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DEA Cross-Efficiency Evaluation Considering Interval Efficiency with Aggressive–Benevolent Formulation

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Abstract

In DEA cross-efficiency, information aggregation of self-evaluation and peer-evaluation has been one of the main concerns. The inherent variability of decision-makers' preferences in peer-evaluation, whether expressed aggressively or benevolently within the secondary goal programming, can yield divergent peer-efficiency scores. Therefore, interval efficiency for considering different preferences of decision-makers and group opinions is used to represent the variable efficiency range. In this paper, comparison norms for the advantage degree between any two interval efficiencies are defined, and a pairwise advantage-degree matrix is developed. By aggregating this matrix through a dominance score, a unique ranking of DMUs can be obtained. Subsequently, a synthetic evaluation model, ICE-AB, is proposed for DEA cross-efficiency evaluation. To demonstrate the effectiveness, discriminatory capability, and practical applicability of ICE-AB, a benchmark numerical example, a differentiating numerical case, and an empirical application to major Chinese new energy vehicle manufacturers are provided. Comparative analysis and ranking results show that the new model not only preserves different peer-evaluation information but also obtains the unique ranking results of DMUs under uncertain attitudes of decision-makers.

Keywords: interval cross-efficiency matrix; benevolent formulation; aggressive formulation; advantage degree

1. Introduction

Data Envelopment Analysis (DEA) is a prominent tool for measuring the relative efficiency of Decision-Making Units (DMUs), and its fundamental aim is to assess the efficiency of these units in converting multiple inputs into multiple outputs. In the classic CCR-DEA model based on the "efficient frontier", more than one DMU may be efficient with the same efficiency score, which makes multiple efficient DMUs indistinguishable and non-ranked [1]. To overcome this weakness, cross-efficiency DEA evaluates DMUs by averaging efficiency scores from multiple angles [2]. The core of cross-efficiency evaluation is to appraise each DMU from both self-evaluation and peer-evaluations. Self-evaluation means a DMU optimizes its input and output weights to obtain maximum efficiency, and peer-evaluation means a DMU's efficiency is derived from the input and output weights selected by other DMUs [3]. Therefore, cross-efficiency is the integration of self-efficiency and peer-efficiencies.

Nevertheless, efficiency scores in traditional cross-efficiency evaluation are subject to uncertainty due to the inherent non-uniqueness in optimal solutions of linear programming. Therefore, many improvements were provided to overcome the non-uniqueness. As one of

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the effective solutions for guaranteeing the uniqueness of peer-efficiencies, the secondary goal was first provided by Doyle and Green [4] based on aggressive or benevolent formulation. The benevolent formulation aims to maximize the aggregated peer-efficiency scores of other evaluated DMUs, whereas the aggressive formulation aims to minimize the cumulative peer-efficiency scores of other evaluated DMUs. Furthermore, Wang and Chin [5] modified the definition of ideal DMUs and presented several secondary aggressive and benevolent goals. However, the single benevolent or aggressive attitude of decision-makers might lead to drastic outcomes at the extremes. To avert the emergence of extreme outcomes, certain neutral strategies have been proposed in prior literature. Wang and Chin [5] proposed a "neutral" DEA model, and the choice of weights depended on maximizing the relative contribution of outputs using a max–min formulation, which effectively reduced the number of zero weights for outputs. Carrillo and Jorge [6] continued pursuing the neutral strategy, by which each DMU obtained its optimal input and output weights without considering its effect on peer-evaluations. Especially, concepts of ideal and anti-ideal units have been used in the article by Carrillo and Jorge [6]. Along this line, Shi et al. [7] provided a new neutral cross-efficiency evaluation method that regarded the ideal virtual frontier and anti-ideal virtual frontier as evaluation criteria.

Indeed, a dilemma arises in obtaining exact benevolent, neutral, or aggressive attitudes from decision-makers. As the DEA methodology evolves, researchers are consistently delving into inventive approaches to improve cross-efficiency evaluation. In these improvements, an efficiency score represented by an interval number can offer a more realistic representation of uncertain attitudes of decision-makers. The cross-efficiency matrix of interval numbers is a good choice for representing uncertain peer-efficiencies. Without directly considering the aggressive or benevolent preferences of decision-makers, Yang et al. [8] constructed a cross-efficiency matrix composed of interval numbers, and the cross-efficiency scores from the benevolent and aggressive formulations are the upper and lower bounds of the interval number, respectively. A new extension dependent function for three-parameter interval numbers was proposed to perform uncertain analysis of decision data [3,9]. Obviously, a three-parameter interval number can be used to represent cross-efficiency scores from benevolent, neutral, or aggressive attitudes of decision-makers. Many extended interval numbers are discussed, such as interval grey numbers, interval-valued intuitionistic fuzzy numbers [10,11]. Zhang et al. [12] further extended this line of research by integrating fuzzy possibilistic means with prospect theory.

In conventional cross-efficiency models, a DMU's self-evaluation score is obtained through primary goal programming, which often assigns extreme weights to certain inputs or outputs to maximize efficiency. This might lead to the neglect of relevant variables. To mitigate this issue, Jahanshahloo et al. [13] proposed a revised secondary goal that promotes symmetry in weight selection, ensuring more balanced and reliable efficiency scores. Lin et al. [14] proposed an iterative method that ensures unique input and output weights while minimizing zero weights, without imposing any prior weight restrictions. Hong and Jeong [15] developed heuristic-based cross-efficiency ranking methods that avoid linear programming (LP) and proposed a systematic framework to evaluate and compare the consistency of DEA-related full ranking approaches. More recently, Zhu et al. [16] coupled entropy and AHP in an iterative loop that prevents zero weights and stabilizes peer scores.

