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# *REFRACTIVE INDEX CHANGE PRODUCED BY THE ELECTRO-OPTICAL EFFECT IN $LiTaO_3$*

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## *ABSTRACT:*

Electro-optically induced refractive-index change in regular and reverse-poled $LiTaO_3$ has been calculated. Specifically we determine how index change depends on electric field magnitude and direction for y-propagating extra-ordinary modes. To accomplish this, changes in index-ellipsoid shape and orientation are found by the use of a numerical eigenvalue procedure to diagonalize the impermeability tensor; then the refractive index is calculated by the use of a vector reference-frame transformation and a small perturbation approximation. A general formula is inferred from calculations for specific field directions. Electro-optic coefficients for reserve-poled $LiTaO_3$ are obtained by application of a tensor reference-frame transformation to those of $LiTaO_3$.

## *1.INTRODUCTION*

We present numerical calculations of the electro-optically induced change in refractive index for y-propagating extraordinary modes in regular and reverse-poled $LiTaO_3$. These calculations were done in order to gain insight into the operation of an integrated photonic electric-field sensor that has no metallic electrodes [1,2] Specifically, we sought to determine whether the electro-optic mechanism, which produces the sensor's intended linear response to z-polarized fields, might also produce undesired secondary responses to x- and y-polarized fields. If large enough, such orthogonal-component responses could hamper the sensor's use for field-direction measurement .The possibility of x- and y- responses is suggested by the presence of electro-optic tensor elements that have magnitudes comparable to that of $r_{33}$ (responsible for the z-response). Since analytical methods for index-change calculation are conveniently applied only to specific combinations of electric-field direction and tensor form [ 3 ] , we use a numerical method that works for general field direction and general electro-optic tensors.This method is not difficult to implement and generates a complete description of the shape and the orientation changes of the index ellipsoid, in addition, with only slight modification, it can produce the index change for both normal modes associated with an arbitrary propagation direction through a general biaxial medium. Since many properties of reverse-poled $LiTaO_3$ have not yet been experimentally determined, we assume a form for its electro-optic tensor.

## 2. THEORY

The extraordinary mode of refractive index and wave-normal unit vector are labeled $n_2$ and **S**, respectively. Since **S** points in the y-direction, the zero-field value of $n_2$ equals $n_e$. In the presence of an electric field, $n_2$-$n_e$ provides the desired TM-mode index-change approximation.

The value of $n_2$ depends on the direction of **S** relative to the index ellipsoid and on the index-ellipsoid shape, which is specified by the principal refractive indices [3 ]. In general, when an electric field is applied, the principal refractive indices change, and the index ellipsoid changes its orientation relative to the crystal axes. Because **S** remains fixed, the ellipsoid orientation change is equivalent to a change in the direction of **S** relative to the ellipsoid. To determine $n_2$, the new principal refractive indices and the new ellipsoid orientation are found; then the direction of **S** relative to the modified ellipsoid is determined, and finally, the newprincipal refractive indices and the new relative **S** direction are used to calculate $n_2$.

The primary axes of the index ellipsoid are always parallel to the axes of the principal dielectric frame, so finding the orientation of this frame relative to the crystal frame provides a method of determining the ellipsoid orientation. The principal dielectric frame is defined to be the reference frame in which the permittivity tensor $\varepsilon$ is a diagonal matrix [3]. Since the impermeability tensor $\eta$ equals $\varepsilon_0 \varepsilon^{-1}$, and since the inverse of a diagonal matrix is also diagonal the principal dielectric frame is also the reference frame in which $\eta$ is diagonal. When an electric field is applied to a uniaxial material characterized by $n_o$ and $n_e$, the impermeability tensor is given by [3]

$$\eta = \begin{vmatrix} 1/n_0^2 + \Delta\eta_1 & \Delta\eta_6 & \Delta\eta_5 \\ \Delta\eta_6 & 1/n_0^2 + \Delta\eta_2 & \Delta\eta_4 \\ \Delta\eta_5 & \Delta\eta_4 & 1/n_e^2 + \Delta\eta_3 \end{vmatrix} \qquad (1)$$

