [33].

By referring to Figure 1a, we used an oscilloscope to monitor the drive signals output by the high-voltage amplifier circuit at three frequencies: 2300 Hz, 1200 Hz, and 600 Hz. We quantified the measured amplitude differences and deviations in the phase differences between X2 and Y2 relative to their input signals, denoted as ΔA_{X2-Y2} and $\Delta Phase difference_{X2-Y2}$, respectively. As depicted in Figure 7a, the amplitude difference ΔA_{X2-Y2} exhibited an overall increasing trend, fluctuating within the range of -0.1 V to +0.9 V. Furthermore, as presented in Figure 7b, the deviations in phase differences $\Delta Phase difference_{X2-Y2}$ ranged from -1° to +0.6°.



Figure 7. The characteristics of the drive source errors at three different frequencies. (a) Amplitude difference ΔA_{X2-Y2} versus the input amplitude ratio curves. The input amplitude $V_{X2} = V_{Y2}$. (b) Phase difference deviation $\Delta Phase \ difference_{X2-Y2}$ versus input phase difference curves. The input amplitude $V_{X2} = V_{Y2} = 0.2 \ V_{max}$.

To assess the impact of the low-pass filter circuit on the drive source error, we performed tests on the X3 channels. The frequency response curves of the low-pass filter circuit are illustrated in Figure 8a,b. The input amplitude $V_{X2} = 0.2 V_{max}$, $\varphi_{X2} = 0^{\circ}$. The results indicated that the measured output amplitude ratio $\frac{A_{X3}}{A_{X2}}$ consistently exceeded 0.97, and the measured phase difference $\Delta \varphi_{X3-X2}$ varied between -1° and -15° .

To establish a reference for the Lissajous trajectory, the drive signal parameters were set as follows: $V_X = 0.2 V_{max}$, $V_Y = 0.2 V_{max}$, $f_X = 2300$ Hz, $f_Y = 1200$ Hz, $\frac{f_X}{f_Y} = \frac{m}{n} = \frac{23}{12}$, $\varphi_X = 0^\circ$, $\varphi_Y = 0^\circ$, and $m\varphi_Y - n\varphi_X = 0^\circ$ and were generated by the signal generator. The reference Lissajous trajectory was observed by an oscilloscope, as illustrated in Figure 9a,b.

The parameters were also set and output by the measurement and control circuit for testing the Lissajous trajectory. Figure 9c,d display the uncompensated Lissajous trajectory, exhibiting a reduction in the X axis amplitude. The measured amplitudes for X3 and Y3 axes were 13.3 V and 15.72 V, and the amplitude difference ΔA_{X3-Y3} was approximately -2.4 V, and the phase difference $\Delta \varphi_{X3-Y3}$ was approximately -6.3° . The calculated

 $m\varphi'_Y - n\varphi'_X \neq 0^\circ$, which resulted in a deviation from the reference Lissajous trajectory. Figure 9e,f show the compensated Lissajous trajectory. The X axis amplitude increased by 2.4 V, such that the amplitude difference $\Delta A_{X3-Y3} = 0$ V. The Y axis phase remained unchanged, while the X axis phase adjusted by $\Delta \varphi_X = 6.3^\circ$, such that $m\varphi'_Y - n(\varphi'_X + \Delta \varphi_X) = 0^\circ$. These results indicated that the compensated Lissajous trajectory aligned with the reference Lissajous trajectory.



Figure 8. Low-pass filter circuit frequency response characteristics. Frequency f_{X2} vary from 50 Hz to 2600 Hz in 50 Hz increments. (a) Amplitude frequency response curve. (b) Phase frequency response curve.



Figure 9. The overall and zoomed-in diagrams of the Lissajous trajectories. (**a**) The reference Lissajous trajectory. (**b**) The zoomed-in view of the white box in (**a**). (**c**) The uncompensated measured Lissajous trajectory. (**d**) The zoomed-in view of the white box in (**c**). (**e**) The compensated measured Lissajous trajectory. (**f**) The zoomed-in view of the white box in (**e**).