Ranking interval numbers is key in the multiple attribute decision-making approach and has been studied by numerous scholars. Ye et al. [17] defined the advantage-degree function based on the symmetry axis and the length of the interval number to rank interval numbers. For ranking interval-valued intuitionistic linguistic sets (IVILSs), Qin et al. [18] develop an improved ranking technique based on a novel triangular divergence distance

measure. Jafari and Ehsanifar [19] extended the multi-attribute decision-making method by introducing an interval arithmetic-based ranking index to handle multiple attribute decision-making problems with interval data. Andjelković et al. [20] proposed a DEA approach using Interval-Valued Fuzzy Rough Numbers (IFRNs) and an enhanced Weighted Aggregated Sum Product Assessment (WASPAS) method to improve DMU ranking under interval uncertainty.

Despite numerous studies on interval cross-efficiency, most focus on intervals derived from fuzzy input–output data, while few address how to directly rank interval cross-efficiencies obtained without assuming decision-makers' preferences for aggressive or benevolent strategies. Yang et al. [8] utilized the stochastic multi-criteria acceptability analysis to rank all DMUs with an interval cross-efficiency matrix. Wang et al. [21] implemented ranking of DMUs based on possibility distribution functions, and Li et al. [22] applied a double-stochastic preference analysis to fuse multiple evaluator views into a single interval-robust ranking. In this paper, we discuss an alternative ranking method for interval efficiency with an aggressive–benevolent formulation, which not only preserves all peer-evaluation information but also obtains unique ranking results of DMUs under uncertain attitudes of decision-makers.

The remainder of this paper is organized as follows. Section 2 introduces a traditional cross-efficiency matrix, the secondary goal with benevolent and aggressive formulations, and basic operations of interval numbers. Moving forward to Section 3, we define six possible relations between two interval numbers and introduce the advantage-degree norm for assessing their relative superiority. Subsequently, Section 4 constructs an alternative ranking model for DMUs based on the interval cross-efficiency matrix, and three examples and a comparative analysis are given to illustrate the effectiveness and validation of our method. Finally, some conclusions and insights are provided in Section 5.

2. Preliminary

2.1. Traditional Cross-Efficiency Matrix

In the traditional cross-efficiency model, the ultimate efficiency of a DMU is deduced from both self-evaluation and peer-evaluation [2]. Suppose there are n DMUs with m inputs and s outputs. The cross-efficiency matrix, E^{Cross}, composed of self-efficiency scores and peer-efficiency scores, is defined by

$$E^{Cross} = \begin{bmatrix} E_{11} & E_{12} & \cdots & E_{1n} \\ E_{21} & E_{22} & \cdots & E_{2n} \\ \vdots & \vdots & & \vdots \\ E_{n1} & E_{n2} & \cdots & E_{nn} \end{bmatrix}, \quad (1)$$

where diagonal and off-diagonal cells are self-efficiency scores and peer-efficiency scores, respectively. E_{jj} ($j = 1, 2, \ldots, n$) can be interpreted as the self-evaluation efficiency of DMU_j, and E_{jk} ($j \neq k$) is the peer-evaluation efficiency score of DMU_k from DMU_j. The ultimate efficiency of a DMU is computed by averaging its self-efficiency and its peer-efficiencies from other DMUs.

In the traditional cross-efficiency model, the non-uniqueness of optimal solutions of linear programming results in non-uniqueness of cross-efficiency scores, which makes evaluation results uncertain. Additionally, averaged evaluation suffers several significant drawbacks, summarized in [23].

2.2. Benevolent Formulation and Aggressive Formulation

Traditional cross-efficiency evaluation may be uncertain due to the non-uniqueness of the optimal solution of an LP. Therefore, the studies by Sexton et al. [2] and by Doyle and Green [4,24] all suggested the secondary goal to guarantee the uniqueness of peer-efficiencies from a benevolent or aggressive viewpoint. Suppose there are n DMUs with m inputs and s outputs. For any DMU_j (j = 1, 2, …, n), x_{ij} (i = 1, 2, …, m) represents the ith input of DMU_j, and y_{rj} (r = 1, 2, …, s) is the rth output of DMU_j. Let E_{jj} be the self-evaluation efficiency of DMU_j. The four benevolent options for DMU_j [4] are

(I) Max E_{jj}, min $\frac{1}{n-1}\sum_{k\neq j}(\sum_{r=1}^{s} u_{rj}y_{rk} - \sum_{i=1}^{m} v_{ij}x_{ik})$,

(II) Max E_{jj}, min $\sum_{k\neq j}(\sum_{r=1}^{s} u_{rj}y_{rk}) / \sum_{k\neq j}(\sum_{i=1}^{m} v_{ij}x_{ik})$,

(III) Max E_{jj}, min $\frac{1}{n-1}\sum_{k\neq j}(\sum_{r=1}^{s} u_{rj}y_{rk} / \sum_{i=1}^{m} v_{ij}x_{ik})$,

(IV) Max E_{jj}, min $(\sum_{r=1}^{s} u_{rj}y_{rk} / \sum_{i=1}^{m} v_{ij}x_{ik}) \forall k \neq j$,

where maximization is treated as the primary goal, and minimization as the secondary goal.

Similarly, we can obtain the four aggressive options for any DMU_j $(j = 1, 2, \ldots, n)$, and they are

(I) Max E_{jj}, max $\frac{1}{n-1}\sum_{k\neq j}(\sum_{r=1}^{s} u_{rj}y_{rk} - \sum_{i=1}^{m} v_{ij}x_{ik})$,

(II) Max E_{jj}, max $\sum_{k\neq j}(\sum_{r=1}^{s} u_{rj}y_{rk}) / \sum_{k\neq j}(\sum_{i=1}^{m} v_{ij}x_{ik})$,

(III) Max E_{jj}, max $\frac{1}{n-1}\sum_{k\neq j}(\sum_{r=1}^{s} u_{rj}y_{rk} / \sum_{i=1}^{m} v_{ij}x_{ik})$,

(IV) Max E_{jj}, max $(\sum_{r=1}^{s} u_{rj}y_{rk} / \sum_{i=1}^{m} v_{ij}x_{ik}) \forall k \neq j$,

where the first maximization is treated as the primary goal, and the secondary maximization as the secondary goal in each case.

