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POSITIVE ALMOST PERIODIC SOLUTION FOR IMPULSIVE NICHOLSON'S BLOWFLIES MODEL WITH LINEAR HARVESTING TERM ON THE BOUNDED DOMAIN

Zhijian Yao

Department of Mathematics and Physics, Anhui University of Architecture,
Hefei, Anhui 230601, China, zhijianyao@126.com

Abstract- In this paper, impulsive Nicholson's blowflies model with linear harvesting term is studied. By using the contraction mapping fixed point theorem, we obtain sufficient conditions for the existence of a unique positive almost periodic solution. In addition, the exponential convergence of positive almost periodic solution is investigated.

Key Words- Nicholson's blowflies model, Impulse, Almost periodic solution, Exponential convergence

1. INTRODUCTION

The dynamic behaviors of biological models are very important research topics. In 1980, Gurney [1] proposed the following delay differential equation

$$x'(t) = -ax(t) + bx(t-\tau)e^{-\beta x(t-\tau)}$$

to describe the population of the Australian sheep-blowfly and to agree with the experimental data obtained by Nicholson in [2]. Since this equation explains Nicholson's data of blowfly more accurately, this model and its modifications have been now referred to as the Nicholson's blowflies model. The theory of Nicholson's blowflies model has made a remarkable progress[3-10,16-19,21-23].

The assumption that the environment is constant is rarely the case in real life. When the environmental fluctuation is taken into account, a model must be nonautonomous. Due to the various seasonal effects of the environmental factors in real life situation, it is rational and practical to study the biological system with periodic coefficients or almost periodic coefficients. Many authors [4,6,7,10,16-18] have studied nonautonomous differential equations with periodic coefficients of the above Nicholson's blowflies model and its generalized models. Recently, L. Berezansky [9] pointed out an open problem: How about the dynamic behaviors of the Nicholson's blowflies model with linear harvesting term.

In the natural biological systems, there exist many impulsive phenomena. If impulsive factors are introduced into biological models, the models must be governed by impulsive differential equations. The theory of impulsive differential equation has been well developed [11-13].

In this paper, motivated by the above mentioned facts, we will study the following impulsive Nicholson's blowflies model with linear harvesting term

$$\begin{cases} x'(t) = -a(t)x(t) + \sum_{i=1}^{n} b_i(t)x(t-\tau)e^{-\beta_i(t)x(t-\tau)} - H(t)x(t-\tau), & t \neq t_k, \\ \Delta x(t_k) = c_k x(t_k) + h_k, & t = t_k, \end{cases} \tag{1.1}$$

where $t_k \in R, t_k < t_{k+1}, k \in Z$, $\lim_{k\to\pm\infty} t_k = \pm\infty$, $\Delta x(t_k) = x(t_k^+) - x(t_k^-)$, $x(t_k^-) = x(t_k)$, $a(t), b_i(t), \beta_i(t), H(t) \in PC(R, R)$ and $a(t), b_i(t), \beta_i(t), H(t)$ are positive bounded almost periodic functions $(i = 1, 2, \cdots, n)$, $PC(R,R) = \{x(t) \mid x: R \to R, x(t)$ is continuous for $t \neq t_k, x(t_k^+), x(t_k^-)$ exist and $x(t_k^-) = x(t_k)\}$. The admissible initial condition associated with equation (1.1) is

$$x(t) = \phi(t) > 0 \text{ for } t \in [-\tau, 0], \ \tau > 0, \quad \phi \in BPC\left([-\tau,0], R^+\right),$$

where $BPC\left([-\tau,0], R^+\right) = \{\phi \mid \phi: [-\tau,0] \to R^+$ is bounded piecewise left continuous function with points of discontinuity of the first kind $\}$.

A function $x(t)$ is called the solution of equation (1.1) if the function $x(t)$ is defined on $[-\tau, +\infty)$ and satisfying (1.1) for $t \geq 0$. For a given initial function $\phi \in BPC\left([-\tau,0], R^+\right)$, by [15] we know that (1.1) has a unique solution $x(t) = x(t;\phi)$ defined on $[-\tau, +\infty)$ and satisfying the initial condition: $x(t;\phi) = \phi(t)$ for $t \in [-\tau, 0]$.

In the study of biological systems, an important problem is concerned with the existence of positive periodic solutions or positive almost periodic solutions. Many authors have investigated the existence of positive periodic solution by using Krasnoselskii cone fixed point theorem and Mawhin's coincidence degree theory.

The almost periodicity is closer to the reality of biological systems. In this paper, we aim to obtain sufficient conditions that guarantee the existence of unique positive almost periodic solution of model (1.1) by using contraction mapping fixed point theorem. We also investigate the exponential convergence of positive almost periodic solution by means of Liapunov functional. For the impulsive Nicholson’s blowflies model with linear harvesting term, we give answers to the open problem proposed in [9] by L. Berezansky. The results of this paper are valuable in applications, which complement the previously obtained results in [3-10,16-19,21-23].

2. PRELIMINARIES

We denote $B = \{t_k \mid t_k \in R, t_k < t_{k+1}, k \in Z, \lim_{k\to\pm\infty} t_k = \pm\infty\}$.