where

$$\Delta\eta_\alpha = \sum_{k=1}^{3} r_{\alpha k} E_k \qquad (2)$$

In eq. (2) $r_{\alpha k}$ represents an element of the 6x3 contracted form of the linear electrooptic tensor, and $E_1$, $E_2$ and $E_3$ are the x, y and z component of E, respectively. For convenience, the field vector will be specified as

$$\mathbf{E} = E\mathbf{U}_E \qquad (3)$$

where E is the field strength and $\mathbf{U}_E$ is a unit vector that specifies direction.

Eq. ( 1 ) indicates that when an electric field is applied, the crystal-frame expression of $\eta$ can become nondiagonal; in such a case, the crystal frame is no longer the principal dielectric frame. As shown below, since $\eta$ is always real and symmetric, there must exist an orthogonal reference frame with the property that the expression of $\eta$ in that frame is diagonal. This frame is called the new principal dielectric frame, and its axes are labeled x ', y ' and z ' to distinguish them from the crystal-frame axes x, y and z.

When expressed in the new principal dielectric frame, the impermeability tensor has the form

$$\eta' = \begin{vmatrix} 1/n_x'^2 & 0 & 0 \\ 0 & 1/n_y'^2 & 0 \\ 0 & 0 & 1/n_z'^2 \end{vmatrix} \qquad (4)$$

where $n_x'$ , $n_y'$ and $n_z'$ are the new principal refractive indices. Note that $\eta'$ is used to distinguish the $x'$ $y'$ $z'$ - frame expression in eq. ( 4 ) from the xyz-frame expression in eq. (1). The new principal refractive indices completely specify the index ellipsoid's modified shape.

The positive directions of the $x'$, $y'$ and $z'$ axes are specified by three mutually orthogonal unit basis vectors labeled $\mathbf{U}_1$, $\mathbf{U}_2$ and $\mathbf{U}_3$, respectively. These vectors completely describe the index ellipsoid's shifted orientation . To help visualize this orientation, the components of $\mathbf{U}_1$, $\mathbf{U}_2$ and $\mathbf{U}_3$,, which are expressed relative to the crystal frame, can be used to calculate three angles, $\theta$, $\phi$ and $\varphi$ ; the first two specify the + $z'$ - axis direction and the third indicates the + $x'$-axis direction within the plane perpendicular to the $z'$ - axis.

To calculate ellipsoid shape and orientation, we make use of the fact that $\mathbf{U}_1$, $\mathbf{U}_2$ and $\mathbf{U}_3$ are eigenvectors of $\eta$ and the fact that $n_x'$ , $n_y'$ and $n_z'$ equal the square roots of the reciprocals of the corresponding eigenvalues. These facts result from application of linear algebra theory [4]. The eigenvalue equation for $\eta$ is

$$\eta \mathbf{P} = \gamma \mathbf{P} \qquad (5)$$

where $\gamma$ represents an eigenvalue and $\mathbf{P}$ represents the corresponding unit magnitude eigenvector. Assuming that $\mathbf{P}$ is a unit vector is valid because any scalar multiple of an eigenvector is also an eigenvector. Since $\eta$ is a 3x3 real symmetric matrix, it has three real eigenvalues $\gamma_1$, $\gamma_2$ and $\gamma_3$ and the corresponding unit eigenvectors $\mathbf{P}_1$, $\mathbf{P}_2$ and $\mathbf{P}_3$ are mutually orthogonal. In addition, the Shur decomposition of $\eta$ can be written as

$$\eta = \mathbf{P}\, \mathbf{D}\, \mathbf{P}^T \qquad (6)$$

where the 3x3 marrices, $\mathbf{D}$, $\mathbf{P}$ and its transpose, $\mathbf{P}^T$ are defined as

$$\mathbf{D} = \begin{vmatrix} \gamma_1 & 0 & 0 \\ 0 & \gamma_2 & 0 \\ 0 & 0 & \gamma_3 \end{vmatrix}, \quad \mathbf{P} = \begin{bmatrix} \mathbf{P}_1 & \mathbf{P}_2 & \mathbf{P}_3 \end{bmatrix}, \quad \mathbf{P}^T = \begin{bmatrix} \mathbf{P}_1 \\ \mathbf{P}_2 \\ \mathbf{P}_3 \end{bmatrix} \qquad (7)$$

