Mathematical & Computational Applications, Vol. 3, No. 2, pp. 113-125, 1998

EVALUATION OF CERTAIN CLASS OF EULERIAN INTEGRALS OF THE MULTIVARIABLE H-FUNCTION

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Abstract-In the present paper we evaluate a number of key Eulerian integrals involving the H- function of several complex variables. Our general Eulerian integral formulas are shown to provide the key formulae from which numerous other potentially useful results for various families of generalized hypergeometric functions of several variables can be derived. Few particular cases are also considered.

1. INTRODUCTION, PRELIMINARIES AND DEFINITIONS

Well known Eulerian Beta integral

$$B(\alpha,\beta)=\int_0^1 t^{\alpha-1}(1-t)^{\beta-1}dt=\frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}, \tag{1.1}$$

(where $\operatorname{Re}(\alpha)$, $\operatorname{Re}(\beta)>0$)

can be rewritten (by suitably manoeuvred) in the form

$$\int_a^b (t-a)^{\alpha-1}(b-t)^{\beta-1}dt=(b-a)^{\alpha+\beta-1}B(\alpha,\beta), \tag{1.2}$$

(where $\operatorname{Re}(\alpha),\operatorname{Re}(\beta)>0$; $a\neq b$)
Since

$$(ut+v)^{\gamma}=(au+v)^{\gamma}\sum_{m=0}^{\infty}\frac{(-\gamma)_m}{m!}\left\{-\frac{(t-a)}{au+v}\right\}^m, \tag{1.3}$$

where

$$\left|\frac{(t-a)u}{au+v}\right|<1\ ;\ t\in[a,b]$$

With the help of (1.2) we obtain [8, p. 301 (2.2.6.1)]

$$\int_a^b (t-a)^{\alpha-1}(b-t)^{\beta-1}(ut+v)^{\gamma}\,dt$$
$$=(b-a)^{\alpha+\beta-1}(au+v)^{\gamma}B(\alpha,\beta)\,{}_2F_1\left({\alpha,-\gamma \atop \alpha+\beta};-\frac{(b-a)u}{au+v}\right), \tag{1.4}$$

where Re(α) ,Re(β)>0; $\left|\arg\left(\frac{bu+v}{au+v}\right)\right| \leq \pi - \in (0 < \in < \pi); b \neq a$ and $2F_1$ occuring on the RHS of (1.4) is a Gaussian hypergeometric function

Recently Srivastava and Hussain [24] made use of (1.4) in order to evaluate Eulerian integrals of the multivariable H- function and hence generalize the Eulerian integrals in terms of an H-function of two variables by Saxena and Nishimoto [10]. In the present paper we evaluated general Eulerian integrals which not only contain the results of Srivastava and Hussain [24] but also give many more interesting key formulas.

The computation of fractional derivatives (and fractional integrals) of special functions of one and more variables is important because of usefulness of these results, such as in evaluation of the series and integrals [5,27] derivation of generating functions [19, chapter 5] and the solution of differential and integrals equations [22, chapter 3; 3, 5, 6, 21]. Motivated by these and many others avenues of applications, many workers obtained several fractional derivative formulas involving different special functions [4, 10, 20, 21, 23 - 27]

In the latter section of the present paper, it is shown that the general Eulerian integrals proved can also easily be stated as a fractional integral formula involving familiar (fractional) differintegral operator ${}_{\alpha}D_x^{\mu}$ defined by [4, 7, 11]

$$ {}_{\alpha}D_x^{\mu}f(x) = \begin{cases} \frac{1}{\Gamma(-\mu)}\int_{\alpha}^{x}(x-t)^{-\mu-1}f(t)dt, (\alpha \in R; Re(\mu) \cdot 0), \\ \frac{d^m}{dx^m}\,{}_{\alpha}D_x^{\mu-m}f(x), (0 \leq Re(\mu) \cdot m, m \in N). \end{cases} \tag{1.5} $$

provided that the integral exists. In fact , when α=0, the operator

$$ D_x^{\mu} \equiv {}_{0}D_x^{\mu} \quad (\mu \in C), \tag{1.6} $$

corresponds to the classical Riemann - Liouville fractional derivative (or integral) of order μ (or -μ). When $\alpha \to \infty$ the equation (1.5) may be identified with the definition of the familiar Weyl fractional derivative (or integral) of order μ (or -μ) [1, Chapter 13]

In this paper we evaluate general Eulerian integrals involving H- function of several complex variables, which was defined by Srivastava and Panda [16, p. 271 (4.1) et. seq.] and studied systematically by them [14-18]. For this multivariable H- function, we adopt the contracted notations (due essentially to Srivastava and Panda (16)) which are used in monograph of Srivastava et. al. [18, p.251 (c.1)]. Thus following the various conventions and notations explained fairly fully given by Srivastava et. al. [14-18, 20, 23].

Let

$$ H[z_1,\ldots\ldots z_r] \equiv H^{0,n:m_1,n_1;\ldots\ldots;m_r,n_r}_{p,q:p_1,q_1;\ldots\ldots;p_r,q_r}\left[\begin{matrix} z_1 \\ \vdots \\ z_r \end{matrix}\left|\begin{matrix} (a_j;\alpha_j^1,\ldots\alpha_j^{(r)})_{1,p}:(c_j^1;\gamma_j^1)_{1,p_1};\ldots\ldots;(c_j^{(r)},\gamma_j^{(r)})_{1,p_r} \\ (b_j;\beta_j^1,\ldots,\beta_j^{(r)})_{1,q}:(d_j^{(1)},\delta_j^{(1)})_{1,q_1};\ldots;(d_j^{(r)},\delta_j^{(r)})_{1,q_r} \end{matrix}\right.\right] \tag{1.7} $$

denote the H-function of r complex variables $z_1........z_r$, here for convenience $(a_j;\alpha_j^1,....,\alpha_j^{(r)})$ abbreviates the p-member array $(a_1;\alpha_1^1.....\alpha_1^{(r)})$......., $(a_p;\alpha_p^1,....\alpha_p^{(r)})$; (1.8) while $(c_j^{(i)}, r_j^{(i)})_{1,p_i}$ abbreviates the array of p_i pairs of parameters

$$(c_1^{(i)}, r_1^{(i)}),.....,(c_{p_i}^{(i)}, r_{p_i}^{(i)}); (i = 1,....,r) \tag{1.9}$$

and so on. Suppose , as usual, that the parameters

$$\begin{cases} a_j, j = 1,\ldots, p; c_j^{(i)}, j = 1,\ldots, p_i; \\ b_j, j = 1,\ldots, q; d_j^{(i)}, j = 1,\ldots q_i; \forall i \in \{1,\ldots, r\}, \end{cases} \tag{1.10}$$

are complex number and the associated coefficients

$$\begin{cases} \alpha_j, j = 1,.......,p; r_j^{(i)}, j = 1,........, p_i; \\ \beta_j, j = 1,..., q; \delta_j^{(i)}, j = 1,...q_i; \forall i \in \{1,..., r\}, \end{cases} \tag{1.11}$$

are positive real numbers such that .

