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EFFECT OF FORWARD SPEED ON SHIP ROLLING AND STABILITY

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Abstract-Ship stability is evaluated at zero speed most of the time. Majority of the stability criteria is also based on the behavior of ships at standstill. However unlike fixed offshore platforms, ships are on the move due to their nature of operation. Therefore a ship's hydrostatic and hydrodynamic characteristics undergo changes because of the varying underwater volume, centers of buoyancy and gravity and pressure distribution. This work deals with the effects of forward speed on ship stability and motions, particularly on rolling motion in synchronous beam waves. An equation of nonlinear roll motion is chosen to calculate roll responses of a test vessel in beam seas. The speed of advance is incremented and roll responses are determined at each speed interval. Various characteristics of GZ curve for the selected test vessel are altered systematically to observe their effects on roll responses along with the forward speed. Several computer programs are also employed to handle colossal mathematical manipulations.

Keywords-Speed, Nonlinear Rolling, Stability, Ship Motions, Ship Safety

1. INTRODUCTION

Roll motion of ships is the center of interest among six modes of motion drawing considerable attention due to its disreputable role in maritime casualties. There has been a substantial amount of literature published on roll motion and prediction of ship roll responses both theoretically and experimentally. However it is somewhat hard to find many studies on ship stability considering the ship as a moving object within this context. That is, the influence of forward speed is somehow undermined or neglected on intact and roll motion stability even in the majority of existing stability standards.

The present work aims at addressing the importance of forward speed on what is so-called the 'dynamic ship stability' in beam waves. In order to predict a ship's steady state roll responses rather realistically, a nonlinear equation of roll motion has been utilized. This equation is then solved at each speed increment. A 50-m. test vessel is chosen to determine roll responses with changing forward speed and intact stability characteristics such as the metacentric height and angle of vanishing stability. A computer program called STAB is developed to handle repetitive roll response calculations easily for each case.

Sobolev and Obrastsov [1], showed that the forward speed affected intact statical ship stability notably. Furthermore, Obrastsov [2,3] conducted a series of model experiments to determine the changes in righting moments and suggested a method to calculate righting moments of a moving ship. Besides the variation of GZ curve, damping characteristics (both linear and nonlinear) of the vessel are also continuously changed with increasing speed of advance. A form of roll motion analysis of ship models was carried out at zero speed by Wright and Marshfield [4], to observe the susceptibility of roll motion to metacentric height and vanishing angle of stability. This work intends to broaden the previous work of Wright and Marshfield [4] incorporating forward speed into the assessment of overall ship stability and using an alternative mathematical model for the nonlinear roll motion.

2. NONLINEAR ROLL MOTION

Linear roll motion equation is relatively straightforward and well understood in all aspects and can be found in any basic ship stability textbook. Nonlinearities enter the equation mainly through damping and restoring terms assuming that the changes are small in the inertia term. As far as the external moments are concerned, regular sinusoidal beam waves are selected to account for it. There can be found roll models in the literature, which formulize the external moment to represent irregular seas as well. Nevertheless, linear wave representation has been stated to give satisfactory simulation results by a number of researchers. In general, the governing equation for nonlinear roll motion may be formulated mathematically as follows, [5,6,7]:

$$I\ddot{\phi} + M_D(\dot{\phi}, \phi) + M_R(\phi) = M_w(t) \qquad (1)$$

Where, M_D represents the damping term and M_R and M_W represent the righting and exciting moments respectively. If the damping moment is split into linear and nonlinear components and the inertia, restoring and exciting moments are replaced by their explicit counterparts, Eq.(1) takes the following form:

$$\left(I_{xx} + \delta I_{xx}\right)\ddot{\phi} + M_L\dot{\phi} + M_N\dot{\phi}\left|\dot{\phi}\right| + \Delta\left(C_1\phi + C_3\phi^3 + C_5\phi^5\right) = \omega_e^{\,2}\alpha_m I_{xx}\cos\omega_e t \qquad (2)$$

In Eq.(2) the squared velocity term is shown as $\dot{\phi}\left|\dot{\phi}\right|$ so that, when $\dot{\phi}$ changes sign this term also changes sign in order that the damping moment will always oppose the motion, [8,9]. When both sides of Eq. (2) is divided by (I_{xx} + δI_{xx}), and the mathematical expressions for coefficients C_1, C_3, C_5 are substituted respectively, it turns into a nondimesional second degree nonlinear differential equation as follows [10,11,12];

$$\ddot{\phi} + m_L\dot{\phi} + m_N\dot{\phi}\left|\dot{\phi}\right| + \omega_\phi^{\,2}\phi + r_3\phi^3 + r_5\phi^5 = \lambda\omega_e^{\,2}\alpha_m\cos\omega_e t \qquad (3)$$