The diagonal element of $\mathbf{D}$ are the eigenvalues of $\eta$, and the column vectors of $\mathbf{P}$ ( row vectors of $\mathbf{P}^T$) are the corresponding unit eigenvectors. A square matris whose column (or row) vectors form a set of mutually orthogonal unit vectors is always an orthogonal matris, Since P is such a matris, premultiplying eq.( 6 ) by $\mathbf{P}^T$ and then postmultiplying by P yields.

$$\mathbf{P}^T \eta . \mathbf{P} = \mathbf{D} \qquad (8)$$

The impermeability tensor is second rank , and $\eta$ and $\eta'$ are its expressions in different reference frames.Therefore , according to the secont-rank reference-frame transformation rule [5,6], the xyz expression is transformed into the $x'$ $y'$ $z'$ expression by using

$$\eta_{ij}' = \sum_{m=1}^{3}\sum_{n=1}^{3} \tau_{im}\tau_{jn}\eta_{mn} \qquad (9)$$

where $\tau$ is the appropriate 3x3 basis transformation matrix.In matrix notation this is written as

$$\eta' = \tau\, \eta \tau^T \qquad (10)$$

By definition, the row vectors of $\tau$ are the original frame expression of the transformed frame unit basis vectors In eqs. (9) and (10), the original frame is xyz and the transformed frame is $x'$ $y'$ $z'$, so $\tau$ is given by

$$\tau = \begin{pmatrix} \mathbf{U}_1 \\ \mathbf{U}_2 \\ \mathbf{U}_3 \end{pmatrix} = \begin{bmatrix} U_{1x} & U_{1y} & U_{1z} \\ U_{2x} & U_{2y} & U_{2z} \\ U_{3x} & U_{3y} & U_{3x} \end{bmatrix} \tag{11}$$

Assuming that the unit eigenvectors of $\eta$ are in fact the new principal dielectric frame basis vectors yields

$$\mathbf{U}_1=\mathbf{P}_1 \qquad \mathbf{U}_2=\mathbf{P}_2, \qquad \mathbf{U}_3=\mathbf{P}_3 \tag{12}$$

These assumptions together with eq. (11) and the $\mathbf{P}^T$ expression from eq. (7) imply that $\tau$ equals $\mathbf{P}^T$, so replacing $\tau$ in eq. (10) with $\mathbf{P}^T$ and then making use of eq. (8) leads to

$$\eta' = \tau\eta\tau^T = \mathbf{P}^T\eta\mathbf{P} = \mathbf{D} \tag{13}$$

This implies that $\eta'$, like **D**, is diagonal, so the assumptions in eq. (12) are correct. Equating the $\eta'$ and D expression from eqs.(4) and (7), respectively, and solving for $n'_x$, $n'_y$ and $n'_z$ reveals that they equal the square roots of the reciprocals of $\gamma_1$, $\gamma_2$ and $\gamma_3$ respectively.

Theory and methods for finding matrix eigenvalues and eigenvectors are detailed in the reference [4] and suitable software packages are readily available [7]. It has been calculated eigenvalues by the use of the general cubic equation procedure [8] to solve for the characteristic polynominal roots; eigenvectors were then found by the use of Gauss elimination with complete pivoting and appropriate choice of arbitrary coustants during back substitation.Whatever method is used, the present application requires some special considerations:

1.The eigenvalues must be determined to a high degree of numerical accuracy (e.g.14 digits). This is necessary because realistically sized electric fields produce index changes that are very small compared with the index values themselves.

2.The eigenvectors must be normalized to unit magnitude, so that they can be used as basis vectors.

