Mathematical and Computational Applications

MDPI

Article

Light Robust Goal Programming

Emmanuel Kwasi Mensah * and Matteo Rocca

Department of Economics, Università degli Studi dell'Insubria, Via Monte Generoso 71, 21100 Varese, Italy; matteo.rocca@uninsubria.it

* Correspondence: ekmensah@uninsubria.it

Received: 10 September 2019; Accepted: 25 September 2019; Published: 28 September 2019

Abstract: Robust goal programming (RGP) is an emerging field of research in decision-making problems with multiple conflicting objectives and uncertain parameters. RGP combines robust optimization (RO) with variants of goal programming techniques to achieve stable and reliable goals for previously unspecified aspiration levels of the decision-maker. The RGP model proposed in Kuchta (2004) and recently advanced in Hanks, Weir, and Lunday (2017) uses classical robust methods. The drawback of these methods is that they can produce optimal values far from the optimal value of the "nominal" problem. As a proposal for overcoming the aforementioned drawback, we propose light RGP models generalized for the budget of uncertainty and ellipsoidal uncertainty sets in the framework discussed in Schöbel (2014) and compare them with the previous RGP models. Conclusions regarding the use of different uncertainty sets for the light RGP are made. Most importantly, we discuss that the total goal deviations of the decision-maker are very much dependent on the threshold set rather than the type of uncertainty set used.

Keywords: goal programming (GP); robust optimization; robust goal programming (RGP); light robust goal programming (LRGP); multi-criteria decision making (MCDM)

1. Introduction

A decision-making process implies the need to face conflicts, whether it is a management strategy, government policy, firm resource allocation, or individual budget planning; a certain course of action usually involving multiple conflicting objectives or criteria has to be taken [1]. Multicriteria decision analysis has been a widespread decision tool for problems involving multiple and usually conflicting objectives or criteria. For instance, the multicriteria optimization is deemed as the ideal setting to analyze portfolio optimization problems in the sense of Markowitz, in which a pair objective—the risk function and the expected return function—are minimized and maximized, respectively, to achieve a portfolio goal [2]. In multicriteria decision making (MCDM) problems, there is usually an infinite number of efficient solutions due to the conflicts among objectives.

The goal programming (GP) technique, along with its variants—lexicographic GP, weighted GP, and fuzzy GP—is a well-known method that is commonly used to optimize multiple goals and derive efficient solutions for the decision-maker (DM). The GP provides a special compromise multi-criteria framework by which the DM can optimize multiple, conflicting objectives and concurrently achieve satisfiable solutions by minimizing the deviations of objectives from aspiration levels or goals set [3]. GP has obtained huge success in engineering, management, and social science problems [4]. However, the standard GP approaches deal with deterministic goals that are precisely defined while practical scenarios involving uncertainties in decision problems are ignored. One of the basic assumptions in mathematical programming including the GP is that the exact value of the input data is fixed and known in advance. This assumption can, however, be violated in many situations arising when real-world problems are considered. This can be due to the fact that the parameters used in the model are just estimates of real parameters or more generally to the effect of uncertainty affecting some parameters. When uncertainty is taken into account, an optimal solution with respect to the nominal values of the parameters can be suboptimal (or even infeasible) according to the actual parameters. Hence, small uncertainty in the input data can make the nominal optimal solution completely meaningless from a practical viewpoint.

The robust optimization (RO) is an approach that is widely used to deal with optimization problems in which the parameters have uncertain values. The RO has gained a great interest among academics and practitioners as an important concept in uncertain linear programming since the seminal paper of [5]. Other approaches in the literature include the stochastic approach and the fuzzy method (e.g., [6,7]). One of the drawbacks of this RO approach is that the solutions provided can be suboptimal if compared with the solution of the so-called nominal problem, i.e., a problem without uncertainty in which the parameter values are fixed (for instance, to some point estimation). We clarify formally the concept of nominal problems in Section 2. Although classical RO approaches are still relevant in practice, in recent time, Fischetti and Monaci [8] proposed a light RO approach intended to focus on the robust but quality solution of the optimization problem. A "quality solution" or a "good solution" (e.g., [9–11]) of an RO problem is intended as a solution for which the optimal value of the robust objective function is close to the optimal value of the objective function in the nominal problem. In the literature (see [5,12–14]), the term "conservative solution" is used with reference to the difference between the optimal values of the robust problem and the nominal problem. When the difference between the two values is high (i.e., the solution of the robust problem is highly suboptimal), it is said that such a solution is very conservative. Hence, to say that a solution is a "quality solution" is equivalent to say that it is slightly conservative.

Although the RO was born as an approach to optimization problems with a single objective [15], it has been extended in the literature to address MCDM problems, for example, the multiobjective portfolio optimization [2,16] and the data envelopment analysis [14,17,18]. One growing area of research in this field is the robust goal programming (RGP), which combines the RO with the GP technique. The RGP was first introduced by [19], and its potential use in multi-criteria linear programming problems is discussed in [20]. RGP has been applied, for example, to the multi-objective portfolio selection problem in [16], the capital budgeting problem in [21], and a transportation problem, specifically, the United States Transportation Command's (USTRANSCOM) liner rate-setting problem in [22]. Despite this, the RGP methodology remains underdeveloped and less applied, specifically when considering different robust concepts and variants of GP techniques. In fact, since the model proposal of [19], only recently in [13] was the conceptual foundation of the RGP touched upon heavily, deepening the application to MCDM problems. Hanks et al. [13] proposed norm-based uncertainty sets using cardinality-constrained robustness and strict robustness via ellipsoidal uncertainty and compared their approach with the interval-based approach in [19]. However, their robust models have the aforementioned drawback—they can be highly suboptimal if compared to the solution of the nominal problem, e.g., their quality is low or, equivalently, they are very conservative. See [23] for discussion on reducing the conservatism of the RO.

The aim of this paper is to overcome this drawback in the RGP models. To this extent, we introduce a model that we call light robust goal programming which gathers the features of light robustness introduced in [8,12] and and "Γ-*robustness*" proposed in [13,19]. The light robustness approach addresses the conservatism of the RGP by setting a limit to the deterioration of the objective value compared to the nominal solution. We focus our proposed model on two arbitrary sets, i.e., the budget of uncertainty of [12] and the ellipsoidal uncertainty sets of [24,25]. As a main result, we show that the new model's solution is "not too far" from optimality from the nominal GP model or—in the aforementioned terminology—is a quality solution or, equivalently, a less conservative solution if compared with the solutions obtained with the approaches in [13,19]. Furthermore, the proposed models are generalized to include modeling uncertainties in the hard constraint of the GP model that were left unconsidered in previous studies.

The outline of the paper is the following. In Section 2, we provide a thorough review of the GP method, the RO concepts, and the RGP approaches. This section also includes a review of the RGP models in [13,19]. Section 3 introduces the proposed light RGP models. We perform a numerical comparison in Section 4 and conclude the paper in Section 5.

2. Preliminaries

In this section, we provide a brief introduction to the GP technique, the RO approach, and the RGP. The section also reviews the RGP methodology in the literature and sets the pace for a new approach in the next section.

2.1. The Goal Programming Method

GP arose from the study of the executive compensation plan in [26], where the authors sought to minimize the total deviations between realized goals and expected goals. Since then, the GP has become by far the most popular technique used in dealing with decision problems with a multiplicity of objectives due to its versatility and the underlying concept of satisfying solution to decision situations. The literature evidence to the bulk of applications of the GP technique to MCDM can be found, e.g., in [4,27].

