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GENERAL FERMION TRIANGLE AMPLITUDE IN THE COUPLING OF SPIN-1 PARTICLES AND WARD IDENTITIES

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Abstract- The general off mass-shell one-loop amplitude for the fermion triangle diagrams is calculated, in theories with γ_5 couplings for spin-1 particles. Properties of the Bose symmetry and Ward identities for the couplings of neutral particles are investigated. The VVA and AAA triangle diagrams are obtained from our general form. All resulting amplitudes are consistent with Ward identities. The amplitude for the coupling of three identical spin-1 particles is given. This amplitude obeys Bose symmetry and Ward identities for anomaly free theory.

Key Words- Fermion triangle amplitude, Gauge bosons, Ward identities, Bose symmetry.

1. INTRODUCTION

It is well known that dimensional regularization method [1] preserves gauge invariance. It is also known that the extension of γ_5 to n-dimensions is ambiguous and contraversial in literature (see for a review Ref. [2] and Refs within). For triangle diagrams with one or more factors of γ_5, the problems of γ_5 in dimensional regularization were investigated in several subsequent attempts [3,4,5,6]. It was checked that totally anticommuting γ_5 is consistent with Ward identities for diagrams with an even numbers of γ_5's and for diagrams with mixed loops containing both boson and fermion propagators. For diagrams with one or three γ_5 matrices, the Adler-Bell-Jackiw anomalies [7] appear in the axial vector Ward identities and vanish after summation over internal fermions, if the theory is anomaly free (see, e.g. [8]).

In this paper we adopt the modified version of anticommuting γ_5 in n-dimensions given in Ref. [2] (see Appendix below). We calculate the most general amplitude for the fermion triangle diagram which produces contributions to the couplings of spin-1 particles at the one loop approximation. Our general diagram $\Pi^{\mu\nu\rho}$ contains all possible graphs with one or more γ_5's factors. All resulting amplitudes obtained from $\Pi^{\mu\nu\rho}$ satisfy vector Ward identities. For the axial vector Ward identities the ABJ anomaly part appears. We finally obtain the general amplitude of fermion triangle diagram for neutral spin-1 particles coupling $\Pi_0^{\mu\nu\rho}(p,r)$, which satisfies Bose symmetry for two vertices $\mu \leftrightarrow \nu$. For three identical neutral spin-1 particles coupling we have to symmetrize $\Pi_0^{\mu\nu\rho}$ to get the proper Bose symmetry in each pair of vertices

$(\mu \leftrightarrow \upsilon, \mu \leftrightarrow \rho,\ \upsilon \leftrightarrow \rho)$. The final resulting amplitude $\Gamma_0^{\mu\nu\rho}(p,r)$ obeys Bose symmetry required and satisfies axial vector Ward identities for anomaly free theories. The $\Pi_0^{\mu\nu\rho}(p,r)$ and $\Gamma_0^{\mu\nu\rho}(p,r)$ can be used to obtain the $Z\gamma\gamma$, $ZZ\gamma$ and ZZZ couplings, in the electroweak theory.

The paper is organized as follows: In the next section the general form $\Pi^{\mu\upsilon\rho}(p,r)$ is calculated. In section 3 we introduce the three point Green's functions, VVA and AAA diagrams, produced from $\Pi^{\mu\upsilon\rho}(p,r)$. In section 4 we show that Ward identities are satisfied for all investigated diagrams. In section 5, the general amplitude for the fermion triangle diagram of neutral spin-1 particles, which satisfies Bose symmetry and Ward identities, is given.

2- THE MOST GENERAL FORM FOR THE FERMION TRIANGLE DIAGRAMS

We calculate the general amplitude, off mass-shell, for the fermion triangle diagram fig.1 which gives the necessary contribution to the coupling between any three spin-1 particles at one loop order.

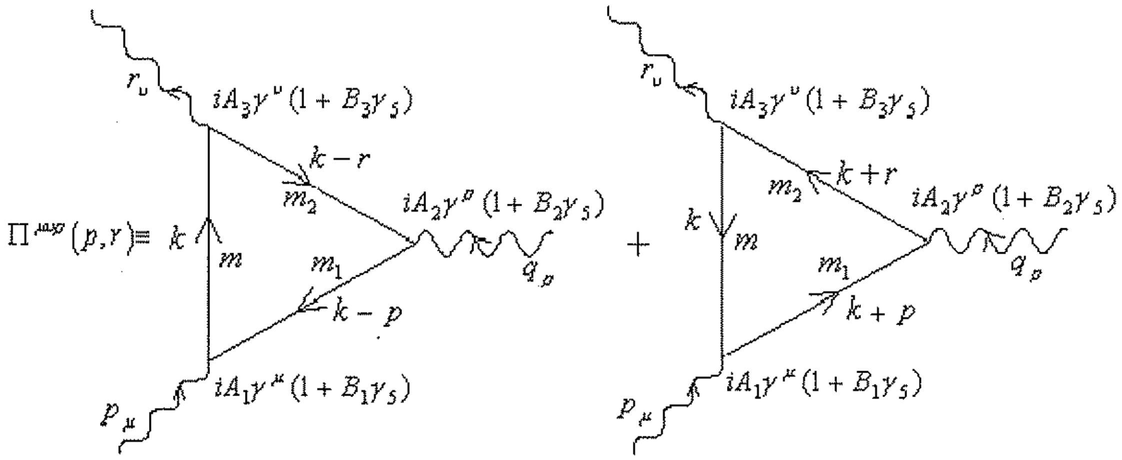


Fig.1: The most general fermion triangle diagram $\Pi^{\mu\upsilon\rho}(p,r)$ for the coupling of spin-1 particles at one loop approximation.

