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STRESS INTENSITY FACTORS FOR A RING–SHAPED CRACK IN A TRANSVERSELY ISOTROPIC THICK LAYER

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Abstract- The problem of a ring-shaped crack in an infinitely long thick layer was considered in this study. The problem was formulated for a transversely isotropic material by using integral transform technique under uniform load. Due to the geometry of the configuration, Hankel integral transform technique was chosen and the problem was reduced to a singular integral equation. This integral equation was solved numerically by using Gaussian Quadrature Formulae and the values were evaluated for discrete points. Stress intensity factors were obtained by using these values. They were tabulated for various ring–shaped crack sizes and transversely isotropic materials.
Keywords- Transversely isotropic, ring-shaped crack, stress intensity factor.

1. INTRODUCTION

Fiber reinforced composite materials and having hexagonal crystals materials such as magnesium, barium-titanate and graphite-epoxy are called transversely isotropic. These types of materials are considered some researches [1-4]. Chen has investigated stress intensity factors in a finite length cylinder with a circumferential crack [5]. The griffith crack problem was studied by Das et al [6]. The distribution of stress in a transversely isotropic cylinder containing penny-shaped crack was studied by Parhi and Atsumi [7]. Dahan [8, 9] has investigated the stress intensity factors and stress distribution in a transversely isotropic solid containing a penny shaped crack. Singular stresses in a transversely isotropic circular cylinder with circumferential edge crack were examined by Atsumi and Shindo [10]. Konishi [11, 12] studied crack problems in transversely isotropic strip and medium. Fildiş and Yahşi [13] determined stress intensity factors for an infinitely long transversely isotropic solid cylinder containing a ring-shaped cavity. Arın and Erdogan [14] investigated penny shaped crack in an elastic layer bonded to dissimilar half spaces. R. Erdol and F. Erdogan considered a thick-walled cylinder with an axisymmetric internal or edge crack problem [15]. Numerical calculation of stress intensity factors in functionally graded materials was studied by Anlas et al [16].

In this study, the governing elasticity equation for the transversely isotropic axisymmetric problem in cylindrical coordinates was obtained in terms of a Love type stress function. Hankel transform was applied on the stress function because of the geometry of the configuration and boundary conditions. The stress function was expressed in terms of the governing equation. Once the governing integral equation was obtained, the problem can be solved for any axisymmetric quasistatic external load. This load may be mechanical, thermal, or residual in nature. Using the boundary conditions, the problem reduced to a singular integral equation. Using the Gaussian Quadrature this singular integral equation was solved. Then the stress intensity factors at the crack tips were determined.

The numerical results were obtained for axial loading and illustrated by graphs for various ring-shaped crack sizes.

2. BASIC FORMULATION

Consider the axisymmetric elasticity problem for a transversely isotropic layer shown in Figure1.

The equilibrium and the compatibility equations are expressed as

$$\frac{\partial\sigma_r}{\partial r}+\frac{\partial\sigma_{rz}}{\partial z}+\frac{\sigma_r-\sigma_\theta}{r}=0 \qquad \frac{\partial\sigma_{rz}}{\partial r}+\frac{\partial\sigma_z}{\partial z}+\frac{\sigma_{rz}}{r}=0 \tag{1,2}$$

$$\varepsilon_r-\varepsilon_\theta-r\frac{\partial\varepsilon_\theta}{\partial r}=0 \qquad \frac{\partial^2\varepsilon_r}{\partial z^2}+\frac{\partial^2\varepsilon_z}{\partial r^2}-\frac{\partial^2\gamma_{rz}}{\partial z\partial r}=0 \tag{3,4}$$