The basic idea of the GP technique is to set up specific goals g_t for each objective function $f_t(\boldsymbol{x})$, $t = 1, \ldots, T$. Then, the total deviation $\sum_{t=1}^{T}|d_t|$ where d_t is the deviation from the goal g_t for the t-th objective function is minimized. For simplicity of computation, mostly, the absolute deviation is split into positive and negative deviations (respectively d_t^+ and d_t^-) such that $|d_t| = d_t^+ - d_t^-$. The deviations can be defined as $d_t^+ = \max(f_t(x) - g_t, 0)$ and $d_t^- = \max(g_t - f_t(x), 0)$. The positive and the negative deviations underscore the over- and the underachievement of goals for each t-th goal of the objective function. Overachievement of the DM goals subject to constraints is attained if $d_t^+ > 0$, while underachievement means $d_t^- > 0$. When the deviations are driven to zero, the goals of the model are achieved [7,19]. A typical GP model for a min-type cost function is formulated as follows:

$$z^* = \text{Min} \sum_{t=1}^{T} w_t(d_t^+ + d_t^-) \quad \text{s.t.} \tag{1}$$

$$f_t(\boldsymbol{x}) + d_t^- - d_t^+ = g_t \quad \forall t \in T \tag{2}$$

$$\boldsymbol{Ax} \leq \boldsymbol{b} \tag{3}$$

$$\boldsymbol{x} \geq 0,\ d_t^-,\ d_t^+ \geq 0 \quad \forall t \in T \tag{4}$$

where $\boldsymbol{x} \in \mathbb{R}^n$ is a vector of decision variables, and w_t is the penalty weight for missing the goal. In the sequel, we assume implicitly that the penalty weight is equal for all deviations, hence we set $w_t = 1$ for all t. The objective function then minimizes the sum of the positive and the negative deviations for each goal. Constraint (2) computes the respective positive and negative deviation from each goal. Constraint (3) relates to additional constraints in the decision space that is not related to the DM goals. It is important to note that the achievement function is a key element to variants' consideration of goals and the priorities attached to them by the decision-maker. As indicated in [28], the GP method results are sensitive to the type of function used. Different variants of the achievement function, namely, the traditional weighted GP, the pre-emptive lexicographic GP, and the Chebyshev MINMAX GP, which reflect different preferences structures have been introduced.

2.2. RO and Concepts

Uncertainty in the parameters of MCDM problems including the GP can lead to inaccurate and unreliable decisions that are sometimes meaningless from a practical point of view [12,25,29]. The RO enables us to overcome this problem since it focuses on finding the worst-case performance for all feasible realization of the uncertain parameters. The robust solution is obtained through an altenative reformulation of the uncertain problem called the robust counterpart. A general form of the robust counterpart to the uncertain linear program is given as:

$$\begin{array}{c}\mathrm{Min}_x \left[\mathrm{Max}_{(A,\ b,\ c)\in\mathcal{U}}\ \boldsymbol{c}^{\mathrm{t}}\boldsymbol{x}\right] \\ \text{s.t.}\end{array} \tag{5}$$

$$\boldsymbol{Ax} \leq \boldsymbol{b} \quad \forall(\boldsymbol{A},\ \boldsymbol{b},\ \boldsymbol{c}) \in \mathcal{U} \tag{6}$$

where $(\boldsymbol{A}, \boldsymbol{b},\ \boldsymbol{c})$ are uncertain technological coefficients, a right-hand side vector, and a cost coefficient vector that take values in the uncertainty set $\mathcal{U} \subseteq \mathbb{R}^n$. The robust problem re-casted in this form produces a solution that is the best possible in the worst-case scenario. If we consider an element $(\boldsymbol{A}_0,\ \boldsymbol{b}_0, \boldsymbol{c}_0) \in \mathcal{U}$ we can consider the so-called nominal problem:

$$\begin{array}{c}\mathrm{Min}_x \left[c_0^t\boldsymbol{x}\right] \\ \text{s.t.}\end{array} \tag{7}$$

$$\boldsymbol{A}_0\boldsymbol{x} \leq \boldsymbol{b}_0 \tag{8}$$

The vector $(\boldsymbol{A}_0,\ \boldsymbol{b}_0, \boldsymbol{c}_0)$ can be interpreted as a point estimation of the parameters' value. Clearly, the true value of the parameters is unknown, and such uncertainty is taken into account by assuming that they can take values in the uncertainty set $\mathcal{U}$. As indicated in [30], the robust counterpart is the main precursor of several RO concepts, e.g., strict robustness, interval-based/cardinality constrained robustness, norm-based robustness, adjustable robustness, and recoverable robustness. Here, we provide but a few of these concepts.

2.2.1. Strict Robustness

Strict robustness follows the pessimistic view of maximizing the worst-case over *all* scenarios in the uncertainty set. It was introduced in [31] and significantly extended in [9]. It is the highest form of robustness with guaranteed feasibility for all $(\boldsymbol{A}, \boldsymbol{b},\ \boldsymbol{c}) \in \mathcal{U}$ in that the probability of violating the i-th constraint is zero. Mathematically, a solution $\boldsymbol{x}$ to the optimization problem max $\{\boldsymbol{c}^{\mathrm{t}}\boldsymbol{x} :\ \boldsymbol{A}(\xi)\boldsymbol{x} \leq \boldsymbol{b}\}$ where ξ denotes an uncertain parameter is called *strictly robust* if it is feasible for all scenarios in $\mathcal{U}$, i.e., if $\boldsymbol{A}(\xi)\boldsymbol{x} \leq \boldsymbol{b} \quad \forall\xi \in\ \mathcal{U}$. The notion of strict robustness is plausible in some applications, such as constructing a very stable bridge or nuclear power plants; however, the idea to hedge against all scenarios in most other applications could be counterproductive. For example, in the robust formulation for the timetabling for public transport, being strictly robust provides a less practically applicable timetable, since it is difficult to meet all announced arrival and departure times even with high buffer times considered in the uncertainty set [32]. In fact, applying strict robustness can lead to a very conservative solution. An approach that provides less conservative solutions is due to [24], who introduced the ellipsoidal uncertainty set in which the "most likely" values of the uncertain parameters are scaled down by the DM via the size of the ellipsoid. Further in [25], the authors combined the interval uncertainty set of [31] with the ellipsoid to provide a robust model that is less conservative and practically reliable.

2.2.2. "Γ-robustness" or Budget of Uncertainty

Bertsimas and Sim [12] introduced the concept of "Γ-*robustness*" predicated on the fact that not all uncertain parameters will simultaneously take their worst-case values; instead, up to Γ of the coefficients that are allowed to change are protected against. The Bertsimas and Sim approach thus relaxes the assumption of feasibility *for all* $(\boldsymbol{A}, \boldsymbol{b},\ \boldsymbol{c}) \in \mathcal{U}$ to feasibility *for some* $(\boldsymbol{A}, \boldsymbol{b},\ \boldsymbol{c}) \in \mathcal{U}$ in model (5) and (6), restricting the number of coefficients allowed to change from the nominal values. The approach leads to the concept of cardinality constrained robustness [30]. Thus, for the uncertain matrix $\boldsymbol{A} = \left(a_{ij}\right)$

where each $a_{ij},\ j \in J_i$ and $J_i = \left\{ j \middle| \hat{a}_{ij} > 0 \right\}$ is the set of coefficients in row i that are subject to uncertainty, and each a_{ij} lies in the interval $\left[a_{ij} - \hat{a}_{ij},\ a_{ij} + \hat{a}_{ij}\right]$, denote by $n = |N|$ and $m = |M|$ the number of variables and constraints in the linear programming (LP) model, respectively. The robust counterpart is obtained by replacing each row $i \in M$ of Constraint (6) by the new constraint:

$$\sum_{j=1}^{n} a_{ij}x_j + \max_{\{S_i | S_i \subseteq J_i,\ |S| = \Gamma_i\}} \{\sum_{j \in S} \hat{a}_{ij}|x_i|\} \leq b_i \quad \forall i \in M \ . \tag{9}$$

where, at most, Γ_i of the parameters in the i-th constraint are "budgeted" for the uncertainty to guarantee robustness for the solution. The parameter $\Gamma_i \in [0, |J_i|]$ determines the quality of the solutions of the model. $\Gamma_i = 0$ implies no robust solution is sought for the model. A solution is very conservative (i.e., not a quality solution) when all the parameters take their worst-case values and lead to a solution much worse than the nominal solution or even infeasibility, i.e., if $\Gamma_i = |J_i|$. In this case, the robust solution is equivalent to the strict robustness proposed in [31]. Values of $\Gamma_i \in (0, |J_i|)$ lead to less conservative solutions. Nothwithstanding, the robust solution is always feasible, i.e., given that not all the parameters will assume their worst-case values, and up to Γ_i of the parameters are allowed to change. On the other hand, if more than Γ_i of the uncertain parameters change, the robust solution will still be feasible with a probability bounded by an exponential term. Note here that the robust concept in [12] is built on the principles similar to those in [25], where the model has an analogous probability of constraint violation. It can be shown that the cardinality constrained approach used in [12] is similar to strict robustness using the convex hull of the cardinality-constrained uncertainty set wherefore the approach can lead to conservative solutions [32].