Definition 1. ([13]) The set of sequences $\{t_k^i \mid t_k^i = t_{k+i} - t_k, k, i \in Z, t_k \in B\}$ is said to be uniformly almost periodic if for arbitrary $\varepsilon > 0$ there exists relatively dense set of ε-almost periods common for any sequences.

Definition 2. ([13]) A function $x(t) \in PC(R,R)$ is said to be almost periodic, if :

(i) The set of sequences $\{t_k^i \mid t_k^i = t_{k+i} - t_k, k, i \in Z, t_k \in B\}$ is uniformly almost periodic.

(ii) For any $\varepsilon > 0$ there exists real number $\delta > 0$ such that if the points t' and t'' belong to one and the same interval of continuity of $x(t)$ and satisfy the inequality $|t' - t''| < \delta$, then $|x(t') - x(t'')| < \varepsilon$.

(iii) For any $\varepsilon>0$ there exists relatively dense set T such that if $\omega\in T$, then $|x(t+\omega)-x(t)|<\varepsilon$ for $t\in R$ satisfying $|t-t_k|>\varepsilon, k\in Z$.
The elements of T are called ε-almost periods.

In this paper, for any bounded function $f(t)$, we denote

$\overline{f}=\sup_{t\in R} f(t)$, $\underline{f}=\inf_{t\in R} f(t)$.

We make the following assumptions:

(C_1) The bounded almost periodic functions $a(t), b_i(t), \beta_i(t), H(t)$ satisfy $0<\underline{a}\le a(t)\le\overline{a}$, $0<\underline{b_i}\le b_i(t)\le\overline{b_i}$, $0<\underline{\beta_i}\le\beta_i(t)\le\overline{\beta_i}$, $0<\underline{H}\le H(t)\le\overline{H}$ $(i=1,2,\cdots,n)$.

(C_2) The set of sequences $\{t_k^i \mid t_k^i=t_{k+i}-t_k, k,i\in Z, t_k\in B\}$ is uniformly almost periodic, and $\inf_{k\in Z}|t_{k+1}-t_k|=\delta_0>0$.

(C_3) The sequence $\{c_k\}$ is almost periodic and $-1<c_k\le 0$, $k\in Z$.

(C_4) The sequence $\{h_k\}$ is almost periodic and there exists constant $m>0$, such that $0<h_k\le m$, $k\in Z$.

Consider equation

$$\begin{cases} x'(t)=-a(t)x(t), & t\ne t_k,\\ \Delta x(t_k)=c_k x(t_k), & t=t_k. \end{cases} \tag{2.1}$$

The Cauchy matrix $W(t,s)$ of (2.1) is defined as follows: ([13])

$$W(t,s)=\begin{cases} \exp\left(-\int_s^t a(u)du\right), & t_{k-1}<s\le t\le t_k,\\ \prod_{i=j}^{k}(1+c_i)\exp\left(-\int_s^t a(u)du\right), & t_{j-1}<s\le t_j\le t_k<t\le t_{k+1}. \end{cases}$$

Equation (2.1) with initial condition $x(t_0)=x_0$ has a unique solution $x(t;t_0,x_0)=W(t,t_0)x_0$.

Lemma 1. ([13]) Let the conditions $(C_1)-(C_4)$ hold. Then for each $\varepsilon>0$, there exist $\varepsilon_1>0, \varepsilon_1<\varepsilon$, relatively dense sets T of positive real numbers and Q of natural numbers , such that the following relations are fulfilled:

(i) $|a(t+\omega)-a(t)|<\varepsilon$, $|b_i(t+\omega)-b_i(t)|<\varepsilon$, $|\beta_i(t+\omega)-\beta_i(t)|<\varepsilon$, $|H(t+\omega)-H(t)|<\varepsilon$, $t\in R, \omega\in T$;

(ii) $|c_{k+q}-c_k|<\varepsilon$, $q\in Q, k\in Z$;

(iii) $|h_{k+q}-h_k|<\varepsilon$, $q\in Q, k\in Z$;

(iv) $\left|t_k^q-\omega\right|<\varepsilon_1$, $q\in Q, \omega\in T, k\in Z$.

Lemma 2. ([13]) Let the condition (C_2) be satisfied. Then for each $L>0$, there exists a positive integer N , such that $i(s,t)\le N(t-s)+N$, where $i(s,t)$ is the number of the points t_k in the interval (s,t) of length L.

By Lemma 2, we get the following Lemma 3.

Lemma 3. Let the condition (C_2) be satisfied. Then for $L=1$, there exists a positive integer P, such that $i(s,t)\le 2P$, where $i(s,t)$ is the number of the points t_k in the interval (s,t) of length 1.

Lemma 4. Let the conditions $(C_1)-(C_3)$ be satisfied. Then, the Cauchy matrix $W(t,s)$ of (2.1) satisfies $0<W(t,s)\le e^{-\underline{a}(t-s)}$, $t\ge s$, $t,s\in R$.

Proof. Because $W(t,s)$ is expressed as follows

$$W(t,s)=\begin{cases}\exp\left(-\int_s^t a(u)du\right), & t_{k-1}<s\le t\le t_k,\\ \prod_{i=j}^{k}(1+c_i)\exp\left(-\int_s^t a(u)du\right), & t_{j-1}<s\le t_j\le t_k<t\le t_{k+1}.\end{cases}$$

From the condition (C_3) $-1<c_k\le 0$, $k\in Z$, we have $0<1+c_k\le 1$, $\forall k\in Z$.