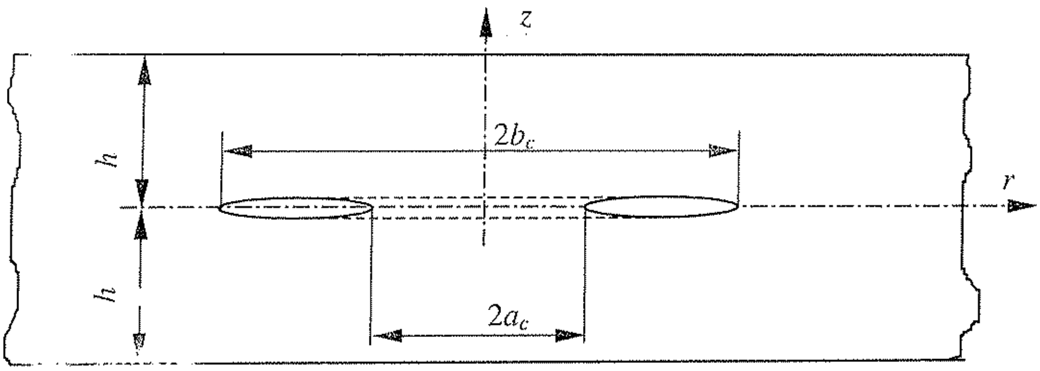

Figure 1. Geometry of the problem

For transversely isotropic bodies the strain components can be written as [17]

$$\varepsilon_r=a_{11}\sigma_r+a_{12}\sigma_\theta+a_{13}\sigma_z \qquad \varepsilon_\theta=a_{12}\sigma_r+a_{11}\sigma_\theta+a_{13}\sigma_z \tag{5,6}$$

$$\varepsilon_z=a_{13}\sigma_r+a_{13}\sigma_\theta+a_{33}\sigma_z \qquad \gamma_{rz}=a_{44}\sigma_{rz} \tag{7,8}$$

Where

$$\varepsilon_r=\frac{\partial u_r}{\partial r} \qquad \varepsilon_\theta=\frac{u_r}{r} \qquad \varepsilon_z=\frac{\partial w}{\partial z} \qquad \gamma_{rz}=\frac{\partial u_r}{\partial z}+\frac{\partial w}{\partial r} \tag{9,..,12}$$

Where a_{ij} $(i,j=1..4)$ elastic constants for the materials considered in this paper are given in Table 1. The numerical values of the moduli c_{ij} $(i,j=1..4)$ for magnesium and barium-titanate found by Huntington [18] were used in this study.

Table 1. Elastic constants (in 1/GPa)

Material	a_{11}	a_{12}	a_{13}	a_{33}	a_{44}
Magnesium	22.1	-7.7	-4.9	19.7	60.3
Barium-titanate	8.15	-2.96	-1.94	6.75	183.1
Graphite epoxy	135.9	-45.4	-0.297	11.5	241.3

In terms of stress function $\phi(r, z)$, the stresses may be expressed as

$$\sigma_r = -\frac{\partial}{\partial z}\left(\frac{\partial^2\phi}{\partial r^2} + \frac{b}{r}\frac{\partial\phi}{\partial r} + a\frac{\partial^2\phi}{\partial z^2}\right) \qquad \sigma_0 = -\frac{\partial}{\partial z}\left(b\frac{\partial^2\phi}{\partial r^2} + \frac{1}{r}\frac{\partial\phi}{\partial r} + a\frac{\partial^2\phi}{\partial z^2}\right) \quad (13,14)$$

$$\sigma_z = \frac{\partial}{\partial z}\left(c\frac{\partial^2\phi}{\partial r^2} + \frac{c}{r}\frac{\partial\phi}{\partial r} + d\frac{\partial^2\phi}{\partial z^2}\right) \qquad \sigma_{rz} = \frac{\partial}{\partial r}\left(\frac{\partial^2\phi}{\partial r^2} + \frac{1}{r}\frac{\partial\phi}{\partial r} + a\frac{\partial^2\phi}{\partial z^2}\right) \quad (15,16)$$

$$u_r = -(1-b)(a_{11} - a_{12})\left(\frac{\partial^2\phi}{\partial r\partial z}\right) \qquad w = a_{44}\left(\frac{\partial^2\phi}{\partial r^2} + \frac{1}{r}\frac{\partial\phi}{\partial r}\right) + (a_{33}d - 2a_{13}a)\frac{\partial^2\phi}{\partial z^2} \quad (17,18)$$

Where the constants a, b, c, and d are

$$\frac{a_{13}(a_{11} - a_{12})}{a_{11}a_{33} - a_{13}^2}, \qquad b = \frac{a_{13}(a_{13} + a_{44}) - a_{12}a_{33}}{a_{11}a_{33} - a_{13}^2} \quad (19,20)$$

$$c = \frac{a_{13}(a_{11} - a_{12}) + a_{44}a_{11}}{a_{11}a_{33} - a_{13}^2}, \qquad d = \frac{a_{11}^2 - a_{12}^2}{a_{11}a_{33} - a_{13}^2} \quad (21,22)$$