In Fig.1 the vertices are denoted generally by $iA_1\gamma^\mu(1+B_1\gamma_5)$, $iA_2\gamma^\rho(1+B_2\gamma_5)$, and $iA_3\gamma^\nu(1+B_3\gamma_5)$. The constants $A_i, B_i, i=1,2,3$ are general coupling constants, which can be defined depending on the kind of the spin-1 particle which couples with the fermion vertex. Inside the loop there are three different fermions of masses m, m_1, m_2 . Therefore the general $\Pi^{\mu\upsilon\rho}(p,r)$ fig.1 gives us four different kinds of diagrams (see fig.2). So, we have VVV diagram, VAA diagrams fig. 2a,b, which give zero contribution to $\Pi^{\mu\upsilon\rho}(p,r)$ because of cancellation between each other, VVA diagrams fig.2c and *AAA* diagram fig.2d which give non-zero contribution to the $\Pi^{\mu\upsilon\rho}(p,r)$ see the formula (2.2).

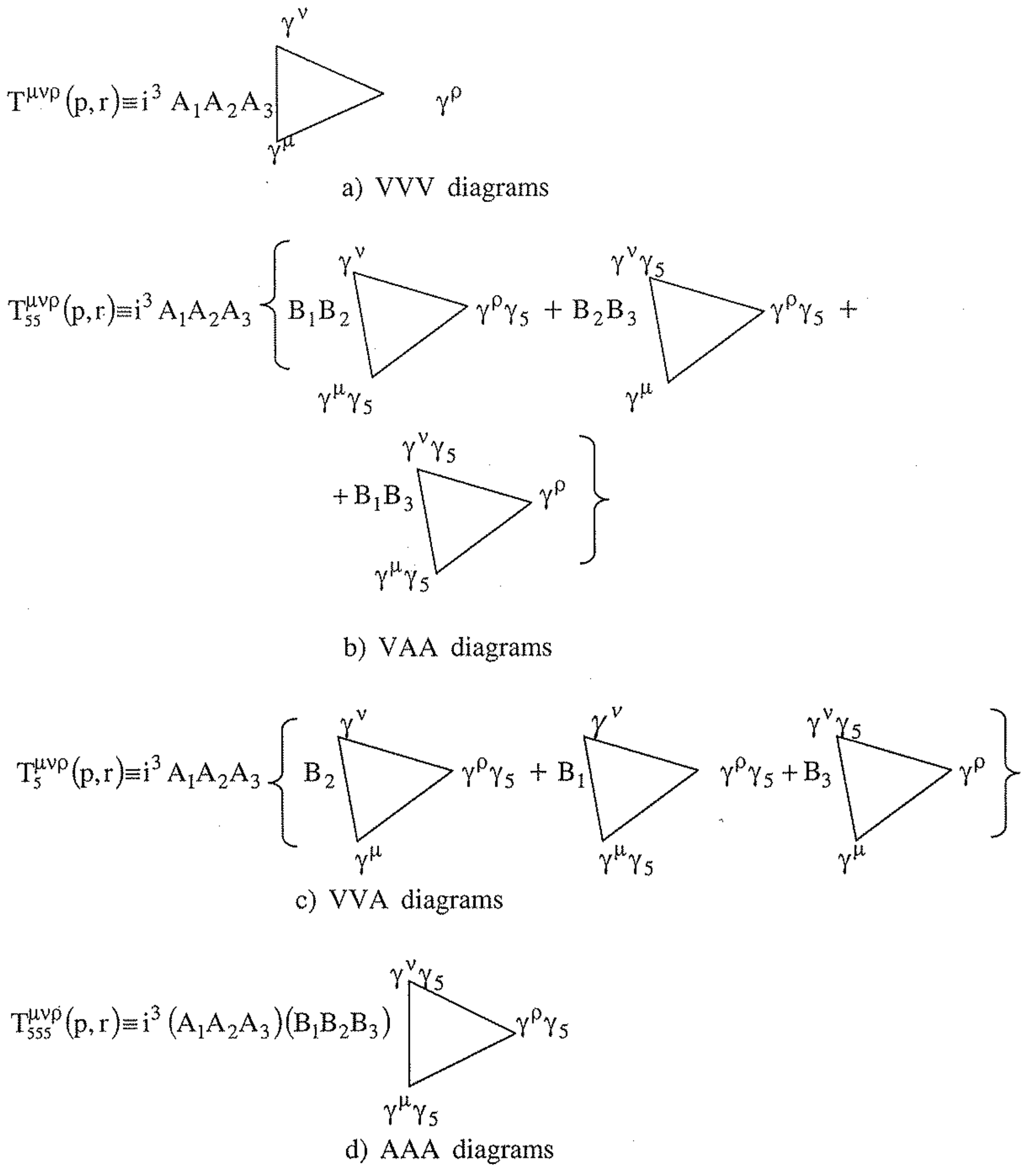


Fig.2: All kinds of diagrams exist in the general diagram Fig.1.

All graphs in fig.2 occur, for example, in the electroweak theory in the couplings between Z-boson, W-boson and photon. The diagrams fig.2c,d give non-zero contribution to ZZZ, $ZZ\gamma$ and $Z\gamma\gamma$ vertex functions.

Using Feynman parameterization and taking into account the t'Hooft-Veltman definition of the γ_5 in n-dimension [1] or the modified version of the anticommuting γ_5 [2], we find (p, r and q are the momenta of any spin-1 particles as shown in Fig.1):

$$\Pi^{\mu\nu\rho} = -i^6 \int \frac{d^n k}{(2\pi)^n} \mathrm{Tr}\left[A_1\gamma^\mu (1+B_1\gamma_5) \frac{1}{\hat{k}-\hat{p}-m_1+i\in} A_2\gamma^\rho (1+B_2\gamma_5) \right.$$
$$\left. \frac{1}{\hat{k}-\hat{r}-m_2+i\in} A_3\gamma^\nu (1+B_3\gamma_5) \frac{1}{\hat{k}-m+i\varepsilon} \right]$$
$$-i^6 \int \frac{d^n k}{(2\pi)^n} \mathrm{Tr}\left[A_1\gamma^\mu (1+B_1\gamma_5) \frac{1}{\hat{k}-m+i\in} A_3\gamma^\nu (1+B_3\gamma_5) \right.$$
$$\left. \frac{1}{\hat{k}+\hat{r}-m_2+i\in} A_2\gamma^\rho (1+B_2\gamma_5) \frac{1}{\hat{k}+\hat{p}-m_1+i\varepsilon} \right] \tag{2.1}$$