2.3. Interval Numbers

An interval number is a closed subset of the real numbers. Next, we give basic definitions of interval numbers and operations among interval numbers [25].

Definition 1. *An interval number a is defined as $a = [a^L, a^U] = \{a | a^L \leq a \leq a^U, a^L, a^U \in R\}$, where a^L and a^U are the lower and upper bounds of a, respectively. Especially, if $a^L = a^U$, then a degenerates into a real number also called a degenerate interval number.*

Definition 2. *Length of an interval number a is defined as $a = a^U - a^L$.*

Definition 3 [25]. *Given two interval numbers $a = [a^L, a^U]$, $b = [b^L, b^U]$, and a real number λ, then*

(I) $a \pm b = [a^L + b^L, a^U + b^U]$,

(II) $\lambda a = \begin{cases} [\lambda a^L, \lambda a^U], & \lambda \geq 0 \\ [\lambda a^U, \lambda a^L], & \lambda < 0 \end{cases}$.

Definition 4 [25]. *Let $a = [a^L, a^U]$ and $b = [b^L, b^U]$; then $a = b$, if $a^L = b^L$ and $a^U = b^U$.*

Definition 5 [17]. *Let $a = [a^L, a^U]$ and $b = [b^L, b^U]$; $a \succ b$ denotes that a is superior to b. Define $P(a \succ b)$ as the advantage degree of $a \succ b$ satisfying (I) $P(a \succ b) \in [0, 1]$, (II) $P(a \succ b) + P(b \succ a) = 1$, and (III) $P(a \succ a) = 0.5$.*

3. Cross-Efficiency Matrix with Interval Numbers

3.1. Comparison of Interval Numbers

According to the principle of mean probability distribution, the non-intersecting part of two interval numbers is comparative, whereas the intersecting part is non-comparative. Therefore, we discuss how to deduce $P(a \succ b)$ based on different relations of two interval numbers (shown in Figure 1).

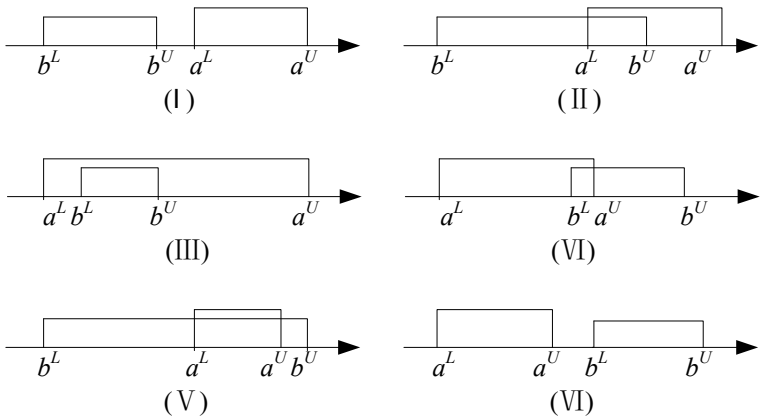


Figure 1. Relations between a and b.

Definition 6. *Let $a = [a^L, a^U]$, $b = [b^L, b^U]$, and $P_{ab} = P(a \succ b)$ be the advantage degree of $a \succ b$. For two interval numbers a and b, $|a \cup b|$ denotes the length of the smallest interval covering both a and b, namely, $|a \cup b| = max(a^U, b^U) - min(a^L, b^L)$. Thus, $|a \cup b|$ is the length of their union-covering interval.*

(I) If $b^U \leq a^L$, then $P_{ab} = 1$;

(II) If $b^L \leq a^L < b^U < a^U$, then $P_{ab} = \frac{1}{|a \cup b|}\left[(a^L - b^L) + \frac{1}{2}(b^U - a^L) + (a^U - b^U)\right]$;

(III) If $a^L < b^L \leq b^U \leq a^U$, then $P_{ab} = \frac{1}{|a \cup b|}\left[\frac{1}{2}(b^U - b^L) + (a^U - b^U)\right]$;

(IV) If $a^L < b^L \leq a^U < b^U$, then $P_{ab} = \frac{1}{|a \cup b|}\left[\frac{1}{2}(a^U - b^L)\right]$;

(V) If $b^L \leq a^L \leq a^U \leq b^U$, then $P_{ab} = \frac{1}{|a \cup b|}\left[(a^L - b^L) + \frac{1}{2}(a^U - a^L)\right]$;

(VI) If $a^L \leq a^U < b^L \leq b^U$, then $P_{ab} = 0$.

If $P_{ab} > 0.5$, then $a \succ b$, whereas if $P_{ab} < 0.5$, then $b \succ a$. Specifically, if $P_{ab} = 0.5$, then $a = b$. According to advantage degrees of interval numbers, n interval cross-efficiencies can be ranked.

We then aim to prove $P(a \succ b) + P(b \succ a) = 1$. Here, $P(b \succ a)$ is obtained by interchanging the roles of a and b and then applying the corresponding formula. There are six cases in total.

(I) a lies entirely to the right of b

$$P(a \succ b) = 1.$$

For (b, a), b lies entirely to the left of a. Thus, Case (VI) applies, and

$$P(b \succ a) = 0.$$

Therefore, $P(a \succ b) + P(b \succ a) = 1$.

(II) $b^L \leq a^L < b^U < a^U$

$$P(a \succ b) = \frac{1}{|a \cup b|}\left[(a^L - b^L) + \frac{1}{2}(b^U - a^L) + (a^U - b^U)\right]$$

For (b, a), the condition becomes $b^L < a^L \leq b^U < a^U$. Hence, Case (IV) applies:

$$P(b \succ a) = \frac{1}{|a \cup b|}\left[\frac{1}{2}(b^U - a^L)\right].$$

The numerator of $P(a \succ b) + P(b \succ a)$ is

$$\begin{aligned} &= (a^L - \mathrm{b}^L) + \tfrac{1}{2}(b^U - a^L) + (a^U - b^U) + \tfrac{1}{2}(b^U - a^L) \\ &= (a^L - b^L) + (b^U - a^L) + (a^U - b^U) \\ &= a^U - b^L \\ &= |a \cup b|. \end{aligned}$$

Therefore, $P(a \succ b) + P(b \succ a) = 1$.