$$\Lambda_2 \equiv \sum_{j=1}^{p}\alpha_j^{(i)} - \sum_{j=1}^{q}\beta_j^{(i)} + \sum_{j=1}^{p_i}\gamma_j^{(i)} - \sum_{j=1}^{q_i}\delta_j^{(i)} \leq 0, \tag{1.12}$$

and

$$\Omega_i \equiv -\sum_{j=n+1}^{p}\alpha_j^{(i)} - \sum_{j=1}^{q}\beta_j^{(i)} + \sum_{j=1}^{n_i}\gamma_j^{(i)} - \sum_{j=n_i+1}^{p_i}\gamma_j^{(i)} + \sum_{j=1}^{m_i}\delta_j^{(i)} - \sum_{j=m_i+1}^{q_i}\delta_j^{(i)} > 0, \forall i \in \{1,...r\}, \tag{1.13}$$

where the integers n,p,q,m_i,n_i,p_i,q_i are constrained by the inequalities $0\leq n \leq p$, $q\geq 0$, $1\leq m_i \leq q_i$, $0\leq n_i \leq p_i$ $(i=1,...r)$ and the equality (1.12) holds true for suitably restricted values of the complex variables $z_1,.....,z_r$

Then it is known that the multiple Mellin - Barnes contour integral [18, p.251 (c.1)] representing the multivariable H- function (1.7) converges absolutely, under the condition (1.13), when

$$|\arg(z_i)| < \frac{1}{2}\pi\Omega_i ; (\forall\ i \in \{1,....,r\}), \tag{1.14}$$

the point $z_i=0$ $(i=1,...,r)$ and various exceptional parameter value being tacitly excluded. Furthermore, we have [1, p.131 equation (1.9)].

$$H[z_1,....z_r] = \begin{cases} 0(|z_1|^{\xi_1}........|z_r|^{\xi_r}), (\max\{|z_1|,......,|z_r|\} \to 0), \\ 0(|z_1|^{\eta_r}...|z_r|^{\eta_r}), (n = 0, \min\{|z_1|,....|z_r|\} \to \infty), \end{cases} \tag{1.15}$$

where (with $i=1,....,r$)

$$\begin{cases} \xi_i = \min\{Re(d_j^{(i)})/\delta_j^{(i)}\}, (j=1,\dots,m_i), \\ \eta_i = \max\{Re(c_j^{(i)}-1)/\gamma_j^{(i)}\}, (j=1,\dots n_i), \end{cases} \tag{1.16}$$

provided that each of the inequalities (1.12), (1.13) and (1.14) holds true.

Throughout the present paper, we assume that the convergence (and existence) conditions corresponding appropriately to the ones detailed above are satisfied by each of the various H-functions involved in our formulas which are presented in the following sections.

2. THE GENERAL EULERIAN INTEGRALS OF THE MULTIVARIABLE H - FUNCTION

In this section, we shall prove our main general Eulerian integrals involving the H-function of r complex variables:

$$\int_a^b (t-a)^{\alpha-1}(b-t)^{\beta-1}(u_1t+v_1)^{r_1}(u_2t+v_2)^{-r_2}(y_1t+z_1)^{\delta_1}(y_2t+z_2)^{-\delta_2}.$$

$$.H\begin{bmatrix} z_1(u_1t+v_1)^{\rho_1}(u_2t+v_2)^{-\rho_1'}(y_1t+z_1)^{\sigma_1}(y_2t+z_2)^{-\sigma_1'} \\ \vdots \\ z_r(u_1t+v_1)^{\rho_r}(u_2t+v_2)^{-\rho_r'}(y_1t+z_1)^{\sigma_r}(y_2t+z_2)^{-\sigma_r'} \end{bmatrix} .dt$$

$$=(b-a)^{\alpha+\beta-1}(au_1+v_1)^{r_1}(au_2+v_2)^{-r_2}(by_1+z_1)^{\delta_1}(by_2+z_2)^{-\delta_2}$$

$$\text{x} \sum_{\ell_1,\ell_2,\ell_3,\ell_4=0}^{\infty} \frac{B(\alpha+\ell_1+\ell_3, \beta+\ell_2+\ell_4)}{\ell_1!\,\ell_2!\,\ell_3!\,\ell_4!}$$

$$\text{x}\left\{\frac{(b-a)u_1}{au_1+v_1}\right\}^{\ell_1}\left\{-\frac{(b-a)y_1}{by_1+z_1}\right\}^{\ell_2}\left\{-\frac{(b-a)u_2}{au_2+v_2}\right\}^{\ell_3}\left\{\frac{(b-a)y_2}{ay_2+z_2}\right\}^{\ell_4}$$

$$H^{0,n+4:m_1,n_1;\dots;m_r,n_r}_{p+4,q+4:p_1,q_1;\dots;p_r,q_r}\left[\begin{matrix} z_1(au_1+v_1)^{\rho_1}(au_2+v_2)^{-\rho_1'}(by_1+z_1)^{\sigma_1}(by_2+z_2)^{-\sigma_1'} \\ \vdots \\ z_r(au_1+v_1)^{\rho_r}(au_2+v_2)^{-\rho_r'}(by_1+z_1)^{\sigma_1}(by_2+z_2)^{-\sigma_r'} \end{matrix}\right|$$

$$(-r_1;\rho_1,\dots,\rho_r),(-\delta_1;\sigma_1,\dots,\sigma_r),(1-r_2-b_3;\rho_1',\dots,\rho_r'),(1-\delta_2-b_4;\sigma_1',\dots,\sigma_r'),$$
$$(-r_1+b_1;\rho_1,\dots,\rho_r),(-\delta_1+b_2;\sigma_1,\dots,\sigma_r),(1-r_2;\rho_1',\dots,\rho_r'),(1-\delta_2;\sigma_1',\dots,\sigma_r')$$

$$\left.\begin{matrix}(a_j;\alpha_j',\dots,\alpha_j^{(r)})_{1,p}:(c_j',r_j')_{1,p_1};\dots;(c_j^{(r)},r_j^{(r)})_{1,p_r} \\ (b_j;\beta_j',\dots,\beta_j^{(r)})_{1,q}:(d_j',\delta_j')_{1,q_1};\dots;(d_j^{(r)},\delta_j^{(r)})_{1,q_r}\end{matrix}\right] \tag{2.1}$$

provided (in addition to the appropriate Convergence and existence Conditions) that

$\min\{\rho_i,\rho_i',\sigma_i,\sigma_i'\}>0$, $(i=1,\dots,r)$; $\min\{Re(\alpha), Re(\beta)\}>0$; $b\neq a$;

$$\max\left\{\left|\frac{(b-a)u_1}{au_1+v_1}\right|,\left|\frac{(b-a)u_2}{au_2+v_2}\right|,\left|\frac{(b-a)y_1}{by_1+z_1}\right|,\left|\frac{(b-a)y_2}{by_2+z_2}\right|\right\}<1$$

$$\int_a^b (t-a)^{\alpha-1}(b-t)^{\beta-1}(u_1t+v_1)^{r_1}(u_2t+v_2)^{-r_2}(y_1t+z_1)^{\delta_1}(y_2t+z_2)^{-\delta_2}.$$

$$* H^*\begin{bmatrix} z_1(u_1t+v_1)^{\rho_1}(u_2t+v_2)^{-\rho_1'}(y_1t+z_1)^{\sigma_1}(y_2t+z_2)^{-\sigma_1'} \\ \vdots \\ z_r(u_1t+v_1)^{\rho_r}(u_2t+v_2)^{-\rho_r'}(y_1t+z_1)^{\sigma_r}(y_2t+z_2)^{-\sigma_r'} \end{bmatrix}.dt$$