From (2.1) after some rather lengthly but straightforward manipulations we obtain $\Pi^{\mu\upsilon\rho}(p,r)$as:

$$\Pi^{\mu\nu\rho}(p,r) = \left[f_1(r+p)^\mu + f_2(r-p)^\mu\right] p_\alpha r_\beta \in^{\nu\rho\beta\alpha} + \left[f_3(r+p)^\nu + f_4(r-p)^\nu\right] p_\alpha r_\beta \in^{\mu\rho\beta\alpha}$$
$$+ \left[f_5(r+p)_\alpha + f_6(r-p)_\alpha\right] \in^{\mu\nu\alpha\rho} \tag{2.2}$$

where,

$$f_l = \eta \int_0^1 dx \int_0^x dy \, \frac{w_l}{D_3(x,y)}, \quad l = 1,2,,3,4.,$$

$$f_l = \int_0^1 dx \int_0^x dy \left\{ \eta \left[\frac{w_l}{D_3(x,y)} + a_l \ln D_3(x,y) \right] + \frac{b_l}{D_3(x,y)} \right\}, \quad l = 5,6.,$$

$w_1 = 4y(x-1)$, $w_2 = 4y(1+x-2y)$,

$w_3 = 4(x-1)(x-y)$, $w_4 = 4(x-y)(x-2y-1)$,

$w_5 = 2r^2(x-y)\left[(x-1)^2 + y(2-x)\right] + 2p^2 y\left[1-y(2-x)\right] + 4r\cdot p\left[y(x-2)(x-y)\right]$,

$w_6 = 2r^2\left[y\left(1-4x^2+5xy-2y^2\right) + x\left(x^2-1\right)\right] + 2p^2 y\left[1+xy-2y^2\right]$
$+4r\cdot p\, y(x^2-3xy+2y^2)$,

$a_5 = 2(2-3x)$, $a_6 = 6(2y-x)$, $b_5 = 2(\xi_1+\xi_3)(x-1) + 2\xi_2 x$,

$b_6 = 2\left[\xi_1(x-2y-1) + \xi_2(x-2y) + \xi_3(x-2y+1)\right]$,

$$\eta = \frac{1}{8\pi^2} A_1A_2A_3\left(B_1+B_2+B_3+B_1B_2B_3\right),$$

$$\xi_1 = \frac{1}{8\pi^2} A_1A_2A_3\left(B_1-B_2-B_3+B_1B_2B_3\right) m m_1,$$

$$\xi_2 = \frac{1}{8\pi^2} A_1 A_2 A_3 \left(B_2 - B_3 - B_1 + B_1 B_2 B_3\right) m_1 m_2,$$

$$\xi_3 = \frac{1}{8\pi^2} A_1 A_2 A_3 \left(B_3 - B_1 - B_2 + B_1 B_2 B_3\right) m m_2,$$

and

$$D_3(x,y) = p^2 y(1-y) + r^2 (x-y)(1-x+y) + 2rp\, y(y-x) + m^2 (x-1) - m_1^2 y + m_2^2 (y-x) + i\in$$

In (2.2) Schouten's identity [11] was used as:

$$X^\mu \in^{\nu\rho\sigma\tau} - X^\nu \in^{\mu\rho\sigma\tau} - X^\rho \in^{\nu\mu\sigma\tau} - X^\sigma \in^{\nu\rho\mu\tau} - X^\tau \in^{\nu\rho\sigma\mu} = 0, \tag{2.3}$$

which is valid for any four-vector X^μ. The formula (2.1) can produce the amplitude which contributes to the coupling of any three spin-1 particles (neutral or charged, e.g. in electroweak theory we can get $ZZZ,\cdots$ or $W^+W^-Z,\cdots$, ... etc) at the one loop approximation.

In the present work, we study the fermion triangle diagrams which contribute to the couplings of the neutral spin-1 particles, using the most general form (2.2).

3- THE THREE POINT GREEN FUNCTIONS VVA AND AAA -DIAGRAMS:

The 1PI Green functions [2] in Fig. 2c,d are essential to check the general form $\Pi^{\mu\nu\rho}(p,r)$. From (2.2) we obtain the VVA and AAA Green functions as:

3.1 The $T^{\mu\nu\rho}_{5(I)}(p,r)$– diagram

From our general form (2.2) putting $B_1 = B_3 = 0$ and $m = m_1 = m_2 = m_i$ we obtain $T^{\mu\nu\rho}_{5(I)}(p,r)$, Fig. 3, as

$$T^{\mu\nu\rho}_{5(I)}(p,r) = \left[f_1^I (r+p)^\mu + f_2^I (r-p)^\mu\right] p_\alpha r_\beta \in^{\nu\rho\beta\alpha} + \left[f_3^I (r+p)^\nu + f_4^I (r-p)^\nu\right] p_\alpha r_\beta \in^{\mu\rho\beta\alpha}$$
$$+ \left[f_5^I (r+p)_\alpha + f_6^I (r-p)_\alpha\right] \in^{\mu\nu\alpha\rho} \tag{3.1}$$

where,

$$f_l^I = 2\chi \int_0^1 dx \int_0^x dy\, \frac{w_l}{D_3(x,y)}, \qquad l = 1,2,3,4,$$

$$f_l^I = \chi \int_0^1 dx \int_0^x dy \left[\frac{w_l}{D_3(x,y)} + a_l\right], \qquad l = 5,6$$