For a crack in a thick layer, it is necessary to select a Love type potential function $\phi(r,z)$ of the form

$$\phi(r,z) = \int_0^\infty \lambda\left(m_1 e^{s_1\lambda z} + m_2 e^{s_2\lambda z} + m_3 e^{s_3\lambda z} + m_4 e^{s_4\lambda z}\right)J_0(\lambda r)d\lambda \quad (23)$$

with

$$s_{1,4} = -\sqrt{\frac{(a+c) \pm \sqrt{(a+c)^2 - 4d}}{2d}} \qquad s_{2,3} = \sqrt{\frac{(a+c) \pm \sqrt{(a+c)^2 - 4d}}{2d}} \quad (24,25)$$

where $J_0(\lambda r)$ is the Bessel function of first kind. We find with the help of (13-18) that

$$\sigma_{rz} = \int_0^\infty \lambda^4 \begin{pmatrix} (1-as_1^2)m_1 e^{s_1\lambda z} + (1-as_2^2)m_2 e^{s_2\lambda z} \\ +(1-as_3^2)m_3 e^{s_3\lambda z} + (1-as_4^2)m_4 e^{s_4\lambda z} \end{pmatrix} J_1(\lambda r)d\lambda \quad (26)$$

$$\sigma_r(r,z) = \int_0^\infty \lambda^4 \begin{bmatrix} (1-as_1^2)m_1 s_1 e^{s_1\lambda z} + (1-as_2^2)m_2 s_2 e^{s_2\lambda z} \\ +(1-as_3^2)m_3 s_3 e^{s_3\lambda z} + (1-as_4^2)m_4 s_4 e^{s_4\lambda z} \end{bmatrix} J_0(\lambda r)d\lambda$$
$$-\frac{1-b}{r}\int_0^\infty \lambda^3 \begin{bmatrix} m_1 s_1 e^{s_1\lambda z} + m_2 s_2 e^{s_2\lambda z} \\ +m_3 s_3 e^{s_3\lambda z} + m_4 s_4 e^{s_4\lambda z} \end{bmatrix} J_1(\lambda r)d\lambda \quad (27)$$

$$\sigma_z = \int_0^\infty \lambda^4 \begin{pmatrix} \left(s_1^3 d - s_1 c\right) m_1 e^{s_1 \lambda z} + \left(s_2^3 d - s_2 c\right) m_2 e^{s_2 \lambda z} \\ + \left(s_3^3 d - s_3 c\right) m_3 e^{s_3 \lambda z} + \left(s_4^3 d - s_4 c\right) m_4 e^{s_4 \lambda z} \end{pmatrix} J_0(\lambda r) d\lambda \tag{28}$$

$$w = \int_0^\infty \lambda^3 \begin{pmatrix} \left((a_{33} d - 2a_{13} a) s_1^2 - a_{44}\right) m_1 e^{s_1 \lambda z} \\ + \left((a_{33} d - 2a_{13} a) s_2^2 - a_{44}\right) m_2 e^{s_2 \lambda z} \\ + \left((a_{33} d - 2a_{13} a) s_3^2 - a_{44}\right) m_3 e^{s_3 \lambda z} \\ + \left((a_{33} d - 2a_{13} a) s_4^2 - a_{44}\right) m_4 e^{s_4 \lambda z} \end{pmatrix} J_0(\lambda r) d\lambda \tag{29}$$

$$\frac{\partial w}{\partial r} = \left[\int_0^\infty \lambda^4 \begin{pmatrix} \left(a_{44} - (a_{33} d - 2a_{13} a) s_1^2\right) m_1 e^{s_1 \lambda z} \\ + \left(a_{44} - (a_{33} d - 2a_{13} a) s_2^2\right) m_2 e^{s_2 \lambda z} \\ + \left(a_{44} - (a_{33} d - 2a_{13} a) s_3^2\right) m_3 e^{s_3 \lambda z} \\ + \left(a_{44} - (a_{33} d - 2a_{13} a) s_4^2\right) m_4 e^{s_4 \lambda z} \end{pmatrix} J_1(\lambda r) d\lambda \right] \tag{30}$$

The perfectly isotropic materials can not be analyzed by making use of the formulation given in this problem.