2.2.3. Light Robustness

A more relaxed condition for robustness that allows the user to control the quality of the solutions was given in [8]. The robust concept introduced, known as *light robustness*, relies on a solution in which the objective function value is not much different from the objective function value of the nominal problem. The light robustness approach is structured in two stages to ensure a less conservative robust solution. The first stage considers a solution for the nominal problems (7) and (8). Subsequently, the constraints are relaxed for local violations that are absorbed by slack variables γ_i, acting as measures of infeasibility for each constraint $i \in M$ of the nominal problem. Hence, Constraint (9) becomes:

$$\sum_{j=1}^{n} a_{ij}x_j + \max_{\{S_i | S_i \subseteq J_i,\ |S| = \Gamma_i\}} \{\sum_{j \in S} \hat{a}_{ij}|x_i|\} + \gamma_i \leq b_i \quad \forall i \in M \ . \tag{10}$$

The second stage minimizes all the possible infeasibilities γ_i in the objective function:

$$\text{Min} \sum_{i \in M} w_i \gamma_i \tag{11}$$

where w_i are the weights for the possibly different scales of constraints. Since the goal of the light robustness is to guarantee a balance between the quality (optimality) and the feasibility (robustness) of the solution, an additional constraint:

$$\sum_{j=1}^{n} c_j x_j \leq (1 + \rho) z^* \tag{12}$$

imposing a maximum worsening of the optimal solution of the nominal problem is added. The parameter ρ is fundamental in the trade-off between the quality (optimality) and the feasibility of the solution; $\rho = 0$ corresponds to the nominal problem where robustness is only considered to break ties among equivalent optimal solutions, whereas $\rho = \infty$ implies that the nominal objective function is not considered at all.

It should be clear now that the light robust models (10)–(12) rely on the robust approach of [12]. On the other hand, the model can be formulated without it. Fischetti and Monaci [8] justified this with some heuristic methods. In an earlier application, Fischetti et al. [8] showed that light robustness, among others, is most suitable for railway timetabling problems where delays are encountered and a certain "quality and most reliable" solution is required for constraint violation. This has also been the case for some applications in similar contexts, for example, the timetabling information for public transport in [10], the robust airport runway scheduling problem in [33], and the potential line planning of public transport in [34]. Light robustness has also found usefulness in RO applications such as the master surgery scheduling problem in [35] and the multi-period multi-product aggregate production planning in [36]. Schöbel [11] in recent times extended the idea of [8] to the notion of generalized light robustness wherein optimization problems with arbitrary uncertainty sets can be applied.

2.3. The RGP Approach

The RGP methodology combines RO and GP techniques to solve decision problems with a multiplicity of objectives in an uncertain environment. Here, any of the RO concepts mentioned above can be used with any variants of the GP. In the RGP, the goals of the DM that are marked by uncertain aspiration levels are confined to an uncertainty set and optimized with respect to their feasible realization in an uncertainty set to obtain a robust solution. Before proposing a new robust concept for the RGP modeling, in the subsections that follow, we provide a review of the models in [1,2], highlighting their limitations.

2.3.1. Kuchta's RGP Approach

Kuchta [19] introduced the robust approach to the GP technique in a single objective LP, where she assumed that uncertain variations pertain only to the cost coefficients in the original objective function of the LP. This approach translates uncertainty analysis to the goal constraints in which the nominal cost values, c_{tj}, are assumed to vary within the interval $\left[c_{tj} - \delta_{tj},\ c_{tj} + \delta_{tj}\right]$, where δ_{tj} is a possible deviation from the nominal cost value that accounts for the negative influence in the attainment of goals. Kuchta [19] adopted a robust methodology based on the interval-based uncertainty sets with cardinality-constraints developed in [12]. She supposed that each cost coefficient is subject to deviation and hence introduced the concept of K robust solution; $K = (k_t)_{t=1}^{T}$ where $k_t,\ t = 1, \ldots, T$ is any integer such that the level of robustness of the RGP model or the optimal value of the total deviation is determined by the DM using the parameter k_t.

The K robust methodology introduced in [12] is the following min-type model:

$$\begin{array}{c}\text{Min } \sum_{t=1}^{T} d_t^+ \\ \text{s.t.}\end{array} \tag{13}$$

$$\sum_{j=1}^{n} c_{tj}x_j + \sum_{j\in G_t} p_{tj} + k_t z_t + d_t^- - d_t^+ = g_t \quad \forall t \in T \tag{14}$$

$$z_t + p_{tj} \geq \delta_{tj}x_j \quad \forall t \in T,\ \forall j \in J_t \tag{15}$$

$$\boldsymbol{Ax} \leq \boldsymbol{b} \tag{16}$$

$$x_j,\ d_t^-,\ d_t^+ \geq 0 \quad \forall t \in T, \forall j \in N \tag{17}$$

$$z_t, p_{tj} \geq 0 \quad \forall t \in T,\ \forall j \in J_t \tag{18}$$

where z_t and $p_{tj},\ j \in J_t$ are robust variables obtained through the dual formulation of constraint second term in the left-hand side of Constraint (9), and J_t is the index for the uncertain goals of the DM. Let $\hbar^t = \sum_{t=1}^{T}|J_t|$ represent the total number of uncertain data in the goal constraints. The robust model (13)–(18) is pessimistic, as it searches for the worst-optimal value of the total deviation from the goals. The parameter k_t indicates how many of the coefficients in the t-th constraint are allowed to be changed and determines how conservative the solution can be. The objective function (13) is specific on minimizing the sum of positive deviations from goals $t \in T$. Constraint (14) computes the respective positive and negative deviations from each goal $t \in T$ for a given solution x_j. It also contains the protection function $\sum_{j \in G_t} p_{tj} + k_t z_t$ with robustness parameter k_t to the uncertain cost, which is bounded for each combination of decision variables and goals via Constraint (15). Constraints (17) and (18) impose nonnegativity on all decision variables. The model involves $t + t\hbar^t$ constraints and is computationally tractable irrespective of the number of coefficients subject to deviation.

2.3.2. Hanks et al.'s RGP Approach

Recently, Hanks et al. [13] broadened the scope of the RGP construct by proposing models that extend the robust methodology of [19] to different uncertainty sets. They proposed three RGP models. The first two models relate to cardinality-constrained robustness via norm-based uncertainty sets using the L_1-norm and the L_2-norm, see [37]. Particularly, considering that the robust counterpart of the L_1-norm, $\sum_{j=1}^{n} \left|\delta_{tj} x_j\right|$, $\forall t \in T$ accounts for data uncertainty given some deviational value δ_{tj}, in [13], the authors proposed the following L_1-norm-based RGP model:

$$\begin{array}{c}\text{Min } \sum_{t=1}^{T} d_t^+ \\ \text{s.t.}\end{array} \tag{19}$$

$$\sum_{j=1}^{n} c_{tj} x_j + p_t + d_t^- - d_t^+ = g_t \quad \forall t \in T \tag{20}$$

$$p_t \geq \sum_{j \in S_t} \left|\delta_{tj} x_j\right| + \left|f_t \delta_{tq} x_{tq}\right| \qquad \forall\ S_t \in J_t,\ |S_t| = k_t,\ q \in J_t \backslash S_t,\ t \in T \tag{21}$$

$$\boldsymbol{Ax} \leq \boldsymbol{b} \tag{22}$$

$$x_j,\ d_t^-,\ d_t^+ \geq 0 \quad \forall t \in T, \forall j \in N \tag{23}$$

where S_t is the set of indices of all coefficients subject to deviation in goal $t = 1, \ldots, T$ where $S_t \subset N$ and $f_t = k_t - k_t$, is the remainder value when k_i is non-integer. Hanks [13] showed that model (19)–(23) is equivalent to model (13)–(18). This is apparent since the robustness of both models is obtained through cardinality-constraints via interval uncertainty. On the contrary, while model (19)–(23) imposes a total penalty of $\sum_{j \in G_t} p_{tj} + k_t z_t$ for the variability in each goal, model (19)–(23) identifies a total goal-specific penalty p_t through every combination of $\delta_{tj} x_j$ in some given k_t, as indicated in Constraint (21). Observe that Constraint (21) applies L_1-norm uncertainty sets to the possible set of coefficient subject to parametric uncertainty (i.e., $S_t \in J_t$, $|S_t| = k_t$, $q \in J_t \backslash S_t$, $t \in T$). The penalty difference is observed in Constraints (14) and (20) of the two models through a combination of $\sum_{j \in S_t} \left|\delta_{tj} x_j\right| + \left|f_t \delta_{tq} x_{tq}\right|$ of the latter. Moreover, it is important to note that model (19)–(23) has higher complexity compared to model (13)–(18), given that the former uses $t + \hbar^t \sum_{t=1}^{T} (\hbar^t - k_t) \binom{\hbar^t}{k_t}$ constraints to obtain the same optimal solution as the latter. The second model of [13] relates to the RGP using the L_2-norm. The model is obtained by replacing Constraint (21) with the following constraint:

$$p_t \geq \sqrt{\sum_{j \in S_t} \left(\delta_{tj} x_j\right)^2 + \left(f_t \delta_{tq} x_{tq}\right)^2} \qquad \forall\ S_t \in J_t,\ |S_t| = k_t,\ q \in J_t \backslash S_t,\ t \in T \tag{24}$$

through which the model becomes a second-order cone programming. Unlike Constraint (21), the penalty p_t, $t \in T$ is induced via the greatest lower bound over every combination of possible subsets of variables taking on uncertainty for each goal.