$w_1 = y(x-1)$, $w_2 = y(1+x-2y)$, $w_3 = (x-1)(x-y)$,

$w_4 = x(x-1-3y) + y(1+2y)$,

$$w_5 = r^2 (x-y)\left[(x-1)^2 + y(2-x)\right] + p^2 y\left[1 - y(2-x)\right] + 2r\cdot p\, y(x-2)(x-y) - m_i^2 (x-2)$$

$$w_6 = r^2 \left[y(1-4x^2+5xy-2y^2) + x(x^2-1)\right] + p^2 y\left[1 + y(x-2y)\right]$$

$$+2r\cdot p\,y(x-y)(x-2y)-m_i^2(x-2y)$$

$$a_5 = (2-3x)\ln D_3(x,y)\,,\ a_6 = 3\,(2y-x)\ln D_3(x,y)\,,$$

and

$$\chi = \frac{1}{4\pi^2} A_1 A_2 A_3\, B_2\,,$$

$$D_3(x,y) = p^2 y(1-y) + r^2 (x-y)(1-x+y) + 2rp\, y(y-x) + m_i^2\,.$$

$T^{\mu\nu\rho}_{5(I)}(p,r) \equiv i^3 (A_1A_2A_3B_2) \times$

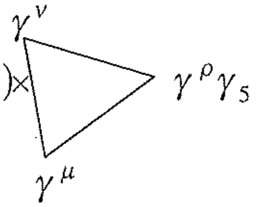


Fig.3: diagram obtained from (2.2) which is consistent with Ward identities.

Using a similar procedure as in Ref [5] (see also Ref. [2]) one gets

$$d = -\,p^2 d_1 - rp\, d_2\ ,\ \tilde{d} = -\,p^2 \tilde{d}_1 - rp\, \tilde{d}_2, \tag{3.2}$$

where

$$d = f_5^I + f_6^I,\ d_1 = f_1^I - f_2^I,\ d_2 = f_1^I + f_2^I,\ \tilde{d} = f_6^I - f_5^I,\ \tilde{d}_1 = f_3^I + f_4^I,$$

$$\tilde{d}_2 = f_3^I - f_4^I \tag{3.3}$$

Then

$$f_5^I = -\frac{1}{2}\Big[\, p^2 d_1 + r^2 \tilde{d}_1 + r\cdot p\,(d_2 + \tilde{d}_2)\Big],\ f_6^I = \frac{1}{2}\Big[\, r^2 \tilde{d}_1 - p^2 d_1\ \Big] \tag{3.4}$$

Similarly, Putting $B_2 = B_3 = 0$ and $m = m_1 = m_2 = m_i$ into (2.2) we get $T^{\mu\nu\rho}_{5(II)}(p,r)$, Fig. 4,

$T^{\mu\nu\rho}_{5(II)}(p,r) \equiv i^3 (A_1A_2A_3B_1) \times$

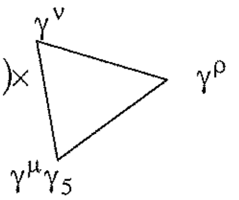


Fig.4: diagram obtained from (2.2) which is consistent with Ward identities.

And so, Putting $B_1 = B_2 = 0$ and $m = m_1 = m_2 = m_i$ into (2.2) we get $T^{\mu\nu\rho}_{5(III)}(p,r)$ (see Fig. 5)

$T^{\mu\nu\rho}_{5(II)}(p,r) \equiv i^3 (A_1A_2A_3B_3) \times$

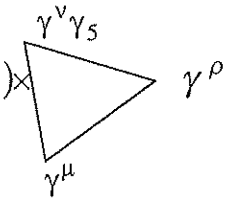


Fig. 5: *VVA* diagram which is consistent with Ward identities.

3.2 The $\tilde{T}_{555}^{\mu\nu\rho}(p,r)$– diagram

To get the diagram with three γ_5 Fig. 6, we take only the terms in (2.2) which contracted with $i^{3'}(A_1A_2A_3)(B_1B_2\,B_3)$, then we get

$$\tilde{T}_{555}^{\mu\nu\rho}(p,r)=\left[\tilde{f}_1(r+p)^{\mu}+\tilde{f}_2(r-p)^{\mu}\right]p_{\alpha}\,r_{\beta}\in^{\nu\rho\beta\alpha}+\left[\tilde{f}_3(r+p)^{\nu}+\tilde{f}_4(r-p)^{\nu}\right]p_{\alpha}\,r_{\beta}\in^{\mu\rho\beta\alpha}$$
$$+\left[\tilde{f}_5(r+p)_{\alpha}+\tilde{f}_6(r-p)_{\alpha}\right]p_{\alpha}\,r_{\beta}\in^{\mu\nu\alpha\rho}$$

(3.5)

where,

$\tilde{f}_l=B_1B_3\,f_l^{I}$, $l=1,2,3,4$

and

$$\tilde{f}_5=B_1B_3\,f_5^{(I)}+4\chi B_1B_3\;m_i^2\int_0^1 dx\int_0^x dy\,\frac{1}{D_3(x,y)}(x-1),$$

$$\tilde{f}_6=B_1B_3\,f_6^{(I)}+4\chi B_1B_3\;m_i^2\int_0^1 dx\int_0^x dy\,\frac{1}{D_3(x,y)}(x-2y),$$

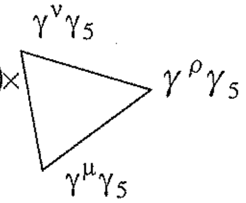
$\tilde{T}_{555}^{\mu\nu\rho}(p,r)\equiv i^3(A_1A_2A_3)(B_1B_2B_3)\times$ 

Fig. 6: *AAA* diagram which satisfies axial vector Ward identities.