Thus, it follows that $0<\prod_{i=j}^{k}(1+c_i)\le 1$.

Hence we get

$$0<W(t,s)\le \exp\left(-\int_s^t a(u)du\right)\le \exp\left(-\int_s^t \underline{a}du\right)=e^{-\underline{a}(t-s)},\qquad t\ge s,\quad t,s\in R.$$

By the Lemma 3 in [11], we have the following Lemma 5.

Lemma 5. Let the conditions $(C_1)-(C_3)$ be satisfied. Then for any $\varepsilon>0$, $t\ge s$, $t,s\in R$, $|t-t_k|>\varepsilon, |s-t_k|>\varepsilon, k\in Z$, there exist relatively dense set T of ε-almost periods of the function $a(t)$ and positive constants $E>0, \eta>0$, such that for $\omega\in T$ it follows that

$$|W(t+\omega,s+\omega)-W(t,s)|\le \varepsilon E e^{-\frac{\eta}{2}(t-s)}.$$

Let $X=\{x(t)\in PC(R,R)$ is almost periodic function$\}$ with the norm $\|x\|=\sup_{t\in R}|x(t)|$, then X is Banach space.

It is easy to verify that $x(t)$ is the solution of equation (1.1) if and only if $x(t)$ is the solution of the following integral equation

$$x(t)=\int_{-\infty}^{t}W(t,s)\left[\sum_{i=1}^{n}b_i(s)x(s-\tau)e^{-\beta_i(s)x(s-\tau)}-H(s)x(s-\tau)\right]ds+\sum_{t_k<t}W(t,t_k)h_k.$$

We define operator $A: X\to X$,

$$(Ax)(t)=\int_{-\infty}^{t}W(t,s)\left[\sum_{i=1}^{n}b_i(s)x(s-\tau)e^{-\beta_i(s)x(s-\tau)}-H(s)x(s-\tau)\right]ds+\sum_{t_k<t}W(t,t_k)h_k.$$

It is clear that $x(t)\in PC(R,R)$ is the almost periodic solution of equation (1.1) if and only if x is the fixed point of operator A.

Let

$$M=\frac{2mP}{1-e^{-\underline{a}}}+\frac{1}{\underline{a}e}\sum_{i=1}^{n}\frac{\overline{b_i}}{\underline{\beta_i}}.$$

We make assumptions:

(S_1) $\frac{1}{\underline{a}}\left(\overline{H}+\sum_{i=1}^{n}\overline{b_i}\right)<1$,

(S_2) $\overline{H} < \sum_{i=1}^{n} \underline{b_i} e^{-\overline{\beta}_i M}$,

(S_3) $c_k M + h_k \le 0$, $k \in Z$.

It is easy to check the global existence of the positive solution $x(t) = x(t;\phi)$ for equation (1.1) defined on $[-\tau, +\infty)$ with the admissible initial condition $x(t;\phi) = \phi(t) > 0$ for $t \in [-\tau, 0]$.

Let

$$U^0 = \left\{\phi \mid \phi \in BPC\left([-\tau,0], R^+\right), 0 < \phi(t) < M, t \in [-\tau, 0]\right\}.$$

Now we prove that every solution $x(t) = x(t;\phi)$ of equation (1.1) with initial function $\phi \in U^0$ is positive and bounded.

Lemma 6. Assume that $(C_1)-(C_4)$ hold. If (S_3) is satisfied, then every solution $x(t)$ of equation (1.1) with initial function $\phi \in U^0$ satisfies

$$0 < x(t) < M \quad \text{for all} \quad t > 0.$$

Proof. For $t \in [-\tau, 0]$, $x(t) = \phi(t) \in U^0$ and $0 < \phi(t) < M$. Hence there must exist an interval $(0, T_1) \subset (0, +\infty)$ such that $x(t) > 0$ for $t \in (0, T_1)$.

For impulsive point $t_k \in (0, T_1)$, if $0 < x(t_k) < M$, then

$$x(t_k^+) = (1 + c_k) x(t_k) + h_k < (1 + c_k) M + h_k \le M \quad \text{and} \quad x(t_k^+) = (1 + c_k) x(t_k) + h_k > 0,$$

which implies $0 < x(t_k^+) < M$.

We claim that $0 < x(t) < M$ for $t \in (0, T_1)$. (2.2)

Suppose the claim (2.2) is not true, then there must exist a $t_1^* \in (0, T_1)$ such that $x(t_1^*) = M$, $x'(t_1^*) \ge 0$ and $0 < x(t) < M$ for $0 < t < t_1^*$.