The third model proposed in [13] considers strict robustness using an ellipsoidal uncertainty set of [24]. The RGP model is presented as follows:

$$\begin{array}{c}\text{Min } \sum_{t=1}^{T} d_t^+ \\ \text{s.t.}\end{array} \tag{25}$$

$$\sum_{j=1}^{n} c_{tj}x_j + \theta_t\left(\sqrt{\sum_{j=1}^{n} \delta_{tj}^2 x_j^2}\right) + d_t^- - d_t^+ = g_t \quad \forall t \in T \tag{26}$$

$$\boldsymbol{Ax} \le \boldsymbol{b} \tag{27}$$

$$0 \le \theta_t \le \sqrt{\mathscr{k}^t} \quad \forall t \in T \tag{28}$$

$$x_j,\ d_t^-,\ d_t^+ \ge 0 \quad \forall t \in T, \forall j \in N \tag{29}$$

Similar to the RGP models (19)–(23) and (13)–(18), the RGP model (25)–(29) controls the deviation using the robustness parameter θ_t, as specified in Constraint (26) and bounded via Constraint (28) as in [24]. Specifically, Constraint (26) computes the positive and the negative deviations from each goal with the *strict* assumption that every coefficient c_{tj} will deviate from its respective nominal value. Although varying θ_t allows for less-than maximum and more-than maximum deviations, model (25)–(29) assumes that each coefficient subject to deviation takes on the maximum value in the uncertainty set, thus, the strict robustness approach adopted for this model can be very conservative for the DM defined goals [10].

3. The Proposed Light RGP

A critical view of the theoretical RGP model proposed in [19] indicates that, although the budget uncertainty concept of [12] presumes that not all the goals will simultaneously take their worst-case values, the optimal solution can be worse. Unfortunately, the RGP models of [13], which intended to extend the robust framework of [19], rather tends to be too conservative and pessimistic to the achievement of the specific goals of the decision-maker due to the *strict* robustness concepts used. Strict RGP models hedge against all scenarios of uncertain goals in the uncertainty set but at a higher price, namely, the optimal objective value in the robust goal often increases drastically. In other words, the quality of the optimal goal is highly affected in that the DM has to give up so much optimality in the nominal problem in order to ensure robustness [25]. It is worth mentioning that recent advances in the RO literature have shifted to solutions that are not too conservative and the price paid for robustness is not so high [24,28].

For this reason, this section considers a different RGP model that is much less conservative, and the quality of the robust goals is assured. The concept follows the *light robustness* proposed in [8] and generalized in [11] for uncertain programs with arbitrary uncertainty sets. Here in known as light robust goal programming (LRGP). The objective of the LRGP is to balance the quality of the solution with respect to the total goal deviations and the robustness of the solution with respect to the uncertain goals. The LRGP models differ from previous works in light of the following contributions:

- Contrary to the conservative models provided in the literature (c.f., [13,16,19,21,22]), we present LRGP models generalized to two arbitrary sets—the budget of uncertainty and the ellipsoidal uncertainty set. The model formulation made with these uncertainty sets provides robust solutions among those which are "not too far" from optimality for the nominal GP model.
- We generalize the RGP to include modeling uncertainties in the hard constraint of the GP model. Dealing with uncertain hard constraints is perhaps a novelty in RGP literature that, to the best of our knowledge, has not been considered before. In other words, our review of the methodological RGP papers indicates that, to date, coefficients susceptible to deviation are only the goals and not any of the other hard constraints, $\boldsymbol{Ax} \le \boldsymbol{b}$. The consideration of the uncertainties in the hard constraints is necessary to obtain complete robust solutions in problems where both constraint parameters and goals are imprecisely defined.

3.1. LRGP via Γ-Robustness

Consider a LRGP model using the budget of uncertainty of [12]. As remarked in [8], the original concept of the light robustness is heavily dependent on the robust concept of [12], where the interval-based uncertainty set is used with the additional assumption that no more than Γ_i coefficients are expected to change to their worst-case values in constraint i. Assume further from Section 2.3.1 that the uncertain constraint parameter $\widetilde{a}_{ij}$ takes value in the interval $\left[a_{ij} - \hat{a}_{ij},\ a_{ij} + \hat{a}_{ij}\right]$. Analogous to [19], K robust solution, the maximum values for the budget of uncertainty are $\Gamma_t \in [0, |J_t|]$ and $\Gamma'_i \in [0, |J_i|]$ where $\mathscr{k}^i = \sum\limits_{i=1}^{m} |J_i|$ represents the total number of uncertain data in the hard constraints. Note that Γ_t and Γ'_i allow the DM to control robustness via the goal and the hard constraints, as is considered in the models of [8,12]. We introduce slack variables γ_t for $t = 1, \ldots, T$ and γ'_i for $i = 1, \ldots, M$ where T and M are set indices for the goals and the decision variables pertaining to the hard constraint. These variables define the robustness level with respect to parameter uncertainties and control the model infeasibility. Therefore, the objective of the LRGP_Γ is to minimize the sum of γ_t and γ'_i in the following model:

$$\begin{gathered}\text{LRGP}_\Gamma = \text{Min} \sum_{t \in T} \sum_{i \in M} \left(\gamma_t + \gamma'_i\right) \\ \text{s.t.}\end{gathered} \tag{30}$$

$$\sum\nolimits_{t=1}^{T} d_t^+ \leq (1+\rho) z^* \tag{31}$$

$$\sum\nolimits_{j=1}^{n} c_{tj} x_j + \sum_{j \in G_t} p_{tj} + \Gamma_t z_t - \gamma_t + d_t^- - d_t^+ = g_t \quad \forall t \in T \tag{32}$$

$$\sum\nolimits_{j=1}^{n} a_{ij} x_j + \sum_{j \in J_i} q_{ij} + \Gamma'_i w_i - \gamma'_i \leq b_i \quad \forall i \in M \tag{33}$$

$$z_t + p_{tj} \geq \delta_{tj} x_j \quad \forall t \in T,\ \forall j \in J_t \tag{34}$$

$$w_i + q_{ij} \geq \sigma_{ij} x_j \quad \forall i \in M,\ \forall j \in J_i \tag{35}$$

$$\gamma_t,\ \gamma'_i \geq 0 \quad \forall t \in T,\ \forall i \in M \tag{36}$$

$$x_j,\ d_t^-,\ d_t^+ \geq 0 \quad \forall i \in M, \forall j \in N \tag{37}$$

$$z_t, p_{tj} \geq 0 \quad \forall t \in T,\ \forall j \in J_t \tag{38}$$

$$w_i, q_{ij} \geq 0 \quad \forall i \in M,\ \forall j \in J_i \tag{39}$$

The main differences between model (30)–(39) and the previous models given in (13)–(18), (19)–(23), and (25)–(29) occur in the objective function (30) and Constraints (31)–(33). A degree of relaxation for strict robustness is made by allowing for local violation of the t-th constraint (32) and the i-th constraint (33), where γ_t and γ'_i are then used to recover and deal with any possible infeasibility issues. Hence, the objective function is designed to control robustness and minimize possible infeasibility of the model to changes in the aspiration levels of the DM and other parameter uncertainties. Moreover, to ensure the quality of the generated solution, the generalized RGP model (30)–(39) requires first an optimal solution z^* to the nominal GP problem in (1)–(4) as the reference scenario. Constraint (31) therefore ensures that an acceptable deviation of the generated solution from the nominal solution is made through the parameter ρ. Constraint (32) identifies a total goal penalty $\sum_{j \in G_t} p_{tj} + \Gamma_t z_t - \gamma_t$ which, in contrast to the model of [19], includes the additional penalty term γ_t, specifically for the violation of the cost parameter c_{tj} from its nominal value. Constraints (34) and (35) are a combination of bounded terms for the robust variables. Finally, Constraints (36)–(39) impose nonnegativity on all decision variables.