$$=(b-a)^{\alpha+\beta-1}(au_1+v_1)^{r_1}(au_2+v_2)^{-r_2}(by_1+z_1)^{\delta_1}(by_2+z_2)^{-\delta_2}$$

$$x \sum_{\ell_1,\ell_2,\ell_3,\ell_4=0}^{\infty} \frac{B(\alpha+\ell_1+\ell_3,\ \beta+\ell_2+\ell_4)}{\ell_1!\,\ell_2!\,\ell_3!\,\ell_4!}$$

$$\left\{-\frac{(b-a)u_1}{au_1+v_1}\right\}^{\ell_1}\left\{\frac{(b-a)y_1}{by_1+z_1}\right\}^{\ell_2}\left\{\frac{(b-a)u_2}{au_2+v_2}\right\}^{\ell_3}\left\{-\frac{(b-a)y_2}{ay_2+z_2}\right\}^{\ell_4}$$

$$H^{0,n+4\,:n_1,m_1;\ldots;n_r,m_r}_{q+4,p+4\,:q_1,p_1;\ldots;q_r,p_r}\left[\begin{matrix} z_1^{-1}(au_1+v_1)^{-\rho_1}(au_2+v_2)^{\rho_1'}(by_1+z_1)^{-\sigma_1}(by_2+z_2)^{\sigma_1'} \\ \vdots \\ z_r^{-1}(au_1+v_1)^{-\rho_r}(au_2+v_2)^{\rho_r'}(by_1+z_1)^{-\sigma_r}(by_2+z_2)^{\sigma_r'} \end{matrix}\right|$$

$$\begin{matrix}(1+r_1-b_1;\rho_1,\ldots,\rho_r),(1+\delta_1-b_2;\sigma_1,\ldots,\sigma_r),(r_2;\rho_1',\ldots,\rho_r'),(\delta_2;\sigma_1',\ldots,\sigma_r'),\\ (1+r_1;\rho_1,\ldots,\rho_r),(1+\delta_1;\sigma_1,\ldots,\sigma_r),(r_2+b_3;\rho_1',\ldots,\rho_r'),(\delta_2+b_4;\sigma_1',\ldots,\sigma_r'),\\ (1-b_j;\beta_j',\ldots,\beta_j^{(r)})_{1,q}:(1-d_j',\delta_j')_{1,q_1};\ldots;(1-d_j^{(r)},\delta_j^{(r)})_{1,q_r}\\ (1-a_j;\alpha_j',\ldots,\alpha_j^{(r)})_{1,p}:(1-c_j',r_j')_{1,p_1};\ldots;(1-c_j^{(r)},r_j^{(r)})_{1,p_r}\end{matrix}\Bigg] \tag{2.2}$$

provided (in addition to the appropriate Convergence and existence Conditions) that

$\min\{\rho_i,\rho_i',\sigma_i,\sigma_i'\}>0,\ (i=1,\ldots,r);$
$\min\{Re(\alpha), Re(\beta)\}>0;\ b\neq a;$

$$\max\left\{\left|\frac{(b-a)u_1}{au_1+v_1}\right|,\left|\frac{(b-a)u_2}{au_2+v_2}\right|,\left|\frac{(b-a)y_1}{by_1+z_1}\right|,\left|\frac{(b-a)y_2}{by_2+z_2}\right|\right\}<1$$

and

$$H^*[z_1,\ldots,z_r]=H[z_1,\ldots,z_r]\Big|_{n=0}$$

Proof of (2.1): We first replace the multivariable H- function on the LHS by its Mellin-Barnes contour integral [18, p.251 (c.1)] collect the powers of (u_1t+v_1), (u_2t+v_2), (y_1t+z_1), (y_2t+z_2), and apply the binomial expansion (1.3). We then use the Eulerian integral (1.2) and interpret the resulting Mellin - Barnes contour integral as an H-function of r variables. We arrive at (2.1 [For Y'_s use

$$(yt+z)^{\delta}=(by+z)^{\delta}\sum_{\ell=0}^{\infty}\frac{(-\delta)_\ell}{\ell!}.\left\{\frac{(b-t)y}{by+z}\right\}, \tag{2.3}$$

where

$$\left|\frac{(b-t)y}{by+z}\right| < 1; t \in [a,b] \Bigg]$$

the sufficient conditions of validity of the integral (2.1), which we stated with (2.1), would follow by appealing to the principle of analytic continuation).

Proof of (2.2) : Proof of (2.2) is almost same as that of (2.1) stated above and then set $\xi_i=-s_i$ $(i=1,\ldots,\gamma)$ where $\xi_1,\ldots,\xi_\gamma$ denote the variables of the aforementioned Mellin-Barnes contour integral.

3. APPLICATIONS INVOLVING GENERALIZED HYPERGEOMETRIC FUNCTIONS

Each generalized Eulerian integral formulas (2.1) and (2.2) has manifold generality. By specializing the various parameters and variables involved, these formulas (and indeed their several variations obtained by letting any desired number of exponents:

$\rho_1,\ldots,\rho_r\,;\,\sigma_1,\ldots,\sigma_r\,;\,\rho_1',\ldots,\rho_r'\,;\,\sigma_1',\ldots,\sigma_r'\,;$

decrease to zero in such a manner that both the sides of resulting equation exist) can suitably applied to derive the corresponding results involving a remarkably wide variety of useful functions (or product of several such functions) which are expressible interms of the E, F, G and H- functions of one, two and more variables. Say, if we put n=p=q=0, the multivariable H- function on the LHS of (2.1) and (2.2) would immediately reduce to the product of r different Fox's H- functions. Various special cases of Fox's H-function can be seen in a monograph of Mathai and Saxena [2, p. 145-159]. Thus it can easily be derived Eulerian integrals involving any of these simpler special functions desired.

(i) In (2.1) and (2.2), replacing ρ_i', σ_i', by$(-\rho_i')$, $(-\sigma_i')$ and ρ_i, σ_i, by$(-\rho_i)$, $(-\sigma_i)$; $(i=1,\ldots,r)$

respectively, we get the following elegant general Eulerian integrals :

$$\int_a^b (t-a)^{\alpha-1}(b-t)^{\beta-1}(u_1t+v_1)^{r_1}(u_2t+v_2)^{-r_2}(y_1t+z_1)^{\delta_1}(y_2t+z_2)^{-\delta_2}.$$

$$* H\begin{bmatrix} z_1(u_1t+v_1)^{\rho_1}(u_2t+v_2)^{\rho_1'}(y_1t+z_1)^{\sigma_1}(y_2t+z_2)^{\sigma_1'} \\ \vdots \\ z_r(u_1t+v_1)^{\rho_r}(u_2t+v_2)^{\rho_r'}(y_1t+z_1)^{\sigma_r}(y_2t+z_2)^{\sigma_r'} \end{bmatrix}.dt$$

$$=(b-a)^{\alpha+\beta-1}(au_1+v_1)^{r_1}(au_2+v_2)^{-r_2}(by_1+z_1)^{\delta_1}(by_2+z_2)^{-\delta_2}$$