3. FORMULATION AND SOLUTION OF THE PROBLEM

Let the ring-shaped crack be embedded in the mid-plane of thick layer. The material of the layer is a transversely isotropic elastic-plastic. In practice, the upper and lower surfaces are stress free.

$$\sigma_z(r, \pm h) = 0 \qquad \sigma_{rz}(r, \pm h) = 0 \tag{31,32}$$

On the plane $z = 0$, it is required that

$$\sigma_{rz}(r, 0) = 0 \qquad \infty > r > 0 \tag{33}$$

$$w(r, 0) = 0 \qquad a > r > 0 \tag{34}$$

$$w(r, 0) = 0 \qquad \infty > r > b \tag{35}$$

$$\sigma_z(r, 0) = -p(r) \qquad b_c > r > a_c \tag{36}$$

Where the pressure $p(r)=p_0$ (constant) is prescribed on the crack faces.

For determining the unknowns m_i $(i=1..4)$ in (23) we need four independent equations.Boundary condition (31) is substituted into (28), similarly (32) and (33) are substituted in (26) so, three equations are derived as

$$A_{11} m_1 + A_{12} m_2 + A_{13} m_3 + A_{14} m_4 = 0 \tag{38}$$

$$A_{21} m_1 + A_{22} m_2 + A_{23} m_3 + A_{24} m_4 = 0 \tag{39}$$

$$A_{31} m_1 + A_{32} m_2 + A_{33} m_3 + A_{34} m_4 = 0 \tag{40}$$

Where

$$A_{1i} = (s_i^3 d - s_i c) e^{s_i h \lambda}, \; A_{2i} = (1 - a s_i^2) e^{s_i h \lambda}, \; A_{3i} = (1 - a s_i^2) \qquad i=1,\ldots,4 \tag{41-43}$$

It is convenient to reduce the mixed boundary condition to an integral equation. The integral equation will be singular. In order to avoid a strong singularity in the resulting equation, it is necessary to introduce a new function as the derivative of the displacement $w(r, z)$, rather than the displacement. The new unknown function will be defined as follows.

$$G(r)=\frac{\partial w}{\partial r} \tag{44}$$

$$w=\int_0^{a_c}G(r)dr+\int_{a_c}^{b_c}G(r)dr+\int_{b_c}^{\infty}G(r)dr \tag{45}$$

From boundary conditions (34) and (35), condition below must be satisfied

$$\int_{a_c}^{b_c}G(r)dr=0 \tag{46}$$

Equation (30) is reduced as

$$\int_0^{\infty}\lambda^4\begin{pmatrix}\left(a_{44}-(a_{33}d-2a_{13}a)s_1^{\,2}\right)m_1e^{s_1\lambda z}\\+\left(a_{44}-(a_{33}d-2a_{13}a)s_2^{\,2}\right)m_2e^{s_2\lambda z}\\+\left(a_{44}-(a_{33}d-2a_{13}a)s_3^{\,2}\right)m_3e^{s_3\lambda z}\\+\left(a_{44}-(a_{33}d-2a_{13}a)s_4^{\,2}\right)m_4e^{s_4\lambda z}\end{pmatrix}J_1(\lambda r)d\lambda=G(r) \tag{47}$$

If $B(\lambda)$ is defined as

$$\begin{aligned}B(\lambda)=&\left(a_{44}-(a_{33}d-2a_{13}a)s_1^{\,2}\right)m_1e^{s_1\lambda z}+\left(a_{44}-(a_{33}d-2a_{13}a)s_2^{\,2}\right)m_2e^{s_2\lambda z}\\&+\left(a_{44}-(a_{33}d-2a_{13}a)s_3^{\,2}\right)m_3e^{s_3\lambda z}+\left(a_{44}-(a_{33}d-2a_{13}a)s_4^{\,2}\right)m_4e^{s_4\lambda z}\end{aligned} \tag{48}$$

Equation (47) is reduced as,

$$\int_0^{\infty}\lambda^4B(\lambda)J_1(\lambda r)d\lambda=G(r) \tag{49}$$

By using inverse Hankel transform technique

$$B(\lambda)=\frac{1}{\lambda^3}\int_0^{\infty}\rho G(\rho)J_1(\lambda\rho)d\rho \tag{50}$$