4. WARD IDENTITIES AND THE TRIANGLE FERMION DIAGRAMS

To find the canonical Ward identities for the diagrams Fig. 2c, d, we use the same procedure as in Ref. [2]. Thus, we get for the 1PI Green functions in Fig. 2-c, d the next Ward identities:

4a). Green functions for VVA diagrams (Fig. 2c)

$$p_{\mu}\,T_{5(I)}^{\mu\nu\rho}(p,r)=0,\; r_{\nu}\,T_{5(I)}^{\mu\nu\rho}(p,r)=0 \tag{4.1}$$

$$q_{\rho}\,T_{5(I)}^{\mu\nu\rho}(p,r)=2m\,T_{5(I)}^{\mu\nu}(p,r) \tag{4.2}$$

$$r_{\nu}\,T_{5(II)}^{\mu\nu\rho}(p,r)=0,\; q_{\rho}\,T_{5(II)}^{\mu\nu\rho}(p,r)=0 \tag{4.3}$$

$$p_{\mu}\,T_{5(II)}^{\mu\nu\rho}(p,r)=2m\,T_{5(II)}^{\nu\rho}(p,r) \tag{4.4}$$

$$p_{\mu}\,T_{5(III)}^{\mu\nu\rho}(p,r)=0,\; q_{\rho}\,T_{5(III)}^{\mu\nu\rho}(p,r)=0 \tag{4.5}$$

$$-r_{\nu}\,T_{5(III)}^{\mu\nu\rho}(p,r)=2m\,T_{5(III)}^{\mu\rho}(p,r) \tag{4.6}$$

4b). Green functions for AAA diagram (Fig. 2d)

$$p_\mu \tilde{T}^{\mu\nu\rho}_{555}(p,r)=2m_i \tilde{T}^{\nu\rho}_{555}(p,r) \tag{4.7}$$

$$-r_\nu \tilde{T}^{\mu\nu\rho}_{555}(p,r)=2m_i \tilde{T}^{\mu\rho}_{555}(p,r) \tag{4.8}$$

$$q_\rho \tilde{T}^{\mu\nu\rho}_{555}(p,r)=2m_i \tilde{T}^{\mu\nu}_{555}(p,r) \tag{4.9}$$

Now, we will check the vector and the axial vector Ward identities (4.1) to (4.9) for the diagrams in Fig. 3 to Fig. 6. From (3.1) we have

$$p_\mu T^{\mu\nu\rho}_{5(I)}(p,r)=\left[p^2 d_1+r\cdot pd_2+d\right]p_\alpha r_\beta \in^{\nu\rho\beta\alpha} \tag{4.10}$$

$$-r_\nu T^{\mu\nu\rho}_{5(I)}(p,r)=\left[-r^2\tilde{d}_1-r\cdot p\tilde{d}_2+\tilde{d}\right]p_\alpha r_\beta \in^{\mu\rho\beta\alpha} \tag{4.11}$$

We see that from (3.2) the vector Ward identities (4.1) for the diagram, $T^{\mu\nu\rho}_{5(I)}(p,r)$, with one γ_5 are fulfilled which agrees with Ref. [2, 5]. For the axial vector Ward identity (4.2) we get

$$\begin{aligned} q_\rho T^{\mu\nu\rho}_{5(I)}(p,r)&=-4\chi m_i^2 p_\alpha r_\beta \in^{\mu\nu\beta\alpha}\int_0^1 dx\int_0^x dy\frac{1}{D_3(x,y)}-2\chi p_\alpha r_\beta \in^{\mu\nu\beta\alpha} \\ &=2m_i T^{\mu\nu}_{5(I)}(p,r)-2\chi p_\alpha r_\beta \in^{\mu\nu\beta\alpha}, \end{aligned} \tag{4.12}$$

where $T^{\mu\nu}_{5(I)}(p,r)$ is given by Appendix (A.6). The second term in (4.12) is the mass independent part which is known as the ABJ anomaly [7], this term has the ability to destroy unitarity and renormalizability [12]. But in an anomaly free theory this anomaly part vanishes and unitarity and renormalizability are restored [12].

Similarly, for diagrams $T^{\mu\nu\rho}_{5(II)}$ and $T^{\mu\nu\rho}_{5(III)}$ the vector Ward identities (4.3) and (4.5) are satisfied. The axial vector Ward identities (4.4) and (4.6) are satisfied for anomaly free theories. The axial vector Ward identities are:

$$\begin{aligned} p_\mu T^{\mu\nu\rho}_{5(II)}(p,r)&=-4\chi_1 m_i^2 p_\alpha r_\beta \in^{\nu\rho\beta\alpha}\int_0^1 dx\int_0^x dy\frac{1}{D_3(x,y)}-\chi_1 p_\alpha r_\beta \in^{\nu\rho\beta\alpha} \\ &=2m_i T^{\nu\rho}_{5(II)}(p,r)-\chi_1 p_\alpha r_\beta \in^{\nu\rho\beta\alpha}, \end{aligned} \tag{4.13}$$

$$\begin{aligned} -r_\nu T^{\mu\alpha\rho}_{5(III)}(p,r)&=+4\chi_2 m_i^2 p_\alpha r_\beta \in^{\mu\rho\beta\alpha}\int_0^1 dx\int_0^x dy\frac{1}{D_3(x,y)}+\chi_2 p_\alpha r_\beta \in^{\mu\rho\beta\alpha} \\ &=2m_i T^{\mu\rho}_{5(III)}(p,r)+\chi_2 p_\alpha r_\beta \in^{\mu\rho\beta\alpha}, \end{aligned} \tag{4.14}$$

where $T^{\nu\rho}_{5(II)}(p,r)$ and $T^{\mu\rho}_{5(III)}(p,r)$ are given by Appendix (A.7) and (A.8) respectively. For the diagram in Fig. 6, using (3.7) we get

$$\begin{aligned} q_\rho \tilde{T}^{\mu\nu\rho}_{555}(p,r)&=-4\chi B_1 B_3 m_i^2 p_\alpha r_\beta \in^{\mu\nu\beta\alpha}\int_0^1 dx\int_0^x dy\frac{(2x-1)}{D_3(x,y)}-2\chi B_1 B_3 p_\alpha r_\beta \in^{\mu\nu\beta\alpha} \\ &=2m_i \tilde{T}^{\mu\nu}_{555}(p,r)-2\chi B_1 B_3 p_\alpha r_\beta \in^{\mu\nu\beta\alpha}, \end{aligned} \tag{4.15}$$

$$p_\mu \tilde{T}^{\mu\nu\rho}_{555}(p,r)=4\chi B_1 B_3 m_i^2 p_\alpha r_\beta \in^{\nu\rho\beta\alpha} \int_0^1 dx \int_0^x dy \frac{(2x-2y-1)}{D_3(x,y)}$$
$$=2m_i \tilde{T}^{\nu\rho}_{555}(p,r), \tag{4.16}$$
$$-r_\nu \tilde{T}^{\mu\nu\rho}_{555}(p,r)=-4\chi B_1 B_3 m_i^2 p_\alpha r_\beta \in^{\mu\rho\beta\alpha} \int_0^1 dx \int_0^x dy \frac{(2y-1)}{D_3(x,y)}$$
$$=2m_i \tilde{T}^{\mu\rho}_{555}(p,r). \tag{4.17}$$