3.2. LRGP via Ellipsoidal Uncertainty Set

This section considers a special case of the generalized LRGP to the ellipsoidal uncertainty set. Let $\mathcal{U}_1^t = \left\{ \boldsymbol{C} = \boldsymbol{P}^0 + \sum_{l=1}^{N} u_l \boldsymbol{P}^l :\ \|u^t\|_2 \leq 1,\ u^t \in \mathbb{R}^T \right\}$ where $\boldsymbol{P}^l = \left(p^l\right),\ l = 0, \ldots, T$ are $t \times n$ matrices, and $\|\cdot\|_2$ denotes the Euclidean norm (analogously to [11], this norm is independent of the particular norm chosen for generalized light robustness). The set $\mathcal{U}_1^t$ is defined for perturbation of the DM goals.

A finite mathematical program for the defined uncertainty set is obtained if the RGP problem is solved with parameter u instead of the cost matrix c_{tj} [24]. To this end, we let $\boldsymbol{R}^t$ contain all the t-th rows of all matrices $\boldsymbol{P}^1, \ldots, \boldsymbol{P}^N$. Then, for $c_{tj} \in \mathcal{U}_1^t$, following [11], the goal constraint $\sum_{j=1}^{n} c_{tj}x_j - \gamma_t + d_t^- - d_t^+ = g_t$ translates to $\boldsymbol{P}_t^o x + \|\boldsymbol{R}^t x\|_2 - \gamma_t + d_t^- - d_t^+ = g_t$ for all $u \in \mathbb{R}^N$ with $\|u^t\|_2 \leq 1$. Similarly, let the $m \times n$ matrix $S^l,\ l = 0, \ldots, k$ in $\mathcal{U}_2^i = \left\{\boldsymbol{A} = \boldsymbol{Q}^0 + \sum_{l=1}^{N} \mu_l \boldsymbol{Q}^l :\ \|\mu^i\|_2 \leq 1,\ \mu^i \in \mathbb{R}^N\right\}$ be defined for the uncertain parameters in the hard constraint, where $\boldsymbol{S}^i$ is the i-th rows of the matrices $\boldsymbol{Q}^1, \ldots, \boldsymbol{Q}^N$. We obtain the constraint $\boldsymbol{Q}_i^o x + \|\boldsymbol{S}^i x\|_2 - \gamma_i' \leq b_i$ for all $\mu^i \in \mathbb{R}^N$ with $\|\mu^i\|_2 \leq 1$. The generalized LRGP using the ellipsoidal set is thus equivalent to the following program:

$$\begin{aligned} \text{LRGP}_e = \text{Min} \left(\|\gamma_t\| + \|\gamma_i'\|\right) \\ \text{s.t.} \end{aligned} \tag{40}$$

$$\sum_{t=1}^{T} d_t^+ \leq (1+\rho)z^* \tag{41}$$

$$\boldsymbol{P}_t^o x + \|\boldsymbol{R}^t x\|_2 - \gamma_t + d_t^- - d_t^+ = g_t \quad \forall t \in T \tag{42}$$

$$\boldsymbol{Q}_i^o x + \|\boldsymbol{S}^i x\|_2 - \gamma_i' \leq b_i \quad \forall i \in M \tag{43}$$

$$\gamma_t,\ \gamma_i' \geq 0 \quad \forall t \in T,\ \forall i \in M \tag{44}$$

$$x_j,\ d_t^-,\ d_t^+ \geq 0 \quad \forall i \in M, \forall j \in N \tag{45}$$

Similar to model (30)–(39), Equation (40) minimizes the sum of the arbitrary norm of the infeasibilities of the goal and the hard constraints. Constraint (41) controls the nominal quality by the parameter ρ. Constraints (42) and (43) apply ellipsoidal uncertainty to allow a deviation of the goals in the t-th constraint and uncertain parameters in the i-th constraint, respectively, wherein the infeasibility induced in the constraints by such deviations is controlled by γ_t and γ_i'. Similarly, Constraints (44)–(45) impose nonnegativity on all decision variables.

The computational tractability and the number of constraints involved in the LRGP models are next compared to the previously discussed models. This is summarized in Table 1. Clearly, the proposed generalized RGP models (30)–(39) and (40)–(45) are computationally tractable and easily implementable, which is not the case for all the models of [13]. That is, the tractability of the RGP models via the L_1-norm and the L_2-norm is nonlinear and increasingly computationally demanding. The computational tractability of these models is not assured as indicated in [13]; see Table 1. Generally, when considering the merits and the shortcomings of the RGP models discussed in this paper, it is interesting to note that Kuchta's model results in the same number of constraints regardless of the number of coefficients subject to uncertainty. This is in contrast to Hanks' [13] models using norm-based uncertainty sets, since the number of possible constraints depends on the conservatism level k_t. However, while the latter case may be computationally efficient to the extremely risk-averse or the risk-seeking DM, it may not be ideal, as the complexity of the model is directly influenced by the k_t-values and can therefore be intractable, at least practically.

Table 1. Dimensions of the robust goal programming (RGP) models.

Author	Robust Concepts/Uncertainty Set	# of Constraints	Computational Tractability	Model Structure
[19]	Cardinality constrained via interval	$t + t\hbar^t$	Tractable	Linear
[13]	Cardinality constrained via L_1-norm	$t + \hbar^t \sum_{t=1}^{T} (\hbar^t - k_t)\binom{\hbar^t}{k_t}$	Not assured	Nonlinear
√	Cardinality constrained via L_2-norm	$t + \hbar^t \sum_{t=1}^{T} (\hbar^t - k_t)\binom{\hbar^t}{k_t}$	Not assured	Nonlinear
√	Strict robustness via ellipsoid	$2t$	Tractable	Nonlinear
This paper	Light robustness via Γ-robustness	$t(\hbar^t + 1) + m(\hbar^t + 1) + 1$	Tractable	Linear
√	Light robustness via ellipsoid	$t + m + 1$	Tractable	Nonlinear

Specifically, the RGP formulation presented in (30)–(39) is linear and deterministically feasible under the assumptions of [12] and the minimization of γ_t and γ'_i if the parameters c_{tj} and a_{ij} respectively deviate from their nominal values or whenever an optimal solution z^* to the nominal scenario exists. Note that the model (30)–(39) involves constraints and variables that are polynomial in the size of the input. Moreover, the RGP model (40)–(45) can be solved as a second-order cone programming, given that $\|\cdot\|$ in (40) is a Euclidean norm or any norm with ellipsoid as a unit block [3,14]. Contrasting the RGP model (25)–(29) to the proposed model (40)–(45), although the presence of θ_t in the former model controls the risk preference of the DM that could be introduced in the latter model, such risk is initially allowed for by constraint violation in (42) and (43), which is then minimized in the objective function. In summary, the proposed models in this paper (i) generalize robustness to the DM goal problem under polyhedral and ellipsoidal based uncertainty set, (ii) derive quality robust solutions to the DM goals, (iii) are less computationally expensive and fully tractable, and (iv) are applicable to general multi-criteria decision problems.

4. Computational Study

To verify the proposed model, we consider a hypothetical example used in [13,19]. The goal with this example is to demonstrate the performance of our new model in terms of DM's conservatism compared to the models of the aforementioned authors. The problem statement of the example is given as follows:

A company manufactures three divisible products. Let $x_j,\ j = 1,\ 2,\ 3$ *denote the amount of the respective products to be manufactured in the coming period. Here is the matrix* $c_{ij},\ i = 1,\ 2,\ 3,\ 4$ *and* $j = 1,\ 2,\ 3$*, where:*

a) c_{1j} $(j = 1,\ 2,\ 3)$ *represents the most possible (normal) amount of material needed to manufacture the* jth product.

b) c_{2j} $(j = 1,\ 2,\ 3)$ *represents the most possible (normal) amount of human work needed to manufacture the* jth product.

c) c_{3j} $(j = 1,\ 2,\ 3)$ *represents the most possible (normal) amount of machine time needed to manufacture the* *jth product.*

d) c_{4j} $(j = 1,\ 2,\ 3)$ *represents the most possible (normal) selling price of the* jth product (multiplied by −1 due to minimization problem).