3.4. Cross-Coupling Error between the MEMS Mirror Axes

The two-dimensional MEMS mirror utilizes biaxial mechanical vibration to precisely control the reflector's position. It employs a double-axis orthogonal support beam structure. As such, machining process errors, including support beam width, unequal elasticity, and

mass asymmetry, can lead to inter-axis cross-coupling interference [34–36]. In [37], the presence of asymmetry along both axes, resulted in unequal elasticity, leading to the generation of coupling forces. The Equation describing the coupling force along the Y axis is as follows:

$$F_y = \frac{\sin 2\alpha}{2} (k_{xx} - k_{yy}) \cdot x, \tag{6}$$

where α represents the angle between the elastic and inertial principal axis, k_{xx} and k_{yy} are the stiffness coefficients along the X and Y axes, respectively, and variable 'x' denotes the displacement of the X axis support beam. The coupling force along the Y axis induces the deflection of its support beams around the X axis, subsequently driving the scanning mirror's deflection and generating a trajectory component in the Y axis.

Given the symmetry and similarities between the MEMS mirror's X and Y axes, we conducted the test on the X axis. Five sets of sinusoidal signals with different amplitudes V_X and frequencies f_X were applied to the X axis, while the amplitude of Y axis was set to zero. PSD was used to capture the trajectory of the MEMS mirror and convert it into electrical signals. The specific parameters of the X and Y signals were observed in the YT mode of the oscilloscope, and the trajectory was observed in the XY mode, as illustrated in Figure 10. In the first set of waveforms (a), (b), and (c), the amplitude was fixed at $V_X = 0.1 V_{max}$ and the frequencies were set to 2300 Hz, 1200 Hz, and 600 Hz, respectively. In the second set of waveforms (a), (d), and (e), the frequency was fixed at $f_X = 2300$ Hz, and the amplitudes V_X were set to 0.1 V_{max} , 0.2 V_{max} , and 0.3 V_{max} , respectively.



Figure 10. Cross-coupling error characteristics of the MEMS mirror's X axis. Yellow line: X axis. Blue line: Y axis. They indicate the results of the PSD waveform signals from the X and Y axes captured in the oscilloscope's YT mode at different drive conditions. (**a**,**b**) $V_X = 0.1 V_{max}$, $f_X = 2300$ Hz. (**c**,**d**) $V_X = 0.1 V_{max}$, $f_X = 1200$ Hz. (**e**,**f**) $V_X = 0.1 V_{max}$, $f_X = 600$ Hz. (**g**,**h**) $V_X = 0.2 V_{max}$, $f_X = 2300$ Hz. (**i**,**j**) $V_X = 0.3 V_{max}$, $f_X = 2300$ Hz.

As depicted in Figure 10a–c, when the amplitude was fixed at $V_X = 0.1 V_{max}$, the trajectory captured in the oscilloscope's YT Mode exhibited a component in the Y direction [38], with respective amplitudes of 8.8 mV, 9.8 mV, and 10.4 mV. These frequencies aligned with the X axis operating frequency, and the coupling error amplitude tended to decrease as the signal frequency increased. These variations in coupling interference amplitude were attributed to the amplitude frequency characteristics of the MEMS mirror, as discussed in Section 3.1. Specifically, the MEMS mirror exhibited low-pass characteristics, causing an increased amplitude attenuation and a decrease in the oscillation angle along the X axis with higher frequencies. Consequently, this attenuation weakened the coupling forces acting on the Y axis, resulting in reduced interference in the Y axis deflection. This reduction was manifested as a decrease in the amplitude component along the Y axis, consistent with experimental results.

As depicted in Figure 10a,d,e, when the frequency was fixed at $f_X = 2300$ Hz, there was a component in the Y direction, with respective amplitudes of 8.8 mV, 15.6 mV, and 22.6 mV. The coupling error amplitude tended to increase as the driving signal amplitude increased and the MEMS mirror vibration amplitude increased. As the amplitude of the MEMS mirror's X axis vibration increased, the amplitude of coupling errors in the Y axis also rose. This increase in coupling interference aligned with the experimental observations as the X axis amplitude continued to increase. The single axis cross-coupling error characteristics presented above revealed an inter-axis coupling interference phenomenon when the MEMS mirror was in operation. The coupling error was found dependent on the driving signal parameters, with a signal-to-noise ratio of approximately 40 dB.