3.If two eigenvalues are equal or their separation becomes comparable to the numerical accuracy, then the eigenvector method must ensure that the corresponding eigenvectors remain distinct and orthogonal (because $\eta$ is real symmetric).

The order in which the eigenvalue -eigenvector pairs are calculated depends on the particular method used and is not necessarily the desired $x'$, $y'$, $z'$ sequence.Therefore the correspondence between the eigenvectors and the $x'y'z'$-frame axes must be established. This can be done by assigning the labels $\mathbf{U}_1$, $\mathbf{U}_2$ and $\mathbf{U}_3$ to the eigenvectors that are most closely parallel to the crystal-frame x, y and z axes, respectively. In addition, after labeling if the $i^{th}$ component of $\mathbf{U}_i$ is negative, $\mathbf{U}_i$ is multiplited by –1; this garantees that the $x'y'z'$ frame is right handed.

To find the direction of S relative to the modified index ellipsoid, the new principal dielectric frame components of S are calculated.To distinguish these components from the crystal-frame components, the notation $S'$ is used. In the current problem, the extraordinary mode propagates in the +y direction, so the xyz components are

$$\mathbf{S} = (S_1, S_2, S_3) = (0,1,0) \tag{14}$$

The reference-frame transformation rule for vectors [4] yields the corresponding $x'y'z'$ components as

$$\mathbf{S}' = \begin{pmatrix} S_1' \\ S_2' \\ S_3' \end{pmatrix} = \tau\mathbf{S} = \begin{bmatrix} U_{1x} & U_{1y} & U_{1z} \\ U_{2x} & U_{2y} & U_{2z} \\ U_{3x} & U_{3y} & U_{3z} \end{bmatrix} \begin{pmatrix} S_1 \\ S_2 \\ S_3 \end{pmatrix} \tag{15}$$

where $\tau$ is the $xyz \rightarrow x'y'z'$ transformation matrix defined in eq. (11). From hereon, the prime notation for S is only used when it is necessary to distinguish between the two sets of components.

The direction of **S** relative to the $x'y'z'$ frame is specified by two angles, $\xi$ and $\phi_s$. Their definitions are shown in figure 1. The value of $\xi$ specifies the angle from the $x'y'$ plane to **S**; this ange ranges from $-90^0$ to $+90^0$. The value of $\phi_s$ specifies the angle from the $x'$ axis $x'y'$ plane projection of **S**; this angle ranges from $-180^0$ to $180^0$.

Numerical values for $\xi$ and $\phi_s$ are calculated with the $\mathbf{S}'$ components from eq. (15). Since **S** is always a unit vector, the formula for $\xi$ is

$$\xi = \text{arcSin}(S)'_3 \tag{16}$$

The value of $\phi_s$ is calculated as follows; if $S_1'$ and $S_2'$ are both zero, then $\phi_s$ is set equal to zero; otherwise, the cosine and the sine of $\phi_s$ are calculated from

$$\begin{aligned} Cos\phi_s &= s_1'\left(s_1'^2 + s_2'^2\right)^{-\frac{1}{2}} \\ Sin\phi_s &= s_2'\left(s_1'^2 + s_2'^2\right)^{-\frac{1}{2}} \end{aligned} \tag{17}$$

and then $\phi_s$ is found with

$$\phi_s = \begin{cases} arcSinSin\phi_s, & Cos\phi_s \geq \sqrt{1/2}, \\ arcCosCos\phi_s, & |Cos\phi_s| < \sqrt{1/2}, \quad Sin\phi_s \geq 0 \\ -arcCosCos\phi_s, & |Cos\phi_s| < \sqrt{1/2}, \quad Sin\phi_s < 0 \\ 180 - arcSinSin\phi_s, & Cos\phi_s \leq -\sqrt{1/2}, \quad Sin\phi_s \geq 0 \\ -180 - arcSinSin\phi_s, & Cos\phi_s \leq -\sqrt{1/2}, \quad Sin\phi_s < 0 \end{cases} \tag{18}$$

The index ellipsoid can be used to find both the extraordinary–mode index $n_2$ and the ordinary-mode index $n_1$. The intersection of the index ellipsoid with the plane perpendicular to **S** and passing through the origin forms and ellipse, which is called the index ellipse. The length of the index-ellipse axes equal $2n_1$, and $2n_2$. To find the lengths, we approximate the orientation of the index ellipse by making use of the fact that **S** is in the xy-plane and the fact that the dielectric perturbation is small.