$$\text{x} \sum_{\ell_1,\ell_2,\ell_3,\ell_4=0}^{\infty} \frac{B(\alpha+\ell_1+\ell_3,\, \beta+\ell_2+\ell_4)}{\ell_1!\,\ell_2!\,\ell_3!\,\ell_4!}$$

$$\left\{\frac{(b-a)u_1}{au_1+v_1}\right\}^{\ell_1}\left\{-\frac{(b-a)y_1}{by_1+z_1}\right\}^{\ell_2}\left\{\frac{(b-a)u_2}{au_2+v_2}\right\}^{\ell_3}\left\{-\frac{(b-a)y_2}{by_2+z_2}\right\}^{\ell_4}$$

$$H^{0,n+4:m_1,n_1;\dots;m_r,n_r}_{p+4,q+4:p_1,q_1;\dots;p_r,q_r}\left[\begin{array}{c} z_1(au_1+v_1)^{\rho_1}(au_2+v_2)^{\rho_1'}(by_1+z_1)^{\sigma_1}(by_2+z_2)^{\sigma_1'} \\ \vdots \\ z_r(au_1+v_1)^{\rho_r}(au_2+v_2)^{\rho_r'}(by_1+z_1)^{\sigma_r}(by_2+z_2)^{\sigma_r'} \end{array}\right|$$

$(-r_1;\rho_1,\dots,\rho_r),(-\delta_1;\sigma_1,\dots,\sigma_r),(r_2;\rho_1',\dots,\rho_r'),(\delta_2;\sigma_1',\dots,\sigma_r'),$
$(-r_1+b_1;\rho_1,\dots,\rho_r),(-\delta_1+b_2;\sigma_1,\dots,\sigma_r),(r_2+b_3;\rho_1',\dots,\rho_r'),(\delta_2+b_4;\sigma_1',\dots,\sigma_r'),$

$$\left.\begin{array}{l}(a_j;\alpha_j',\dots,\alpha_j^{(r)})_{1,p}:(c_j',\gamma_j')_{1,p_1};\dots;(c_j^{(r)},\gamma_j^{(r)})_{1,p_r}\\(b_j;\beta_j',\dots,\beta_j^{(r)})_{1,q}:(d_j',\delta_j')_{1,q_1};\dots;(d_j^{(r)},\delta_j^{(r)})_{1,q_r}\end{array}\right] \tag{3.1}$$

$$\int_a^b (t-a)^{\alpha-1}(b-t)^{\beta-1}(u_1t+v_1)^{r_1}(u_2t+v_2)^{-r_2}(y_1t+z_1)^{\delta_1}(y_2t+z_2)^{-\delta_2}.$$

$$*H\left[\begin{array}{c} z_1(u_1t+v_1)^{-\rho_1}(u_2t+v_2)^{-\rho_1'}(y_1t+z_1)^{-\sigma_1}(y_2t+z_2)^{-\sigma_1'} \\ \vdots \\ z_r(u_1t+v_1)^{-\rho_r}(u_2t+v_2)^{-\rho_r'}(y_1t+z_1)^{-\sigma_r}(y_2t+z_2)^{-\sigma_r'} \end{array}\right].dt$$

$$=(b-a)^{\alpha+\beta-1}(au_1+v_1)^{r_1}(au_2+v_2)^{-r_2}(by_1+z_1)^{\delta_1}(by_2+z_2)^{-\delta_2}$$

$$x\sum_{\ell_1,\ell_2,\ell_3,\ell_4=0}^{\infty}\frac{B(\alpha+\ell_1+\ell_3,\beta+\ell_2+\ell_4)}{\ell_1!\ell_2!\ell_3!\ell_4!}$$

$$\left\{-\frac{(b-a)u_1}{au_1+v_1}\right\}^{\ell_1}\left\{\frac{(b-a)y_1}{by_1+z_1}\right\}^{\ell_2}\left\{-\frac{(b-a)u_2}{au_2+v_2}\right\}^{\ell_3}\left\{\frac{(b-a)y_2}{ay_2+z_2}\right\}^{\ell_4}$$

$$H^{0,n+4:m_1,n_1;\dots;m_r,n_r}_{p+4,q+4:p_1,q_1;\dots;p_r,q_r}\left[\begin{array}{c} z_1(au_1+v_1)^{-\rho_1}(au_2+v_2)^{-\rho_1'}(by_1+z_1)^{-\sigma_1}(by_2+z_2)^{-\sigma_1'} \\ \vdots \\ z_r(au_1+v_1)^{-\rho_r}(au_2+v_2)^{-\rho_r'}(by_1+z_1)^{-\sigma_r}(by_2+z_2)^{-\sigma_r'} \end{array}\right|$$

$(1+r_1-b_1;\rho_1,\dots,\rho_r),(1+\delta_1-b_2;\sigma_1,\dots,\sigma_r),(1-r_2+b_3;\rho_1',\dots,\rho_r'),$
$(1+r_1;\rho_1,\dots,\rho_r),(1+\delta_1;\sigma_1,\dots,\sigma_r),(1-r_2;\rho_1',\dots,\rho_r'),$
$(1-\delta_2-b_4;\sigma_1',\dots,\sigma_r'),(a_j;\alpha_j',\dots,\alpha_j^{(r)})_{1,p}:$
$(1-\delta_2;\sigma_1',\dots,\sigma_r'),(b_j;\beta_j',\dots,\beta_j^{(r)})_{1,q}$

$$\left.\begin{array}{l}(c_j^1,\gamma_j^1)_{1,p_1};\dots;(c_j^{(r)},\gamma_j^{(r)})_{1,p_r}\\(d_j^1,\delta_j^1)_{1,q_1};\dots;(d_j^{(r)},\delta_j^{(r)})_{1,q_r}\end{array}\right], \tag{3.2}$$

$$\int_a^b (t-a)^{\alpha-1}(b-t)^{\beta-1}(u_1t+v_1)^{r_1}(u_2t+v_2)^{-r_2}(y_1t+z_1)^{\delta_1}(y_2t+z_2)^{-\delta_2}.$$

$$H^{0,0:m_1,n_1;\dots;m_r,n_r}_{p,q:p_1,q_1;\dots;p_r,q_r}\left[\begin{array}{c} z_1(tu_1+v_1)^{\rho_1}(tu_2+v_2)^{\rho_1'}(ty_1+z_1)^{\sigma_1}(ty_2+z_2)^{\sigma_1'} \\ \vdots \\ z_r(tu_1+v_1)^{\rho_r}(tu_2+v_2)^{\rho_r'}(ty_1+z_1)^{\sigma_r}(ty_2+z_2)^{\sigma_r'} \end{array}\right|$$

$$\left.\begin{array}{l}(a_j;\alpha_j^1,\dots,\alpha_j^{(r)})_{1,p}:(c_j^1,\gamma_j^1)_{1,p_1};\dots;(c_j^{(r)},\gamma_j^{(r)})_{1,p_r}\\(b_j;\beta_j^1,\dots,\beta_j^{(r)})_{1,q}:(d_j^1,\delta_j^1)_{1,q_1};\dots;(d_j^{(r)},\delta_j^{(r)})_{1,q_r}\end{array}\right]\,dt$$

$$=(b-a)^{\alpha+\beta-1}(au_1+v_1)^{r_1}(au_2+v_2)^{-r_2}(by_1+z_1)^{\delta_1}(by_2+z_2)^{-\delta_2}$$

