NUMERICAL STUDY OF COLLISION EFFICIENCY OF DUST PARTICLES

Sabit Adanur¹ and Sayavur I. Bakhtiyarov²

¹Textile Engineering Department, Auburn University, AL 36849-5327

²Space Power Institute, 231 Leach Center, Auburn University, AL 36849-5320

Abstract-The collision efficiency of bound sand particles is studied numerically. The fourth-order Runge - Kutta method was used to solve two ordinary differential equations.

1. INTRODUCTION

The efficient filtration of dust is directly related to physical characteristics of the dust particles. Collisions between the moist dust particles take place at the filtration devices because of the different velocities for particles of various sizes. As a result of coalescence during successive collisions, the size of a dust aggregate grows and its flow velocity increases with time. The larger size and the higher velocity of the aggregate cause an increase in the rate of collisions. In this paper the collision efficiency of moist dust particles is studied numerically using the fourth-order Runge - Kutta method.

2. THEORETICAL ANALYSIS

Consider the collisions between a dust aggregate of diameter D with velocity u_1 and smaller dust particles of diameter d with velocity u_2 . The dust aggregate is moving toward the small particles at a speed $u_r = u_1 - u_2$ (Figure 1). The small particles contained within a cylinder of diameter D ahead of the aggregate would collide with it. The air flow around the aggregate has a tendency to carry the particles away from the central axis. Therefore only the particles contained within a cylinder of diameter D_c ($D_c < D$) can make the collision. The ratio of the cross sectional areas of these two cylinders is called the collision efficiency,

$$E = S_c / S_1 ,$$

where $S_c = \pi D_c^2 / 4$ and $S_1 = \pi D^2 / 4$.

Then the collision efficiency,

$$E = (D_c / D)^2$$

Figure 1. Collision of a particle with an aggregate.

Based on the collision efficiency, the growth rate of a dust aggregate in a jet of given dust bulk porosity can be calculated if the fraction of collisions leading to coalescence is known. We assume that $d \ll D$, so that the presence of the small particles does not alter the flow around the aggregate, and the changes in diameter and velocity of the aggregate are negligible within a period when a small particle has traveled through a distance $D/2$. A potential flow around the aggregate is assumed. If a flow separation takes place, the flow ahead of the aggregate is not affected significantly. A cylindrical coordinate system attached to the center of the aggregate is used. The axis z is chosen in the direction of gravitational acceleration g . Assume at time t the position of an individual dust particle is at (z, r) with a velocity $U(u, v)$ and, at the same location an air velocity is $U_f(u_f, v_f)$. While moving through a fluid of density ρ_f and dynamic viscosity μ the following forces act on the particle:

- (i) The gravitational force acting in the positive z direction

$$F_g = m g = \rho V g = \pi d^3 \rho g / 6, \quad (1)$$

where m , V and ρ are mass, volume and density of the particle, respectively.

- (ii) The buoyant force (the weight of the fluid displaced by the particle)

$$F_b = -m_f g = -\pi d^3 \rho_f g / 6. \quad (2)$$

- (iii) The force on an accelerating particle associated with the fluid motion (Lamb, 1932)

$$F_a = -\pi d^3 \rho_f (du/dt)/12. \quad (3)$$

(iv) Drag force resulting from integrating the shear stresses throughout the body

$$F_d = c \pi d^2 \rho_f u^2 / 8, \quad (4)$$

where c is a drag coefficient, which is a function of the particle shape and the Reynolds number $Re = u d \rho_f / \mu$. At Reynolds numbers from 1,000 to 500,000 the drag coefficient for smooth spherical particles is 0.6 (Goldstein, 1938). At low Reynolds numbers ($Re < 0.2$) the drag force can be determined by Stokes equation,

$$F_d = 3 \pi d \mu u. \quad (5)$$

- (v) The wave drag is caused when the particle speed is comparable to the speed of sound in a fluid medium. At low subsonic speeds the wave drag can be ignored.
- (vi) Magnus force. Symmetrical bodies such as spheres fall vertically with the same velocity in any orientation. For non-symmetrical bodies moving through a fluid an orienting force exists. This force termed Magnus force is caused by fluid velocity differences on opposite sides of body. According to the Bernoulli equation pressure on the particle side with lower fluid velocity will be more than the pressure on the particle side with higher fluid velocity. This pressure difference causes a Magnus force. One would intuitively conclude that the radial segregation of solid particles in gas-particle suspension flow could be caused by Magnus force. For small particles the Magnus force can be ignored.

