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UNCERTAINTY ANALYSIS OF CRYOGENIC TURBINE EFFICIENCY

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Abstract-A procedure for estimating uncertainty in the hydraulic efficiency of cryogenic turbines is presented. A case study is performed based on the test data from a cryogenic turbine testing facility. The effects of uncertainties in the measurements of temperature, pressure, and generator power on the turbine hydraulic efficiency are studied and the uncertainty in turbine efficiency is estimated to be ±0.20%. About 79% of the uncertainty is determined to come from the uncertainty in generator power measurement. This uncertainty in turbine efficiency is believed to be reasonable and acceptable for the testing of cryogenic turbines.

1. INTRODUCTION

Natural gas should be liquefied before it can be transported across long distances. Liquefaction of natural gas is done in liquefied natural gas (LNG) liquefaction plants based on non-conventional refrigeration cycles. These plants are rather complicated with numerous systems interacting with each other [1]. These plants use great amounts of process energy. Therefore, minimizing operating costs is closely associated with minimizing process energy consumption while producing the same desired output [2, 3].

In the conventional liquefaction plants, the high pressure LNG is expanded in a throttling valve, also called Joule-Thompson valve, following high pressure condensation process. The object of throttling is to decrease the pressure of LNG to the levels manageable for economic transportation and allow the refrigeration cycle to be completed. A throttling process is essentially a constant enthalpy process and heat transfer to the fluid is negligible. From a thermodynamic point of view, a throttling valve can be replaced with a turbine. This way, the same pressure drop can be achieved while producing power. However, this replacement is often neither practical nor economical. A cryogenic turbine is recently developed and tested by a private company for the replacement of the throttling valve in LNG liquefaction plants. An investigation of this cryogenic turbine revealed that replacing the throttling valve with the cryogenic turbine can save an LNG liquefaction plant about half a million dollars a year in electricity costs [4].

Cryogenic turbines operate based on the hydraulic turbine principals. A hydraulic turbine extracts energy from a fluid which possesses high head. The head represents the energy of a unit weight of the fluid. Energy transfer occurs between the fluid in motion and a rotating shaft due to dynamic action, which results in changes in pressure and fluid momentum [5]. The fluid is liquid water in hydraulic turbines and a cryogenic liquid in cryogenic turbines.

The cryogenic turbine is a radial inflow reaction turbine with an induction generator mounted on an integral shaft. The entire unit including the turbine and generator is submerged in the cryogenic liquid. In applications where a high power output is required, a radial turbine runner is more effective than a mixed flow or axial runner since the energy transfer between the fluid and the runner is enhanced by the reduction in radius along the fluid path through the runner. In addition, radial turbine runners have the advantage of lower run-away speeds when compared to mixed flow or axial runners. To expand high differential pressures, multi-stage turbines with multiple identical runners are utilized [6, 1].

In cases of variable head differences and flow rates, variable speed hydraulic turbines have been used in hydropower plants [7, 8]. The peak efficiency is maintained by changing running speeds. The cryogenic turbine tested is also a variable speed turbine. A wide range of peak efficiency can be maintained at various head differences and flow rates with a fixed turbine geometry [1].

Performance of a cryogenic turbine is best expressed by its so called hydraulic efficiency, which is the fraction of work potential entering the turbine that is actually converted to work. When reporting the hydraulic efficiency of this newly developed cryogenic turbine using the test data, it is essential to know the uncertainty associated with its determination. Performing an uncertainty analysis of the cryogenic turbine efficiency using the test data provided by the testing facility is what we intend to do in this paper.

2. UNCERTAINTY ANALYSIS

Experimental results will involve some level of uncertainty because of inaccuracies associated with measurement equipment, random variation in the measured parameters, and approximations in data reduction relations. All these individual uncertainties eventually translate into uncertainty in the final results [9]. The object of an uncertainty analysis can be different. In our case it is to demonstrate the validity of the turbine efficiency results.