For regular and reverse-poled $LiTaO_3$ both $n_0$ and $n_e$ equal~2, and the largest electro-optic-coefficient magnitudes are approximately $5x10^{-11}$ m/V. The small size of the dielectric perturbation has two implications:

1.The angle $\theta$ between the z and $z'$- axes is small. Since $\theta$ is also the angle between the xy and the $x'y'$ planes and because **S** is in the xy plane, the angle $\xi$ from the $x'y'$ plane to **S** is also small.

2.The quantity $|n_x'-n_y'|$ is much smaller than $|n_y'-n_z'|$, so the crystal remains almost uniaxial. This holds true for E up to approximately $4\times10^6$ V/m; at this field strength, the largest $|\Delta\eta_6|$ is still 100 times smaller than $\left|\left(\frac{1}{n_e^2}\right)-\left(\frac{1}{n_0^2}\right)\right|$.

To find approximate formulas for $n_1$ and $n_2$ we assume that, for small $\xi$, the orientation of the almost-uniaxial index ellipse is approximately the same as that of the uniaxial index ellipse. Let the intersection of the $x'y'$ plane with the index ellipsoid be called the equatorial ellipse. Let the plane formed by S and the $z'$- axis be referred to as the longitudinal plane and let the intersection of this plane with the index ellipsoid be called the longitudinal ellipse. For uniaxial crystals the $2n_1$ and the $2n_2$ axes of the index ellipse are always in the equatorial and the longitudinal planes, respectively. Typically, when an electric field is applied $n_x'$ does not equal $n_y'$ so the crystal becomes biaxial. The index ellipsoid has always mirror symmetry about the $x'y'$ plane, so if S is in that plane, the index-ellipse axes must be parallel to the equatorial and the longitudinal planes. Therefore if s is close to the $x'y'$ plane ($\xi$ small), we assume that the index-ellipse axes are almost parallel to the equatorial and the longitudinal planes.

This implies that the index-ellipse vertices are approximately located on the equatorial and the longitudinal ellipses. Therefore approximate index values are given by

$$n_1 \approx n_x' n_y' \left(n_y'^2 Sin^2\phi_s + n_x'^2 Cos^2\phi_s\right)^{-\frac{1}{2}} \tag{19}$$

$$n_2 \approx n_{xy} n_z' \left(n_z'^2 Sin^2\xi + n_{xy}^2 Cos^2\xi\right)^{-\frac{1}{2}} \tag{20}$$

where $n_{xy}$ is the distance from the origin to the points of intersection between the longitudinal and the equatorial ellipses; $n_{xy}$ is given by

$$n_{xy} = n_x' n_y' \left(n_x'^2 Sin^2\phi_s + n_y'^2 Cos^2\phi_s\right)^{-\frac{1}{2}} \tag{21}$$

These equations are all polar-coordinate equations for ellipses in each case, the variable on the left is the distance from the origin and the trigonometric function argument is the polar angle within the plane that contains the ellipse.

Expression (19) traces out the equatorial ellipse starting from its $+y'$ -axis intercept at $\phi_s=0$. Expression (20) traces out the longitudinal ellipse starting from its $+z'$-axis intercept at $\xi=0$. Equation (21) traces out the equatorial ellipse starting from its $-x'$-axis intercept at $\phi_s=0$.