The equations of motion of particle with mass m , including all the forces discussed above, are given by

$$m (d^2 z / dt^2) = mg - m_f g - m_f (d^2 z / dt^2) / 2 - F_{Rz}, \quad (6)$$

$$m (d^2 r / dt^2) = - m_f (d^2 r / dt^2) / 2 + F_{Rr}, \quad (7)$$

where F_{Rz} and F_{Rr} are components of the resultant fluid dynamic force acting on the particle. The magnitude and the direction of this force is determined by the velocity of the fluid relative to the velocity of the particle

$$U_r(u, v) = U_f(u_f, v_f) - U(u, v). \quad (8)$$

If the relative velocity U_r makes an angle ϕ with the r -axis,

$$U_r \sin \phi = u_f - u, \quad (9)$$

$$U_r \cos \phi = v_f - v, \quad (10)$$

Then the components of the resultant fluid dynamic force become (Figure 1)

$$F_{Rz} = F_R \sin \phi = \pi \rho_f d^2 c (u_f - u) U_r / 8, \quad (11)$$

$$F_{Rf} = F_R \cos \varphi = \pi \rho_f d^2 c (v_f - v) U_r / 8. \quad (12)$$

Substituting equations (11) and (12) into equations (6) and (7) we obtain

$$d^2 z / dt^2 = [(1 - \alpha) g - 0.75 \alpha c (u_f - u) U_r / d] / \beta, \quad (13)$$

$$d^2 r / dt^2 = 0.75 \alpha c (v_f - v) U_r / d \beta, \quad (14)$$

where $\alpha = m_f / m = \rho_f / \rho$ and $\beta = 1 + m_f / 2m = 1 + \rho_f / 2\rho$.

In the moving coordinate system chose, the gravitational acceleration does not appear. Also
 $u = dz / dt$ and $v = dr / dt$.

Then

$$d^2 z / dt^2 = - 0.75 \alpha c (u_f - dz / dt) U_r / d \beta, \quad (15)$$

$$d^2 r / dt^2 = 0.75 \alpha c (v_f - dr / dt) U_r / d \beta. \quad (16)$$

The fluid velocity components u_f and v_f of a potential flow can be determined as (Schlichting, 1987),

$$u_f = u_r [1 + D^3 (r^2 / 2 - z^2) / 8 (r^2 + z^2)^{2.5}], \quad (17)$$

$$v_f = - u_r 3 D^3 z r / 16 (r^2 + z^2)^{2.5}. \quad (18)$$

Substituting the fluid velocity components in equations (15) and (16), and nondimensionalizing them we have

$$d^2 Z / d\tau^2 = - 0.75 \gamma \xi c [1 + (\delta^2 / 2 - Z^2) / (\delta^2 + Z^2)^{2.5} - U] (D / d), \quad (19)$$

$$d^2 \delta / d\tau^2 = - 0.75 \gamma \xi c [- 1.5 \delta Z / (\delta^2 + Z^2)^{2.5} - V] (D / d), \quad (20)$$

where the following dimensionless variables were used:

$$Z = 2 z / D, \quad \delta = 2 r / D, \quad \tau = 2 t u_r / D, \quad \gamma = \alpha / \beta,$$

$$U = u / u_r, \quad V = v / u_r, \quad U_f = u_f / u_r, \quad V_f = v_f / u_r,$$

$$\xi = U_r / u_r = [(U_f - U)^2 - (V_f - V)^2]^{0.5}.$$

By introducing a new variables ψ and ζ , the equations (19) and (20) can be written in the form of four first-order equations,

$$d Z / d \tau = \psi, \quad (21)$$

$$d \psi / d \tau = -0.75 \gamma \xi c [1 + (\delta^2/2 - Z^2)/(\delta^2 + Z^2)^{2.5} - \psi](D/d), \quad (22)$$

$$d \delta / d \tau = \zeta, \quad (23)$$

$$d \zeta / d \tau = -0.75 \gamma \xi c [-1.5 \delta Z/(\delta^2 + Z^2)^{2.5} - \zeta](D/d). \quad (24)$$