Consider the result of R to be a function of n measured variables x_1, x_2, ..., x_n as

$$R = f(x_1, x_2, \ldots\ldots, x_n) \tag{1}$$

Denoting uncertainty (error) by E, the effect of error in measured variables x_1, x_2,.....,x_n on the final calculated result R in the case of uncorrelated error is given by

$$E_R = \left[\sum_{i=1}^{n}\left(E_{x_i}\frac{\partial R}{\partial x_i}\right)^2\right]^{1/2} \tag{2}$$

This is a general relation that is applicable to any relation. It is also called the most probable uncertainty. The error in this relation is the absolute error, and it should include both the bias and precision (random) errors. The precision error is lack of precision in the measurements and the bias error is estimated maximum fixed error. All uncertainty estimates in Equation (2) should be based on same confidence level, usually 95%. This means that when an uncertainty is estimated, the actual error will be less than the estimated uncertainty, 95% of the times. The confidence level in the uncertainty in the result, R, will be the same as the confidence levels of the uncertainties in the x_i's. In Equation (2), the individual terms are squared before they are added, and the larger terms dominate the result. This provides a method of reducing the overall uncertainty by reducing a single large error rather than several small ones.

The maximum uncertainty on the result R can be estimated by

$$E_R = \sum_{i=1}^{n} \left| E_{x_i} \frac{\partial R}{\partial x_i} \right| \qquad (3)$$

This formula assumes that all error terms will become positive simultaneously at the extreme value of the uncertainty interval. Equation (3) will produce an unreasonably high estimate of E_R. The most probable uncertainty, Equation (2), is a better estimate for uncertainty [9], and it will be used throughout this paper.

3. UNCERTAINTY IN TURBINE EFFICIENCY

The hydraulic efficiency of the cryogenic turbine is defined as the ratio of the actual power output from the generator to the maximum power output under ideal conditions. Note that by using generator power output instead of turbine power output, the generator efficiency is accounted for in the efficiency. The hydraulic efficiency is determined from

$$\varepsilon = \frac{\dot{W}_{gen}}{SG \times \rho_{water} \times g \times \dot{V} \times TDH} \qquad (4)$$

where $\dot{W}_{gen}$ (kW) is the power output from the generator, SG is the specific gravity, g (m/s^2) is the acceleration due to gravity, $\dot{V}$ (m^3/s) is volume flow rate, and TDH (m) is total dynamic head. The specific gravity is defined as the ratio of the density of the cryogenic liquid to the density of liquid water, which is taken to be $\rho_{water} = 1000$ kg/m^3.

The effect of errors in generator power, specific gravity, volume flow rate, and total dynamic head on the turbine efficiency can be determined using Equation (2)

$$E_\varepsilon = \left[\left(\frac{\partial \varepsilon}{\partial \dot{W}_{gen}} E_{\dot{W}_{gen}} \right)^2 + \left(\frac{\partial \varepsilon}{\partial SG} E_{SG} \right)^2 + \left(\frac{\partial \varepsilon}{\partial \dot{V}} E_{\dot{V}} \right)^2 + \left(\frac{\partial \varepsilon}{\partial TDH} E_{TDH} \right)^2 \right]^{1/2} \qquad (5)$$

where all errors denoted by E are absolute errors. Note that the mean values should be used when substituting values into the derivatives.

The confidence level of all uncertainty estimates is made to the 95%. Reference [10] recommends 95% confidence level for uncertainty analysis. Note that Equation (2) is valid for the case of uncorrelated errors. It will be seen later in this study that errors in the measurement of temperature are used to determine the error in specific gravity and errors in specific gravity and pressure are used to determine the error in volume flow rate. Also, errors in the measurement of pressure, specific gravity, and volume flow rate are used to determine the error in total dynamic head. Finally, errors in generator power, specific gravity, volume flow rate, and total dynamic head are used to determine the error in turbine efficiency. All these calculation steps are followed in their logical order, and thus the error correlations between the variables are successfully taken care of.

The mean values for variables of interest are obtained from four sets of test data provided by the testing facility. They are listed in Table 1.

Errors in the measurement of temperature and pressure at various states are provided by the testing facility as follows:

Error in temperature at the turbine inlet, $E_{T,in} = \pm 0.04°C$
Error in temperature at the turbine outlet, $E_{T,out} = \pm 0.33°C$
Error in pressure drop (across the valve), $E_{\Delta P} = \pm 0.24\%$
Error in pressure at the turbine inlet, $E_{P,in} = \pm 0.097\%$
Error in pressure at the turbine outlet, $E_{P,out} = \pm 0.53\%$

Table 1. Four sets of data from testing of turbine and mean values for parameters.