As $n_x'$ approches $n_y'$, the formulas above become more accurate. Because of the index ellipsoid's mirror symmetry about the $x'y'$ plane the formulas are exact whenever $\xi$ equals zero. The index ellipsoid has also mirror symmetry about the $x'z'$ and $y'z'$ planes so the formulas are also exact when $\phi_s$ equals either $0^0$, $180^0$, or $\pm90^0$. For a given nonzero $\xi$, the worst-case errors occur when $\phi_s$ is in the vicinity of either $\pm45^0$ or $\pm135^0$. As E goes to zero, $n_x'$ approaches $n_y'$ and $\xi$ goes to zero, so the approximation errors decrease rapidly. Exact $n_1$ and $n_2$ values can be obtained by solution of the Fresnel equation, which is a quadratic in $n^2$ with coefficients that are functions of $n_x'$, $n_y'$, $n_z'$ and the components of $S'$ [3]. The approximate $n_1$ and $n_2$ values have been compared with the Fresnel equation roots. Even for field strengths as large as $4\times10^6$ V/m the worst-case $n_1$ and $n_2$ error magnitudes are much smaller than $|n_1-n_0|$ and $|n_2-n_e|$, respectively. In addition, as E is described, the errors go to zero faster than the index changes do.

Since $\xi$ is small and $|n_y'-n_x'|$ remains much smaller than $|n_y'-n_z'|$, the variation of $n_{xy}$ with $\phi_s$ has little effect on the value of $n_2$. As a result, $(n_2-n_e)$ depends primarily on $\xi$ and

($n_z'$-$n_c$) and has only a weak dependence on $\phi_s$, ($n_x'$-$n_0$) and ($n_y'$-$n_0$). These last three quantities strongly influence the ordinary-mode index change ($n_1$-$n_0$) but, since the proton-exchange waveguides do not support TE modes, the ordinary-mode change does not affect the guided-mode index. For regular $LiTaO_3$ the contracted form of the crystal frame (xyz) expression of the electro-optic tensor is

$$r_{rej} = \begin{vmatrix} 0 & -r_{22} & r_{13} \\ 0 & r_{22} & r_{13} \\ 0 & 0 & r_{33} \\ 0 & r_{51} & 0 \\ r_{51} & 0 & 0 \\ -r_{22} & 0 & 0 \end{vmatrix} \qquad (22)$$

where the four independent elements have numerical values of

$$r_{13} = 9\text{x}10^{-12}\ \text{m/V}\ ,\ r_{22} = 7.5\text{x}10^{-12}\ \text{m/V}\ ,\ r_{33} = 32\text{x}10^{-12}\ \text{m/V}\ ,\ r_{51} = 37.2\text{x}10^{-12}\ \text{m/V},\ [3] \qquad (23)$$

These are the constant stress (T) values. For reverse-poled $LiTaO_3$, we assume that the contracted form of the xyz expression of the electro-optic tensor is

$$r_{rev} = \begin{vmatrix} 0 & r_{22} & -r_{13} \\ 0 & -r_{22} & -r_{13} \\ 0 & 0 & -r_{33} \\ 0 & -r_{51} & 0 \\ -r_{51} & 0 & 0 \\ r_{22} & 0 & 0 \end{vmatrix} \qquad (24)$$

where $r_{13}$, $r_{22}$, $r_{33}$ and $r_{51}$ are the same as in eq. (23). The reasoning that leads to this assumption can be explained as follows.

Etching experiments [ 9,10 ] reveal that the y and the z-axes in reverse-poled $LiTaO_3$ point opposite to their corresponding axes in regular $LiTaO_3$. Whether the reverse-poled x-axis also points opposite its regular counterpart is irrelevant. This is because reverse-poled $LiTaO_3$ has a mirror- symmetric plane perpendicular to its x-axis. This mirror plane requires that the electro-optic tensor remain invariant when the direction of the x-axis is reversed. Having the regular and the reverse-poled x-axis point in opposite directions is the mathematically simpler alternative, so we assume that all three reverse-poled axis point opposite to their regular counterparts. Here the reverse-poled axis are labeled $x'$, $y'$, $z'$, and the $x'y'z'$ frame is called the reverse-poled crystal-frame expression of the reverse-poled tensor be designated as $r'_{rev}$. We assume that $r'_{rev}$ is identical to the $r'_{rej}$ defined by eqs. (22) and (23).