Thus,

$$\begin{aligned} x'(t_1^*) &= -a(t_1^*)x(t_1^*) + \sum_{i=1}^{n} b_i(t_1^*) x(t_1^* - \tau) e^{-\beta_i(t_1^*) x(t_1^* - \tau)} - H(t_1^*) x(t_1^* - \tau) \\ &< -a(t_1^*)x(t_1^*) + \sum_{i=1}^{n} b_i(t_1^*) x(t_1^* - \tau) e^{-\beta_i(t_1^*) x(t_1^* - \tau)} \\ &\le -\underline{a} M + \sum_{i=1}^{n} \overline{b}_i x(t_1^* - \tau) e^{-\underline{\beta_i} x(t_1^* - \tau)}. \end{aligned} \tag{2.3}$$

Since the function $g_i(u) = u e^{-\underline{\beta_i} u}$, $u \in [0, +\infty)$ reaches its maximum $\frac{1}{\underline{\beta_i} e}$ at $u = \frac{1}{\underline{\beta_i}}$,

then we have
$$x(t_1^* - \tau) e^{-\underline{\beta_i} x(t_1^* - \tau)} \le \frac{1}{\underline{\beta_i} e}. \tag{2.4}$$

By (2.3) and (2.4), we have

$$x'(t_1^*) < -\underline{a} M + \sum_{i=1}^{n} \left(\overline{b}_i \frac{1}{\underline{\beta_i} e} \right) = -\underline{a} \left(\frac{2mP}{1 - e^{-\underline{a}}} + \frac{1}{\underline{a} e} \sum_{i=1}^{n} \frac{\overline{b_i}}{\underline{\beta_i}} \right) + \frac{1}{e} \sum_{i=1}^{n} \frac{\overline{b_i}}{\underline{\beta_i}} = -\frac{2mP\underline{a}}{1 - e^{-\underline{a}}} < 0,$$

which contradicts $x'(t_1^*) \ge 0$. So the claim (2.2) is true.

Hence, $0 < x(t) < M$ for $t \in (0, T_1)$.

Thus, there must exist an interval $[T_1, T_2) \subset [T_1, +\infty)$ such that $x(t) > 0$ for $t \in [T_1, T_2)$.

By the above similar arguments, we claim that

$$0<x(t)<M \quad \text{for} \quad t\in[T_1,T_2). \tag{2.5}$$

Suppose the claim (2.5) is not true, then there must exist a $t_2^*\in[T_1,T_2)$ such that $x(t_2^*)=M$, $x'(t_2^*)\ge 0$ and $0<x(t)<M$ for $T_1\le t<t_2^*$.

Thus,

$$\begin{aligned} x'(t_2^*) &= -a(t_2^*)x(t_2^*)+\sum_{i=1}^{n} b_i(t_2^*)x(t_2^*-\tau)e^{-\beta_i(t_2^*)x(t_2^*-\tau)}-H(t_2^*)x(t_2^*-\tau) \\ &< -a(t_2^*)x(t_2^*)+\sum_{i=1}^{n} b_i(t_2^*)x(t_2^*-\tau)e^{-\beta_i(t_2^*)x(t_2^*-\tau)} \\ &\le -\underline{a}\,M+\sum_{i=1}^{n}\overline{b}_i x(t_2^*-\tau)e^{-\underline{\beta}_i x(t_2^*-\tau)}. \end{aligned} \tag{2.6}$$

Note that

$$x(t_2^*-\tau)e^{-\underline{\beta}_i x(t_2^*-\tau)}\le \frac{1}{\underline{\beta}_i e}. \tag{2.7}$$

By (2.6) and (2.7) , we have

$$x'(t_2^*)<-\underline{a}M+\sum_{i=1}^{n}\left(\overline{b}_i\frac{1}{\underline{\beta}_i e}\right)=-\underline{a}\left(\frac{2mP}{1-e^{-\underline{a}}}+\frac{1}{\underline{a}e}\sum_{i=1}^{n}\frac{\overline{b}_i}{\underline{\beta}_i}\right)+\frac{1}{e}\sum_{i=1}^{n}\frac{\overline{b}_i}{\underline{\beta}_i}=-\frac{2mP\underline{a}}{1-e^{-\underline{a}}}<0,$$

which contradicts $x'(t_2^*)\ge 0$. So the claim (2.5) is true. Hence, $0<x(t)<M$ for $t\in[T_1,T_2)$.

Repeating the above similar steps, we have $0<x(t)<M$ on intervals $[T_2,T_3),[T_3,T_4),\cdots,[T_n,T_{n+1}),\cdots$. That means $0<x(t)<M$ for all $t>0$. The proof of Lemma 6 is complete.

3. EXISTENCE OF POSITIVE ALMOST PERIODIC SOLUTION

Let

$$\Omega=\{x\mid x\in X, 0\le x(t)\le M, t\in R\}.$$

Theorem 1. Assume that $(C_1)-(C_4)$ hold. If (S_1) and (S_2) are satisfied, then equation (1.1) has a unique almost periodic positive solution in Ω.

Proof. By Lemma 4, we have $0<W(t,t_k)\le e^{-\underline{a}(t-t_k)}$ for $t_k<t$.

This and Lemma 3 imply that

$$\sum_{t_k<t}W(t,t_k)\le\sum_{t_k<t}e^{-\underline{a}(t-t_k)}=\sum_{j=0}^{+\infty}\left(\sum_{t-j-1\le t_k<t-j}e^{-\underline{a}(t-t_k)}\right)\le\sum_{j=0}^{+\infty}\left(\sum_{t-j-1\le t_k<t-j}e^{-\underline{a}j}\right)\le\sum_{j=0}^{+\infty}\left(2Pe^{-\underline{a}j}\right)=\frac{2P}{1-e^{-\underline{a}}}.$$

Firstly, we prove that $A\Omega\subset\Omega$.