	T_{in} (°C)	T_{out} (°C)	SG_{in}	SG_{out}	ΔP (kPa)	P_{in} (kPa)	P_{out} (kPa)	$\dot{V}$ (m^3/s)	TDH (m)	$\dot{W}_{gen}$ (kW)	ε (%)
Test 1	-50.12	-49.64	0.5935	0.5927	141.89	1014.15	26.82	0.047	173.60	27.90	59.30
Test 2	-49.30	-48.81	0.5921	0.5913	116.25	1130.12	25.03	0.042	193.91	24.60	51.77
Test 3	-48.56	-47.81	0.5909	0.5896	134.65	1502.99	25.79	0.046	258.80	31.60	46.36
Test 4	-50.44	-49.83	0.5941	0.930	165.34	1596.62	28.48	0.050	273.48	44.40	55.47
Mean Value	-49.61	-49.02	0.5927	0.5917	139.53	1310.97	26.53	0.046	224.95	32.13	53.26

The errors given include only the bias errors since it is determined by the testing facility that precision errors were extremely small, and therefore they could be neglected. The error in temperature reading at the turbine exit is much greater than that at the turbine inlet. This is because a silicon diode thermometer was used to measure the temperature at the turbine inlet whereas a thermocouple was used to measure the temperature at the turbine exit. The details of the estimation of measurement errors are not within the scope of this paper. Cryogenic temperature measurements and their accuracy have been the topic of several studies [11, 12, 13, 14]. They discuss in detail the cryogenic thermometry and methods for estimating errors.

The errors in temperature are absolute errors while the errors in pressure are given as percentages. We now convert the percentage pressure errors to absolute units using their mean values as shown in Table 1.

$$E_{\Delta P} = E_{\Delta P}(\%) \times \Delta P = (0.0024)(139.53\,\text{kPa}) = 0.33\,\text{kPa}$$
$$E_{P_{in}} = E_{P_{in}}(\%) \times P_{in} = (0.00097)(1310.97\,\text{kPa}) = 1.27\,\text{kPa}$$
$$E_{P_{out}} = E_{P_{out}}(\%) \times P_{out} = (0.0053)(26.53\,\text{kPa}) = 0.14\,\text{kPa}$$

In addition to the temperature and pressure error data, generator power errors for individual test points are provided to be

Test 1	$E_{\dot{W}_{gen}} = \pm 0.16\,\text{kW}$
Test 2	$E_{\dot{W}_{gen}} = \pm 0.11\,\text{kW}$
Test 3	$E_{\dot{W}_{gen}} = \pm 0.08\,\text{kW}$
Test 4	$E_{\dot{W}_{gen}} = \pm 0.08\,\text{kW}$
average	$E_{\dot{W}_{gen}} = \pm 0.11\,\text{kW}$

Next, we determine the uncertainties in specific gravity, volume flow rate, and total dynamic head.

3.1. Specific Gravity Uncertainty

Specific gravity has different values at the turbine inlet and at the turbine exit. They are calculated from the relations

$$SG_{in} = 0.001692\,(T_{@SG} - T_{in}) + SG \tag{6}$$
$$SG_{out} = 0.001692\,(T_{@SG} - T_{out}) + SG \tag{7}$$

where SG is a known specific gravity at a given liquid temperature T. Theoretically, an infinite number of SG and T combinations satisfy Equation (6) and Equation (7). Here following combination is provided to be used in these formulas.

$SG = 0.5929$ at $T = -49.76°C$

T_{in} (°C) and T_{out} (°C) are the temperatures of the liquid measured at the turbine inlet and at the turbine exit, respectively. This relation successfully accounts for the dependence of specific gravity with temperature. This empirical formula was obtained by correlating data. The constant, 0.001692, that appears in the equation must have the unit of 1/°C so that the formula is dimensionally consistent. The uncertainties in specific gravity at the turbine inlet and exit are determined by applying Equation (2) to be

$E_{SG,in} = \pm 0.00009$
$E_{SG,out} = \pm 0.00057$

Specific gravity, as used in the turbine efficiency relation, is the average of the two specific gravity values. That is,

$$SG = \frac{SG_{in} + SG_{out}}{2} \tag{8}$$

Finding the mean value of SG using Equation (8) and then applying Equation (2) to Equation (8) to determine the error in SG, we find