The electro-optic tensor is third rank so according to the third rank transformation rule [ 5,6 ], $r'_{rev}$ is transformed into $r_{rev}$ by using

$$\left(r_{rev}\right)_{ijk} = \sum_{m=1}^{3}\sum_{n=1}^{3}\sum_{p=1}^{3} \tau_{im}\tau_{jn}\tau_{kp}\left(r'_{rev}\right)_{mnp} \qquad (25)$$

where $\tau$ is the 3x3 basis transformation matrix. The row vectors of $\tau$ are the original frame expressions of the transformed frame basis vectors. In eq. (25) the original frame is $x'y'z'$ and the transformed frame is xyz. Since corresponding axes of these frames point in opposite directions, $\tau$ is given by

$$\tau = \begin{vmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{vmatrix} \qquad (26)$$

because the off-diagonal elements of $\tau$ equal zero, the only nonzero term on the right-hand side of eq. (25) is the one for which m = i , n = j and p = k, so

$$\left(r_{rev}\right)_{ijk} = \tau_{ii}\tau_{jj}\tau_{kk}\left(r'_{rev}\right)_{ijk} \qquad (27)$$

Since all the diagonal elements of $\tau$ equal –1, the $\tau_{ii}\tau_{jj}\tau_{kk}$ term always equals –1. As a result corresponding expanded-form elements of $r_{rev}$ and $r'_{rev}$ are negatives of each other. This same relationship applies to the contracted-form elements, so assuming that $r'_{rev}$ has the value specified by eqs. (22) and (23) implies that $r_{rev}$ takes the value given by eqs. (23) and (24).

## *3. NUMERICAL RESULTS*

For various electric-field directions in regular and reverse-poled $LiTaO_3$ it has been calculated the y-propagating extraordinary-mode index change ($n_2$-$n_e$) as a function of field strength (E). Here we present results obtained for fields applied along the +x, +y, and +z directions and along the $45^0$ directions of the xy, xz and yz planes. From these results, we infer a general second-order relationship between ($n_2$-$n_e$) and the field vector **E**. The six field directions listed above are chosen because they maximize, respectively, the magnitudes of terms involving $E_x$ or $E_x^2$, $E_y$ or $E_y^2$ , $E_z$ or $E_z^2$ and $E_xE_y$, $E_xE_z$ and $E_yE_z$. For all calculations, $n_0$ and $n_e$ are 2.3 and 2.2, respectively, and the independent electro-optic coefficient magnitudes are those given in eq. (23).

Numerical calculations were made with floating point arithmetic with 15 decimal digits of precision. For a field of $4x10^6$ V/m in the +z direction (Table 1) the regular and the reverse-poled $n'_z - n_e$ values are equal in magnitude and opposite in sign. In both cases $\xi$ equals zero, so ($n_2$-$n_e$) = ($n'_2 - n_e$). For both materials as E is decreased, the magnitude of ($n_2$-$n_e$) decreases in proportion to E and agrees with the $-\left(\frac{1}{2}\right)n_e^3 r_{33}E_z$ formula to at least 3 significant digits over the entire range of fie strengths. For the same field in the –x-direction (Table 1) the regular and the reverse-poled $n'_z - n_e$ values are equal. In both cases, $\xi$ is so small that ($n_2$-$n_e$) decreases in proportion to $E^2$. This holds to more than 3 significant digits over the entire field-strength range, so an $E_x^2$ term exists, but no significant $E_x$ term is apperent.

