Mathematical & Computational Applications, Vol. 8, No. 3, pp. 369-376, 2003

EVALUATION OF CONTROL METHODS ON A STRUCTURAL SYSTEM

Nurkan Yagiz and Cuneyt Ertal
Istanbul University, Faculty of Engineering, Department of Mechanical Engineering,
34850 Avcilar, Istanbul, Turkey
nurkany@istanbul.edu.tr, certal@istanbul.edu.tr

Abstract- In this study, PID, LQR and sliding mode controllers are designed as active seismic isolation devices on a multi degrees of freedom structure. At first PID control is applied as a conventional method. Since this control method can be applied easily and well known widely, it has an important role in control applications. Secondly, a LQR is designed to suppress the vibrations of the structural system. But these methods are not insensitive to parameter changes. Since the model might have uncertainties and/or parameter changes, sliding mode control has been introduced because of its well-known robust character and superior performance. Additionally, this control method can easily be applied to non-linear systems. The simulated system has four stories. A disturbance input representing earthquake is applied to the base. At the end of the study, the time history of the storey displacements, control voltage and frequency response of the both uncontrolled, PID, LQR and sliding mode controlled structures have been presented and results have been compared.

Keywords- Structural system, PID, LQR, sliding mode control.

1. INTRODUCTION

Vibration control of buildings has become more important recently. Developments were started with using passive control methods. Rubber bearing system was one of good vibration isolation system because of lower cost and high damping effects at decreasing level of vibration amplitude [1],[2]. Then the application of semi-active control methods were became widespread. In 1999 Watanabe used Bidirectional Active Dynamic Vibration Absorber to control vertical and horizontal vibrations of a base-isolated building [3]. Because actual buildings have non-linearities, using active and semi active schemes which are based on sliding mode control method was suggested [4].

The aim of this study is to apply PID, LQR and non-chattering sliding mode control to suppress the vibrations of structural systems and compare the results. Chattering may cause damages if not prevented. A chattering free sliding mode control application has been realized by Sabanovic [5]. Static output feedback controllers using only the measured information from a limited number of sensors installed at strategic locations were developed. Yang *et al.* showed that using static feedback controllers with sliding mode control method has robust character with successful performance [6]. Recent days, sliding mode control method has been applied at flight control, robot control and power systems control.

2. MATHEMATICAL MODEL OF THE STRUCTURAL SYSTEM

In this study, the building used has four stories. The physical system has been shown in Figure 1. m_1, m_2 and m_3 are masses of each storey respectively. m_0 is the mass of the base. x_o, x_1, x_2 and x_3 are the horizontal displacements of them. All springs and dampers are acting in horizontal directions. The system parameters are presented at the Appendix.

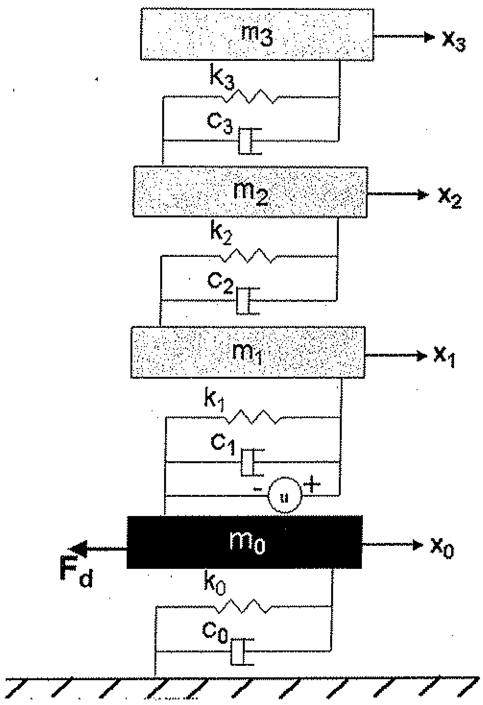

Figure 1. Structural System

The equation of motion is given by

$$M\ddot{\underline{X}} + C\dot{\underline{X}} + K\underline{X} = \underline{F}_d + \underline{F}_u \tag{1}$$