where;

$$\omega_\phi^{\,2} = \frac{\Delta GM}{I_{xx} + \delta I_{xx}} \qquad (3a)$$

$$r_3 = \frac{4\omega_\phi^2}{\phi_v^2}\left[\frac{3A_{\phi v}}{GM\phi_v^2} - 1\right] \quad (3b)$$

$$r_5 = -\frac{3\omega_\phi^2}{\phi_v^4}\left[\frac{4A_{\phi v}}{GM\phi_v^2} - 1\right] \quad (3c)$$

$$m_L = \frac{M_L}{I_{xx} + \delta I_{xx}} \quad (3d)$$

$$m_N = \frac{M_N}{I_{xx} + \delta I_{xx}} \quad (3e)$$

Numerical values for the linear and nonlinear damping terms, namely m_L and m_N are obtained from the results of model test conducted at AMTE(H) (Admiralty Marine Technology Establishment, Hasler) with varying speed values [4].

In Eq.(3), number of ship's dynamic and environmental parameters besides damping are included in the analysis of ship rolling in beam waves through both the characteristics of GZ curve and the external moment on the right hand side of the equation. The righting arms curve is approximated as a quintic polynomial $GZ(\phi)=C_1\phi+C_3\phi^3+C_5\phi^5$. The coefficients of the quintic polynomial can be determined by using various characteristics of the GZ curve such as metacentric height GM, angle of vanishing stability ϕ_v, and area under the curve up to the angle of vanishing stability $A_{\phi v}$, [13]. Natural frequency of the ship, ω_ϕ and the encountering frequency ω_e are the other constituents of the equation.

Obviously, the closed form solution of the aforementioned nonlinear differential equation of roll motion does not exist. Nonetheless, there are number of approximate methods available to solve such equations. Perturbation method, method of averaging and harmonic balance may be given as examples of the approximate methods. In this work, the generalized form of the Duffing's method is utilized to solve Eq.(3). The very general form of the Duffing's equation can be stated as follows, Stoker [14]:

$$\ddot{\phi} + C\dot{\phi} + (\alpha\phi + \beta\phi^3 + \gamma\phi^5) = H\cos\omega t - G\sin\omega t \quad (4)$$

Where $Z = (H^2 + G^2)^{1/2}$ is the amplitude of the exciting force taken as constant, however the ratio H/G has to be determined. It is assumed that coefficients C, G, H, α and β are all small of order γ. Having performed necessary manipulations, the solution of Eq. (4) renders the following expression:

$$0.39 r_5^2\phi_a^{10} + 0.94 r_3 r_5 \phi_a^8 + \left[1.25 r_5\left(\omega_\phi^2 - \omega_e^2\right) + 0.56 r_3^2 + 0.063 m_N^2\omega_e^2\right]\phi_a^6$$
$$+\left[1.5 r_3\left(\omega_\phi^2 - \omega_e^2\right) + 0.5 m_L m_N \omega_e^2\right]\phi_a^4 + \left[\left(\omega_\phi^2 - \omega_e^2\right)^2 + m_L^2\omega_e^2\right]\phi_a^2 - \left(\lambda\alpha_m\omega_e^2\right)^2 = 0 \quad (5)$$

The above equation is a tenth order polynomial of the roll amplitude ϕ_a. Evidently, the solution of it yields ten roots of which some are complex conjugates for a given frequency of encounter. The real roots are the ones plotted against the encountering frequencies to signify the roll amplitudes [15,16,17,18].

3. CALCULATION PROCEDURE

For the numerical solution of Eq.(5), certain parameters pertinent to the ship and environmental characteristics have to be known or determined beforehand. In this analysis, the first three items governed by the GZ curve are thus directly obtained from the righting arms curve for each case. The natural frequency of roll on the other hand is obtained by a simple relation taking into account the ship's beam and metacentric height multiplied by a certain coefficient. The added mass moment of inertia is assumed to account for 25% of the mass moment of inertia, thus the nondimensional inertia term λ, can be calculated as 0.8. The maximum wave slope is defined as $\alpha_m=2\pi H_w/L_w$ where H_w represents the wave height of a linear long-crested beam wave whereas L_w is the wave length assuming that L_w is large compared to the breadth of the vessel. For all cases, $\alpha_m=0.15$ is selected which corresponds to approximately a 7-m. wave height in linear beam waves. Linear and nonlinear damping terms for the test vessel are supplied by Wright and Marshfield [4] based on the model test results for a 50-m. ship, Fig.1.

