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TRANSPORT COEFFICIENTS OF NUCLEAR MATTER AT HIGH TEMPERATURES

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Abstract-High temperature coefficients of viscosity and heat conductivity of nuclear matter are calculated on the basis of Chapman-Enskong approximation in the dilute gas limit It is observed that increasing temperatures produce significant changes in coefficients.
Keywords- Nuclear matter, Transport coefficients

1. INTRODUCTION

The properties of nuclear matter can be classified as equilibrium (like equation of state) and non-equilibrium (like transport coefficients) properties. To describe transport properties of a fermi system such as nuclear matter , one needs a transport equation which describes the non-equilibrium process of the system [1-4]. Such an equation was first derived by Boltzmann and modified Uehling-Uhlenbeck to include Pauli blocking term [5].

To evaluate transport coefficients in the high temperature region, Boltzmann equation may be used since mean field effects and Pauli blocking are negligible so that the system can be viewed as a dilute gas [6]. Transport properties of nuclear matter at high temperatures have recently been studied by Malfliet [6]. In this paper we study in Boltzmann statistics limit. We derive the expressions for viscosity and heat conductivity coefficients by using Chapman-Enskong method and mean free path approximation with the hard spheres model. The viscosity and heat conductivity coefficients obtained in this models are compared.

2. CALCULATION OF THE COEFFICIENT OF HEAT CONDUCTIVITY OF NUCLEAR MATTER

The heat conductivity involves the energy transport in nuclear matter [4]. The heat conductivity κ is given by Fourier's law [7],

$$q_i = \kappa \frac{\partial T}{\partial \chi_i} \tag{1}$$

The heat flux can be expressed as

$$q_i = \frac{m}{2}\int d^3U U_i U^2 f^{(0)}\phi \tag{2}$$

$$q_i = \frac{m}{2} n\left(\frac{\alpha}{\pi}\right)^{\frac{3}{2}} \frac{1}{T}\frac{\partial T}{\partial \chi_\mu}\chi_i \int d^3U U_i U_\mu U^2\left(\alpha U^2 - \frac{5}{2}\right)e^{-\alpha U^2} \tag{3}$$

where α is $\frac{m}{2kT}$. From equation (3) ; one may write

$$q_i = \frac{5nk}{4\alpha}\chi_1 \frac{\partial T}{\partial \chi_i} \tag{4}$$

$$\kappa = -\frac{5nk}{4\alpha} \frac{\int d^3Ue^{-\alpha U^2} U_\mu U_\mu \left(\alpha U^2 - \frac{5}{2}\right)^2}{\int d^3Ue^{-\alpha U^2} U_\mu \left(\alpha U^2 - \frac{5}{2}\right) C\left[U_\mu - \left(\alpha U^2 - \frac{5}{2}\right)\right]} \quad . \tag{5}$$

Calculating integral values in equation (5), for the heat conductivity coefficient ; we obtain

$$\kappa = \frac{25}{16} C_v (\pi m k T)^{\frac{1}{2}} \frac{1}{\int dGG^7 e^{-G^2} Q(g)} \tag{6}$$

where $Q(g) = 2\pi \int \sin^3\theta I(g,\theta) d\theta$, C_v=3k/2m and G=g(m/4kT). Using the Chapman-Enskong method with the hard spheres model of the particles one gets also density independent expression as

$$\kappa = \frac{75}{64\sigma}\left(\frac{\pi T}{m}\right)^{\frac{1}{2}} \tag{7}$$

2.1 Mean Free Path Approximation

The elementary mean free path arguments [1] yield for the heat conductivity coefficient that,

$$\kappa = \frac{1}{3} n c u \ell \quad . \tag{8}$$

Here we have taken $\ell = (n\sigma)^{-1}$, u=$(3T/m)^{1/2}$ and c=3/2 where ℓ denotes mean free path, n nucleon number density, c specific heat per nucleon, σ nucleon-nucleon cross section and u average velocity. Using this mean free path arguments [1] one gets

$$\kappa = \frac{1}{2\sigma}\left(\frac{3T}{m}\right)^{1/2} \tag{9}$$

3. CALCULATION OF THE COEFFICENT OF VISCOSITY OF NUCLEAR MATTER

The viscosity involves the transport of the momentum in nuclear matter, viscosity η is given by Fourier and Newton laws [6].