where $\underline{X} = [\, x_0 \; x_1 \; x_2 \; x_3 \,]^T$, $\underline{F}_d = [\, -F_d \; 0 \; 0 \; 0]^T$ and $\underline{F}_u = [\, -F_u \; F_u \; 0 \; 0]^T$. F_d denotes a force representing the effect of earthquake acting on the structure. Control force F_u is produced by linear motor. M, C and K are the mass, damping and stiffness matrices. The equation of the linear motor is:

$$R \cdot i + K_e(\dot{x}_1 - \dot{x}_0) = u \tag{2}$$

u and i are the voltage and current of the armature coil respectively. u is the control voltage input. R and K_e are the resistance value and induced voltage constant of the armature coil. The current of the armature coil and control force have the following relation:

$$F_u = K_f \cdot i \tag{3}$$

K_f is the thrust constant. The inductance of the armature coil is neglected. By combining the equations (1) through (3) and arranging them, it is also possible to get the governing equations in state space form.

3. THE CONTROLLER DESIGN

In this study, three control methods are considered. These are conventional PID and LQR control together with chattering free sliding mode control.

3.1. The PID Design

PID control has been used in industry widely. The control input u(t) is obtained as follows:

$$u(t) = K\left[\underline{e}(t) + \frac{1}{\tau_i}\int_0^t \underline{e}(t)dt + \tau_d \frac{d\underline{e}(t)}{dt}\right] \tag{4}$$

K, τ_i and τ_d are proportionality constant, integral time and derivative time respectively. These values are obtained using Ziegler-Nicholes method [7].

3.2. The LQR Design

The performance index in order to calculate optimum control input by taking into account all state variables:

$$J = \frac{1}{2}\int_0^\infty \left[q_1 x_1^2 + q_2 x_2^2 + q_3 x_3^2 + q_4 x_4^2 + q_5 x_5^2 + q_6 x_6^2 + q_7 x_7^2 + q_8 x_8^2 + Ru^2\right]dt \tag{5}$$

or in matrix form,

$$J = \frac{1}{2}\int_0^\infty \left[\underline{x}^T(t)\,[Q]\,\underline{x}(t) + \underline{u}^T(t)[R]\underline{u}(t)\right]dt \tag{6}$$

where the weighting matrix,

$$[Q] = \begin{bmatrix} q_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & q_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & q_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & q_4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & q_5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & q_6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & q_7 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & q_8 \end{bmatrix} \tag{7}$$

All weighting variables q_i and [R] are taken to be positive to make the performance index positive definite. From control law for LQR, optimal control input is given by:

$$\underline{u} = [G_{opt}]\underline{x} = [R]^{-1}[B]^{T}[P]\,\underline{x} \tag{8}$$

where P is the solution of the following matrix Riccati equation:

$$[A]^{T}[P] - [P][B][R]^{-1}[B]^{T}[P] + [P][A] + [Q] = 0 \tag{9}$$

It is assumed that system is controllable and all state variables are observable.

3.3. The Sliding Mode Controller Design

Sliding Mode Control Theory has been applied to many non-linear systems. The main idea is to bring the error on sliding surface such that system is on sliding surface and insensitive to the disturbances and parameter changes. If the system is defined in state space form as:

$$\dot{\underline{x}} = \underline{f}(x) + [B]\cdot\underline{u} \tag{10}$$

[B] is the control input matrix. The sliding surface $\underline{\sigma}$ (x,t) can be expressed as:

$$\underline{S} = \{\, \underline{x} : \underline{\sigma}(x,t) = 0 \,\} \tag{11}$$

In order to obtain a stable solution of the system, the error must stay on this surface, as shown in Figure 2. The sliding surface equation for the control of the system can be selected as follows:

$$\underline{\sigma}(x,t) = [G]\cdot(\underline{x}_{ref} - \underline{x}) = [G]\cdot\underline{e} \tag{12}$$