If boundary condition (34,35) is substituted in (30) and using (46) we obtain fourth equation following (38), (39) and (40) as

$$A_{41}m_1+A_{42}m_2+A_{43}m_3+A_{44}m_4=B(\lambda) \tag{51}$$

Where

$$A_{4i}=\left(a_{44}-(a_{33}d-2a_{13}a)s_i^{\,2}\right)\qquad i=1,\ldots,4 \tag{52}$$

m_i (i=1..4) coefficients can be obtain in terms of $B(\lambda)$ by using (38), (39), (40) and (51). Under condition (46), m_i (i=1..4) coefficients are substituted in boundary condition (36)

$$p(r)=\int_0^{\infty}\lambda\begin{Bmatrix}(s_1^{\,3}d-s_1c)m_1+(s_2^{\,3}d-s_2c)m_2\\+(s_3^{\,3}d-s_3c)m_3+(s_4^{\,3}d-s_4c)m_4\end{Bmatrix}\int_a^b\rho G(\rho)J_1(\lambda\rho)d\rho J_0(\lambda r)d\lambda \tag{53}$$

If $M(\lambda)$ is defined as follow

$$M(\lambda)=(s_1^{\,3}d-s_1c)m_1+(s_2^{\,3}d-s_2c)m_2+(s_3^{\,3}d-s_3c)m_3+(s_4^{\,3}d-s_4c)m_4 \tag{54}$$

(53) is reduced as

$$p(r)=\int_0^{\infty}\lambda M(\lambda)\int_{a_c}^{b_c}\rho G(\rho)J_1(\lambda\rho)d\rho J_0(\lambda r)d\lambda \tag{55}$$

For larger values of λ, $M(\lambda)$ converges the M_∞ can be defined as

$$\lim_{\lambda\to\infty} M(\lambda)=\frac{C_4B_1-C_1B_4}{B_1A_4-B_4A_1}=M_\infty \tag{56}$$

Where

$$C_i=s_i^3d-s_ic,\quad B_i=1-as_i^2,\quad A_i=\left(a_{44}-(a_{33}d-2a_{13}a)s_i^2\right)\qquad i=1,4 \quad (57\text{-}59)$$

(55) can be written as

$$p(r)=\int_{a_c}^{b_c}G(\rho)\int_0^\infty \rho\lambda[\{M(\lambda)-M_\infty\}+M_\infty]J_1(\lambda\rho)J_0(\lambda r)d\lambda d\rho \tag{60}$$

After some manipulation, (60) is reduced as

$$-\pi p(r)=\int_{a_c}^{b_c}\left[\frac{M_\infty}{\rho-r}+k(r,\rho)\right]G(\rho)d\rho \tag{61}$$

Where

$$k(r,\rho)=M_\infty k_1(r,\rho)-\pi k_2(r,\rho) \tag{62}$$

$$k_1(r,\rho)=\frac{m(r,\rho)-1}{\rho-r}+\frac{m(r,\rho)}{\rho+r} \tag{63}$$

$$k_2(r,\rho)=\int_0^\infty \rho\lambda\{M(\lambda)-M_\infty\}J_1(\lambda\rho)J_0(\lambda r)d\lambda \tag{64}$$

$$\mathrm{m}(r,\rho)=\begin{cases}E\left(\dfrac{r}{\rho}\right) & r<\rho\\[2ex] \dfrac{r}{\rho}E\left(\dfrac{\rho}{r}\right)+\dfrac{\rho^2-r^2}{r\rho}K\left(\dfrac{\rho}{r}\right) & r>\rho\end{cases} \tag{65}$$

K and E are complete elliptic integrals of first and second kind respectively. Note that (61) has to be solved under the condition (46).

4. NUMERICAL SOLUTION

It is obvious that the first part of the kernel $k_1(r,\rho)$ in (61), has a simple logarithmic singularity when $r=\rho$ in the form of $\log|\rho-r|$. The second part of the kernel, $k_2(r,\rho)$ is bounded in the closed interval $a_c\leq (r,\rho)\leq b_c$. The unknown function $G(\rho)$ is infinite but integrable at $\rho=\pm1$, therefore the solution is of the form [19].

$$G(\rho)=\Phi(\rho)[(\rho-a_c)(b_c-\rho)]^{-1/2} \tag{66}$$

A standard numerical technique can be used to find out the unknown function $G(\rho)$ [19]. To be able to apply the numerical solution technique to the singular integral equation, it should be normalized. Normalization is carried out by the following quantities:

$$r=\frac{b_c-a_c}{2}\eta+\frac{b_c+a_c}{2}\qquad \rho=\frac{b_c-a_c}{2}\tau+\frac{b_c+a_c}{2} \tag{67,68}$$

Equations (61) and (46) become,

$$\frac{1}{\pi}\int_{-1}^{1}\left[\frac{M_\infty}{\tau-\eta}+K(\tau,\eta)\right]G(\tau)d\tau=-f(\eta) \qquad \int_{-1}^{1}G(\eta)d\eta=0 \tag{69,70}$$

where

$$K(\tau,\eta)=\frac{b_c-a_c}{2}k(\tau,\eta) \tag{71}$$

Since $G(\tau)$ has an integrable singularity,

$$G(\tau)=(1-\tau^2)^{-1/2}F(\tau) \tag{72}$$

may be written.