(III) a contains b $(a^L < b^L \leq b^U \leq a^U)$

$$P(\mathrm{a} \succ \mathrm{b}) = \frac{1}{|a \cup b|}(\frac{1}{2}(\mathrm{b}^\mathrm{U} - b^L) + (a^U - b^U)), |a \cup b| = a^U - a^L$$

For (b, a), b is contained in a. Hence, Case (V) applies:

$$P(b \succ a) = \frac{1}{|a \cup b|}\left[(b^L - a^L) + \frac{1}{2}(b^U - b^L)\right]$$

The numerator of $P(\mathrm{a} \succ \mathrm{b}) + P(\mathrm{b} \succ \mathrm{a})$ is

$$\begin{aligned} &= \tfrac{1}{2}(b^U - b^L) + (a^U - b^U) + (b^L - a^L) + \tfrac{1}{2}(b^U - b^L) \\ &= (b^U - b^L) + (a^U - b^U) + (b^L - a^L) \\ &= a^U - a^L \\ &= |a \cup b|. \end{aligned}$$

Therefore, $P(a \succ b) + P(b \succ a) = 1$.

(IV) $a^L < b^L \leq a^U \leq b^U$

$$P(a \succ b) = \frac{1}{|a \cup b|}\left[(\frac{1}{2}(a^U - b^L)\right],\ |a \cup b| = b^U - a^L$$

For (b, a), the condition is

$$a^L < b^L \leq a^U < b^U$$

Hence, Case (II) applies:

$$P(b \succ a) = \frac{1}{|a \cup b|}\left[(b^L - a^L) + \frac{1}{2}(a^U - b^L) + (b^U - a^U)\right]$$

The numerator of $P(a \succ b) + P(b \succ a)$ is

$$\begin{aligned} &= \tfrac{1}{2}(a^U - \mathrm{b}^L) + (b^L - a^L) + \tfrac{1}{2}(a^U - b^L) + (b^U - a^U) \\ &= (a^U - b^L) + (b^L - a^L) + (b^U - a^U) \\ &= b^U - a^L \\ &= |a \cup b|. \end{aligned}$$

Therefore, $P(\mathrm{a} \succ \mathrm{b}) + P(\mathrm{b} \succ \mathrm{a}) = 1$.

(V) b contains a $(b^L \leq a^L \leq a^U \leq b^U)$

$$P(a \succ b) = \frac{1}{|a \cup b|}\left[(a^L - b^L) + (\frac{1}{2}(a^U - a^L)\right], \ |a \cup b| = b^U - b^L$$

For (b, a), a is contained in b. Hence, Case (III) applies:

$$P(b \succ a) = \frac{1}{|a \cup b|}\left[\frac{1}{2}(a^U - a^L) + (b^U - a^U)\right]$$

The numerator of $P(a \succ b) + P(b \succ a)$ is

$$\begin{aligned} &= (a^L - b^L) + \tfrac{1}{2}(a^U - a^L) + \tfrac{1}{2}(a^U - a^L) + (b^U - a^U) \\ &= (a^U - b^L) + (b^U - a^U) \\ &= b^U - b^L \\ &= |a \cup b|. \end{aligned}$$

Therefore, $P(a \succ b) + P(b \succ a) = 1$.

(VI) a lies entirely to the left of b

$$P(a \succ b) = 0$$

For (b, a), b lies entirely to the right of a. Thus, Case (I) applies, and $P(b \succ a) = 1$.
Therefore, the sum equals $P(a \succ b) + P(b \succ a) = 1$.
Hence, the complementarity condition $P(a \succ b) + P(b \succ a) = 1$ holds in all six cases.

3.2. Interval Cross-Efficiency Matrix with Aggressive–Benevolent Formulation

In cross-efficiency evaluation with a single aggressive formulation or benevolent formulation, there exists a dilemma in obtaining precise benevolent or aggressive attitudes from decision-makers. So cross-efficiency can be seen as an interval number, with the score from the aggressive formulation as the lower bound and the score from the benevolent formulation as the upper bound. The cross-efficiency matrix of the interval number [8] is defined as

$$\widetilde{E} = \begin{bmatrix} E_{11} & E_{12} & \cdots & E_{1n} \\ E_{21} & E_{22} & \cdots & E_{2n} \\ \vdots & \vdots & & \vdots \\ E_{n1} & E_{n2} & \cdots & E_{nn} \end{bmatrix} = \begin{bmatrix} [E_{11}^L, E_{11}^U] & [E_{12}^L, E_{12}^U] & \cdots & [E_{1n}^L, E_{1n}^U] \\ [E_{21}^L, E_{21}^U] & [E_{22}^L, E_{22}^U] & \cdots & [E_{2n}^L, E_{2n}^U] \\ \vdots & \vdots & & \vdots \\ [E_{n1}^L, E_{n1}^U] & [E_{n2}^L, E_{n2}^U] & \cdots & [E_{nn}^L, E_{nn}^U] \end{bmatrix}, \quad (2)$$

where each cell in the matrix represents an interval number, diagonal cells $E_{ii} = [E_{ii}^L, E_{ii}^U]$ $(i = 1, 2, ..., m)$ are special cases where $E_{ii}^L = E_{ii}^U$, and off-diagonal cells $E_{jk} = \left[E_{jk}^L, E_{jk}^U\right]$ $(j \neq k)$, are interval numbers with the lower bound E_{jk}^L and the upper bound E_{jk}^U.

4. Interval Cross-Efficiency Evaluation with Aggressive–Benevolent Formulation

4.1. Model Construction

Interval cross-efficiency evaluation with aggressive–benevolent formulation, ICE-AB for short, will be constructed in the model construction section. Suppose there are n DMUs with m inputs and s outputs. For the DMU_j (j = 1, 2, . . ., n), the ICE-AB method is defined by the following steps.