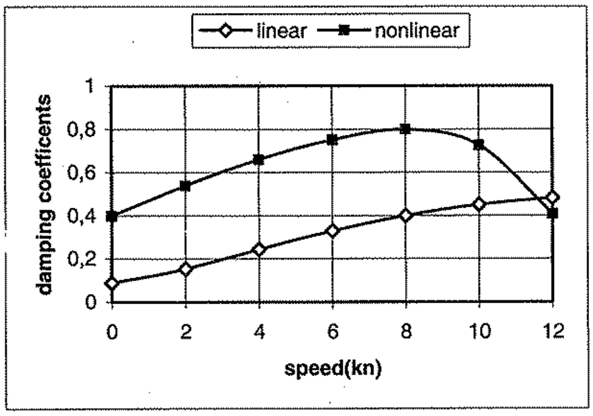


Fig. 1. Linear and nonlinear damping coefficients.

The computer program automatically handles the calculations itself and inputs the coefficients of the fifth order polynomial representing GZ curve. Having entered all the parameters into the program, the steady state roll responses are determined against a range of frequency of encounter for the resonance condition, which is considered to cause the maximum amplitudes of all. The frequency range for the responses lies between 0 and 1.5 rad/sec in general.

4. STABILITY ANALYSIS FOR THE TEST VESSEL

A test vessel having 50-m. length and 8-m. beam is selected to be analyzed. There are mainly four cases considered to investigate the effect of various stability parameters along with the increasing ship speed. Thus, theoretical GZ curves are constructed by keeping one or two parameters of the curve constant while varying the others. For all cases, the ship is assumed to satisfy IMO's intact stability criteria for ships under 100 m. in length which specify certain lower limits for the area under the GZ curve up to and between 30 and 40 degrees and for an initial GM and righting arms at certain heel angles. Four speed values are taken into account for all cases namely, 0 knots, 4 knots, 8 knots and 12 knots. The corresponding Froude number range falls in between 0.023 – 0.28. Maximum local wave slope is assumed to be constant for all cases α_m=0.15.

Case 1: Constant GM=0.45m. and Varying ϕ_v

In this case, Let's consider three different GZ curves with a constant GM=0.45 m. provided that GZ=0.2 m. at 30 degrees of heel as required by the criteria, Fig.2.

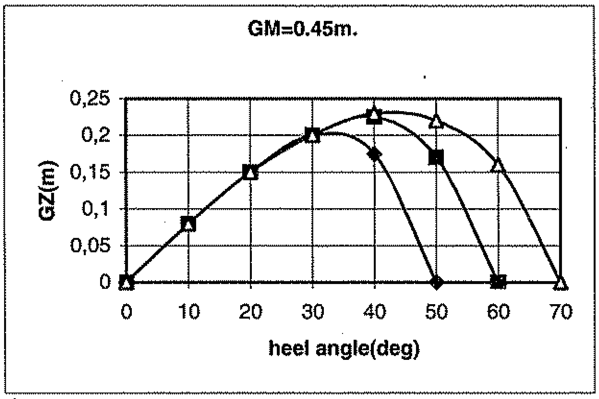


Fig.2. GZ curve for case 1.

The angle of vanishing stability is incremented by 10 degrees starting at 50 degrees, which in turn enlarges the area under the curve proportionally. All three conditions satisfy the IMO's criteria in full. The roll responses are determined for 50, 60 and 70-degree angle of vanishing stability and four different speed values. Natural frequency of roll is found out to be 0.704 rad/sec by the aforementioned procedure. Linear and nonlinear damping coefficients on the other hand, are read off from Fig.1 at corresponding speeds.

Case 2: Constant GM=0.60m. and Varying ϕ_v

This case is very similar to case 1 except the initial GM is increased to 0.60 m. and the rest of the parameters are kept the same and calculations are repeated as described above. For this case, the natural frequency is determined to be 0.81 rad/sec. As shown in Fig.3, the deviation in righting arm values starts at 30 degree of heel. The angle of vanishing stability values ϕ_v values are set to 50, 60 and 70 degrees.

Case 3: Constant ϕ_v=70 deg. and Varying GM

Here, the effect of varying GM on roll responses for advancing speed is sought. Therefore angle of vanishing stability is fixed at 70 degrees and GM intervals are taken as 0.15m., 0.30m. and 0.45m., Fig.4. Unlike the last one (GM=0.45m.), the first two situations do not meet IMO criteria. Natural frequencies are 0.405 rad/sec, 0.574 rad/sec and 0.704 rad/sec respectively, Fig.1.