Where $\tilde{T}^{\mu\upsilon}_{555}(p,r)$, $\tilde{T}^{\upsilon\rho}_{555}(p,r)$ and $\tilde{T}^{\mu\rho}_{555}(p,r)$ are given in Appendix (A.9), (A.10), and (A.11) respectively. The axial vector Ward identities for $\tilde{T}^{\mu\nu\rho}_{555}(p,r)$ are satisfied for anomaly free theory. Note that Ward identities (4.7) and (4.8) are fulfilled directly for (3.7) (no anomaly parts exist, see (4.16) and (4.17)).

5. THE GENERAL AMPLITUDE FOR THE FERMION TRIANGLE DIAGRAM OF THE NEUTRAL SPIN-1 PARTICLES

The amplitude of a triangle fermion diagram with general vertices, Fig. 1, which gives a contribution to the coupling of any chargeless spin-1 particles obtained from (2.2) by taking into account (3.1), (3.5), (3.6), and (3.7) is

$$\Pi_0^{\mu\nu\rho}(p,r)=\left[F_1(r+p)^\mu + F_2(r-p)^\mu\right]p_\alpha r_\beta \in^{\nu\rho\beta\alpha} + \left[F_3(r+p)^\nu + F_4(r-p)^\nu\right]p_\alpha r_\beta \in^{\mu\rho\beta\alpha}$$
$$+\left[F_5(r+p)_\alpha + F_6(r-p)_\alpha\right]\in^{\mu\nu\alpha\rho} \tag{5.1}$$

where,

$$F_k=\sigma \int_0^1 dx \int_0^x dy \frac{1}{D_3^0(x,y)} c_k, \qquad k=1,2,\cdots,6$$

and

$$c_1=(\sigma_1+\sigma_2)y(x-1), \quad c_2=(\sigma_1+\sigma_2)y(1+x-2y), \quad c_3=(\sigma_1+\sigma_2)(x-1)(x-y),$$
$$c_4=(\sigma_1+\sigma_2)\left[x(x+3y-1)+y(2y+1)\right]$$
$$c_5=(\sigma_1+\sigma_2)\left[p^2 y(1-y)+r^2(x-y)(1-x+y)+2r\cdot p\, y(y-x)\right]+2\sigma_1 m_i^2(x-1)$$
$$c_6=(\sigma_1+\sigma_3)p^2 y(1-y)+(\sigma_1+\sigma_4)r^2(x-y)(x-y-1)+\sigma_5(2r\cdot p)y(x-y)+2\sigma_1 m_i^2(x-2y),$$

with

$$\sigma=\frac{1}{2\pi^2}A_1A_2A_3, \quad \sigma_1=B_1B_2B_3, \quad \sigma_2=B_1+B_2+B_3, \quad \sigma_3=B_2+2B_3, \quad \sigma_4=B_2+2B_1,$$
$$\sigma_5=B_1-B_3, \quad D_3^0(x,y)=p^2y(1-y)+r^2(x-y)(1-x+y)+2r\cdot p\,y(y-x)-m_i^2.$$

The general form (5.1) is useful in obtaining the amplitudes for any triangle graph with one or three γ_5. This form automatically satisfies Bose symmetry and Ward identities required (see the final form (5.6)). As we choose p, r as two independent four vector momenta, the form $\Pi_0^{\mu\nu\rho}(p,r)$ possesses the Bose symmetry

$$\Pi_0^{\mu\nu\rho}(p,r)=\Pi_0^{\nu\mu\rho}(-r,-p) \tag{5.2}$$

for two identical chargeless spin-1 particles with momentum p and r. But the amplitude $\Pi_0^{\mu\nu\rho}(p,r)$ has no proper Bose symmetry in the other vertices $(\mu \leftrightarrow \rho)$ and $(\nu \leftrightarrow \rho)$

$$\Pi_0^{\mu\nu\rho}(p,r,q)\neq\Pi_0^{\rho\nu\mu}(q,r,p) \tag{5.3}$$

and

$$\Pi_0^{\mu\nu\rho}(p,r,q)\neq\Pi_0^{\mu\rho\nu}(p,-q,-r) \tag{5.4}$$

To restore Bose symmetry let us define

$$\Gamma_0^{\mu\nu\rho}(p,r)=\frac{1}{6}\Big[\Pi_0^{\mu\nu\rho}(p,r,q)+\Pi_0^{\nu\mu\rho}(-r,-p,q)+\Pi_0^{\rho\nu\mu}(q,r,p)$$
$$+\Pi_0^{\mu\rho\nu}(p,-q,-r)+\Pi_0^{\nu\rho\mu}(-r,-q,p)+\Pi_0^{\rho\mu\nu}(q,-p,-r)\Big] \tag{5.5}$$

which by definition has Bose symmetry in each pair of vertices ($\mu\leftrightarrow\nu, \mu\leftrightarrow\lambda$, and $\nu\leftrightarrow\rho$). This should be the case of the coupling of three identical neutral particles as ZZZ vertex function. Then we obtain