Step 1. Create a primary goal to calculate self-evaluation scores of DMUs. The linear programming for DMU_j is defined by

$$\begin{aligned} \max \quad & E_{jj} = \sum_{r=1}^{s} u_{rj} y_{rj} \\ s.t. \quad & \begin{cases} \sum_{i=1}^{m} v_{ij} x_{ik} - \sum_{r=1}^{s} u_{rj} y_{rk} \geq 0 \ (k = 1, 2, \ldots, \overline{n}) \\ \sum_{i=1}^{m} v_{ij} x_{ij} = 1 \\ v_{ij} \geq 0, u_{rj} \geq 0, r = 1, 2, \ldots, s, i = 1, 2, \ldots, m \end{cases} \end{aligned} \tag{3}$$

Let $\left[U_j^*, V_j^*\right]^T$ be the optimal solution of DMU_j in Equation (3), where $U_j^* = (u_{1j}^*, u_{2j}^*, ..., u_{sj}^*)$ is composed of optimal output weights and $V_j^* = (v_{1j}^*, v_{2j}^*, ..., v_{mj}^*)$ is composed of optimal input weights, and the optimal value E_{jj} is interpreted as the self-evaluation score of DMU_j.

Step 2. Calculate peer-efficiency scores by secondary goal programming under aggressive and benevolent formulations. Suppose E_{jk}^L represents the peer-efficiency of DMU_k from DMU_j under aggressive formulation and E_{jk}^U represents the peer-efficiency under benevolent formulation. We select option IV in both aggressive formulations and benevolent formulations to continue our research. The secondary goal programming under aggressive formulation is defined by

$$\begin{aligned} \min \quad & E_{jk}^L = \sum_{r=1}^{s} u_{rj} y_{rk} \\ s.t. \quad & \begin{cases} \sum_{i=1}^{m} v_{ij} x_{ik} - \sum_{r=1}^{s} u_{rj} y_{rk} \geq 0, \ k = 1, 2, \ldots, n, j \neq k \\ E_{jj} \times \sum_{i=1}^{m} v_{ij} x_{ik} - \sum_{r=1}^{s} u_{rj} y_{rk} = 0 \\ \sum_{i=1}^{m} v_{ij} x_{ik} = 1 \\ v_{ij} \geq 0, u_{rj} \geq 0, r = 1, 2, \ldots, s, i = 1, 2, \ldots, m \end{cases} \end{aligned} \tag{4}$$

Similarly, the secondary goal of benevolent formulation is defined by

$$\begin{aligned} \max \quad & E_{jk}^U = \sum_{r=1}^{s} u_{rj} y_{rk} \\ s.t. \quad & \begin{cases} \sum_{i=1}^{m} v_{ij} x_{ik} - \sum_{r=1}^{s} u_{rj} y_{rk} \geq 0, \ k = 1, 2, \ldots, n, j \neq k \\ E_{jj} \times \sum_{i=1}^{m} v_{ij} x_{ik} - \sum_{r=1}^{s} u_{rj} y_{rk} = 0 \\ \sum_{i=1}^{m} v_{ij} x_{ik} = 1 \\ v_{ij} \geq 0, u_{rj} \geq 0, r = 1, 2, \ldots, s, i = 1, 2, \ldots, m \end{cases} \end{aligned} \tag{5}$$

Obviously, the peer-evaluation efficiency E_{jk}^L and E_{jk}^U must be unique even under different optimal solutions.

Step 3. Construct the cross-efficiency matrix of interval numbers according to primary goal programming and secondary goal programming. We have

$$\widetilde{E} = \begin{bmatrix} E_{11} & E_{12} & \cdots & E_{1n} \\ E_{21} & E_{22} & \cdots & E_{2n} \\ \vdots & \vdots & & \vdots \\ E_{n1} & E_{n2} & \cdots & E_{nn} \end{bmatrix} = \begin{bmatrix} E_{11} & [E_{12}^L, E_{12}^U] & \cdots & [E_{1n}^L, E_{1n}^U] \\ [E_{21}^L, E_{21}^U] & E_{22} & \cdots & [E_{2n}^L, E_{2n}^U] \\ \vdots & \vdots & & \vdots \\ [E_{n1}^L, E_{n1}^U] & [E_{n2}^L, E_{n2}^U] & \cdots & E_{nn} \end{bmatrix}, \tag{6}$$

where diagonal and off-diagonal cells are respectively composed of the self-evaluation scores and peer-evaluation scores. $E_{jj}(j = 1, 2, \ldots, n)$ represents the self-evaluation efficiency, and E_{jk}^{L} and E_{jk}^{U} represent the lower bound and upper bound of the interval number, respectively.

Step 4. Compute the final interval cross-efficiency for each DMU_j (j = 1, 2, …, n) by Definition 3, and we have

$$E_j = [E_j^L, E_j^U] = \frac{1}{n}\sum_{i=1}^{n} E_{ij},\ j = 1, 2, \ldots, n. \tag{7}$$

In Equation (7), when $i = j$, E_{ij} is a real number and $E_{ij} = E_{jj} = [E_{jj}^L, E_{jj}^U] = E_{jj}^L = E_{jj}^U$.

From Step 1 to Step 4, we can obtain the interval cross-efficiency by considering both the aggressive and benevolent views, which provides a more comprehensive assessment of the DMU's performance.

4.2. A Pairwise Advantage-Degree Ranking Method

For a set of interval efficiencies $E = \{E_1, \ldots, E_n\}$, pairwise advantage-degree comparisons (Definition 6) produce an $n \times n$ advantage-degree matrix $P = (p_{ij})_{n\times n} = P(E_i \succ E_j)$, where $P_{ij} \in [0, 1]$, $E_i \in E$, $E_j \in E$ denotes the degree to which E_i is preferred to E_j. To obtain a unique, complete ranking that is robust to the possible non-transitivity of pairwise interval comparisons, we aggregate the matrix into a scalar dominance score per DMU rather than chaining pairwise relations. The procedure is summarized in Algorithm 1.