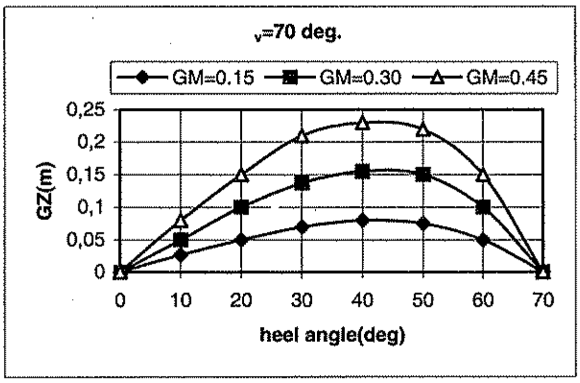


Fig. 3. GZ curve for case 2.

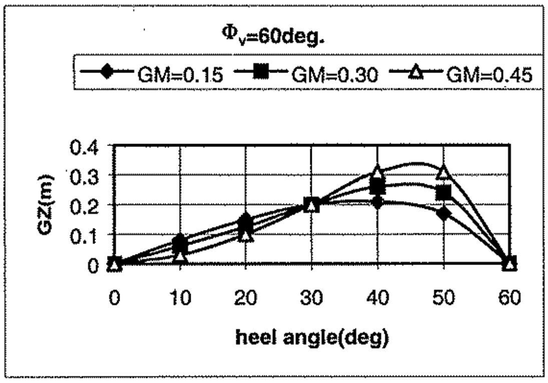


Fig. 4. GZ curve for case 3.

Case 4: Constant ϕ_v=60^0 and GZ=0.20m. (at 30deg.) and Varying GM

This condition evaluates roll responses at a constant 60-degree angle of vanishing stability with three previously mentioned GM values of 0.15m., 0.30m. and 0.45m. This time corresponding natural frequencies of roll are 0.408 rad/sec, 0.572 rad/sec and 0.704 rad/sec respectively. Ones again periodic responses of the model ship are determined for four speed values. Even though case 4 resembles case 3, it is quite different because GZ=0.2m is fixed at 30 degrees satisfying IMO's intact stability requirements, Fig.5.

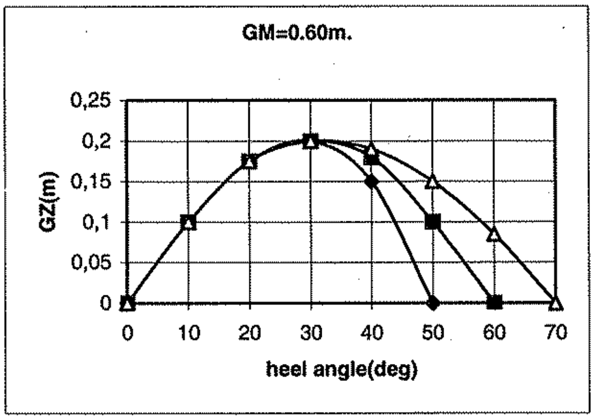


Fig. 5. GZ curve for case 4.

5. EFFECT OF SPEED ON INTACT STABILITY

Obratsov [2] have conducted series of experiments on the subject. The outcome of experiments indicated that the restoring moment varied with increasing speed due to changing pressure distribution on the underwater body of the ship. The change in pressure distribution influences the restoring moment in turn. There may be several factors that affect the pressure distribution such as, the discrepancy between the hydrostatic and hydrodynamic pressure distributions, free surface effects, additional forces on the appendages of a heeled ship and unsymmetrical boundary layer due to heel. It was concluded that the block coefficient played an important role on the restoring moment changes under similar conditions. There is a critical speed at which the restoring moment and the transverse stability is not affected at all. Exceeding this critical speed deteriorates stability. Finally, restoring moment changes are proportional to the metacentric height as well.

Sobolev and Obrastsov [1], further concluded that restoring moment might increase as much as 15-20% by increasing speed. Increasing block coefficient also enlarges the restoring moment. For widely used Froude numbers within the operational limits of ships, righting moment changes are always positive. For slender ships on the other hand, the fluctuations are rather small either positively or negatively. For lower Froude numbers (<0.25), a great portion of the additional restoring moment comes from the vortex component whereas for higher Froude numbers it is due to wave components of the hydrodynamic forces.

As can be deduced from the above-mentioned conclusions, increasing forward speed alone improves intact statical ship stability most of the time. The preceding discussion stated the importance of forward speed in terms of statical characteristics of ship stability. Additionally, the next section evaluates the effects of forward speed on restoring moment and roll motion stability. The GZ curve is assumed to be constant with increasing speed just to observe the effect of other parameters themselves.

6. EFFECT OF SPEED ON ROLL MOTION

In this present work, the effect of forward speed on periodic roll motion responses is implemented by taking into account various stability parameters. Since there are so many factors interacting simultaneously, it is very challenging to simulate the motion of a ship in a seaway. Existing studies on comparative results between the experiment and theory are scarce due to difficulties evolve from obtaining necessary input for the mathematical model and many setbacks in model experiments. It is the contemplation of this assessment to carry out a wide range of recursive and systematic investigations with different parameters.