$$P_{ij} = p\delta_{ij} - 2\eta\left(D_{ij} - \frac{1}{3}\sum_k D_{kk}\delta_{ij}\right) \tag{10}$$

where P_{ij} is momentum flux. Using Chapman-Enskong method can be written [5]

$$P_{ij} = p\overline{U_i U_j} = mn\left(\frac{\alpha}{\pi}\right)^{3/2} \int d^3U U_i U_j e^{-\alpha U^2}$$

$$+mn\left(\frac{\alpha}{\pi}\right)\int d^3UU_iU_je^{-\alpha U^2}2\alpha D_\mu D_\gamma\left(U_\mu U\gamma-\frac{1}{3}\delta_{\mu\gamma}U^2\right)\chi_2 \tag{11}$$

$$P_{ij}=p\delta_{ij}+mn\left(\frac{\alpha}{\pi}\right)^{3/2}2\alpha\int U_z^{\,4}e^{-\alpha U^2}d^3U\left(D_{ij}-D_{\alpha\alpha}\frac{1}{3}\delta_{ij}\right)\chi_2 \tag{12}$$

Using equation (10) and (12) , viscosity coefficient;

$$\eta=-\frac{mn}{2}\frac{\pi^{3/2}}{\alpha^{5/2}}\left(\frac{\alpha}{\pi}\right)^{3/2}\chi_2 \tag{13}$$

where

$$\chi_1=\frac{\int d\overline{U}e^{-\alpha U^2}U_\mu U_\mu\left(\alpha U^2-\frac{5}{2}\right)^2}{\int d\overline{U}e^{-\alpha U^2}U_\mu\left(\alpha U^2-\frac{5}{2}\right)C\left[U_\mu\left(\alpha U^2-\frac{5}{2}\right)\right]} \tag{14a}$$

$$\chi_2=\frac{\int d\overline{U}e^{-\alpha U^2}\left(U_\mu U_\gamma-\frac{1}{3}\delta_{\mu\gamma}U^2\right)\left(U_\mu U_\gamma-\frac{1}{3}\delta_{\mu\gamma}U^2\right)}{\int d\overline{U}e^{-\alpha U^2}\left(U_\mu U_\gamma-\frac{1}{3}\delta_{\mu\gamma}U^2\right)C\left(U_\mu U_\gamma-\frac{1}{3}\delta_{\mu\gamma}U^2\right)} \tag{14b}$$

The calculation of (14b) and hence of η goes along similar with κ, and one finds:

$$\eta=\frac{5}{8}(\pi mkT)^{1/2}\frac{1}{\int_0^\infty dGG^7e^{-G^2}Q(g)} \tag{15}$$

Calculating of integral in equation (15) similar to equation (6) and using the Chapman-Enskong method with the hard spheres model [9,10] of the particles one gets also density independent expression as

$$\eta=\frac{5}{16\sigma}(\pi mT)^{1/2} \tag{16}$$

3.1 Mean Free Path Approximation

According to mean free path approximation viscosity coefficient is given

$$\eta=\frac{1}{3}nmu\ell \tag{17}$$

Using the same units in equation (9) one gets

$$\eta=\left(\frac{1}{\sigma}\right)\left(\frac{1}{3}mT\right)^{1/2} \tag{18}$$

4. CALCULATION OF TRANSPORT COEFFICIENTS OF NUCLEAR MATTER BETWEEN 40-100 MEV

Here, taking $\sigma = 40$mb, $m = 931$MeV/c^2 and T=40-100MeV, According to CEM and MFPA We calculated of transport coefficients of nuclear matter.