$$\times \sum_{\ell_1,\ell_2,\ell_3,\ell_4=0}^{\infty} \frac{B(\alpha+\ell_1+\ell_3, \beta+\ell_2+\ell_4)}{\ell_1!\,\ell_2!\,\ell_3!\,\ell_4!}$$

$$\times \left\{-\frac{(b-a)u_1}{au_1+v_1}\right\}^{\ell_1} \left\{\frac{(b-a)y_1}{by_1+z_1}\right\}^{\ell_2} \left\{-\frac{(b-a)u_2}{au_2+v_2}\right\}^{\ell_3} \left\{\frac{(b-a)y_2}{ay_2+z_2}\right\}^{\ell_4}$$

$$\times H^{0,4\,:m_1,n_1;\ldots;m_r n_r}_{q+4,p+4:q_1,p_1;\ldots;q_r,p_r}\left[\begin{array}{c} z_1^{-1}(au_1+v_1)^{\rho_1}(au_2+v_2)^{\rho_1'}(by_1+z_1)^{\sigma_1}(by_2+z_2)^{\sigma_1'} \\ \vdots \\ z_r^{-1}(au_1+v_1)^{\rho_r}(au_2+v_2)^{\rho_r'}(by_1+z_1)^{\sigma_r}(by_2+z_2)^{\sigma_r'} \end{array}\right|$$

$(1+r_1-b_1;\ \rho_1,\ldots,\rho_r),\ (1+\delta_1-b_2;\sigma_1,\ldots,\sigma_r),\ (1-r_2-b_3;\ \rho_1',\ldots,\rho_r'),$
$(1+r_1;\ \rho_1,\ldots,\rho_r),\ (1+\delta_1;\sigma_1,\ldots,\sigma_r),\ (1-r_2;\rho_1',\ldots,\rho_r'),$

$(1-\delta_2-b_4;\ \sigma_1',\ldots,\sigma_r'),\ (1-b_j;\ \beta_j',\ldots,\beta_j^{(r)})_{1,q}:$
$(1-\delta_2;\ \sigma_1',\ldots,\sigma_r'),\ (1-a_j;\ \alpha_j',\ldots,\alpha_j^{(r)})_{1,p}:$

$$\left.\begin{array}{l} (1-d_j^1,\delta_j^1)_{1,q_1};\ \ldots\ldots;(1-d_j^{(r)},\delta_j^{(r)})_{1,q_r} \\ (1-c_j^1,\gamma_j^1)_{1,p_1};\ \ldots\ldots;(1-c_j^{(r)},\gamma_j^{(r)})_{1,p_r} \end{array}\right], \qquad (3.3)$$

$$\int_a^b (t-a)^{\alpha-1}(b-t)^{\beta-1}(u_1t+v_1)^{r_1}(u_2t+v_2)^{-r_2}(y_1t+z_1)^{\delta_1}(y_2t+z_2)^{-\delta_2}.$$

$$\times H^{0,0\,:m_1,n_1;\ldots;m_r n_r}_{p,q\,:p_1,q_1;\ldots;p_r,q_r}\left[\begin{array}{c} z_1(tu_1+v_1)^{-\rho_1}(tu_2+v_2)^{-\rho_1'}(ty_1+z_1)^{-\sigma_1}(ty_2+z_2)^{-\sigma_1'} \\ \vdots \\ z_r(tu_1+v_1)^{-\rho_r}(tu_2+v_2)^{-\rho_r'}(ty_1+z_1)^{-\sigma_r}(ty_2+z_2)^{-\sigma_r'} \end{array}\right|$$

$$\left.\begin{array}{l} (a_j;\ \alpha_j^1,\ldots,\alpha_j^{(r)})_{1,p}:(c_j^1,\gamma_j^1)_{1,p_1};\ldots\ldots;(c_j^{(r)},\gamma_j^{(r)})_{1,p_r} \\ (b_j;\ \beta_j^1,\ldots,\beta_j^{(r)})_{1,q}:(d_j^1,\delta_j^1)_{1,q_1};\ldots\ldots;(d_j^{(r)},\delta_j^{(r)})_{1,q_r} \end{array}\right] dt$$

$$=(b-a)^{\alpha+\beta-1}(au_1+v_1)^{r_1}(au_2+v_2)^{-r_2}(by_1+z_1)^{\delta_1}(by_2+z_2)^{-\delta_2}$$

$$\times \sum_{\ell_1,\ell_2,\ell_3,\ell_4=0}^{\infty} \frac{B(\alpha+\ell_1+\ell_3, \beta+\ell_2+\ell_4)}{\ell_1!\,\ell_2!\,\ell_3!\,\ell_4!}$$

$$\left\{-\frac{(b-a)u_1}{au_1+v_1}\right\}^{\ell_1} \left\{\frac{(b-a)y_1}{by_1+z_1}\right\}^{\ell_2} \left\{-\frac{(b-a)u_2}{au_2+v_2}\right\}^{\ell_3} \left\{\frac{(b-a)y_2}{by_2+z_2}\right\}^{\ell_4}$$

$$H^{0,4\,:n_1,m_1;\ldots;n_r,m_r}_{q+4,p+4:q_1,p_1;\ldots;q_r,p_r}\left[\begin{array}{c} z_1^{-1}(au_1+v_1)^{-\rho_1}(au_2+v_2)^{-\rho_1'}(by_1+z_1)^{-\sigma_1}(by_2+z_2)^{-\sigma_1'} \\ \vdots \\ z_r^{-1}(au_1+v_1)^{-\rho_r}(au_2+v_2)^{-\rho_r'}(by_1+z_1)^{-\sigma_r}(by_2+z_2)^{-\sigma_r'} \end{array}\right|$$

$(1+r_1-b_1;\ \rho_1,\ldots,\rho_r),\ (1+\delta_1-b_2;\sigma_1,\ldots,\sigma_r),\ (1-r_2-b_3;\ \rho_1',\ldots,\rho_r'),$
$(1+r_1;\ \rho_1,\ldots,\rho_r),\ (1+\delta_1;\sigma_1,\ldots,\sigma_r),\ (1-r_2;\rho_1',\ldots,\rho_r'),$

$(1-\delta_2-b_4;\ \sigma_1',\ldots\ldots,\sigma_r'),\ (1-b_j\,;\ \beta_j',\ldots\ldots,\beta_j^{(r)})_{1,q}$:
$(1-\delta_2\,;\ \sigma_1',\ldots\ldots,\sigma_r'),\ (1-a_j\,;\ \alpha_j',\ldots\ldots,\alpha_j^{(r)})_{1,p}$:

$$\left.\begin{matrix}(1-d_j^1,\delta_j^1)_{1,q_1};\ \ldots\ldots\ldots;(1-d_j^{(r)},\delta_j^{(r)})_{1,q_r}\\(1-c_j^1,r_j^1)_{1,p_1};\ \ldots\ldots\ldots;(1-c_j^{(r)},r_j^{(r)})_{1,p_r}\end{matrix}\right],\qquad(3.4)$$