Case 5: Constant GM=0.45m. and Varying ϕ_v

This condition analyzes the effect of forward speed on nonlinear responses with changing angle of vanishing stability. It is obvious that area under the GZ curve will also increase consequently. The areas under three curves are integrated to be 0.111; 0.147 and 0.181 m.radians corresponding to 50, 60 and 70 deg. angle of vanishing stability respectively. The computer program STAB is run repeatedly at each speed for three different ϕ_v values. The response curves are plotted comparatively with respect to frequency of encounter, Figures 6-8.

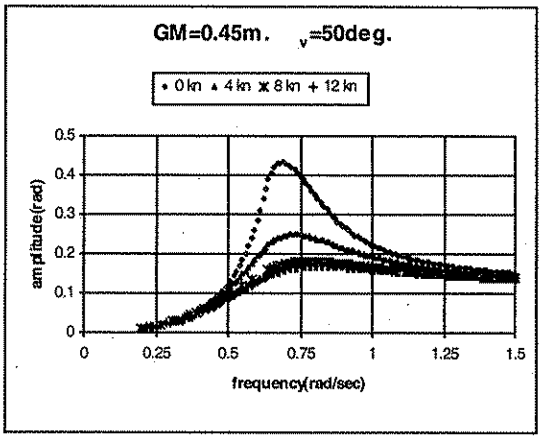


Fig. 6. Roll amplitudes for ϕ_v=50deg.

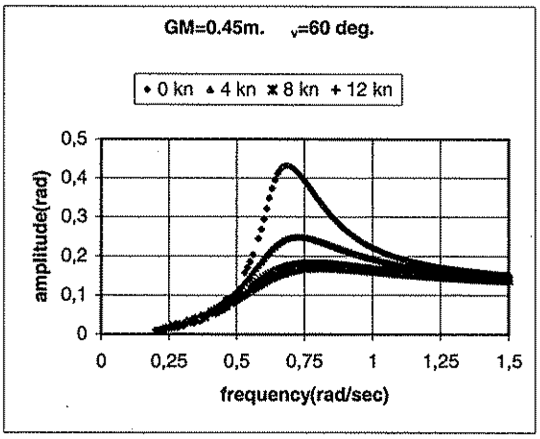


Fig. 7. Roll amplitudes for ϕ_v=60deg.

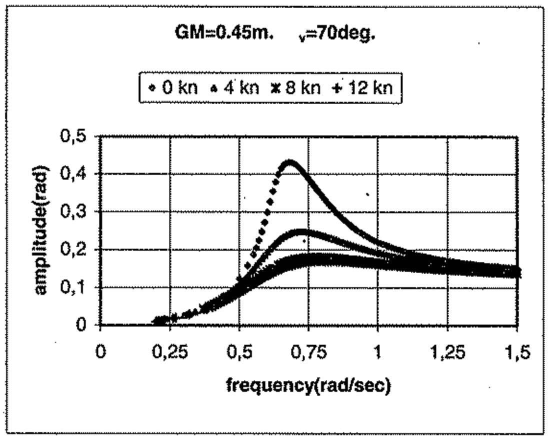


Fig. 8. Roll amplitudes for GM=0.45m. ϕ_v=70deg.

As shown in the figures, there is a decreasing trend in the peak roll amplitudes starting from standstill towards 12-knot speed. The discrepancy between the peak amplitudes is found to be as much as 0.264 radians (about 15 degrees). The maximum peak response is 0.433 radians (about 25 degrees) for the non-moving vessel. The trend implies that increasing speed improves rolling behavior of ships especially at relatively low speeds (from zero to 6 knots). There observed a critical Froude number (0.23) at which responses are no longer susceptible to speed variations for this particular test vessel. What is so-called the 'back bone' curve is also constructed by plotting peak amplitudes against speed for each GZ curve, Fig.9. Variation of peak amplitudes with changing angle of vanishing stability shows almost no variation from 50 through 70 degrees.