Table 2 shows the nominal and the possible deviations as well as the target values for each product in the example. It should be noted that when uncertainty in the data is not present (implying the nominal value c_{ij}), the term "the most possible (normal) amount" is considered to be the coefficient associated with each decision variable x_j, $\forall j \in N$ for each goal $i \in M$. To account for uncertainty, Kuchta [19] assumed that the possible variations of the coefficient are 10% of the nominal values. Thus, mathematically, $\delta_{ij} = \varphi c_{ij}$ where $\varphi = 0.1$ is the uncertainty level. Note that, in regards to the above example, the uncertainties considered for models (30)–(39) and (40)–(45) are those of deviations only and not the hard constraints. The exclusion of the hard constraints (33), (35), and (43) thus makes the numerical comparison with the previous RGP models more plausible.

Table 2. Nominal values and possible deviations for each product.

	j=1		j=2		j=3		**Target Values**
	c_{i1}	δ_{i1}	c_{i2}	δ_{i2}	c_{i3}	δ_{i3}	
$i = 1$	3	0.3	7	0.7	5	0.5	200
$i = 2$	6	0.6	5	0.5	7	0.7	200
$i = 3$	3	0.3	6	0.6	5	0.5	200
$i = 4$	−28	2.8	−40	4	−32	3.2	−1500

In this example, the different k_t-values (also the same as Γ_t-values) of varying scenarios summarized in [13] (see pp 642, Table 3) are used to understand the DM's view on the conservatism of the robust solutions. As indicated in the first column of Table 3, the scenario (0, 0, 0, 0) shows the problem without uncertainty, whereas the scenario (3, 3, 3, 3) shows that all the coefficients of each goal are affected. It also indicates all the models' behaviors at the highest level of uncertainty and the worst value of the problem. Allowing for constraint violations in the case of the light robustness model, the maximum worsening of the optimal robust solution with respect to the optimal nominal solution $z^* = 62.5$ in Constraint (31) is considered at 10%, i.e., $\rho = 0.1$. The results are computed using the GAMS software and the CPLEX solver for the linear models and the GUROBI solver for the nonlinear models, as shown in Tables 3–5.

Table 3. Comparison of Kuchta (interval), Hanks et al. (L_2-norm), and light robust goal programming (LRGP) (Γ-robustness) models.

k_t-Scenarios	Kuchta (Interval)		Hanks et al. (L_2-Norm)		LRGP via Γ-Robustness	
	$\sum_{t=1}^{T} d_t^+$	(x_1,x_2,x_3)	$\sum_{t=1}^{T} d_t^+$	(x_1,x_2,x_3)	$\sum_{t=1}^{T} d_t^+$	(x_1,x_2,x_3)
(0, 0, 0, 0)	62.5	(20.8, 23.0, 0.0)	62.5	(20.8, 23.0, 0.0)	62.5	(20.8, 23.0, 0.0)
(0, 0, 0, 3)	125.0	(41.7, 12.5, 0.0)	106.5	(35.5, 15.6, 0.0)	68.75	(19.6, 23.5, 0.0)
(1, 1, 1, 1)	136.2	(28.2, 19.7, 0.0)	136.2	(28.2, 19.7, 0.0)	68.75	(26.0, 18.2, 1.3)
(1, 1, 1, 3)	172.2	(36.9, 15.8, 0.0)	149.0	(33.7, 16.8, 0.0)	68.75	(31.3, 15.6, 0.0)
(2, 2, 2, 2)	187.3	(56.1, 1.4, 1.0)	158.6	(34.6, 16.2, 0.0)	68.75	(21.3, 20.6, 2.5)
(3, 3, 3, 3)	187.5	(56.8, 1.9, 0.0)	158.6	(34.6, 16.2, 0.0)	68.75	(19.6, 23.5, 0.0)
Mean	161.6	(43.9, 10.3, 0.2)	141.8	(33.3, 16.9, 0.0)	68.75	(23.6, 20.3, 0.8)
St. Dev	29.3	(12.4, 8.3, 0.5)	21.7	(2.9, 1.6, 0.0)	0.0	(5,1, 3.4, 1.1)

Table 4. Effect of regret using LRGP via Γ-robustness.

k_t-Scenarios	Functions	ρ					
		0.0	0.1	0.5	1.0	1.5	2.0
(0, 0, 0, 0)	LRGP$_\Gamma$	0.00	0.00	0.00	0.00	0.00	0.00
	$\sum_{t=1}^{T} d_t^+$	62.5	68.75	87.5	87.5	87.5	87.5
(0, 0, 0, 3)	LRGP$_\Gamma$	150	149.00	145.0	140.14	135.86	131.58
	$\sum_{t=1}^{T} d_t^+$	62.5	68.75	93.75	125	156.25	187.53
(1, 1, 1, 1)	LRGP$_\Gamma$	133.95	112.24	86.30	75.54	73.01.	70.61
	$\sum_{t=1}^{T} d_t^+$	62.5	68.75	93.75	125	156.25	187.53
(1, 1, 1, 3)	LRGP$_\Gamma$	192.29	189.06	178.65	172.47	166.90	161.42
	$\sum_{t=1}^{T} d_t^+$	62.5	68.75	93.75	125	156.25	187.53
(2, 2, 2, 2)	LRGP$_\Gamma$	216.25	204.63	161.15	149.65	144.63	139.89
	$\sum_{t=1}^{T} d_t^+$	62.5	68.75	93.75	125	156.25	187.53
(3, 3, 3, 3)	LRGP$_\Gamma$	216.25	214.88	209.38	202.60	196.27	189.93
	$\sum_{t=1}^{T} d_t^+$	62.5	68.75	93.75	125	156.25	187.53

Table 5. Comparison of Hanks et al. (ellipsoid) and LRGP (Γ-robustness) models.

Theta Values	Hanks et al. (Ellipsoid)		LRGP via Ellipsoid	
	$\sum_{t=1}^{T} d_t^+$	(x_1,x_2,x_3)	$\sum_{t=1}^{T} d_t^+$	(x_1,x_2,x_3)
0.1	70.7	(23.4, 21.4, 0.0)	68.75	(33.2, 18.0, 0.0)
0.5	105.1	(33.6, 15.4, 0.0)	68.75	(33.2, 19.1, 0.0)
1.0	158.6	(34.6, 16.2, 0.0)	68.75	(33.5, 20.3, 0.0)
1.5	215.4	(33.2, 17.2, 2.5)	68.75	(31.6, 20.2, 3.1)
$\sqrt{3}$	241.3	(30.6, 16.9, 5.0)	68.75	(30.1, 19.6, 5.7)
Mean	158.2	(31.1, 17.4, 1.5)	68.75	(32.3, 19.4, 1.8)
St. Dev	71.8	(4.5, 2.3, 2.2)	0.0	(1.4, 0.9, 2.5)

The second to the fourth column of Table 3 show the robust results in [19] and in [13] obtained through cardinality-constrained methods using either the interval-based or the L_2-norm uncertainty sets. The last two columns show the new results from the light robustness. As a first stage feasibility criteria for the LRGP models, we obtain the optimal objective values of the nominal problem $z^* = 62.5$. Note that we assume weights equal to 1 for the GP and all the RGP models. Moreover, we consider only one-sided deviation from a target value that will negatively impact the solution. The solution z^* indicates the lack of consideration of uncertainty for all the RGP models and the LRGP models (30)–(39) and (40)–(45) when $\rho = 0$. From Table 3, the results of the [19] interval-based model and the L_2-norm approach of [13] are identical at the scenario $(1,\ 1,\ 1,\ 1)$. This is due to the equivalency of the formulation of the two models when at least one coefficient of each goal is affected (Note that for all scenarios, the interval-based approach and the L_1-based approach are also equivalent.). It appears that the objective function value of these models increases as the number of coefficients allowed to take on uncertainty increases.

On the contrary, a unique performance level is obtained for all the k_t scenarios using the LRGP models. Here, increasing the conservatism degree leads to an increase in the infeasibility γ_t values. When $\gamma_t = 0$ (keeping ρ constant), the total deviational goals take values between z^* and $(1+\rho)z^*$. On the other hand, increasing ρ considering a fixed scenario reduces the infeasibility. Table 4 shows the infeasibility minimized at different k_t-scenarios. More important is the quality of the robust solution. Figure 1 shows how the different echelon of robustness differs from the nominal deviational goals. It can be observed that the deviation in the number of products made to achieve robust goals is not much sacrificed regardless of scenario when using the LRGP models. In contrast, the total deviations become worse in the robust models of [13,19] as the DM's conservatism in the scenario increases. The methodology to obtain the DM's using the LRGP models out-performs the RGP models of [13,19] in all instances.