(ii) In (3.2), setting $n=p$; $n_1=p_1,\ldots\ldots,n_r=p_r$; $m_1=1,\ldots\ldots,m_r=1$; adjusting gamma factors in $q_1,\ldots,q_r$ so that they run from 1 to $q_1,\ldots\ldots\ldots,1$ to q_r; in numberator applying [9, p.32 (9)]; taking $\delta^1=\delta^{11}=\ldots=\delta^{(r)}=1$ and finally replacing $(1-a_j)$, $(1-b_j)$, $(1-c_j)$, $(1-d_j)$, z's respectively a_j,b_j,c_j,d_j, $(-z)$'s; we get general Eulerian integral involving (Srivastava - Daoust) generalized Lauricella function of several variables [12,p. 454].

$$\int_a^b (t-a)^{\alpha-1}(b-t)^{\beta-1}(u_1t+v_1)^{r_1}(u_2t+v_2)^{-r_2}(y_1t+z_1)^{\delta_1}(y_2t+z_2)^{-\delta_2}$$

$$F^{p:p_1;\ldots;p_r}_{q:q_1;\ldots;q_r}\begin{bmatrix}z_1(au_1+v_1)^{-\rho_1}(au_2+v_2)^{-\rho_1'}(y_1t+z_1)^{-\sigma_1}(y_2t+z_2)^{-\sigma_1'}\\ \vdots\\ z_r(au_1+v_1)^{-\rho_r}(au_2+v_2)^{-\rho_r'}(y_1t+z_1)^{-\sigma_r}(y_2t+z_2)^{-\sigma_r'}\end{bmatrix}dt$$

$$=(b-a)^{\alpha+\beta-1}B(\alpha,\beta)(au_1+v_1)^{r_1}(au_2+v_2)^{-r_2}(by_1+z_1)^{\delta_1}(by_2+z_2)^{-\delta_2}$$

$$F\begin{matrix}p+6:p_1;\ldots\ldots;p_r;0;0;0;0\\q+5:q_1;\ldots\ldots\ldots;q_r;0;0;0;0\end{matrix}\left(\begin{matrix}(-r_1;\rho_1,\ldots\ldots,\rho_r,1,0,0,0),\\ \\(-r_1;\rho_1,\ldots\ldots,\rho_r,0,0,0,0),\end{matrix}\right.$$

$(-\delta_1;\ \sigma_1,\ldots\ldots\ldots\sigma_r,0,1,0,0),\ (r_2;\ \rho_1',\ldots\ldots,\rho_r',0,0,1,0),$
$(-\delta_1;\ \sigma_1,\ldots\ldots\ldots\sigma_r,0,0,0,0),\ (r_2;\ \rho_1',\ldots\ldots,\rho_r',0,0,,0),$

$(\delta_2;\ \sigma_1',\ldots\ldots\ldots\sigma_r',0,0,0,1),\ (a_j;\ \alpha_j',\ldots\ldots,\alpha_j^{(r)},0,0,0,0)_{1,p},$
$(\delta_2;\ \sigma_1',\ldots\ldots\ldots\sigma_r',0,0,0,0),\ (b_j;\ \beta_j',\ldots\ldots,\beta_j^{(r)},0,0,0,0)_{1,q},$

$(\alpha;0,\ldots\ldots,0,1,0,1,0),\ (\beta;0,\ldots\ldots0,0,1,0,1)$:
$(\alpha+\beta;0,\ldots\ldots,0,1,1,1,1)$:

$(c_j',r_j')_{1,p_1}$ $;\ldots\ldots\ldots;(c_j^{(r)},r^{(r)})_{1,p_r}$;______;________;________;________;
$(d_j',\delta_j')_{1,q_1}$ $;\ldots\ldots\ldots;(d_j^{(r)},\delta^{(r)})_{1,q_r}$;______;________;________;________;

$$z_1(au_1+v_1)^{-\rho_1}(au_2+v_2)^{-\rho_1'}(by_1+z_1)^{-\sigma_1}(by_2+z_2)^{-\sigma_1'},\ldots\ldots,$$
$$\vdots$$
$$z_r(au_1+v_1)^{-\rho_r}(au_2+v_2)^{-\rho_r'}(by_1+z_1)^{-\sigma_r}(by_2+z_2)^{-\sigma_r'},$$

$$\left.\left\{-\frac{(b-a)u_1}{au_1+v_1}\right\},\ \left\{-\frac{(b-a)u_2}{au_2+v_2}\right\},\left\{\frac{(b-a)y_1}{by_1+z_1}\right\},\left\{\frac{(b-a)y_2}{by_2+z_2}\right\}\right),\qquad(3.5)$$

(iii) Each of Eulerian integral formula (2.1), (2.2), (3.1), - (3.5) can easily be stated as a fractional integral formula involving the fractional operator ${}_{\alpha}D_x^{\mu}$ defined in (1.5), for b=x ($\alpha \leftrightarrow \beta$) as follows:

$$
\begin{aligned}
&{}_aD_x^{-\alpha}\Big\{(x-a)^{\beta-1}(u_1x+v_1)^{r_1}(u_2x+v_2)^{-r_2}(xy_1+z_1)^{\delta_1}(xy_2+z_2)^{-\delta_2}\\
&\quad H^*\Big[z_1(u_1x+v_1)^{\rho_1}(u_2x+v_2)^{-\rho_1'}(xy_1+z_1)^{\sigma_1}(xy_2+z_2)^{-\sigma_1'},\ldots\ldots,\\
&\quad z_r(u_1x+v_1)^{\rho_r}(u_2x+v_2)^{-\rho'_r}(xy_1+z_1)^{\sigma_r}(xy_2+z_2)^{-\sigma_r'}\Big]\Big\}\\
&=(x-a)^{\alpha+\beta-1}(au_1+v_1)^{r_1}(au_2+v_2)^{-r_2}(xy_1+z_1)^{\delta_1}(xy_2+z_2)^{-\delta_2}\\
&\times\sum_{\ell_1,\ell_2,\ell_3,\ell_4=0}^{\infty}\frac{B(\alpha+\ell_1+\ell_3,\beta+\ell_2+\ell_4)}{\ell_1!\ell_2!\ell_3!\ell_4!\Gamma(\alpha)}\\
&\times\left\{+\frac{(x-a)u_1}{au_1+v_1}\right\}^{\ell_1}\left\{-\frac{(x-a)y_1}{xy_1+z_1}\right\}^{\ell_2}\left\{-\frac{(x-a)u_2}{au_2+v_2}\right\}^{\ell_3}\left\{\frac{(x-a)y_2}{xy_2+z_2}\right\}^{\ell_4}\\
&H^{0,n+4:n_1,m_1;\ldots;n_r,m_r}_{q+4,p+4:q_1,p_1;\ldots;q_r,p_r}\left[\begin{array}{c} z_1(au_1+v_1)^{\rho_1}(au_2+v_2)^{-\rho_1'}(xy_1+z_1)^{\sigma_1}(xy_2+z_2)^{-\sigma_1'}\\ \vdots\\ z_r(au_1+v_1)^{\rho_r}(au_2+v_2)^{-\rho_r'}(xy_1+z_1)^{\sigma_r}(xy_2+z_2)^{-\sigma_r'}\end{array}\right|
\end{aligned}
\tag{3.6}
$$