For $\forall x\in\Omega$, we have

$$
\begin{aligned}
&(Ax)(t)\\
&=\int_{-\infty}^{t}W(t,s)\left[\sum_{i=1}^{n}b_i(s)x(s-\tau)e^{-\beta_i(s)x(s-\tau)}-H(s)x(s-\tau)\right]ds+\sum_{t_k<t}W(t,t_k)h_k\\
&\geq\int_{-\infty}^{t}W(t,s)\left[\sum_{i=1}^{n}\underline{b_i}x(s-\tau)e^{-\overline{\beta_i}M}-\overline{H}x(s-\tau)\right]ds+\sum_{t_k<t}W(t,t_k)h_k\\
&=\int_{-\infty}^{t}W(t,s)\left(\sum_{i=1}^{n}\underline{b_i}e^{-\overline{\beta_i}M}-\overline{H}\right)x(s-\tau)ds+\sum_{t_k<t}W(t,t_k)h_k>0.
\end{aligned}
\tag{3.1}
$$

Again, we get

$$
\begin{aligned}
&(Ax)(t)\\
&=\int_{-\infty}^{t}W(t,s)\left[\sum_{i=1}^{n}b_i(s)x(s-\tau)e^{-\beta_i(s)x(s-\tau)}-H(s)x(s-\tau)\right]ds+\sum_{t_k<t}W(t,t_k)h_k\\
&\leq\int_{-\infty}^{t}W(t,s)\sum_{i=1}^{n}b_i(s)x(s-\tau)e^{-\beta_i(s)x(s-\tau)}ds+m\sum_{t_k<t}W(t,t_k)\\
&\leq\int_{-\infty}^{t}W(t,s)\sum_{i=1}^{n}\overline{b_i}x(s-\tau)e^{-\underline{\beta_i}x(s-\tau)}ds+m\sum_{t_k<t}W(t,t_k).
\end{aligned}
\tag{3.2}
$$

Since the function $g_i(u)=ue^{-\underline{\beta_i}u}$, $u\in[0,+\infty)$ reaches its maximum $\frac{1}{\underline{\beta_i}e}$ at $u=\frac{1}{\underline{\beta_i}}$,

then we have $x(s-\tau)e^{-\underline{\beta_i}x(s-\tau)}\leq\frac{1}{\underline{\beta_i}e}$.

From (3.2), we obtain

$$
\begin{aligned}
(Ax)(t)&\leq\int_{-\infty}^{t}W(t,s)\sum_{i=1}^{n}\frac{\overline{b_i}}{\underline{\beta_i}e}ds+m\sum_{t_k<t}W(t,t_k)\\
&\leq\sum_{i=1}^{n}\frac{\overline{b_i}}{\underline{\beta_i}e}\int_{-\infty}^{t}W(t,s)ds+m\frac{2P}{1-e^{-\underline{a}}}\\
&\leq\sum_{i=1}^{n}\frac{\overline{b_i}}{\underline{\beta_i}e}\int_{-\infty}^{t}e^{-\underline{a}(t-s)}ds+m\frac{2P}{1-e^{-\underline{a}}}\\
&=\frac{1}{\underline{a}}\sum_{i=1}^{n}\frac{\overline{b_i}}{\underline{\beta_i}e}+\frac{2mP}{1-e^{-\underline{a}}}=M.
\end{aligned}
\tag{3.3}
$$

Let $\omega\in T$, by Lemma 1 and Lemma 5, we can deduce that

$|(Ax)(t+\omega)-(Ax)(t)|<K_0\varepsilon,$ where K_0 is a positive constant.

Hence, $(Ax)(t)$ is almost periodic. This and (3.1) (3.3) imply $Ax\in\Omega$.
So we have $A\Omega\subset\Omega$.

Next, we show that A is a contraction mapping.
For $\forall x,y\in\Omega$, we have

$$
\begin{aligned}
\|Ax-Ay\| &= \sup_{t\in R} |(Ax)(t)-(Ay)(t)| \\
&= \sup_{t\in R}\left|\int_{-\infty}^{t} W(t,s)\left[\sum_{i=1}^{n} b_i(s)x(s-\tau)e^{-\beta_i(s)x(s-\tau)} - \sum_{i=1}^{n} b_i(s)y(s-\tau)e^{-\beta_i(s)y(s-\tau)}\right]ds \right.\\
&\quad \left. -\int_{-\infty}^{t} W(t,s)\big(H(s)x(s-\tau)-H(s)y(s-\tau)\big)ds\right| \\
&\le \sup_{t\in R}\left\{\int_{-\infty}^{t} W(t,s)\sum_{i=1}^{n}\left|b_i(s)x(s-\tau)e^{-\beta_i(s)x(s-\tau)} - b_i(s)y(s-\tau)e^{-\beta_i(s)y(s-\tau)}\right|ds \right.\\
&\quad \left. +\int_{-\infty}^{t} W(t,s)\left|H(s)x(s-\tau)-H(s)y(s-\tau)\right|ds\right\} \\
&\le \sup_{t\in R}\left\{\int_{-\infty}^{t} W(t,s)\sum_{i=1}^{n}\overline{b_i}\left|x(s-\tau)e^{-\beta_i(s)x(s-\tau)} - y(s-\tau)e^{-\beta_i(s)y(s-\tau)}\right|ds \right.\\
&\quad \left. +\int_{-\infty}^{t} W(t,s)\overline{H}\left|x(s-\tau)-y(s-\tau)\right|ds\right\} \\
&= \sup_{t\in R}\left\{\int_{-\infty}^{t} W(t,s)\sum_{i=1}^{n}\left(\overline{b_i}\,\frac{1}{\beta_i(s)}\left|\beta_i(s)x(s-\tau)e^{-\beta_i(s)x(s-\tau)} - \beta_i(s)y(s-\tau)e^{-\beta_i(s)y(s-\tau)}\right|\right)ds \right.\\
&\quad \left. +\overline{H}\int_{-\infty}^{t} W(t,s)\left|x(s-\tau)-y(s-\tau)\right|ds\right\}.
\end{aligned}
\tag{3.4}
$$