Algorithm 1. Ranking procedure based on the advantage-degree matrix and dominance scores.

Step 1. For every ordered pair (i, j), $i \neq j$, compute $P_{ij} = P(E_i \succ E_j)$ according to Definition 6. For each diagonal element, set $P_{ii} = 0.5$ by reflexivity. Then, form the advantage-degree matrix $P = (P_{ij})_{n\times n}$.

Step 2. For each DMU_i, compute its dominance score as follows:

$$S_i = \sum_{j=1}^{n} (P_{ij} - 0.5). \tag{8}$$

By construction, $\Sigma_i S_i = 0$, i.e., the scores are zero-sum and directly reflect relative dominance across the group.

Step 3. Rank DMUs in descending order of S_i.

Step 4. (Tie-breaking) If $S_i = S_j$,

(a) Break the tie by the midpoint, $c_k = ({E_k}^U + {E_k}^L)/2$ (larger midpoint ranked higher);
(b) Still tied, by the half-length, $r_k = ({E_k}^U - {E_k}^L)/2$ (smaller radius/sharper efficiency ranked higher);
(c) If still tied, declare a genuine tie between DMU_i and DMU_j.

The dominance score S_i has a transparent interpretation. It measures the net advantage of *DMUi* over all other DMUs, accumulating evidence across all pairwise comparisons. Ranking by S_i yields a complete ordering regardless of whether the underlying pairwise relation is itself transitive, which is the principal advantage of the matrix-based formulation over direct pairwise chaining of interval comparisons.

In terms of complexity, Algorithm 1 requires $O(n^2)$ advantage-degree evaluations and $O(n^2)$ arithmetic operations. The preceding construction steps of ICE-AB additionally

involve $2n$ linear programs (the aggressive and benevolent secondary goals for each DMU), solved here by Python 3.9.7.

The following property guarantees that the dominance-score ranking is consistent with intuitive pairwise preferences whenever those preferences are coherent.

Property 1. (Consistency with pairwise dominance). If $P_{ij} > 0.5$, i.e., DMU_i advantage-dominates DMU_j, then $S_i > S_j$, and DMU_i is ranked strictly above DMU_j by Algorithm 1.

Proof. From Equation (8) and the complementary property $P_{ir} + P_{ri} = 1$ for every r, we have

$$\begin{aligned} S_i - S_j &= \Sigma_r(P_{ir} - 0.5) - \Sigma_r(P_{jr} - 0.5) \\ &= \Sigma_r(P_{ir} - P_{jr}) \\ &= \Sigma_r 2(P_{ir} - 0.5) \text{ (using } P_{jr} = 1 - P_{ri}). \end{aligned} \tag{9}$$

In the term $r = j$, we obtain $2(P_{ir} - 0.5) > 0$ by assumption. For the remaining terms, when pairwise preferences are midpoint-consistent (i.e., $P_{ir} - 0.5$ shares the sign of $m_i - m_r$), every term in (9) has the same sign as $m_i - m_r$, and, therefore, $S_i - S_j > 0$. Hence, DMU_i is ranked strictly above DMU_j. □

Definition 8. (Midpoint-consistency). *An interval set I is midpoint-consistent if, for every pair (i, j),* $P_{ij} > 0.5$ *implies* $m_i \geq m_j$ *and* $P_{ij} = 0.5$ *implies* $m_i = m_j$.

One remark is that when the pairwise relation is itself a strict weak order (e.g., under midpoint-consistency), the matrix-based ranking coincides with the naive pairwise chaining; when the pairwise relation contains an intransitive triple, the matrix-based ranking still yields a well-defined unique ordering via the dominance score, whereas naive chaining may produce a cycle. This robustness is exactly why we adopt the matrix-based formulation. The midpoint-consistency condition holds for all interval sets in our numerical study (Section 4.3) and can be verified in $O(n^2)$ time by a single pairwise scan.

4.3. Comparable Analysis

A simple numerical example from Yang et al. [8] is used to illustrate the effectiveness and reasonableness of our ICE-AB method. Table 1 shows data with three inputs and three outputs.

Table 1. Dataset of seven DMUs.

	DMU_1	DMU_2	DMU_3	DMU_4	DMU_5	DMU_6	DMU_7
x_1	12	19	42	15	45	19	41
x_2	400	750	1500	600	2000	730	2350
x_3	20	70	70	100	250	50	600
y_1	60	139	225	90	253	132	305
y_2	35	41	68	12	145	45	159
y_3	17	40	75	17	130	45	97

Following Step 1 to Step 3, we have self-evaluation efficiencies, peer-evaluation efficiencies, and a cross-efficiency matrix. Table 2 shows the interval cross-efficiency with aggressive formulation and benevolent formulation, where diagonal cells represent self-evaluation scores, and off-diagonal cells represent interval peer-evaluation scores. Obviously, diagonal cells must be real numbers, also called degenerate interval numbers. Compared with traditional peer-efficiency, an interval peer-efficiency comprises the minimum and the maximum of peer-evaluation efficiency scores. When a DMU obtains the

same peer-evaluation scores under aggressive formulation and benevolent formulation, an interval peer-efficiency degenerates into a real number. For example,E_{41} = 0.687, which means DMU_1 obtains the same the peer-efficiency from DMU_4 under different attitudes, including aggressive, neutral, or benevolent formulation.

Table 2. Interval cross-efficiencies of seven DMUs.