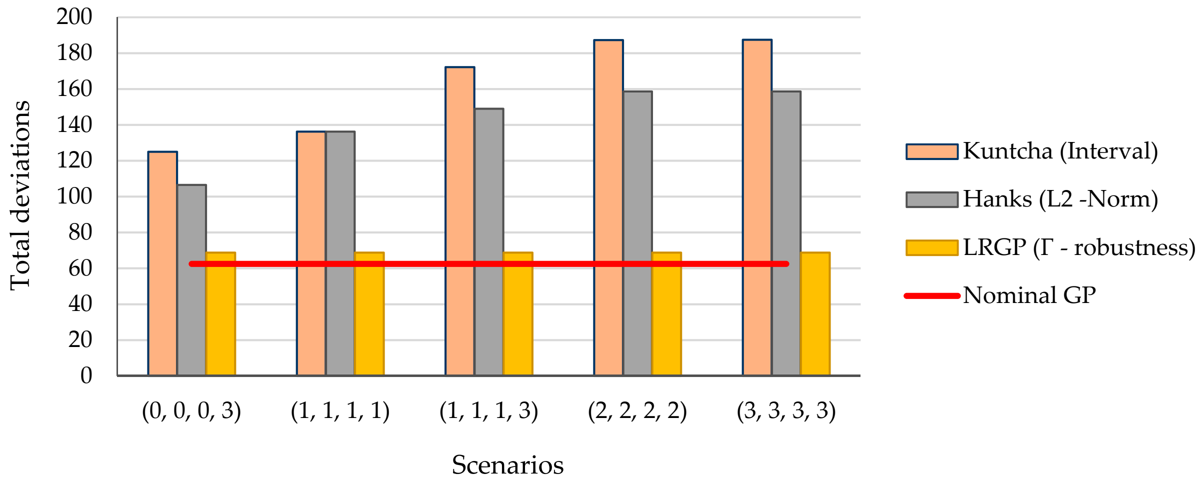

Figure 1. Comparison of total goal deviations.

In Table 5, the LRGP_e is compared with the ellipsoid based RGP model of [13]. We use the same theta values on the LRGP and set $\rho = 0.1$. The result shows that the total goal deviation is very much minimized and closer to the nominal goal deviation at all levels of conservatism compared to the result of the ellipsoid based RGP of [13]. It is also intriguing to note that the total goal deviation (68.75) obtained by the two LRGP models are the same (see Table 3). Figure 2 demonstrates these identical total goal deviations of the LRGP_Γ and the LRGP_e models for different values of ρ for a fixed level of conservatism. The total goal deviations of the LRGP models are thus dependent on the value of ρ rather than the type of uncertainty set used for the robust model. As ρ increases, the total goal deviations increase, indicating a more conservative and less quality solution. This result reechoes the objective of the light robustness, as emphasized in [8]. Thus, the LRGP approach achieves robustness for the uncertain goals of the DM by first enforcing a demanding optimality goal and a certain quality of robustness away from the optimality by a determined threshold level ρ while at the same time dealing with possible infeasibility issues (through the slack variables) for allowing local violations of the constraints apriori.

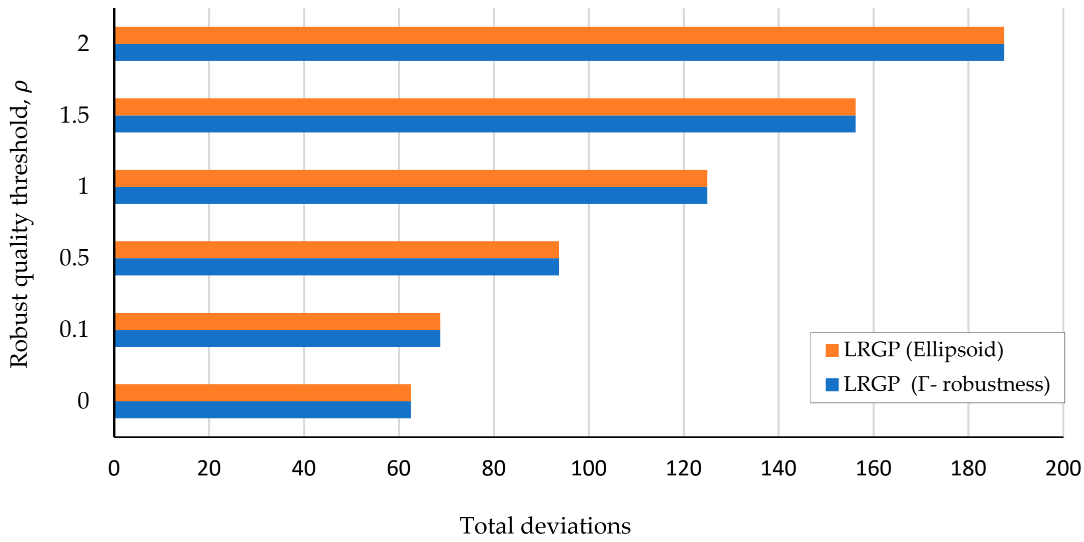

Figure 2. Total goal deviations by LRGP$_\Gamma$ and LRGP$_e$ models.

On the other hand, to see the effect of different uncertainties on the LRGP models, we investigate the different violations of the constraints at $\varphi = 10\%, 20\%, 40\%, 60\%$, and 80%. Since there is no limitation in choosing the values of ρ, here, we select $\rho = 0.2$. Note from Tables 4 and 5 that when ρ is fixed, the DM's total goal deviations for all the products remain the same irrespective of the conservatism of the DM. Figure 3 illustrates the effect of uncertainty on constraint violations and infeasibility between the LRGP via the Γ-robustness and the ellipsoid. In general, increasing the level of uncertainty or the DM's conservatism leads to an increase in model infeasibility. Specifically, while the LRGP seeks to minimize the risk of the infeasibility of robust goals, it is clear from Figure 3 that the infeasibility minimized with the ellipsoid based LRGP model performs better than the LRGP using the Γ-robustness.

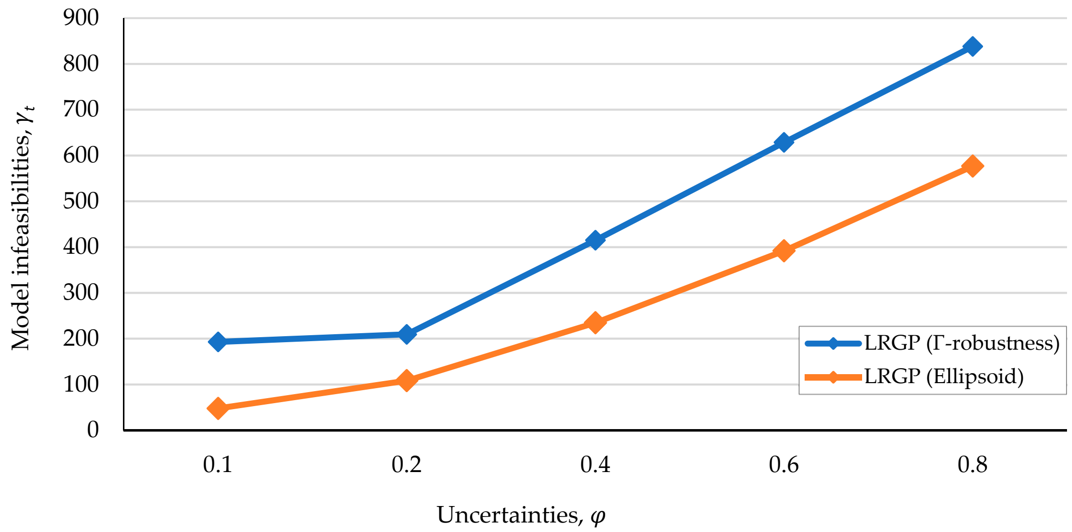

Figure 3. Constraint violated uncertainties and infeasibilities.

5. Conclusions

In this paper, we addressed GP models in an uncertain environment in which the validity and the reliability of the GP technique are determined by—and therefore greatly influenced by—the accuracy of the DM goals. Although the RGP, a recent approach based on RO and GP, has been introduced to deal with such a situation, its methodology is not widely researched. The main methodology offered in the literature—interval-based, norm-based, ellipsoidal uncertainty sets RGP—produces solutions that are too often conservative. In this paper, we propose new RGP models—light RGP generalized on two arbitrary sets, i.e., the budget of uncertainty in [12] and the ellipsoidal uncertainty sets in [24]. Some observations regarding the use of different uncertainty sets for the LRGP are proposed. We also compare our new model with the RGP models developed in the literature and show that our model's solution is much less conservative and is "quality" in terms of the DM goals in that the robust solution is "not too far" from optimality from the nominal GP model. Furthermore, we show that, rather than the specific uncertainty set used, the total goal deviations of the decision-maker are very much dependent on the robust quality threshold set as the trade-off between the quality (optimality) and the feasibility of the robust solution.