Table 1. Heat conductivity coefficient of nuclear matter between 40-100 MeV

T(MeV)	κ_{CEM} (c/fm^2)	κ_{MFPA} (c/fm^2)	$\kappa_{CEM} - \kappa_{MFPA}$ (c/fm^2)
40	0.107	0.044	0.063
42	0.110	0.045	0.065
44	0.112	0.046	0.066
46	0.115	0.047	0.068
48	0.117	0.048	0.069
50	0.120	0.049	0.071
52	0.122	0.050	0.072
54	0.125	0.051	0.074
56	0.127	0.052	0.075
58	0.129	0.053	0.076
60	0.131	0.054	0.077
62	0.133	0.055	0.078
64	0.136	0.056	0.080
66	0.138	0.056	0.082
68	0.140	0.057	0.083
70	0.142	0.058	0.084
72	0.144	0.059	0.085
74	0.146	0.060	0.086
76	0.148	0.061	0.087
78	0.150	0.061	0.089
80	0.152	0.062	0.090
82	0.154	0.063	0.091
84	0.155	0.064	0.091
86	0.157	0.065	0.092
88	0.159	0.065	0.094
90	0.161	0.066	0.095
92	0.163	0.067	0.096
94	0.164	0.067	0.097
96	0.166	0.068	0.098
98	0.168	0.069	0.099
100	0.170	0.070	0.100

Table 2. Viscosity coefficient of nuclear matter between 40-100 MeV

T(MeV)	η_{CEM} (MeV/fm^{2}c)	η_{MFPA} (MeV/fm^{2}c)	η_{MFPA} - η_{CEM} (MeV/fm^{2}c)
40	26.712	27.850	1.138
42	27.371	28.538	1.167
44	28.017	29.211	1.194
46	28.647	29.868	1.221
48	29.263	30.510	1.247
50	29.868	31.140	1.272
52	30.459	31.757	1.298
54	31.038	32.360	1.322
56	31.608	32.955	1.347
58	32.165	33.536	1.371
60	32.719	34.113	1.394
62	33.259	34.677	1.418
64	33.792	35.232	1.440
66	34.315	35.778	1.463
68	34.831	36.315	1.484
70	35.338	36.843	1.505
72	35.840	37.368	1.528
74	36.334	37.883	1.549
76	36.820	38.389	1.569
78	37.302	38.891	1.589
80	37.779	39.389	1.610
82	38.248	39.878	1.630
84	38.712	40.362	1.650
86	39.169	40.838	1,669
88	39.621	41.309	1.688
90	40.068	41.776	1.708
92	40.512	42.238	1.726
94	40.951	42.696	1.745
96	41.386	43.150	1.764
98	41.813	43.595	1.582
100	42.240	44.040	1.800

5. CONCLUSION

It is observed from tables 1 and 2 that heat conductivity and viscosity coefficients increase with increasing temperatures. As can be seen from these tables CEM values greater than MFPA values for heat conductivity coefficient, but smaller for viscosity coefficient. In addition to this, values of (κ_{CEM} - κ_{MFPA}) and (η_{MFPA} - η_{CEM}) increase with increasing temperatures. It should be noted that the present results are valid only in the high temperature region under Boltzmann statistic limit. To make a comparison between the present results and existing calculations let us take T=60 MeV. Then we obtain (see table 1 and 2) κ_{MFPA}=0.054 c/fm^2 and η_{MFPA}=34.113 MeV/fm^{2}c, the hyrodynamical predictions by Danielewicz [8] yield κ_{MFPA}=0.055 c/fm^2, η_{MFPA}=34 MeV/fm^{2}c. However, we should point out that for calculation of transport coefficients of nuclear matter It is necessary to know temperature region and its effects on transport coefficients. Because the kinetic equations which describe the time evolutions of the collision process are different at various temperatures.

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