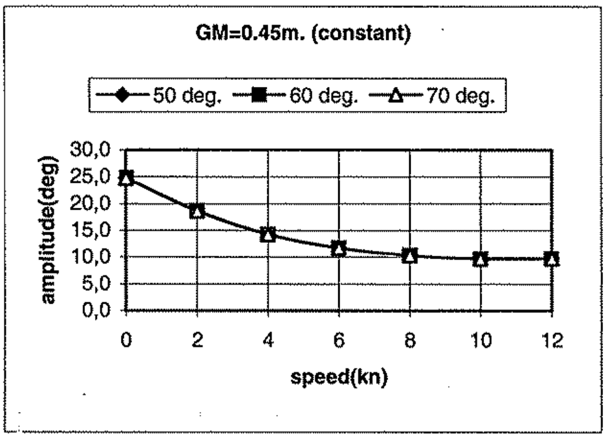


Fig.9. Maximum roll amplitudes vs speed, GM=0.45m

Case 6: Constant GM=0.60m. and Varying ϕ_v

Another configuration regarding the effect of ϕ_v at constant GM=0.60m over varying speed is considered separately. In this case, the previous trend hasn't been changed, roll responses decreased with increasing forward speed, Figures 10-12.

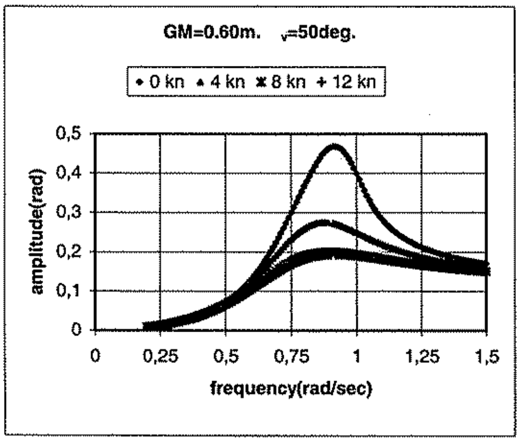


Fig. 10. Roll amplitudes for ϕ_v=50deg.

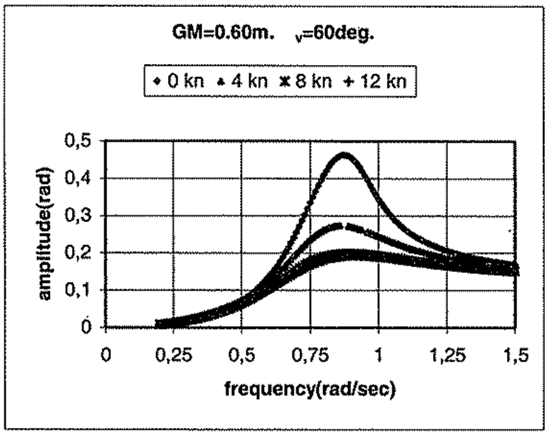


Fig. 11. Roll amplitudes for ϕ_v=60deg.

0.275 Radians (16 degrees) maximum difference in extreme peak amplitudes is also observed. Interestingly, the backbone curves revealed the same character as for GM=0.45m. showing almost no deviation at all, Fig.13.

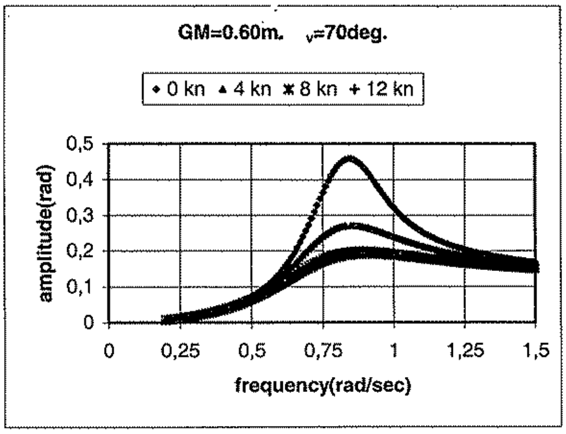


Fig. 12. Roll amplitudes for ϕ_v=70deg.

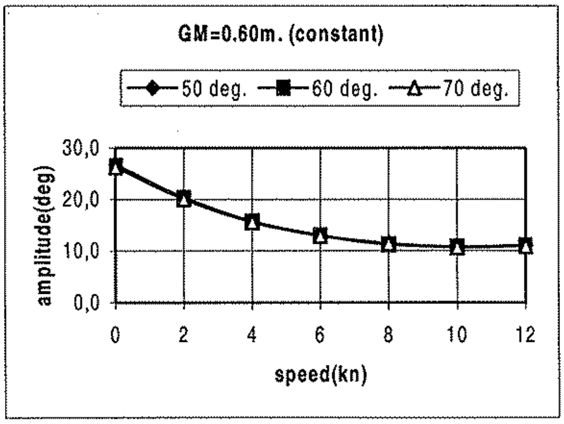


Fig.13. Maximum roll amplitudes vs speed

Case 7: Constant $\phi_v=70^0$ and Varying GM

The angle of vanishing stability is kept constant at 70 degrees for varying GM=0.15m.; 0.30m. and 0.45m., Figures 8, 14 and 15. Thus the corresponding areas under the curves are 0.063; 0.125 and 0.188 m.rad respectively. Only the latter condition meets the IMO's area criteria, however even these results have theoretical significance if not practical in the parametric study. The natural frequencies for this case are 0.405, 0.574 and 0.704 rad/sec.