	DMU_1	DMU_2	DMU_3	DMU_4	DMU_5	DMU_6	DMU_7
DMU_1	1	[0.335, 0.985]	[0.518, 1.000]	[0.069, 0.684]	[0.331, 1]	[0.514, 1]	[0.151, 1]
DMU_2	[0.686, 0.936]	1	[0.735, 0.848]	[0.687, 0.820]	[0.662, 0.921]	[0.951, 1]	[0.604, 1]
DMU_3	[0.793, 1.000]	[0.533, 0.858]	1	[0.151, 0.470]	[0.315, 0.708]	[0.821, 1]	[0.151, 0.294]
DMU_4	0.687	1	0.735	0.820	0.765	0.951	1
DMU_5	[0.490, 1]	[0.699, 0.970]	[0.551, 0.829]	[0.241, 0.672]	1	[0.780, 1]	[0.525, 1]
DMU_6	[0.645, 1]	[0.695, 1]	[0.749, 1]	[0.214, 0.772]	[0.478, 1]	1	[0.246, 1]
DMU_7	[0.634, 1]	[0.556, 1]	[0.418, 0.772]	[0.206, 0.820]	[0.756, 1]	[0.611, 1]	1

Following Step 4, the final interval cross-efficiency scores of DMUs by the ICE-AB method are computed. Subsequently, the ranking of DMUs is given by Algorithm 1. For comparative analysis, the efficiencies from the CCR model and the traditional cross-efficiency method are also given. Related efficiencies and rankings are listed in Table 3.

Table 3. Results from the numerical example.

	CCR Efficiency	CCR Ranking	CE Efficiency	CE Ranking	ICE-AB Efficiency	ICE-AB Ranking	Yang's Ranking
DMU_1	1	1	0.849	3	[0.705, 0.946]	3	3
DMU_2	1	1	0.851	2	[0.688, 0.973]	2	2
DMU_3	1	1	0.767	6	[0.672, 0.883]	6	6
DMU_4	0.82	7	0.506	7	[0.341, 0.722]	7	7
DMU_5	1	1	0.847	4	[0.615, 0.913]	4	5
DMU_6	1	1	0.929	1	[0.804, 0.993]	1	1
DMU_7	1	1	0.782	5	[0.525, 0.899]	5	4

Note: CE represents traditional cross-efficiency; ICE-AB represents interval cross-efficiency with aggressive–benevolent formulation.

Based on the CCR model, the efficiencies cannot fully distinguish DMUs because there are six DEA-efficient DMUs with the same efficiency score of 1. The minimal efficiency from DMU_4 is 0.82.

Based on the traditional cross-efficiency method, six DMUs with the same efficiency of 1 in CCR obtain differentiated efficiency scores. Peer-evaluation reduces the efficiency of DMUs, which reflects group opinions of decision-makers. DMU_6 with the maximal efficiency of 0.929 ranks at first place, whereas DMU_4 with the minimal efficiency of 0.506 is still in last place.

Based on the ICE-AB method, all DMUs obtain final interval cross-efficiency scores. Ranking results from CE and ICE-AB are the same, but efficiency scores from the ICE-AB method can provide a reasonable efficiency range based on different attitudes of decision-makers. DMU_6 with the interval efficiency [0.804, 0.993] still ranks at first place, whereas DMU_4 is still in last place.

Furthermore, traditional cross-efficiency scores may be non-unique because of the non-uniqueness of weights in self-evaluation, which may lead to disparate rankings in evaluations. Notably, the peer-efficiency scores derived from single aggressive or benevolent formulations can exhibit significant disparities. For example, DMU_1 obtains the score 0.849 in traditional cross-efficiency evaluation, whereas it obtains the scores 0.705 and 0.946 in the aggressive peer-evaluation and the benevolent peer-evaluation, respectively. What is more,

different efficiency scores may result in different ranking results for DMUs. For instance, DMU_1 under the traditional cross-evaluation, cross-evaluation with aggressive formulation, and cross-evaluation with benevolent formulation rank in the third place, the second place, and the third place, respectively.

The ICE-AB method constructs interval efficiencies for DMUs based on the aggressive–benevolent formulation and gives a unique ranking result through comparison among interval efficiencies, which not only considers group opinions via cross-evaluation but also avoids the uncertainty of decision-makers' attitudes.

4.4. A Differentiating Case Study

The example in Section 4.3, drawn from Yang et al. [8], reveals that the ICE-AB ranking coincides with that of the traditional point-value cross-efficiency (CE) method (Table 3). A natural question is whether ICE-AB can ever alter the ranking and under what conditions. We argue that ICE-AB diverges from CE precisely when the latter is least informative. When two or more DMUs have nearly identical average peer-evaluations but different spreads in their attitude-induced intervals. In such situations, CE, which ranks by the single midpoint, produces either a tie or a numerically fragile order, whereas ICE-AB exploits the interval width and overlap to deliver a stable, defensible ranking.

Table 4 reports a constructed numerical dataset with seven DMUs, two inputs, and two outputs. The dataset is designed to include several DMUs with relatively close point-valued CE efficiencies, so that the additional interval information generated by the aggressive and benevolent formulations can affect the pairwise dominance structure.

Table 4. Differentiating numerical dataset.

	DMU_1	DMU_2	DMU_3	DMU_4	DMU_5	DMU_6	DMU_7
x_1	2.441	4.752	3.510	9.287	6.398	7.632	8.839
x_2	1.804	2.096	2.389	2.351	3.419	4.848	8.729
y_1	3.216	9.115	7.338	8.138	7.820	12.521	9.035
y_2	2.577	4.204	4.017	5.882	7.377	7.073	12.310

Following Steps 1–4 of the ICE-AB procedure, the CCR efficiencies, traditional CE efficiencies, ICE-AB interval efficiencies, dominance scores, and final rankings are obtained as shown in Table 5.

Table 5. Results from the differentiating numerical example.

	CE Efficiency	CE Ranking	ICE-AB Interval	ICE-AB (S_i)	ICE-AB Ranking
DMU_1	0.7513	6	[0.6695, 0.8234]	−3.2969	6
DMU_2	0.8747	2	[0.8464, 0.9628]	3.3790	3
DMU_3	0.9311	1	[0.8577, 0.9823]	3.9853	1
DMU_4	0.6419	7	[0.6285, 0.8295]	−3.2601	5
DMU_5	0.8714	3	[0.8161, 1.0000]	3.9794	2
DMU_6	0.7627	5	[0.7101, 0.8164]	−3.4705	7
DMU_7	0.8151	4	[0.6616, 0.9042]	−1.3162	4

The traditional CE ranking is as follows: $DMU_3 \succ DMU_2 \succ DMU_5 \succ DMU_7 \succ DMU_6 \succ DMU_1 \succ DMU_4$. By contrast, the ICE-AB ranking based on the dominance score (S_i) is as follows: $DMU_3 \succ DMU_5 \succ DMU_2 \succ DMU_7 \succ DMU_4 \succ DMU_1 \succ DMU_6$.