For the function $g(x)=xe^{-x}$, it is easy to see that $g'(x)=(1-x)e^{-x}$.
Hence, by means of the mean value theorem, we get

$$
\begin{aligned}
&\left|\beta_i(s)x(s-\tau)e^{-\beta_i(s)x(s-\tau)} - \beta_i(s)y(s-\tau)e^{-\beta_i(s)y(s-\tau)}\right| \\
&= \left|(1-\xi)e^{-\xi}\left(\beta_i(s)x(s-\tau)-\beta_i(s)y(s-\tau)\right)\right| \\
&= \left|(1-\xi)e^{-\xi}\right|\left|\beta_i(s)x(s-\tau)-\beta_i(s)y(s-\tau)\right|,
\end{aligned}
\tag{3.5}
$$

where ξ lies between $\beta_i(s)x(s-\tau)$ and $\beta_i(s)y(s-\tau)$.

Since the function $f(x)=\left|(1-x)e^{-x}\right|$, $x\in[0,+\infty)$ has maximum $f_{\max}=1$,
Then we get $\left|(1-\xi)e^{-\xi}\right|\le 1$.

Thus, from (3.5), we have

$$
\begin{aligned}
&\left|\beta_i(s)x(s-\tau)e^{-\beta_i(s)x(s-\tau)} - \beta_i(s)y(s-\tau)e^{-\beta_i(s)y(s-\tau)}\right| \\
&\le \left|\beta_i(s)x(s-\tau)-\beta_i(s)y(s-\tau)\right|.
\end{aligned}
\tag{3.6}
$$

Hence, (3.4) and (3.6) imply that

$$
\begin{aligned}
&\|Ax - Ay\| \\
&\le \sup_{t\in R}\left\{\int_{-\infty}^{t} W(t,s)\sum_{i=1}^{n}\left(\overline{b_i}\,|x(s-\tau)-y(s-\tau)|\right)ds + \overline{H}\int_{-\infty}^{t} W(t,s)|x(s-\tau)-y(s-\tau)|ds\right\} \\
&\le \sup_{t\in R}\left\{\|x-y\|\sum_{i=1}^{n}\overline{b_i}\int_{-\infty}^{t} W(t,s)ds + \overline{H}\,\|x-y\|\int_{-\infty}^{t} W(t,s)ds\right\} \\
&\le \sup_{t\in R}\left\{\|x-y\|\sum_{i=1}^{n}\overline{b_i}\int_{-\infty}^{t} e^{-\underline{a}(t-s)}ds + \overline{H}\,\|x-y\|\int_{-\infty}^{t} e^{-\underline{a}(t-s)}ds\right\} \\
&= \sup_{t\in R}\left\{\|x-y\|\frac{1}{\underline{a}}\sum_{i=1}^{n}\overline{b_i} + \overline{H}\,\|x-y\|\frac{1}{\underline{a}}\right\} \\
&= \frac{1}{\underline{a}}\left(\overline{H}+\sum_{i=1}^{n}\overline{b_i}\right)\|x-y\|.
\end{aligned}
$$

From the condition $\frac{1}{\underline{a}}\left(\overline{H}+\sum_{i=1}^{n}\overline{b_i}\right)<1$, we know that A is a contraction mapping. So the operator A exists a unique fixed point x^* in Ω. Moreover, from the inequality (3.1), we have $(Ax)(t)>0$ for $\forall x\in\Omega$. So the fixed point $x^*\in\Omega$ satisfies $x^* = Ax^* > 0$, which means that x^* is positive. This implies that equation (1.1) exists a unique almost periodic positive solution $x^*(t)$ in Ω satisfying $0 < x^*(t) \le M$. The proof is completed.

4. EXPONENTIAL CONVERGENCE OF POSITIVE ALMOST PERIODIC SOLUTION

Theorem 2. Assume that $(C_1)-(C_4)$ hold. If (S_1), (S_2) and (S_3) are satisfied, then every solution $x(t)$ of equation (1.1) with initial function $\phi\in U^0$ converges exponentially to $x^*(t)$ as $t\to+\infty$, where $x^*(t)$ is the unique almost periodic positive solution of equation (1.1) satisfying $0<x^*(t)\le M$.

Proof. From Theorem 1, we know that equation (1.1) exists a unique almost periodic positive solution $x^*(t)$ satisfying $0<x^*(t)\le M$. Assume the initial function of $x^*(t)$ is $x^*(t)=\psi(t)>0$ for $-\tau\le t\le 0$.