The difference in peak amplitudes goes up with increasing speed as much as 0.263 radians (about 15 degrees) for GM=0.45m. Then the decrease continues in a more gradual manner up to 10 knots. Finally, if the speed is incremented to 12 knots, the responses show almost no change.

When the peak responses are analyzed in terms of increasing GM, it reveals about a 5-degree incline for the highest GM=0.45m compared to the lowest GM=0.15m., Fig.16.

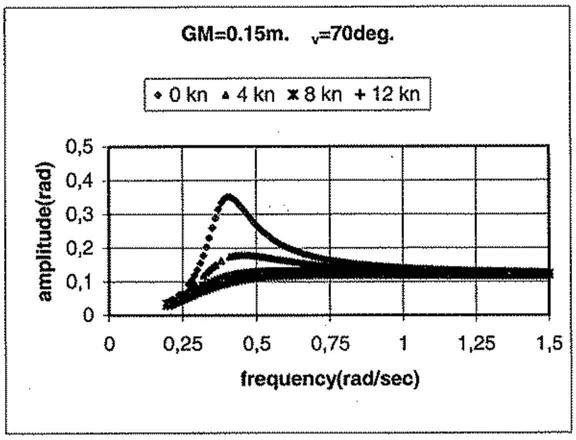


Fig. 14. Roll amplitudes for ϕ_v=70deg.

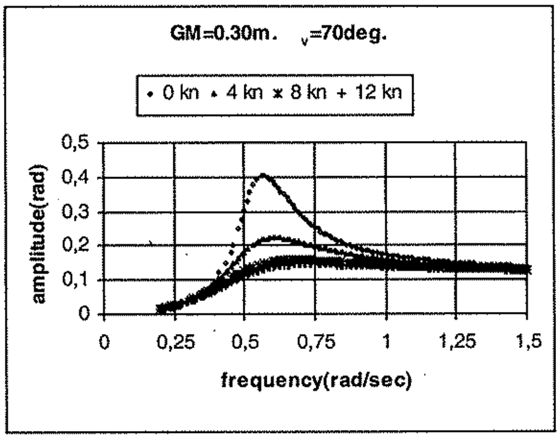


Fig. 15. Roll amplitudes for ϕ_v=70deg.

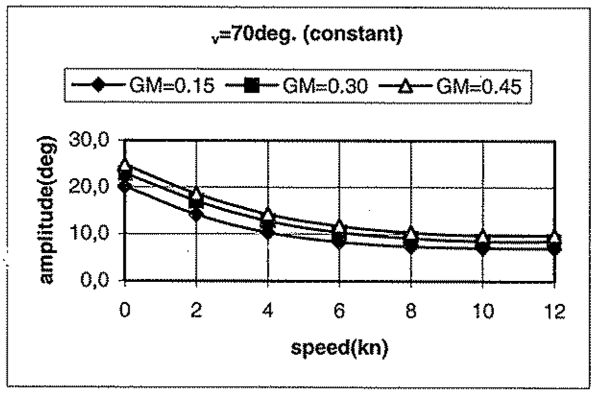


Fig. 16. Maximum roll amplitudes vs speed (70 deg.).

Case 8: Constant ϕ_v=60^0 and GZ=0.20m. (at 30deg.) and Varying GM

Finally, this case is comparable to the previous case except the fixed angle of vanishing stability is lowered to 60 degrees and GZ is also fixed as 0.2m at 30 degrees of heel all satisfying IMO criteria. GM is varied from 0.15m. to 0.45m as was before, Figures 17-19.

The peak amplitude difference is about 15 degrees between zero and highest speeds for each GM value. This time the peak value slightly increased than that of case 7. However, the difference in peaks between GM=0.45m and GM=0.15m. is approximately halved at a particular speed value compared to case 7, Fig.20.

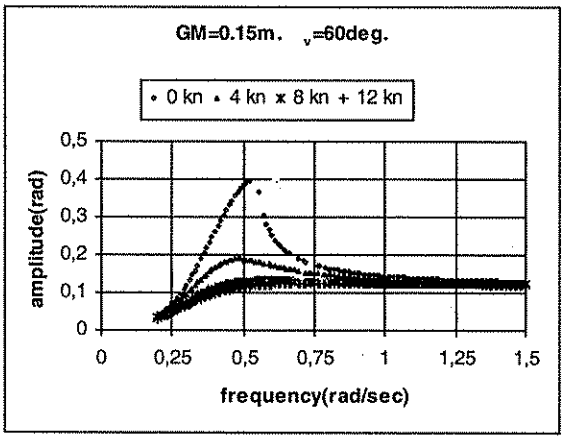


Fig. 17. Roll amplitudes for ϕ_v=60deg.