$$\Gamma_0^{\mu\nu\rho}(p,r)=\left[E_1(r+p)^\mu+E_2(r-p)^\mu\right]p_\alpha r_\beta \in^{\nu\rho\beta\alpha}+\left[E_3(r+p)^\nu+E_4(r-p)^\nu\right]p_\alpha r_\beta \in^{\mu\rho\beta\alpha}$$
$$+\left[E_5(r+p)^\rho+E_6(r-p)^\rho\right]p_\alpha r_\beta \in^{\mu\nu\beta\alpha}+\left[E_7(r+p)_\alpha+E_8(r-p)_\alpha\right]\in^{\mu\nu\alpha\rho}, \tag{5.6}$$

where,

$$E_l=\frac{1}{3}h\int_0^1 dx\int_0^x dy\,\frac{C_l}{D_3^0(x,y)},\quad l=1,2,3,4,5,6,7,8$$

$$C_1=(x-1)(x+y),C_2=(x-1)(x+y)+2y(1-y),$$
$$C_3=(x-1)(2x-y),$$
$$C_4=(x-1)(x-2y)+(x-y)(x-2y-1),C_5=(x-1)(x-2y),$$
$$C_6=(2y-x)^2-x,$$
$$C_7=\left(p^2+r^2\right)y(y-x)+r\cdot p\left[x(1-x)+2y(x-y)\right]+\frac{h_2}{h}m_i^2(3x-2),$$
$$C_8=p^2y(2-x-y)+r^2\left[2x(x-1)+y(y-3x+2)\right]+r\cdot p(x-1)(2y-x)+$$
$$\frac{3h_2}{h}m_i^2(x-2y)$$

with,

$$h=\frac{1}{2\pi^2}A^3B(3+B^2),\quad h_1=\frac{1}{2\pi^2}A^3B(1+B^2),\qquad h_2=\frac{1}{2\pi^2}A^3B^3.$$

We put in (5.1) $A_1=A_2=A_3=A$ and $B_1=B_2=B_3=B$ for three identical external particles coupling.
The formula (5.6) satisfies Bose symmetry for all vertices and satisfies Ward identities as:

$$p_\mu \Gamma_0^{\mu\nu\rho}(p,r)=\left[(E_1-E_2)p^2+(E_1+E_2)r\cdot p+E_7+E_8\right]p_\alpha r_\beta \in^{\nu\rho\beta\alpha}$$
$$=2m_i T_{555}^{\nu\rho}(p,r)+2m_i T_5^{\nu\rho}(p,r)-\frac{1}{2\pi^2}A^3B\left(\frac{1}{3}B^2+1\right)p_\alpha r_\beta \in^{\nu\rho\beta\alpha} \tag{5.7}$$

$$-r_\nu \Gamma_0^{\mu\nu\rho}(p,r)=-\left[(E_3+E_4)r^2+(E_3-E_4)r\cdot p+E_7-E_8\right]p_\alpha r_\beta \in^{\mu\rho\beta\alpha}$$
$$=2m_i T_{555}^{\mu\rho}(p,r)+2m_i T_5^{\mu\rho}(p,r)+\frac{1}{2\pi^2}A^3B\left(\frac{1}{3}B^2+1\right)p_\alpha r_\beta \in^{\mu\rho\beta\alpha} \tag{5.8}$$

$$q_\rho \Gamma_0^{\mu\nu\rho}(p,r)=\left[(E_6-E_5)p^2+(E_6+E_5)r^2-2r\cdot p\,E_6-2E_7\right]p_\alpha r_\beta \in^{\mu\nu\beta\alpha}$$
$$=2m_i T_{555}^{\mu\nu}(p,r)+2m_i T_5^{\mu\nu}(p,r)-\frac{1}{2\pi^2}A^3B\left(\frac{1}{3}B^2+1\right)p_\alpha r_\beta \in^{\mu\nu\beta\alpha} \tag{5.9}$$

where $T_{555}^{\nu\rho}$, $T_5^{\nu\rho}$, $T_{555}^{\mu\rho}$, $T_5^{\mu\rho}$, $T_{555}^{\mu\nu}$, and $T_5^{\mu\nu}$ are given in Appendix by (A.10), (A.7), (A.11), (A.8), (A.9), and (A.6) respectively. Formulae (5.7), (5.8), and (5.9) are the expected correct results for Ward identities of the diagram $\Gamma_0^{\mu\nu\rho}$, since this diagram contains both AAA and VVA diagrams. The ABJ anomaly parts appear which cancel in anomaly free theory [8].

CONCLUSION

The general amplitude for the fermion triangle diagram which can produce any coupling between three spin-1 particles is obtained. For neutral spin-1 particles couplings, we get the general form which gives the VVA and AAA diagrams which are consistent with Ward identities. Our general form for neutral particles possesses Bose symmetry and satisfies Ward identities. In the axial vector Ward identities, Adler-Bell-Jackiw anomalies appear which vanish, however, after summation over internal fermions if the theory is anomaly free. All resulting amplitudes obey the proper Bose symmetry and vector Ward identities required. These amplitudes are consistent with the axial vector Ward identities in the case of anomaly free theory. The general form for neutral spin-1 particles can be used to calculate the $Z\gamma\gamma$, $ZZ\gamma$, and ZZZ amplitudes in the electroweak theory.

APPENDIX

1- γ_5 in n-dimensions

To calculate the fermion triangle diagrams we use the properties of the Dirac matrices γ^μ in n-dimensions

$$\left\{\gamma^\mu,\gamma^\nu\right\}=2g^{\mu\nu},\ g_\mu^\mu=n,\ \mathrm{Tr}\,I=4 \tag{A.1}$$

and the properties of γ^μ in n-dimensions as [2]

$$\{\gamma_5,\gamma^\mu\}=0 \text{ for } \mu=0,1,\cdots,n-1,\ \ \gamma_5^2=1,\mathrm{Tr}\gamma_5=0 \tag{A.2}$$

$$\mathrm{Tr}\{\gamma_5\gamma^\mu\gamma^\nu\gamma^\alpha\gamma^\beta\}=-4i\in^{\mu\nu\alpha\beta} \tag{A.3}$$

and

$$\mathrm{Tr}\{\gamma_5\gamma^\alpha\gamma^\beta\}=0,\ \mathrm{Tr}\left\{\gamma_5\gamma^{\alpha_1}\gamma^{\alpha_2}\cdots\gamma^{\alpha_{2i-1}}\right\}=0 \tag{A.4}$$

$$\begin{aligned}\mathrm{Tr}\{\gamma^\alpha\gamma^\mu\gamma^\beta\gamma^\nu\gamma^\gamma\gamma^\lambda\gamma_5\}=-4i\big[&g^{\gamma\lambda}\in^{\alpha\mu\beta\nu}+g^{\mu\beta}\in^{\alpha\nu\gamma\lambda}+g^{\alpha\mu}\in^{\beta\nu\gamma\lambda}\\&-g^{\beta\alpha}\in^{\mu\nu\gamma\lambda}+g^{\mu\lambda}\in^{\alpha\beta\nu\gamma}-g^{\beta\lambda}\in^{\alpha\mu\nu\gamma}\big].\end{aligned} \tag{A.5}$$