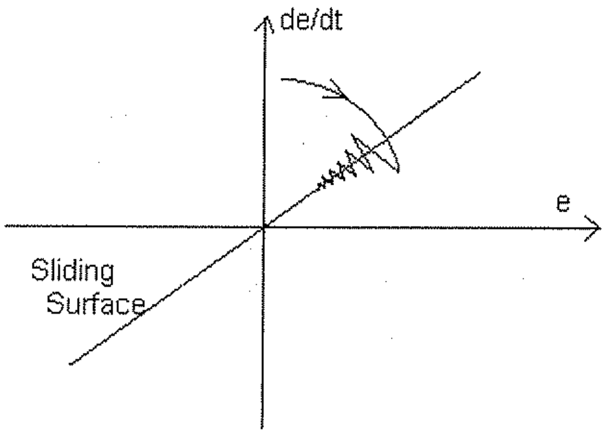

Figure 2. Sliding Surface

In this equation $\underline{x}_{ref}$ represents the state vector of the reference, $\underline{e}$ is the error, and the constant [G] matrix represents the slope of the sliding surface. The first step in design is to select Lyapunov function $\underline{v}$. According to Lyapunov Stability Criteria, Lyapunov function must have a value greater than zero whereas its derivative is smaller than zero. Selecting the function as below makes its value greater than zero:

$$\underline{v} = \underline{\sigma}^T(x,t) \cdot \frac{\underline{\sigma}(x,t)}{2} > 0 \tag{13}$$

In order to have the value of the derivative of Lyapunov Function smaller than zero:

$$\frac{d\underline{v}}{dt} = -\underline{\sigma}^T(x,t) \cdot [\Gamma] \cdot \underline{\sigma}(x,t) < 0 \tag{14}$$

Thus Lyapunov's Stability Criteria has been satisfied. Here $[\Gamma]$ is a constant term matrix having positive values. Making necessary arrangements:

$$\underline{u}(t) = \underline{u}_{eq}(t) + [GB]^{-1}\Gamma \cdot \underline{\sigma}(x,t) \tag{15}$$

Equivalent control $\underline{u}_{eq}$ can be obtained by equating Eq. (14) to the derivative of Eq. (13) and solving for $\underline{u}$. If the knowledge of $\underline{f}(x)$ and [B] are very poor, then the equivalent control calculated will be too far from the actual value. In the literature a number of approaches are proposed for the estimation of $\underline{u}_{eq}$, rather that calculating it. In this study, the approach suggested uses the fact that the equivalent control is the average of the total control. Designing an averaging filter for calculation of the equivalent control as below.

$$\underline{u}_{eq} = \frac{1}{\tau s + 1}\underline{u} \tag{16}$$

This is a low-pass filter. The value of $1/\tau$ gives the cut-off frequency. The logic behind the designing a low pass filter is that low frequencies determine the characteristics of the signal and high frequencies come from unmodeled dynamics.

4. EVALUATION OF THE CONTROL METHODS

Structural system has been tested against 10000 N. of step input to the base starting at fifth second. Figure 3. shows the PID, LQR and SMC controlled and uncontrolled time responses of the first and third stories. It is observed that there is an important improvement when the horizontal displacements of the structure are considered for both the LQR and SMC controlled structures.