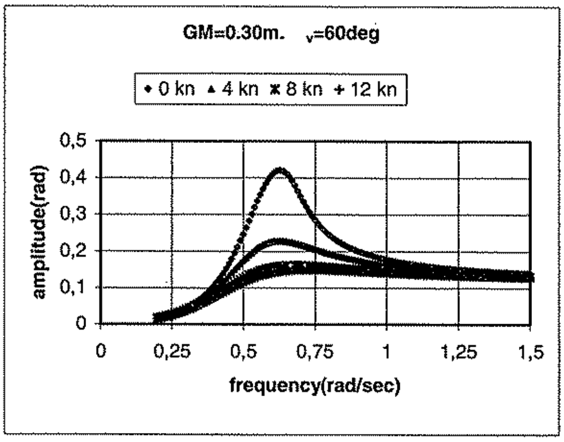
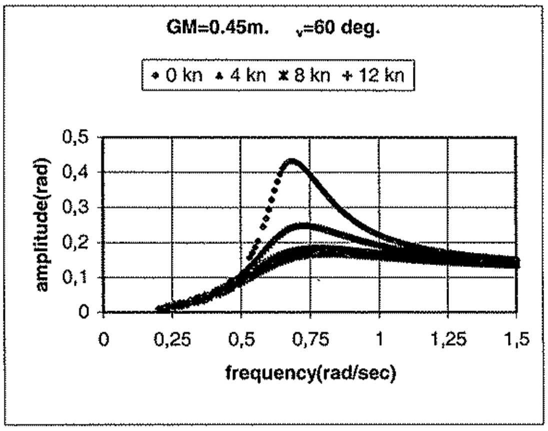


Fig. 18. Roll amplitudes for ϕ_v=60deg.



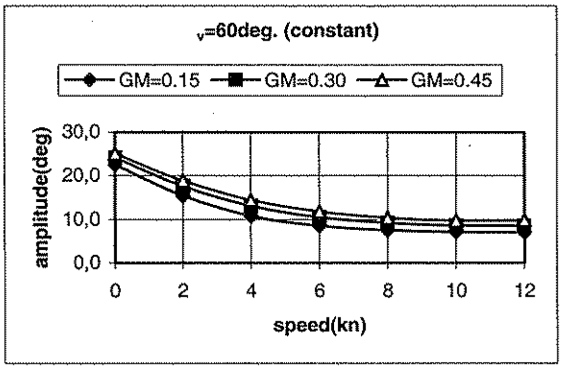


Fig. 19. Roll amplitudes for ϕ_v=60deg.

Fig. 20. Maximum roll amplitudes vs speed .

8. CONCLUSIONS

In the present study, the influence of forward speed on peak roll responses and stability is evaluated by considering the effect of a number of stability parameters. It has been shown earlier that linear and nonlinear roll damping coefficients change with increasing speed. Having utilized this fact in numerous parametric evaluations it has been discovered that for relatively lower speeds, 0-6 knots, the periodic resonance peak roll amplitudes decrease drastically, improving ship's roll motion stability. Then the rate of decrease slows down and reaches a specific value at which amplitudes are no longer affected by changing speed. Around 10 knots, we keep getting almost the same responses even if we speed up the vessel more for all the cases analyzed. That speed value can be considered to be the "critical speed" for this particular vessel. It is the author's belief that the critical speed may vary from ship to ship depending on their individual intact stability and damping characteristics. The hazard of capsizing is much superior when the ship is slowly propagating or is not moving at all as far as the roll damping is concerned. This may especially pose a danger for fishing vessels while trawling.

The influence of vanishing angle of stability seems not to be so imperative since the curves are of the same character up to 30 degrees in case 4 with GM=0.45m even though the areas under the curves are quite different. GM would have an impact on the responses more if the GZ curves varied for lower angles of heel. It seems that the area up to 30-40 degrees plays a more influential role than the entire area.

Finally, the effect of GM on peak amplitudes is somewhat adverse. We obtain smaller responses (case 7 and case 8) for lower GM values with a constant stability range. However, It has to be kept in mind that GM cannot be lower than certain values required by the intact stability standards.

As a topic for future study, changes in statical stability curve with changing speed may be included in the simulation along with the damping variations. It was stated that the effect of speed becomes more evident for Froude numbers greater than 0.22. In the present analysis since one speed value exceeds this limit, the responses may be pronounced to be quite realistic.

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