The fourth-order Runge - Kutta method was used to solve the equations (21) - (24). The fourth-order Runge - Kutta formulas from τ_i to τ_{i+1} are (Kuo, 1972):

$$Z_{i+1} = Z_i + (\Delta_1 Z_i + 2 \Delta_2 Z_i + 2 \Delta_3 Z_i + \Delta_4 Z_i) / 6 \quad (25)$$

$$\psi_{i+1} = \psi_i + (\Delta_1 \psi_i + 2 \Delta_2 \psi_i + 2 \Delta_3 \psi_i + \Delta_4 \psi_i) / 6, \quad (26)$$

$$\delta_{i+1} = \delta_i + (\Delta_1 \delta_i + 2 \Delta_2 \delta_i + 2 \Delta_3 \delta_i + \Delta_4 \delta_i) / 6, \quad (27)$$

$$\zeta_{i+1} = \zeta_i + (\Delta_1 \zeta_i + 2 \Delta_2 \zeta_i + 2 \Delta_3 \zeta_i + \Delta_4 \zeta_i) / 6. \quad (28)$$

The individual terms in the equations (25) - (28) are simulated by following formulas:

$$\Delta_1 Z_i = n \psi_i, \quad (29)$$

$$\Delta_1 \psi_i = n \{-0.75 \gamma \xi c [1 + (\delta^2/2 - Z_i^2)/(\delta^2 + Z_i^2)^{2.5} - \psi_i](D/d)\}, \quad (30)$$

$$\Delta_2 Z_i = n (\psi_i + \Delta_1 \psi_i / 2), \quad (31)$$

$$\Delta_2 \psi_i = n \{-0.75 \gamma \xi c [1 + [\delta^2/2 - (Z_i + \Delta_1 Z_i)^2]/[\delta^2 + (Z_i + \Delta_1 Z_i)^2]^{2.5} - (\psi_i + \Delta_1 \psi_i / 2)](D/d)\}, \quad (32)$$

$$\Delta_3 Z_i = n (\psi_i + \Delta_2 \psi_i / 2), \quad (33)$$

$$\Delta_3 \psi_i = n \{-0.75 \gamma \xi c [1 + [\delta^2/2 - (Z_i + \Delta_2 Z_i)^2]/[\delta^2 + (Z_i + \Delta_2 Z_i)^2]^{2.5} - (\psi_i + \Delta_2 \psi_i / 2)](D/d)\}, \quad (34)$$

$$\Delta_4 Z_i = n (\psi_i + \Delta_3 \psi_i / 2), \quad (35)$$

$$\Delta_4 \psi_i = n \{-0.75 \gamma \xi c [1 + [\delta^2/2 - (Z_i + \Delta_3 Z_i)^2]/[\delta^2 + (Z_i + \Delta_3 Z_i)^2]^{2.5} - (\psi_i + \Delta_3 \psi_i / 2)](D/d)\}, \quad (36)$$

$$\Delta_1 \delta_i = n \zeta_i \quad (37)$$

By introducing a new variables ψ and ζ , the equations (19) and (20) can be written in the form of four first-order equations,

$$d Z / d \tau = \psi, \quad (21)$$

$$d \psi / d \tau = -0.75 \gamma \xi c [1 + (\delta^2/2 - Z^2)/(\delta^2 + Z^2)^{2.5} - \psi](D/d), \quad (22)$$

$$d \delta / d \tau = \zeta, \quad (23)$$

$$d \zeta / d \tau = -0.75 \gamma \xi c [-1.5 \delta Z/(\delta^2 + Z^2)^{2.5} - \zeta](D/d). \quad (24)$$