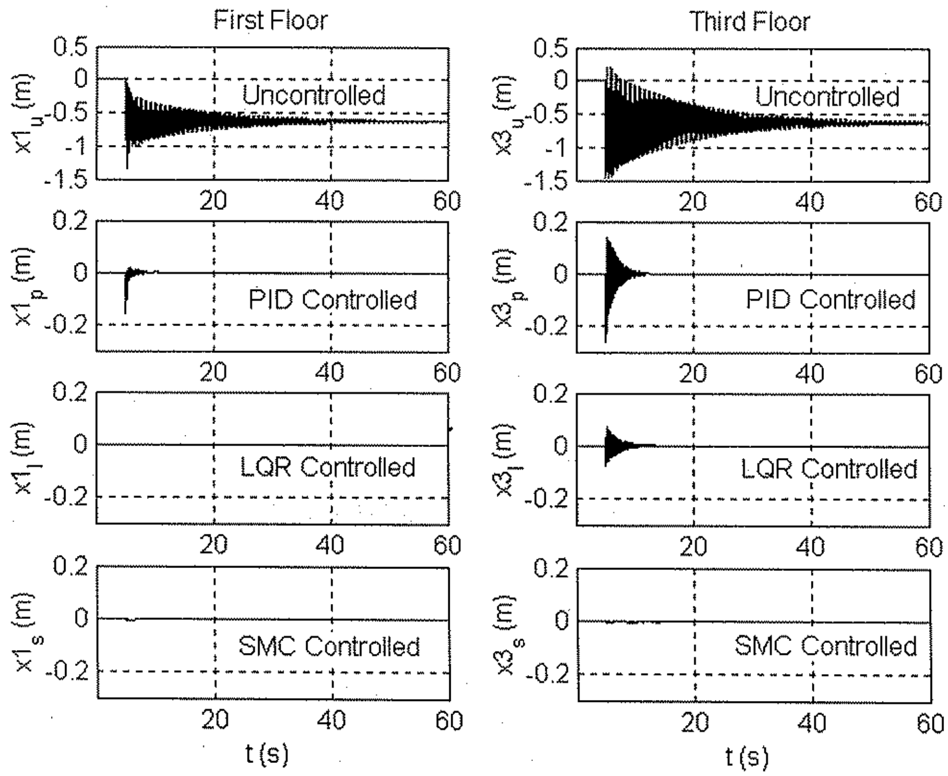

Figure 3. PID, LQR and SMC controlled and uncontrolled time responses of the first and third stories

On the other hand, control voltage inputs are presented in Figure 4.

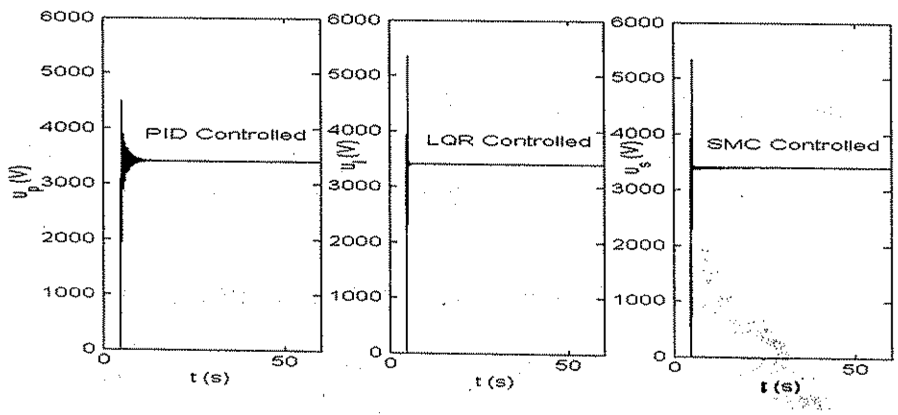

Figure 4. Time history of the control voltages.

Since the system has four degrees of freedom, there are four resonance values. First resonance value belongs to the third storey and around 2.6 Hz . Second resonance value belongs to the first storey and at 7 Hz. Third and fourth resonance values are around 12.5 Hz. coinciding on each other. Figure 5. shows the frequency responses of the first storey and third displacements respectively for both PID, LQR and SMC controlled and uncontrolled cases.