The solution of (69) is determined by using single-valuedness condition in (70) [20].

Substituting (72) into (69) we obtain

$$\frac{1}{\pi}\int_{-1}^{1}\left[\frac{M_\infty}{\tau-\eta}+K(\tau,\eta)\right]\frac{F(\tau)}{(1-\tau^2)^{1/2}}d\tau=-f(\eta) \tag{73}$$

$F(\tau)$ has to be obtained from (73) subjected to the single-valuedness condition,

$$\int_{-1}^{1}\frac{F(\eta)}{(1-\eta^2)^{1/2}}d\eta=0 \tag{74}$$

Equation (73) and (74) can be evaluated by using the Gauss-Chebyshev integration formula [21]. Thus from (73) and (74) we obtain

$$\sum_{k=1}^{n}\frac{1}{n}F(\tau_k)\left[\frac{M_\infty}{\tau_k-\eta_r}+K(\tau_k,\eta_r)\right]=-f(\eta_r) \qquad (r=1,..,n-1) \tag{75}$$

$$\sum_{r=1}^{n}\frac{\pi}{n}F(\eta_r)=0 \tag{76}$$

The collocation points are [22]

$$\tau_k=\cos\left(\frac{2k-1}{2n}\pi\right) \quad (k=1,..,n) \qquad \eta_r=\cos\left(\frac{r\pi}{n}\right) \quad (r=1,..,n-1) \tag{77,78}$$

The set of n simultaneous algebraic equations of (75) and (76) is solved and one can find n of values for $F(\tau_i)$ (i=1..n). In order to determine the stress intensity factors at the inner and outer crack tips, the values of $F(+1)$ and $F(-1)$ are used. The values of $F(+1)$ and $F(-1)$ are the first and the last elements of the set of $F(\tau_i)$.

The Mode I stress intensity factors at the crack tips are defined as

$$k(a_c)=\lim_{r\to a_c}\sqrt{2(a_c-r)}.\sigma_z(r,0) \qquad k(b_c)=\lim_{r\to b_c}\sqrt{2(r-b_c)}.\sigma_z(r,0) \tag{79,80}$$

$k(a_c)$ and $k(b_c)$ can also be expressed in terms of unknown function $G(r)$

$$k(a_c)=\lim_{r\to a_c}M_\infty\sqrt{2(a_c-r)}.G(r)=M_\infty\sqrt{(b_c-a_c)/2}.F(-1) \tag{81}$$

$$k(b_c)=\lim_{r\to b_c}M_\infty\sqrt{2(r-b_c)}.G(r)=-M_\infty\sqrt{(b_c-a_c)/2}.F(+1) \tag{82}$$

The normalized stress intensity factors are [15]

$$k'(a_c) = {k(a_c)} \Big/ {p_0\sqrt{(b_c - a_c)/2}} \qquad k'(b_c) = {k(b_c)} \Big/ {p_0\sqrt{(b_c - a_c)/2}} \tag{83,84}$$

The normalized stress intensity factors are calculated for different transversely isotropic layers contained a ring-shaped crack are tabulated in Tables 2..9.

5. NUMERICAL RESULTS AND DISCUSSION

In this study stress intensity factors were obtained for a crack by using equations (81) and (82). Normalized stress intensity factors for different materials were given in Tables 2-7. In these tables, $k_a=k'(a_c)$ is the stress intensity factor of the inner tip of crack and $k_b=k'(b_c)$ is the outer tip of crack. For these materials k_b increases with a_c/h ratios for fixed b_c/h ratio. Also for fixed b_c/h ratio k_a decreases with a_c/h ratios. It can be seen that the difference of k_a or k_b between the materials considered were very small. Stress intensity factors were effected the kernels k_1 and k_2 in equation (61). k_2 second part of the kernel in equation (61) was related to material constants and smaller than k_1 first part of (61) was related to crack dimensions. Thus, crack dimensions effected the stress intensity factors much more than elastic constants for studied materials. Thus, the difference of stress intensity factors k_a or k_b for these materials are very small.