**Table 1**. For $4x10^6$ V/m Fields directed along the crystal frame axes in regular and reverse-Poled $LiTaO_3$, the y-propagating.

| Field direction | Type | $n'_z - n_e$ | $\xi$ (deg) | $n_2$-$n_e$ |
|---|---|---|---|---|
| +z – axis | Regular | $-6.6 \times 10^{-4}$ | 0 | $-6.6 \times 10^{-4}$ |
| | Reverse | $6.6 \times 10^{-4}$ | 0 | $6.6 \times 10^{-4}$ |
| +x – axis | Regular | $-5.9 \times 10^{-6}$ | $-8.7 \times 10^{-4}$ | $-5.9 \times 10^{-6}$ |
| | Reverse | $-5.9 \times 10^{-6}$ | $-8.7 \times 10^{-4}$ | $-5.9 \times 10^{-6}$ |
| +y – axis | Regular | $-5.9 \times 10^{-6}$ | 0.49 | $-4 \times 10^{-14}$ |
| | Reverse | $-5.9 \times 10^{-6}$ | -0.49 | $-2.8 \times 10^{-14}$ |

For the field of $4x10^6$ V/m in the +y direction (Table 1) the regular and the reverse-poled ($n_z' - n_e$) values are negative and essentially equal each other. The +y direction $\xi$ values have opposite signs and nearly equal magnitudes, which are much larger than the corresponding +x direction magnitudes. Since $n_0$ is greater than $n_e$ and since both materials remain almost uniaxial, the nonzero $\xi$ causes $n_2$ to be greater than $n_z'$. This $n_2$ increase cancels the $n_2$ decrease produced by the negative ($n_z' - n_e$) values. As a result, to the limit of numerical accuracy, both ($n_2$-$n_e$) values equal zero. As E is decreased, the index changes, if any, remain smaller than the numerical resolution, so no significant $E_y$ or $E_y^2$ terms are present.

## *4. CONCLUSION*

The specific field-direction results presented above suggest, that, in general, the index change for an extraordinary mode propagating along the y-axis is given by

$$n_2 - n_e \approx \pm AE_z - BE_x^2 \qquad (28)$$

where $E_z$ and $E_x$ are the z and x components of **E**. In the first term, the minus sign is used for regular $LiTaO_3$ and the plus sign is used for reverse-poled $LiTaO_3$. The values of A and B are $1{,}7x10^{-10}$ m/V and $3.7x10^{-19}$ m/V$^2$, respectively.

The agreement between expression (28) and the numerical procedure has been verified by comparing their results for additional field directions. Expression (28) indicates that, as far as the fundamental electro-optic mechanism is concerned, the $E_x^2$ term is the only potential source of sensor response to non-$E_z$ field components. The $E_x^2$ term produces index changes that are smaller than those produced by the $E_z$ term.

The numerical procedure used to calculate index change is very general. Both the electric-field vector and the electro-optic tensor can be chosen arbitrarily. Replacing $n_0$ and $n_e$ in eq. (1) with $n_x$, $n_y$ and $n_z$ and using the quadratic form of the Fresnel equation instead of eqs. (19)-(21) would allow $n_1$ and $n_2$ to be calculated for an arbitrary **S** direction in a biaxial material. The behaviour of the principal index changes and the ellipsoid-orientation angles can be easily investigated.

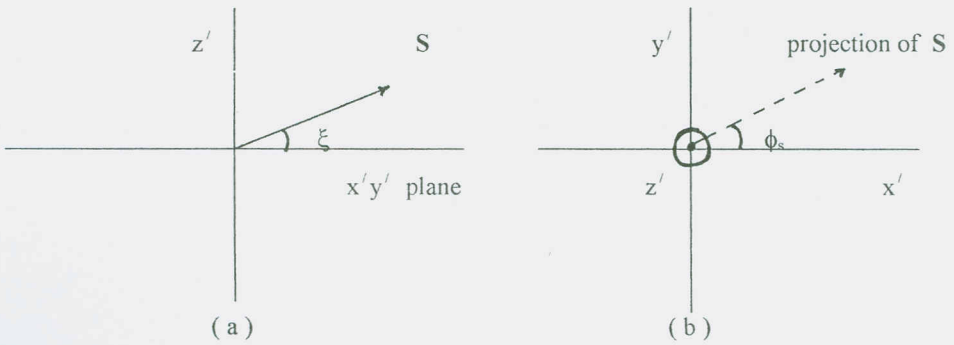


**Figure 1**. Direction of wave-normal vector **S** relative to the new principal dielectric frame: (a) definition of $\xi$, (b) definition of $\phi_s$

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