The fourth-order Runge - Kutta method was used to solve the equations (21) - (24). The fourth-order Runge - Kutta formulas from τ_i to τ_{i+1} are (Kuo, 1972):

$$Z_{i+1} = Z_i + (\Delta_1 Z_i + 2 \Delta_2 Z_i + 2 \Delta_3 Z_i + \Delta_4 Z_i) / 6 \quad (25)$$

$$\psi_{i+1} = \psi_i + (\Delta_1 \psi_i + 2 \Delta_2 \psi_i + 2 \Delta_3 \psi_i + \Delta_4 \psi_i) / 6, \quad (26)$$

$$\delta_{i+1} = \delta_i + (\Delta_1 \delta_i + 2 \Delta_2 \delta_i + 2 \Delta_3 \delta_i + \Delta_4 \delta_i) / 6, \quad (27)$$

$$\zeta_{i+1} = \zeta_i + (\Delta_1 \zeta_i + 2 \Delta_2 \zeta_i + 2 \Delta_3 \zeta_i + \Delta_4 \zeta_i) / 6. \quad (28)$$

The individual terms in the equations (25) - (28) are simulated by following formulas:

$$\Delta_1 Z_i = n \psi_i, \quad (29)$$

$$\Delta_1 \psi_i = n \{-0.75 \gamma \xi c [1 + (\delta^2/2 - Z_i^2)/(\delta^2 + Z_i^2)^{2.5} - \psi_i](D/d)\}, \quad (30)$$

$$\Delta_2 Z_i = n (\psi_i + \Delta_1 \psi_i / 2), \quad (31)$$

$$\Delta_2 \psi_i = n \{-0.75 \gamma \xi c [1 + [\delta^2/2 - (Z_i + \Delta_1 Z_i)^2]/[\delta^2 + (Z_i + \Delta_1 Z_i)^2]^{2.5} - (\psi_i + \Delta_1 \psi_i / 2)](D/d)\}, \quad (32)$$

$$\Delta_3 Z_i = n (\psi_i + \Delta_2 \psi_i / 2), \quad (33)$$

$$\Delta_3 \psi_i = n \{-0.75 \gamma \xi c [1 + [\delta^2/2 - (Z_i + \Delta_2 Z_i)^2]/[\delta^2 + (Z_i + \Delta_2 Z_i)^2]^{2.5} - (\psi_i + \Delta_2 \psi_i / 2)](D/d)\}, \quad (34)$$

$$\Delta_4 Z_i = n (\psi_i + \Delta_3 \psi_i / 2), \quad (35)$$

$$\Delta_4 \psi_i = n \{-0.75 \gamma \xi c [1 + [\delta^2/2 - (Z_i + \Delta_3 Z_i)^2]/[\delta^2 + (Z_i + \Delta_3 Z_i)^2]^{2.5} - (\psi_i + \Delta_3 \psi_i / 2)](D/d)\}, \quad (36)$$

$$\Delta_1 \delta_i = n \zeta_i \quad (37)$$

$$\Delta_1 \zeta_i = n \{ -0.75 \gamma \xi c [-1.5 \delta_i Z / (\delta_i^2 + Z^2)^{2.5} - \zeta_i] (D/d) \}, \quad (38)$$

$$\Delta_2 \delta_i = n (\zeta_i + \Delta_1 \zeta_i / 2), \quad (39)$$

$$\Delta_2 \zeta_i = n \{ -0.75 \gamma \xi c [-1.5 (\delta_i + \Delta_1 \delta_i / 2) Z / ((\delta_i + \Delta_1 \delta_i / 2)^2 + Z^2)^{2.5} - (\zeta_i + \Delta_1 \zeta_i / 2)] (D/d) \}, \quad (40)$$