$E_{SG} = \pm 0.00029$

3.2. Volume Flow Rate Uncertainty

Volume flow rate, as appears in the turbine efficiency relation, is determined from

$$\dot{V} = C_V \sqrt{\frac{\Delta P}{SG_{in}}} \tag{9}$$

where $\dot{V}$ is in m^3/s, ΔP (kPa) is the pressure drop across the valve, and SG_{in} is the specific gravity of the liquid at the turbine inlet. The quantity C_V is called valve flow coefficient, which manufacturers tabulate in their valve brochures. For our case, it is $C_V = 0.003009$. Making sure that this empirical formula is dimensionally consistent we see that it has rather an odd unit, $m^{7/2}/kg^{1/2}$. The values of C_V in the literature increase nearly as the square of the size of the valve [15]. The uncertainty in volume flow rate is determined by applying Equation (2) to be

$E_{\dot{V}} = \pm 0.00005$

3.3. Total Dynamic Head Uncertainty

Total dynamic head, as appears in the turbine efficiency relation, is determined from

$$TDH = \frac{P_{in} - P_{out}}{\rho g} + \frac{Vel_{in}^{\ 2} - Vel_{out}^{\ 2}}{2g} + z_{in} - z_{out} \tag{10}$$

or

$$TDH = \frac{P_{in} - P_{out}}{SG\rho_{water} g} + \frac{\dot{V}^2}{2g}\left(\frac{1}{A_{in}^{\ 2}} - \frac{1}{A_{out}^{\ 2}}\right) + z_{in} - z_{out} \tag{11}$$

where TDH is in m, P_{in} and P_{out} are pressures at the turbine inlet and outlet in kPa, g is in m/s^2, $\dot{V}$ is in m^3/s, A_{in} and A_{out} are turbine inlet and outlet areas in m^2, and z_{in} and z_{out} are elevations at the turbine inlet and outlet in m. The heads due to velocity and elevation change are small, and the first term on the right hand side of Equation (11) dominates the calculation of total dynamic head. The uncertainty in total dynamic head is determined by applying Equation (2) to be

$E_{TDH} = \pm 0.24$

In Table 2, we list the absolute errors in specific gravity, volume flow rate, total dynamic head, and generator power together with their mean values. Now the uncertainty in turbine hydraulic efficiency can be determined using Equation (5). Taking the derivatives and substituting the values in Table 2, we find

$E_{\varepsilon} = \pm 0.0020 = \pm 0.20\%$

That is, the overall uncertainty in the determination of hydraulic efficiency is ±0.20%. Using the mean values in Table 2 and substituting them into hydraulic efficiency formula, Equation (4), we find the mean value of hydraulic efficiency as

$\varepsilon = 0.5316 = 53.16\%$

That is, the hydraulic efficiency of this prototype cryogenic turbine for the four test data considered can be written as

$\varepsilon = 53.16 \pm 0.20\ \%$

Table 2. Mean values and absolute errors for variables that appear in the equation used to calculate hydraulic efficiency.

Variable	Mean value	Absolute error (±)
$\dot{W}_{gen}$ (kW)	32.13	0.11
SG	0.59220	0.00029
$\dot{V}$ (m^3/s)	0.04625	0.00005
TDH (m)	224.95	0.24

Using Equation (3) to estimate maximum uncertainty in hydraulic efficiency, we find

$E_{\varepsilon} = \pm 0.0032 = \pm 0.32\%$

That is the maximum uncertainty in hydraulic efficiency estimated using Equation (3) is about 50 percent greater than the most probable uncertainty estimated using Equation (2).

4. CONCLUSIONS

The effects of uncertainties in the measurements of temperature, pressure, and generator power on the turbine hydraulic efficiency are studied and the uncertainty in turbine efficiency is estimated to be ±0.20%. A study of Equation (5) shows that about 79% of this uncertainty comes from the error in generator power measurement. The errors in volume flow rate, total dynamic head, and specific gravity account for about 10%, 9%, and 2% of the uncertainty in turbine efficiency, respectively. A ±0.20% uncertainty in turbine efficiency is reasonable and acceptable for the testing of cryogenic turbine. The maximum uncertainty in turbine efficiency is estimated to be ±0.32%, which is also acceptable. However, if any attempt is to be made to improve the accuracy of turbine efficiency, it should start with the accuracy of generator power measurement.

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