$$
\begin{aligned}
&{}_aD_x^{-\alpha}\Big\{(x-a)^{\beta-1}(u_1x+v_1)^{r_1}(u_2x+v_2)^{-r_2}(xy_1+z_1)^{\delta_1}(xy_2+z_2)^{-\delta_2}\\
&\quad H^*\Big[z_1(u_1x+v_1)^{\rho_1}(u_2x+v_2)^{-\rho_1'}(xy_1+z_1)^{\sigma_1}(xy_2+z_2)^{-\sigma_1'},\ldots\ldots,\\
&\quad z_r(u_1x+v_1)^{\rho_r}(u_2x+v_2)^{-\rho'_r}(xy_1+z_1)^{\sigma_r}(xy_2+z_2)^{-\sigma_r'}\Big]\Big\}\\
&=(x-a)^{\alpha+\beta-1}(au_1+v_1)^{r_1}(au_2+v_2)^{-r_2}(xy_1+z_1)^{\delta_1}(xy_2+z_2)^{-\delta_2}\\
&\times\sum_{\ell_1,\ell_2,\ell_3,\ell_4=0}^{\infty}\frac{B(\alpha+\ell_1+\ell_3,\beta+\ell_2+\ell_4)}{\ell_1!\ell_2!\ell_3!\ell_4!\Gamma(\alpha)}\\
&\times\left\{-\frac{(x-a)u_1}{au_1+v_1}\right\}^{\ell_1}\left\{\frac{(x-a)y_1}{xy_1+z_1}\right\}^{\ell_2}\left\{\frac{(x-a)u_2}{au_2+v_2}\right\}^{\ell_3}\left\{-\frac{(x-a)y_2}{xy_2+z_2}\right\}^{\ell_4}\\
&\times H^{0,4:n_1,m_1;\ldots;n_r,m_r}_{q+4,p+4:q_1,p_1;\ldots;q_r,p_r}\left[\begin{array}{c} z_1^{-1}(au_1+v_1)^{-\rho_1}(au_2+v_2)^{\rho_1'}(xy_1+z_1)^{-\sigma_1}(xy_2+z_2)^{\sigma_1'}\\ \vdots\\ z_r^{-1}(au_1+v_1)^{-\rho_r}(au_2+v_2)^{\rho_r'}(xy_1+z_1)^{-\sigma_r}(xy_2+z_2)^{\sigma_r'}\end{array}\right].
\end{aligned}
\tag{3.7}
$$

$$
\begin{aligned}
&{}_aD_x^{-\alpha}\Big\{(x-a)^{\beta-1}(u_1x+v_1)^{r_1}(u_2x+v_2)^{-r_2}(y_1x+z_1)^{\delta_1}(y_2x+z_2)^{-\delta_2}\\
&\quad H\Big[z_1(u_1x+v_1)^{\rho_1}(u_2x+v_2)^{\rho_1'}(xy_1+z_1)^{\sigma_1}(xy_2+z_2)^{\sigma_1'},\ldots\ldots,\\
&\quad z_r(u_1x+v_1)^{\rho_r}(u_2x+v_2)^{\rho'_r}(xy_1+z_1)^{\sigma_r}(xy_2+z_2)^{\sigma_r'}\Big]\Big\}\\
&=(x-a)^{\alpha+\beta-1}(au_1+v_1)^{r_1}(au_2+v_2)^{-r_2}(xy_1+z_1)^{\delta_1}(xy_2+z_2)^{-\delta_2}\\
&\times\sum_{\ell_1,\ell_2,\ell_3,\ell_4=0}^{\infty}\frac{B(\alpha+\ell_1+\ell_3,\beta+\ell_2+\ell_4)}{\ell_1!\ell_2!\ell_3!\ell_4!\Gamma(\alpha)}
\end{aligned}
$$

$$\times\left\{\frac{(x-a)u_1}{au_1+v_1}\right\}^{\ell_1}\left\{-\frac{(x-a)z_1}{xy_1+z_1}\right\}^{\ell_2}\left\{\frac{(x-a)u_2}{au_2+v_2}\right\}^{\ell_3}\left\{-\frac{(x-a)y_2}{xy_2+z_2}\right\}^{\ell_4}$$

$$H^{0,n+4:n_1,m_1,\ldots,n_r,m_r}_{p+4,q+4:p_1,q_1;\ldots;p_r,q_r}\begin{bmatrix} z_1(au_1+v_1)^{\rho_1}(au_2+v_2)^{\rho_1'}(xy_1+z_1)^{\sigma_1}(xy_2+z_2)^{\sigma_1'} \\ \vdots \\ z_r(au_1+v_1)^{\rho_r}(au_2+v_2)^{\rho_r'}(xy_1+z_1)^{\sigma_r}(xy_2+z_2)^{\sigma_r'}\end{bmatrix} \tag{3.8}$$

$${}_aD_x^{-\alpha}\Big\{(x-a)^{\beta-1}(u_1x+v_1)^{r_1}(u_2x+v_2)^{-r_2}(xy_1+z_1)^{\delta_1}(xy_2+z_2)^{-\delta_2}$$
$$H\Big[z_1(u_1x+v_1)^{-\rho_1}(u_2x+v_2)^{-\rho_1'}(xy_1+z_1)^{-\sigma_1}(xy_2+z_2)^{-\sigma_1'},\ldots\ldots,$$
$$z_r(u_1x+v_1)^{-\rho_r}(u_2x+v_2)^{-\rho_r'}(xy_1+z_1)^{-\sigma_r}(xy_2+z_2)^{-\sigma_r'}\Big]\Big\}$$

$$=(x-a)^{\alpha+\beta-1}(au_1+v_1)^{r_1}(au_2+v_2)^{-r_2}(xy_1+z_1)^{\delta_1}(xy_2+z_2)^{-\delta_2}$$
$$\times\sum_{\ell_1,\ell_2,\ell_3,\ell_4=0}^{\infty}\frac{B(\alpha+\ell_1+\ell_3,\beta+\ell_2+\ell_4)}{\ell_1!\ell_2!\ell_3!\ell_4!\Gamma(\alpha)}$$

$$\left\{-\frac{(x-a)u_1}{au_1+v_1}\right\}^{\ell_1}\left\{\frac{(x-a)y_1}{xy_1+z_1}\right\}^{\ell_2}\left\{-\frac{(x-a)u_2}{au_2+v_2}\right\}^{\ell_3}\left\{\frac{(x-a)y_2}{xy_2+z_2}\right\}^{\ell_4}\times$$

$$\times H^{0,n+4:n_1,m_1,\ldots,n_r,m_r}_{q+4,p+4:q_1,p_1;\ldots;q_r,p_r}\begin{bmatrix} z_1(au_1+v_1)^{-\rho_1}(au_2+v_2)^{-\rho_1'}(xy_1+z_1)^{-\sigma_1}(xy_2+z_2)^{-\sigma_1'} \\ \vdots \\ z_r(au_1+v_1)^{-\rho_r}(au_2+v_2)^{-\rho_r'}(xy_1+z_1)^{-\sigma_r}(xy_2+z_2)^{-\sigma_r'}\end{bmatrix} \tag{3.9}$$