$$\Delta_3 \delta_i = n (\zeta_i + \Delta_2 \zeta_i / 2), \quad (41)$$

$$\Delta_3 \zeta_i = n \{ -0.75 \gamma \xi c [-1.5 (\delta_i + \Delta_2 \delta_i / 2) Z / ((\delta_i + \Delta_2 \delta_i / 2)^2 + Z^2)^{2.5} - (\zeta_i + \Delta_2 \zeta_i / 2)] (D/d) \}, \quad (42)$$

$$\Delta_4 \delta_i = n (\zeta_i + \Delta_3 \zeta_i / 2), \quad (43)$$

$$\Delta_4 \zeta_i = n \{ -0.75 \gamma \xi c [-1.5 (\delta_i + \Delta_3 \delta_i / 2) Z / ((\delta_i + \Delta_3 \delta_i / 2)^2 + Z^2)^{2.5} - (\zeta_i + \Delta_3 \zeta_i / 2)] (D/d) \}. \quad (44)$$

The collision efficiency was computed according to the following procedure. At the initial ($\tau = 0$) the first particle $A(Z_0, \delta_{10})$ moves along the centerline ($\delta_{10} = 0$) and collides with the aggregate. If collision takes place, an integer parameter $N_1 = +1$. If collision does not take place, $N_1 = -1$. A second particle B initially is at position $(Z_0, \delta_{10} + \Delta \delta_0)$. Solving nonlinear equations (21) - (24) by the fourth-order Runge - Kutta method the second particle trajectory can be determined. If the distance between the center of the second particle and the center of the aggregate

$$\Phi \leq (D + d) / 2,$$

it collides with the aggregate and $N_2 = +1$. If

$$\Phi > (D + d) / 2,$$

the second particles does not collide with the aggregate and $N_2 = -1$.

When $N_1 N_2 > 0$, either both particles collide with the aggregate, or neither can collide with it. If $N_1 N_2 < 0$, only one of the particles collides with the aggregate.

Then we call the second particle "first" and assign the value of $\delta_{10} + \Delta \delta_0$ to the variable δ_{10} . Next particle is C $(Z_0, \delta_{10} + \Delta \delta_0)$. The process of simulations is repeated until $|\Delta \delta_0| < \epsilon$, where ϵ is a specified small number.

3. RESULTS AND DISCUSSION

The program simulated the collision efficiency for the following particle and aggregate parameters:

$$\begin{aligned} D/d &= 5; 10; 20; 40; 60; 80; 100, & \alpha &= 0.386, \\ \beta &= 1.193, & \gamma &= 0.324, & c &= 0.600. \end{aligned}$$

Figure 2. Variation of the collision efficiency coefficient as a function of the aggregate to particle diameters ratio.

The results of simulations show that the collision efficiency E decreases with increasing the value of D/d ratio (Figure 2). The rate of decrease of collision efficiency coefficient is greatest at big aggregate to particle diameters ratios.

We remark that if $D/d \approx 1$, the flow distortions around the particles can not be neglected. It is clear that the numerical solution presented here is incapable of describing the collision efficiency for comparable diameters of the particles and the aggregates.

4. CONCLUSIONS

The collision efficiency of dust particles is studied numerically using the forth-order Runge-Kutta method. The results of simulations indicate that the collision efficiency coefficient decreases with increasing the aggregate to particle diameters ratio. The rate of decrease of collision efficiency coefficient is greatest at big aggregate to particle diameters ratios.

REFERENCES

1. Goldstein, S., Modern Developments in Fluid Dynamics, Vol. 1, Clarendon Press, Oxford, England, 1938.
2. Kuo, S. S., Computer Applications of Numerical Methods, Addison-Wesley, Reading, Mass, 1972.
3. Lamb, H., Hydrodynamics, Cambridge University Press, Cambridge, England, 1932.
4. Merkulov, V. I., Hydrodynamics - Known and Unknown, Nauka, Moscow, 1989.
5. Schlichting, H., Boundary - Layer Theory, Mc Grow-Hill, New York, 1987.