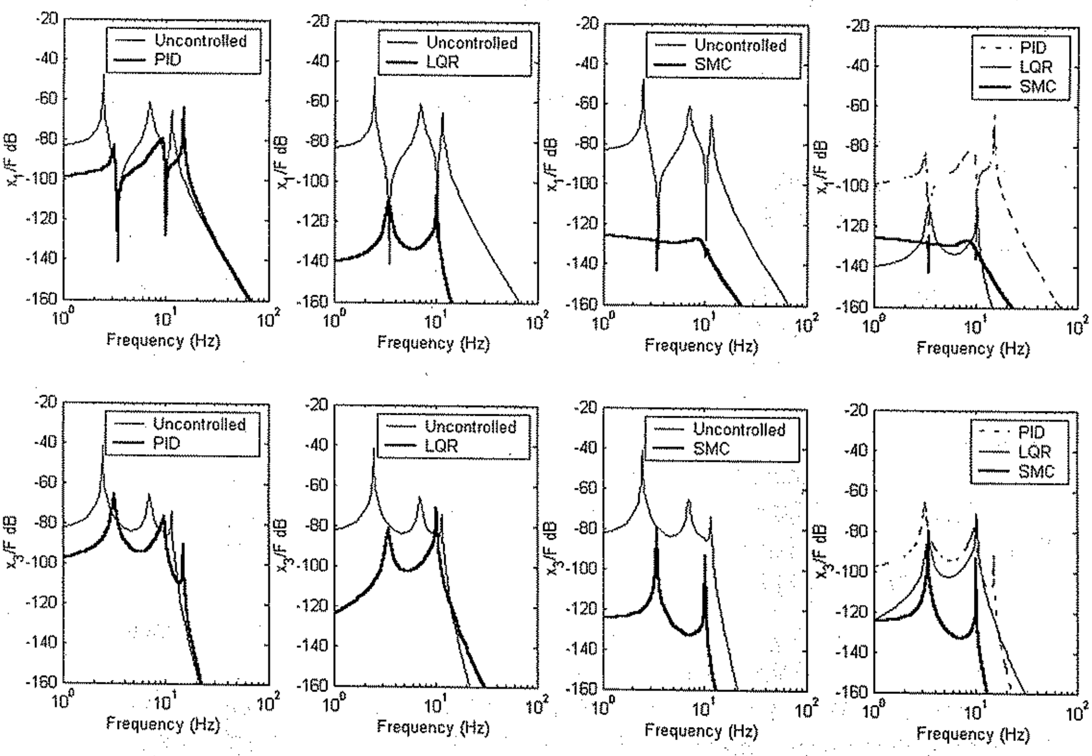

Figure 5. PID, LQR and SMC controlled and uncontrolled frequency responses of the first and third storey.

As expected the upper curves belong to the uncontrolled system. Uncontrolled, PID, LQR and sliding mode controlled frequency responses are plotted for both the first and third stories. A significant improvement in terms of magnitudes have been witnessed particularly for LQR and SMC. Especially at the resonance values of the displacement response of the first storey, there is a satisfactory improvement. On the other hand when PID, LQR and SMC controlled responses are plotted on each other, the superiority of SMC on other controllers is witnessed. This becomes another advantage of SMC besides its robust character and applicability to non-linear systems.

5. CONCLUSION

In this study, PID, LQR and sliding mode controllers have been designed for a multi degrees of freedom structural system separately. Since the damaging effect of earthquakes happens as a result of horizontal vibrations, all degrees of freedom have been assumed only at this direction. System is modeled including the dynamics of linear motor which is used as the active isolator. The structural systems and buildings have uncertainties and their parameters are subject to the changes. Because of its robust character, applicability to non-linear systems and superior performance, the sliding mode control must be preferred to the other methods. Against the disturbances coming from the earth, it is verified that the designed non-chattering sliding mode controller has brought satisfactory seismic isolation.

REFERENCES

1. B.C.Lin, I.G. Tadjbakhsh, A.S. Papageorgiou, G. Ahmadi, Performance of Earthquake Isolation Systems, *Journal of Engineering Mechanics,* **116-2**, 446-461, 1990.
2. S.C. Sinha,G. Li, Optimal design of Base-Isolated Structures with Dynamic Absorbers, *Journal of Engineering Mechanics,* **120-2**, 221-231, 1994.
3. T.Watanabe, K. Yoshida, Y. Fukunishi, *Simultaneous Control of Vertical and Horizontal Vibrations of Base-Isolated Buildings Using Bidirectional Active Dynamic Vibration Absorber*, Proceedings of the 1999 ASME Design Engineering Technical Conferences, Las Vegas, 1999.
4. M.P. Singh, E. E. Matheu, L. E. Suarez, Active ad Semi-Active Control of Structures Under Seismic Excitation, *Earthquake Engineering and Stuctural Dynamics*, **26**, 193-213, 1997.
5. Sabanovic, *Chattering Free Sliding Mode*, in Proceedings of the 1st Turkey Automatic Control Symposium , Istanbul, 1994.
6. J.N. Yang, J.C. Wu, A.K. Agrawal, Sliding Mode Control for Nonlinear and Hysteretic Structures, *Journal of Engineering Mechanics* **121-12**, 1330-1339, 1995.
7. K. Ogata, *Modern Control Engineering*, Prentice-Hall , New Jersey, 1990.

APPENDIX

Parameters of the structural system:

$\mathbf{m_o}$	5 kg	$\mathbf{m_2}$	1.5 kg
$\mathbf{m_1}$	1.7 kg	$\mathbf{m_3}$	2.3 kg
$\mathbf{k_o}$	16 000 N/m	$\mathbf{k_1= k_2= k_3}$	2600 N/m
$\mathbf{c_o}$	100 N.s/m	$\mathbf{c_1= c_2= c_3}$	0.08 N.s/m
$\mathbf{K_f}$	2 N/A	$\mathbf{K_e}$	2 Volt
$\mathbf{R}$	1.5 Ω		