A notable ranking reversal occurs between DMU_2 and DMU_5. Under the traditional CE method, DMU_2 (0.8747) obtains a slightly higher point-valued efficiency than DMU_5 (0.8714), so CE ranks DMU_2 above DMU_5. However, under the ICE-AB method, DMU_5

(3.9794) obtains a higher dominance score than DMU_2 (3.3790). Therefore, ICE-AB ranks DMU_5 above DMU_2.

This ranking reversal is not a defect of the proposed method. Rather, it shows that ICE-AB does not merely reproduce the point-valued CE ranking. Traditional CE summarizes each DMU by a single average efficiency score and, therefore, discards the attitude-induced variation between aggressive and benevolent peer-evaluations. ICE-AB, by contrast, represents each DMU using an interval efficiency and compares DMUs through pairwise advantage degrees. As a result, the final S_i reflects the net advantage of each DMU over all other DMUs under interval uncertainty.

In this example, DMU_2 and DMU_5 have very close CE scores, but their ICE-AB intervals lead to different pairwise dominance relations. DMU_5 has a wider interval and a higher upper bound, and its interval overlaps with those of the leading DMUs. These interval characteristics affect the advantage-degree matrix and result in a larger net dominance score for DMU_5. Therefore, ICE-AB provides a more informative ranking when point-valued CE scores are close and the aggressive–benevolent interval information is relevant.

This example demonstrates that ICE-AB can preserve the traditional CE ranking when the point-valued and interval-based dominance information are consistent, while also allowing a different ranking when the interval information reveals additional dominance relations. Hence, ICE-AB can be viewed as a robust extension of traditional cross-efficiency evaluation rather than a simple replacement of the CE score.

4.5. An Empirical Case Study

In 2025, China's new energy vehicle (NEV) market was highly competitive. Annual sales reached 16.49 million vehicles, and market penetration exceeded 50.8%.

Government agencies and investment institutions need to assess the overall operating efficiency of major automakers to support industrial policymaking and investment decisions.

This section evaluates six representative manufacturers in China's NEV market using the ICE-AB model. The sample covers traditional automakers transitioning to NEVs (BYD, Geely, and Changan) and emerging manufacturers (Leapmotor, XPeng, and Li Auto). The input indicators are fixed assets (RMB 100 million), R&D personnel (persons), and R&D expenditure (RMB 100 million); the output indicators are annual sales (10,000 vehicles) and net profit (RMB 100 million). The data come from the companies' publicly disclosed 2025 financial reports and sales figures.

As shown in Table 6, the empirical application demonstrates how ICE-AB can provide additional and practically interpretable information in a real-world efficiency evaluation. Geely (DMU_2) obtains the highest traditional CE score, and BYD (DMU_1) achieves the highest ICE-AB dominance score. This difference indicates that the final ICE-AB ranking is determined by the relative position of each firm's entire efficiency interval in pairwise advantage-degree comparisons. The upward movement of Li Auto (DMU_6) and the ranking reversal between Leapmotor (DMU_4) and Changan (DMU_3) further show that aggressive–benevolent variation can affect comparative conclusions in actual enterprise data. By contrast, XPeng (DMU_5) remains last under both methods, indicating that ICE-AB can also preserve traditional conclusions when the point-valued and interval-based evidence is consistent. Overall, the application shows that ICE-AB can translate uncertainty in peer-evaluation attitudes into a unique and interpretable ranking, thereby offering a more comprehensive basis for industrial policy and investment assessment.

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Table 6. Input–output data of major Chinese NEV manufacturers in 2025.

Company	Code	Fixed Assets (RMB 100 Million)	R&D Personnel (Persons)	R&D Expenditure (RMB 100 Million)	Annual Sales (10,000 Vehicles)	Net Profit (RMB 100 Million)
BYD	DMU_1	3200	120,000	634	460.24	326.2
Geely	DMU_2	1800	35,000	180	168.78	120
Changan	DMU_3	2000	30,000	160	98.63	110
Leapmotor	DMU_4	280	5200	45	59.66	8
XPeng	DMU_5	350	6800	55	42.94	12
Li Auto	DMU_6	420	7500	113	40.63	35

5. Conclusions

Our research focused on information aggregation of self-evaluation efficiency and peer-evaluation efficiency in DEA cross-efficiency evaluation. As a representation of efficiency, interval efficiency mainly results from uncertainty of decision-makers' attitudes or imprecise or fuzzy input–output data. In peer-evaluation, uncertain attitudes of decision-makers, aggressive or benevolent formulation in secondary goal programming, may result in different peer-efficiencies; therefore, interval efficiency is used to represent the variable efficiency range. Interval efficiency scores can provide a broader and more comprehensive assessment for DMUs. Based on the interval cross-efficiency matrix and advantage degree, a synthetic evaluation method, ICE-AB, is proposed for DEA cross-efficiency evaluation and ranking of DMUs. In addition, a dominance-score ranking method is developed to aggregate pairwise advantage degrees and obtain a unique ranking result. A numerical example and a differentiating case study are given to illustrate the effectiveness and rationality of our ICE-AB method. Comparative analysis and ranking results show that the ICE-AB method not only preserves peer-evaluation information under different decision-making attitudes but also provides a unique and more informative ranking result for DMUs.

Although we demonstrate many advantages of our new model through a benchmark numerical example, a differentiating numerical case, and an empirical application, we claim that our research is a starting point for future studies. Our model mainly aims to address the uncertainty associated with different decision-making attitudes, while uncertainty from fuzzy or imprecise input–output data is another concern in interval efficiency research. Therefore, interval efficiency research still faces many challenges in terms of data mining in interval information, solution algorithms, and practical implementation.

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