2- The three point Green's functions VVP and AAP-diagrams

We calculate separately all necessary Green's functions (VVP and AAP-diagrams) to check Ward identities. Then, using the properties of γ_5 given in appendix-1 and Feynman parameterization. The diagrams in fig A1 calculated as

$$T^{\mu\nu}_{5(I)}(p,r)=-\frac{1}{2\pi^2}A_1A_2A_3B_2\ m_i\int_0^1 dx\int_0^x dy\ \frac{1}{D_3(x,y)}p_\alpha r_\beta\in^{\mu\nu\beta\alpha} \tag{A.6}$$

$$T^{\nu\rho}_{5(II)}(p,r)=-\frac{1}{2\pi^2}A_1A_2A_3B_1\ m_i\int_0^1 dx\int_0^x dy\ \frac{1}{D_3(x,y)}p_\alpha r_\beta\in^{\nu\rho\beta\alpha} \tag{A.7}$$

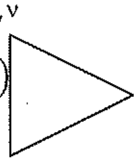
$$T^{\mu\rho}_{5(III)}(p,r)=\frac{1}{2\pi^2}A_1A_2A_3B_3\ m_i\int_0^1 dx\int_0^x dy\ \frac{1}{D_3(x,y)}p_\alpha r_\beta\in^{\mu\rho\beta\alpha} \tag{A.8}$$

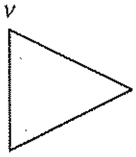
$$\tilde{T}^{\mu\nu}_{555}(p,r)=\frac{-1}{2\pi^2}(A_1A_2A_3)(B_1B_2B_3)\,m_i\int_0^1 dx\int_0^x dy\ \frac{1}{D_3(x,y)}(2x-1)p_\alpha r_\beta\in^{\mu\nu\beta\alpha} \tag{A.9}$$

$$\tilde{T}^{\nu\rho}_{555}(p,r)=\frac{1}{2\pi^2}(A_1A_2A_3)(B_1B_2B_3)\,m_i\int_0^1 dx\int_0^x dy\ \frac{1}{D_3(x,y)}(2x-2y-1)p_\alpha r_\beta\in^{\nu\rho\beta\alpha} \tag{A.10}$$

$$\tilde{T}^{\mu\rho}_{555}(p,r)=\frac{1}{2\pi^2}(A_1A_2A_3)(B_1B_2B_3)\,m_i\int_0^1 dx\int_0^x dy\ \frac{1}{D_3(x,y)}(1-2y)p_\alpha r_\beta\in^{\mu\rho\beta\alpha} \tag{A.11}$$

where $D_3(x,y)$ is the same as in (3.1).

$T^{\mu\nu}_{5(I)}(p,r)=i^3\,(AB_2)$ 

$T^{\nu\rho}_{5(II)}(p,r)=i^3\,(AB_1)$ 

a) b)

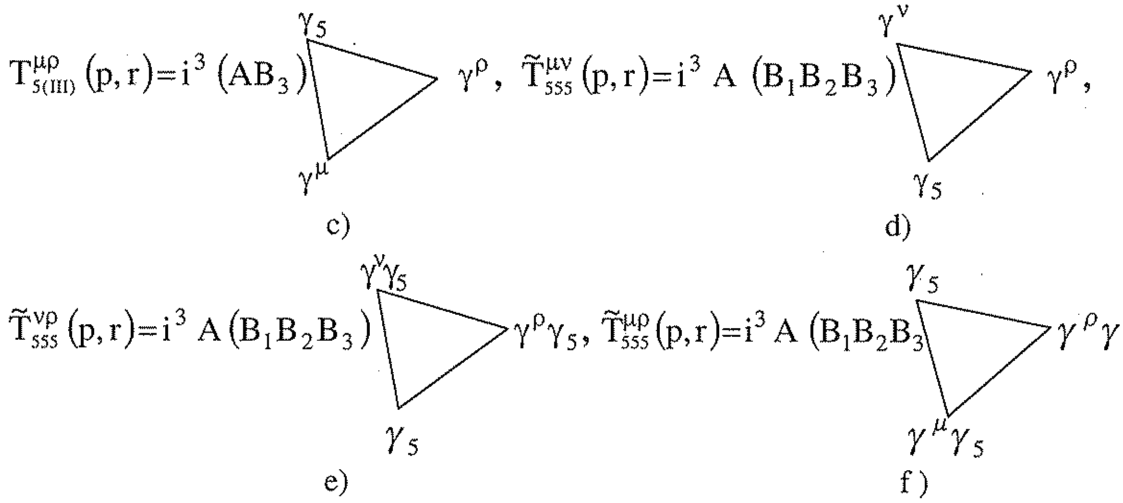


Fig. A1: a), b), c) The VVP diagrams $T^{\mu\nu}_{5(I)}$, $T^{\nu\rho}_{5(II)}$ $T^{\mu\rho}_{5(III)}$; d), e), f) The AAP diagrams $\tilde{T}^{\mu\nu}_{555}$, $\tilde{T}^{\nu\rho}_{555}$, $\tilde{T}^{\mu\rho}_{555}$. All diagrams satisfy Ward identities.

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