Table 2. The normalized stress intensity factors for Magnesium at inner tip of crack "k_a"

$b_c/h\rightarrow$ $a_c/h\downarrow$	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.1	1.13274	1.247464	1.333907	1.391554	1.424142	1.4375	1.437902	1.431023
0.2		1.063146	1.115292	1.151349	1.171421	1.17839	1.176018	1.168094
0.3			1.039559	1.069548	1.087462	1.094453	1.093181	1.086147
0.4				1.028191	1.047651	1.057447	1.058949	1.054372
0.5					1.021348	1.03539	1.040915	1.040141
0.6						1.017232	1.027874	1.030971
0.7							1.014381	1.022703
0.8								1.012259

Table 3. The normalized stress intensity factors for Magnesium at outer tip of crack "k_b"

$b_c/h\rightarrow$ $a_c/h\downarrow$	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.1	0.943745	0.918056	0.892966	0.865859	0.838638	0.813062	0.790534	0.771628
0.2		0.95905	0.930888	0.903375	0.87577	0.849307	0.825113	0.803781
0.3			0.967322	0.939964	0.912605	0.886144	0.860802	0.837513
0.4				0.972692	0.947054	0.921198	0.895934	0.87152
0.5					0.976235	0.953017	0.928588	0.904378
0.6						0.979177	0.957794	0.934638
0.7							0.981446	0.961575
0.8								0.983217

Table 4. The normalized stress intensity factors for Barium titanate at inner tip of crack "k_a"

$b_c/h\rightarrow$ $a_c/h\downarrow$	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.1	1.111225	1.165069	1.184014	1.19568	1.212702	1.235948	1.262539	1.292189
0.2		1.048387	1.065205	1.062803	1.05815	1.061836	1.074419	1.091927
0.3			1.027999	1.028304	1.01484	1.004014	1.002536	1.008384
0.4				1.017318	1.010498	0.992696	0.976727	0.969027
0.5					1.011235	0.999216	0.975454	0.955288
0.6						1.006427	0.989785	0.963616
0.7							1.003585	0.985168
0.8								1.001326

Table 5. The normalized stress intensity factors for Barium titanate at outer tip of crack "k_b"

$b_c/h\rightarrow$ $a_c/h\downarrow$	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.1	0.928879	0.877762	0.834442	0.798164	0.773324	0.755274	0.74089	0.729829
0.2		0.947997	0.898395	0.850984	0.814993	0.788228	0.767724	0.75228
0.3			0.957346	0.908712	0.863223	0.825242	0.795952	0.77535
0.4				0.963189	0.917246	0.869689	0.830219	0.802032
0.5					0.966878	0.920898	0.873672	0.835686
0.6						0.969047	0.924721	0.878452
0.7							0.971386	0.927791
0.8								0.972741

Table 6. The normalized stress intensity factors for Graphite-epoxy at inner tip of crack "k_a"

$b_c/h\rightarrow$ $a_c/h\downarrow$	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.1	1.110816	1.15961	1.166127	1.16112	1.161872	1.17272	1.192015	1.218199
0.2		1.046561	1.057603	1.047054	1.033093	1.027482	1.032593	1.045737
0.3			1.026819	1.024249	1.005851	0.988034	0.979222	0.979545
0.4				1.016739	1.007673	0.985532	0.963812	0.950245
0.5					1.010673	0.996892	0.969809	0.944592
0.6						1.006124	0.988093	0.958445
0.7							1.003194	0.983033
0.8								1.000912

Table 7. The normalized stress intensity factors for Graphite-epoxy at outer tip of crack "k_b"

$b_c/h\rightarrow$ $a_c/h\downarrow$	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.1	0.928224	0.872914	0.824935	0.785985	0.758615	0.738899	0.723924	0.712826
0.2		0.946487	0.894198	0.844479	0.805174	0.775485	0.753087	0.736556
0.3			0.956541	0.906327	0.857842	0.816737	0.784552	0.761365
0.4				0.962669	0.915022	0.865223	0.8226	0.790609
0.5					0.966419	0.91936	0.869586	0.827626
0.6						0.968784	0.92305	0.873852
0.7							0.970982	0.925949
0.8								0.972386

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