$${}_aD_x^{-\alpha}\Big\{(x-a)^{\beta-1}(u_1x+v_1)^{r_1}(u_2x+v_2)^{-r_2}(xy_1+z_1)^{\delta_1}(xy_2+z_2)^{-\delta_2}$$
$$H^*\Big[z_1(u_1x+v_1)^{\rho_1}(u_2x+v_2)^{\rho_1'}(xy_1+z_1)^{\sigma_1}(xy_2+z_2)^{\sigma_1'},\ldots\ldots,$$
$$z_r(u_1x+v_1)^{\rho_r}(u_2x+v_2)^{\rho_r'}(xy_1+z_1)^{\sigma_r}(xy_2+z_2)^{\sigma_r'}\Big]\Big\}$$

$$=(x-a)^{\alpha+\beta-1}(au_1+v_1)^{r_1}(au_2+v_2)^{-r_2}(xy_1+z_1)^{\delta_1}(xy_2+z_2)^{-\delta_2}$$
$$\times\sum_{\ell_1,\ell_2,\ell_3,\ell_4=0}^{\infty}\frac{B(\alpha+\ell_1+\ell_3,\beta+\ell_2+\ell_4)}{\ell_1!\ell_2!\ell_3!\ell_4!\Gamma(\alpha)}$$

$$\times\left\{-\frac{(x-a)u_1}{au_1+v_1}\right\}^{\ell_1}\left\{\frac{(x-a)y_1}{xy_1+z_1}\right\}^{\ell_2}\left\{-\frac{(x-a)u_2}{au_2+v_2}\right\}^{\ell_3}\left\{\frac{(x-a)y_2}{xy_2+z_2}\right\}^{\ell_4}$$

$$\times H^{0,4:n_1,m_1;\ldots;n_r,m_r}_{q+4,p+4:q_1,p_1;\ldots;q_r,p_r}\begin{bmatrix} z_1^{-1}(au_1+v_1)^{\rho_1}(au_2+v_2)^{\rho_1'}(xy_1+z_1)^{\sigma_1}(xy_2+z_2)^{\sigma_1'} \\ \vdots \\ z_r^{-1}(au_1+v_1)^{\rho_r}(au_2+v_2)^{\rho_r'}(xy_1+z_1)^{\sigma_r}(xy_2+z_2)^{\sigma_r'}\end{bmatrix} \tag{3.10}$$

$${}_aD_x^{-\alpha}\Big\{(x-a)^{\beta-1}(u_1x+v_1)^{r_1}(u_2x+v_2)^{-r_2}(xy_1+z_1)^{\delta_1}(xy_2+z_2)^{-\delta_2}$$
$$H^*\Big[z_1(u_1x+v_1)^{-\rho_1}(u_2x+v_2)^{-\rho_1'}(xy_1+z_1)^{-\sigma_1}(xy_2+z_2)^{-\sigma_1'},\ldots\ldots,$$
$$z_r(u_1x+v_1)^{-\rho_r}(u_2x+v_2)^{-\rho_r'}(xy_1+z_1)^{-\sigma_r}(xy_2+z_2)^{-\sigma_r'}\Big]\Big\}$$

$$= (x-a)^{\alpha+\beta-1}(au_1+v_1)^{r_1}(au_2+v_2)^{-r_2}(xy_1+z_1)^{\delta_1}(xy_2+z_2)^{-\delta_2}$$

$$\times \sum_{\ell_1,\ell_2,\ell_3,\ell_4=0}^{\infty} \frac{B(\alpha+\ell_1+\ell_3,\beta+\ell_2+\ell_4)}{\ell_1!\ell_2!\ell_3!\ell_4!\Gamma(\alpha)}$$

$$\times\left\{-\frac{(x-a)u_1}{au_1+v_1}\right\}^{\ell_1}\left\{\frac{(x-a)y_1}{xy_1+z_1}\right\}^{\ell_2}\left\{-\frac{(x-a)u_2}{au_2+v_2}\right\}^{\ell_3}\left\{\frac{(x-a)y_2}{xy_2+z_2}\right\}^{\ell_4}$$

$$\times H^{0,+4:n_1,m_1;\ldots;n_r,m_r}_{q+4,p+4:q_1,p_1;\ldots;q_r,p_r}\begin{bmatrix} z_1^{-1}(au_1+v_1)^{-\rho_1}(au_2+v_2)^{-\rho_1'}(xy_1+z_1)^{-\sigma_1}(xy_2+z_2)^{-\sigma_1'} \\ \vdots \\ z_r^{-1}(au_1+v_1)^{-\rho_r}(au_2+v_2)^{-\rho_r'}(xy_1+z_1)^{-\sigma_r}(xy_2+z_2)^{-\sigma_r'} \end{bmatrix} \quad (3.11)$$

$${}_aD_x^{-\alpha}\{(x-a)^{\beta-1}(u_1x+v_1)^{r_1}(u_2x+v_2)^{-r_2}(xy_1+z_1)^{\delta_1}(xy_2+z_2)^{-\delta_2}$$

$$F^{p:p_1;\ldots;p_r}_{q:q_1;\ldots;q_r}\begin{pmatrix} z_1(u_1x+v_1)^{-\rho_1}(u_2x+v_2)^{-\rho_1'}(xy_1+z_1)^{-\sigma_1}(xy_2+z_2)^{-\sigma_1'} \\ \vdots \\ z_r(u_1x+v_1)^{-\rho_r}(u_2x+v_2)^{-\rho_r'}(xy_1+z_1)^{-\sigma_r}(xy_2+z_2)^{-\sigma_r'} \end{pmatrix}$$

$$= (x-a)^{\alpha+\beta-1}B(\alpha,\beta)(au_1+v_1)^{r_1}(au_2+v_2)^{-r_2}(xy_1+z_1)^{\delta_1}(xy_2+z_2)^{-\delta_2}$$

$$F^{p+6:p_1;\ldots;p_r;0;0;0;0}_{q+5:q_1;\ldots;q_r;0;0;0;0}\Big(z_1(au_1+v_1)^{-\rho_1}(au_2+v_2)^{-\rho_1'}(xy_1+z_1)^{-\sigma_1}(xy_2+z_2)^{-\sigma_1'},$$

$$\ldots\ldots\ldots,\big(z_r(au_1+v_1)^{-\rho_r}(au_2+v_2)^{-\rho_r'}(xy_1+z_1)^{-\sigma_r}(xy_2+z_2)^{-\sigma_r'}, \quad (3.12)$$

$$\left\{-\frac{(x-a)u_1}{au_1+v_1}\right\},\left\{-\frac{(x-a)u_2}{au_2+v_2}\right\},\left\{\frac{(x-a)y_1}{xy_1+z_1}\right\},\left\{\frac{(x-a)y_2}{xy_2+z_2}\right\}\Bigg)$$

Where the multivariable function parameters in (3.6)-(3.12) are precisely the same as those displayed on the RHS of (2.1), (2.2), (3.1)-(3.5) respectively. The condition of validity of Eulerian integral formulas (3.1)-(3.5) and (3.6)-(3.12) can easily be derived from their parent formulas.

(iv)In (3.2), (3.3),(3.5) and (3.9) putting $r_2=\delta_1=\delta_2=\rho_1'=\sigma_1=\sigma_1'=0$; $r_2=\delta_2=\rho_1'=\sigma_1'=0$; we get elegant results (2.1), (2.5), (2.3), (2.6), (3.9)-(3.12), (3.14) of Srivastava and Hussain [24] respectivly. Hence inturn the results of Saxena and Nishimoto [10].

Acknowledgment-The present investigation was supported by U.G.C (India)

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