Further research can focus on the applicability of the proposed models to a real-world situation such as the portfolio selection and the capital budgeting problem, as described respectively in [10,11] or the multi-criteria data envelopment analysis. Until now, the RGP has concentrated on the weighted GP. Therefore, an interesting future study that would be of merit would be to extend and compare the RGP with different GP techniques such as the pre-emptive lexicographic GP and the Chebyshev MINMAX GP.

Author Contributions: Conceptualization: E.K.M. and M.R.; Methodology: E.K.M. and M.R.; Writing original draft: E.K.M.; writing-review and editing: E.K.M. and M.R.; The submission was approved by both authors.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Crespi, G.P.; Kuroiwa, D.; Rocca, M. Robust optimization: Sensitivity to uncertainty in scalar and vector cases, with applications. *Oper. Res. Perspect.* **2018**, *5*, 113–119. [CrossRef]
2. Fliege, J.; Werner, R. Robust multiobjective optimization & applications in portfolio optimization. *Eur. J. Oper. Res.* **2014**, *234*, 422–433.
3. Larbani, M.; Aouni, B. A new approach for generating efficient solutions within the goal programming model. *J. Oper. Res. Soc.* **2011**, *62*, 175–182. [CrossRef]
4. Colapinto, C.; Jayaraman, R.; Marsiglio, S. Multi-criteria decision analysis with goal programming in engineering, management and social sciences: A state-of-the art review. *Ann. Oper. Res.* **2017**, *251*, 7–40. [CrossRef]
5. Ben-Tal, A.; Nemirovski, A. Robust Convex Optimization. *Math. Oper. Res.* **1998**, *23*, 769–805. [CrossRef]
6. Aouni, B.; Martel, J.M.; Hassaine, A. Fuzzy goal programming model: An overview of the current state-of-the art. *J. Multi-Criteria Decis. Anal.* **2009**, *16*, 149–161. [CrossRef]
7. Singh, P.; Kumari, S.; Singh, P. Fuzzy Efficient Interactive Goal Programming Approach for Multi-objective Transportation Problems. *Int. J. Appl. Comput. Math.* **2017**, *3*, 505–525. [CrossRef]
8. Fischetti, M.; Monaci, M. Light robustness. In *Robust and Online Large-Scale Optimization*; Ahuja, R.K., Möhring, R.H., Zaroliagis, C.D., Eds.; Springer: Berlin, Germany, 2009; Volumn 5868, pp. 61–84.
9. Ben-Tal, A.; El Ghaoui, L.; Nemirovski, A. *Robust Optimization*; Princeton University Press: Princeton, NJ, USA, 2009.
10. Goerigk, M.; Schmidt, M.; Knoth, M.; Müller-Hannemann, M.; Schöbel, A. The Price of Strict and Light Robustness in Timetable Information. *Transp. Sci.* **2013**, *48*, 225–242. [CrossRef]
11. Schöbel, A. Generalized light robustness and the trade-off between robustness and nominal quality. *Math. Methods Oper. Res.* **2014**, *80*, 161–191. [CrossRef]
12. Bertsimas, D.; Sim, M. The Price of Robustness. *Oper. Res.* **2004**, *1*, 35–53. [CrossRef]

13. Hanks, R.W.; Weir, J.D.; Lunday, B.J. Robust goal programming using different robustness echelons via norm-based and ellipsoidal uncertainty sets. *Eur. J. Oper. Res.* **2017**, *262*, 636–646. [CrossRef]
14. Mensah, E.K. Robust Optimization in Data Envelopment Analysis: Extended Theory and Applications. Ph.D. Thesis, University of Insubria, Varese, Italy, 2019.
15. Gabrel, V.; Murat, C.; Thiele, A. Recent advances in robust optimization: An overview. *Eur. J. Oper. Res.* **2014**, *235*, 471–483. [CrossRef]
16. Ghahtarani, A.; Najafi, A.A. Robust goal programming for multi-objective portfolio selection problem. *Econ. Model.* **2013**, *33*, 588–592. [CrossRef]
17. Arabmaldar, A.; Jablonsky, J.; Saljooghi, F.H. A new robust DEA model and super-efficiency measure. *Optimization* **2017**, *66*, 723–736. [CrossRef]
18. Toloo, M.; Mensah, E.K. Robust optimization for nonnegative decision variables: A DEA approach. *Comput. Chem. Eng.* **2019**, *127*, 313–325. [CrossRef]
19. Kuchta, D. Robust goal programming. *Control Cybern.* **2004**, *33*, 501–510.
20. Kuchta, D. A concept of a robust solution of a multicriterial linear programming problem. *Cent. Eur. J. Oper. Res.* **2011**, *19*, 605–613. [CrossRef]
21. Ghasemi Bojd, F.; Koosha, H. A robust goal programming model for the capital budgeting problem. *J. Oper. Res. Soc.* **2018**, *69*, 1105–1113. [CrossRef]
22. Hanks, R.W.; Lunday, B.J.; Weir, J.D. Robust goal programming for multi-objective optimization of data-driven problems: A use case for the United States transportation command's liner rate setting problem. *Omega* **2018**, in press. [CrossRef]
23. Liu, P.; Yang, W.; Guo, T. A discussion on the conservatism of robust linear optimization problems. *Optimization* **2016**, *65*, 1641–1650. [CrossRef]
24. Ben-Tal, A.; Nemirovski, A. Robust solutions of uncertain linear programs. *Oper. Res. Lett.* **1999**, *1*, 1–13. [CrossRef]
25. Ben-Tal, A.; Nemirovski, A. Robust optimization of linear programming problems contamined with uncertain data. *Math. Program.* **2000**, *88*, 411–424. [CrossRef]
26. Charnes, A.; Cooper, W.W.; Ferguson, R.O. Optimal Estimation of Executive Compensation by Linear Programming. *Manag. Sci.* **1955**, *1*, 138–151. [CrossRef]
27. Tamiz, M.; Jones, D.; Romero, C. Goal programming for decision making: An overview of the current state-of-the-art. *Eur. J. Oper. Res.* **1998**, *111*, 569–581. [CrossRef]
28. Romero, C. A general structure of achievement function for a goal programming model. *Eur. J. Oper. Res.* **2003**, *153*, 675–686. [CrossRef]
29. Birge, J.; Louveaux, F. *Introduction to Stochastic Programming*, 2nd ed.; Springer: New York, NY, USA, 2001.
30. Bertsimas, D.; Brown, D.B.; Caramanis, C. Theory and Applications of Robust Optimization. *SIAM Rev.* **2011**, *53*, 464–501. [CrossRef]
31. Soyster, A.L. Convex programming with set-inclusive constraints and applications to inexact linear programming. *Oper. Res.* **1973**, *21*, 1154–1157. [CrossRef]
32. Goerigk, M.; Schöbel, A. Algorithm engineering in robust optimization. In *Algorithm Engineering*; Kliemann, L., Sanders, P., Eds.; Springer: Berlin, Germany, 2016; pp. 245–279.
33. Heidt, A.; Helmke, H.; Kapolke, M.; Liers, F.; Martin, A. Robust runway scheduling under uncertain conditions. *J. Air Transp. Manag.* **2016**, *56*, 28–37. [CrossRef]
34. Schöbel, A. Line planning in public transportation: Models and methods. *OR Spectr.* **2012**, *34*, 491–510. [CrossRef]
35. Mannino, C.; Nilssen, E.J.; Nordlander, T.E. A pattern based, robust approach to cyclic master surgery scheduling. *J. Sched.* **2012**, *15*, 553–563. [CrossRef]
36. Rahmani, D.; Zandi, A.; Behdad, S.; Entezaminia, A. A light robust model for aggregate production planning with consideration of environmental impacts of machines. *Oper. Res.* **2019**, 1–25. [CrossRef]
37. Bertsimas, D.; Pachamanova, D.; Sim, M. Robust linear optimization under general norms. *Oper. Res. Lett.* **2004**, *32*, 510–516. [CrossRef]