Suppose $x(t)$ is arbitrary solution of equation (1.1) with initial function $\phi\in U^0$, here $0<\phi(t)<M$ and $x(t)=\phi(t)$ for $-\tau\le t\le 0$.

By Lemma 6, we know $0<x(t)<M$ for all $t>0$.

Consider the function

$$F(x) = x - \underline{a} + \left(\overline{H}+\sum_{i=1}^{n}\overline{b_i}\right)e^{\tau x}, \qquad x\in[0,1].$$

Since $F(0)=\overline{H}+\sum_{i=1}^{n}\overline{b_i}-\underline{a}<0$, then there exists a constant $\lambda\in(0,1)$, such that $F(\lambda)<0$.

That is,

$$\lambda - \underline{a} + \left(\overline{H} + \sum_{i=1}^{n} \overline{b_i} \right) e^{\lambda \tau} < 0 \ . \tag{4.1}$$

Define the Liapunov functional

$$V(t) = \left| x(t) - x^*(t) \right| e^{\lambda t} .$$

For $t \neq t_k$, we have

$$\begin{aligned} &D^+ V(t) \\ &\le \lambda \left| x(t) - x^*(t) \right| e^{\lambda t} - a(t) \left| x(t) - x^*(t) \right| e^{\lambda t} \\ &+ \left| \sum_{i=1}^{n} b_i(t) x(t-\tau) e^{-\beta_i(t) x(t-\tau)} - \sum_{i=1}^{n} b_i(t) x^*(t-\tau) e^{-\beta_i(t) x^*(t-\tau)} - \left(H(t) x(t-\tau) - H(t) x^*(t-\tau) \right) \right| e^{\lambda t}. \end{aligned} \tag{4.2}$$

For $t = t_k$, we have

$$\begin{aligned} V(t_k^+) &= \left| x(t_k^+) - x^*(t_k^+) \right| e^{\lambda t_k} = \left| x(t_k) + c_k x(t_k) + h_k - \left(x^*(t_k) + c_k x^*(t_k) + h_k \right) \right| e^{\lambda t_k} \\ &= \left| 1 + c_k \right| \left| x(t_k) - x^*(t_k) \right| e^{\lambda t_k}, \\ V(t_k) &= \left| x(t_k) - x^*(t_k) \right| e^{\lambda t_k} . \end{aligned}$$

From the condition (C_3) $-1 < c_k \le 0$, we know $\left| 1 + c_k \right| = 1 + c_k \le 1$, hence it implies that $V(t_k^+) \le V(t_k)$.

Let

$$h = M + \sup_{-\tau \le t \le 0} \left| \phi(t) - \psi(t) \right| .$$

For $\forall t \in [-\tau, 0]$, we have

$$V(t) = \left| x(t) - x^*(t) \right| e^{\lambda t} \le \left| x(t) - x^*(t) \right| = \left| \phi(t) - \psi(t) \right| \le \sup_{-\tau \le t \le 0} \left| \phi(t) - \psi(t) \right| < M + \sup_{-\tau \le t \le 0} \left| \phi(t) - \psi(t) \right| = h .$$

Now, we prove that

$$V(t) < h \qquad \text{for all} \quad t > 0 . \tag{4.3}$$

Suppose that (4.3) does not hold true, then there must exist $t^* > 0$, $K^* \in Z^+$ and $t^* \in (t_{K^*-1}, t_{K^*}]$, such that $V(t^*) = h$, $V(t) < h$ for $t < t^*$, and $D^+ V(t) |_{t = t^*} \ge 0$.

It follows from (4.2) that

$$
\begin{aligned}
&0 \le D^{+}V(t)\big|_{t=t^*}\\
&\le \lambda\left|x(t^*)-x^*(t^*)\right|e^{\lambda t^*}-a(t^*)\left|x(t^*)-x^*(t^*)\right|e^{\lambda t^*}\\
&+\left|\sum_{i=1}^{n}b_i(t^*)x(t^*-\tau)e^{-\beta_i(t^*)x(t^*-\tau)}-\sum_{i=1}^{n}b_i(t^*)x^*(t^*-\tau)e^{-\beta_i(t^*)x^*(t^*-\tau)}-\left(H(t^*)x(t^*-\tau)-H(t^*)x^*(t^*-\tau)\right)\right|e^{\lambda t^*}\\
&=\lambda V(t^*)-a(t^*)V(t^*)\\
&+\left|\sum_{i=1}^{n}\left(b_i(t^*)x(t^*-\tau)e^{-\beta_i(t^*)x(t^*-\tau)}-b_i(t^*)x^*(t^*-\tau)e^{-\beta_i(t^*)x^*(t^*-\tau)}\right)-\left(H(t^*)x(t^*-\tau)-H(t^*)x^*(t^*-\tau)\right)\right|e^{\lambda t^*}\\
&\le \lambda V(t^*)-a(t^*)V(t^*)\\
&+\left(\sum_{i=1}^{n}\left|b_i(t^*)x(t^*-\tau)e^{-\beta_i(t^*)x(t^*-\tau)}-b_i(t^*)x^*(t^*-\tau)e^{-\beta_i(t^*)x^*(t^*-\tau)}\right|+\left|H(t^*)x(t^*-\tau)-H(t^*)x^*(t^*-\tau)\right|\right)e^{\lambda t^*}\\
&=\lambda V(t^*)-a(t^*)V(t^*)\\
&+\left(\sum_{i=1}^{n}\frac{b_i(t^*)}{\beta_i(t^*)}\left|\beta_i(t^*)x(t^*-\tau)e^{-\beta_i(t^*)x(t^*-\tau)}-\beta_i(t^*)x^*(t^*-\tau)e^{-\beta_i(t^*)x^*(t^*-\tau)}\right|+\left|H(t^*)x(t^*-\tau)-H(t^*)x^*(t^*-\tau)\right|\right)e^{\lambda t^*}.
\end{aligned}
\tag{4.4}
$$