In practical Lissajous scanning situation, effective drive signals were applied to both MEMS mirror axes. For example, a group of input drive signals paramters were considered: $V_X = V_Y = 0.1 V_{max}$, $f_X = f_Y = 1200$ Hz, and $\varphi_X = 0^\circ$, $\varphi_Y = 90^\circ$. As depicted in Figure 11a,b, the Y axis peak-to-peak signals were compensated by +0.4 V, while the X and Y axes' phase difference was compensated by +0.2°, effectively eliminating the drive source signal error (Section 3.3). Furthermore, by referring to Section 3.4, the Y axis peak-to-peak signals were compensated by +0.16 V, while the X and Y axes phase difference was compensated by -2.6°, as shown in Figure 12a,b, thus eliminating the effect of the MEMS mirror frequency response characteristics. The compensated waveforms and combined trajectory were then obtained, as shown in Figure 13a,b. The theoretical trajectory, representing a circle, closely matched the measured trajectory after eliminating the source signal and MEMS mirror frequency response errors. Additionally, the cross-coupling error (less than 20 mV) exhibited a minimal impact in the practical two-dimensional scanning applications.



Figure 11. Source drive signal waveforms captured in the YT mode. Yellow line: X axis. Blue line: Y axis. (a) Uncompensated X and Y axis waveforms. $V_X = 13.6$ V, $V_Y = 14$ V, $f_X = f_Y = 1200$ Hz. X and Y axes' phase difference of 89.8°. (b) Compensated X and Y axis waveforms. $V_X = 14$ V, $V_Y = 14$ V, $f_X = f_Y = 1200$ Hz. X and Y axes' phase difference of 90°.



Figure 12. Signal waveforms after eliminating the effect of MEMS mirror frequency response characteristics captured in the YT mode. Yellow line: X axis. Blue line: Y axis. (a) Uncompensated X and Y axis waveforms. $V_X = 0.88$ V, $V_Y = 1.04$ V, $f_X = f_Y = 1200$ Hz. X and Y axes' phase difference 92.6°. (b) Compensated X and Y axis waveforms. $V_X = 0.88$ V, $V_Y = 1.04$ V, $f_X = f_Y = 1200$ Hz. X and Y axes' phase difference 90°.



Figure 13. Compensated trajectory waveforms. Yellow line: X axis. Blue line: Y axis. (a) X and Y axis waveforms in the YT mode. $V_X = V_Y = 1.04$ V, $f_X = f_Y = 1200$ Hz. X and Y axes' phase difference 90°. (b) Combined trajectory in the XY mode.

4. Conclusions

This study aimed to analyze the error sources in MEMS-based Lissajous scanning platforms. To provide a theoretical foundation for error analysis and compensation, we introduced Lissajous graph theory and essential signal parameters. Furthermore, we identified four error sources within the measurement and control circuit and developed dedicated experimental methods for quantitative assessment. Finally, error calibration experiments were conducted, to verify the existence of error types, assess their magnitudes, and demonstrate the feasibility of error compensation. Overall, this study offers essential quantitative error references for implementing feedforward control and image reconstruction in MEMS-based Lissajous scanning microscopy.

In forthcoming research endeavors, we aspire to further develop Lissajous scanning miniature nonlinear microscopy using two-dimensional MEMS mirrors. In this case, our upcoming research focus will prioritize high-density scans of Lissajous trajectories, precise circuit synchronization, thus achieving the integration of miniature nonlinear probes for biomedical applications, including label-free endomicroscopic imaging and brain imaging in freely behaving mice. Author Contributions: Conceptualization, X.Z. and L.F.; methodology, X.Z., C.W., Y.H. (Yongxuan Han), J.W. (Junjie Wang), Y.H. (Yanhui Hu), J.W. (Jie Wang) and Q.F.; Validation, X.Z., C.W. and Y.H. (Yongxuan Han); software, X.Z., Y.H. (Yongxuan Han), Y.H. (Yanhui Hu) and J.W. (Jie Wang); writing—original draft preparation, X.Z. and Y.H. (Yongxuan Han); writing—review and editing, X.Z., C.W., Y.H. (Yongxuan Han), A.W., L.F. and X.H.; supervision, A.W., L.F. and X.H.; project administration, A.W., L.F. and X.H.; funding acquisition, A.W., L.F. and X.H. All authors have read and agreed to the published version of the manuscript.

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