Using the mean value theorem, we get

$$
\begin{aligned}
&\left|\beta_i(t^*)x(t^*-\tau)e^{-\beta_i(t^*)x(t^*-\tau)}-\beta_i(t^*)x^*(t^*-\tau)e^{-\beta_i(t^*)x^*(t^*-\tau)}\right|\\
&=\left|(1-\eta)e^{-\eta}\left(\beta_i(t^*)x(t^*-\tau)-\beta_i(t^*)x^*(t^*-\tau)\right)\right|\\
&=\left|(1-\eta)e^{-\eta}\right|\left|\beta_i(t^*)x(t^*-\tau)-\beta_i(t^*)x^*(t^*-\tau)\right|\\
&\le\left|\beta_i(t^*)x(t^*-\tau)-\beta_i(t^*)x^*(t^*-\tau)\right|\\
&=\beta_i(t^*)\left|x(t^*-\tau)-x^*(t^*-\tau)\right|,
\end{aligned}
\tag{4.5}
$$

where η lies between $\beta_i(t^*)x(t^*-\tau)$ and $\beta_i(t^*)x^*(t^*-\tau)$.

Hence, (4.4) and (4.5) imply that

$$
\begin{aligned}
&0 \le D^{+}V(t)\big|_{t=t^*}\\
&\le \lambda V(t^*)-a(t^*)V(t^*)+\left(\sum_{i=1}^{n}\left(b_i(t^*)\left|x(t^*-\tau)-x^*(t^*-\tau)\right|\right)+\left|H(t^*)x(t^*-\tau)-H(t^*)x^*(t^*-\tau)\right|\right)e^{\lambda t^*}\\
&\le \lambda V(t^*)-\underline{a}V(t^*)+\left(\sum_{i=1}^{n}\left(\overline{b_i}\left|x(t^*-\tau)-x^*(t^*-\tau)\right|\right)+\overline{H}\left|x(t^*-\tau)-x^*(t^*-\tau)\right|\right)e^{\lambda t^*}\\
&=\lambda h-\underline{a}h+\left(\sum_{i=1}^{n}\overline{b_i}+\overline{H}\right)\left|x(t^*-\tau)-x^*(t^*-\tau)\right|e^{\lambda t^*}\\
&=\lambda h-\underline{a}h+\left(\sum_{i=1}^{n}\overline{b_i}+\overline{H}\right)\left|x(t^*-\tau)-x^*(t^*-\tau)\right|e^{\lambda(t^*-\tau)}e^{\lambda\tau}\\
&=\lambda h-\underline{a}h+\left(\sum_{i=1}^{n}\overline{b_i}+\overline{H}\right)V(t^*-\tau)e^{\lambda\tau}\\
&<\lambda h-\underline{a}h+\left(\sum_{i=1}^{n}\overline{b_i}+\overline{H}\right)he^{\lambda\tau}\\
&=\left[\lambda-\underline{a}+\left(\sum_{i=1}^{n}\overline{b_i}+\overline{H}\right)e^{\lambda\tau}\right]h.
\end{aligned}
$$

Thus we get $\lambda-\underline{a}+\left(\sum_{i=1}^{n}\overline{b_i}+\overline{H}\right)e^{\lambda\tau}>0$, which contradicts (4.1). So (4.3) holds true.

Hence we have $V(t)=\left|x(t)-x^*(t)\right|e^{\lambda t}<h$ for all $t>0$.

That is $\left|x(t)-x^*(t)\right|<he^{-\lambda t}$ for all $t>0$, which means that $x(t)$ converges exponentially to $x^*(t)$ as $t\to+\infty$. The proof is completed.

5. APPLICATION

Now we give an example to illustrate our results.

Consider equation

$$\begin{cases} x'(t)=-\left(90+\sin\sqrt{3}t\right)x(t)+\left(2+\cos\sqrt{5}t\right)x(t-\tau)e^{-\left(\frac{1}{4}+\frac{1}{100}\sin\sqrt{2}t\right)x(t-\tau)} \\ \qquad +\left(\frac{1}{10}+\frac{1}{40}\sin\sqrt{5}t\right)x(t-\tau)e^{-\left(4+\cos\sqrt{2}t\right)x(t-\tau)}-\left(\frac{1}{10}+\frac{1}{20}\sin\sqrt{3}t\right)x(t-\tau), \qquad t\neq t_k, \\ \Delta x(t_k)=c_kx(t_k)+h_k, \qquad t=t_k, \end{cases} \tag{5.1}$$

where $a(t)=90+\sin\sqrt{3}t, b_1(t)=2+\cos\sqrt{5}t, \beta_1(t)=\frac{1}{4}+\frac{1}{100}\sin\sqrt{2}t$,

$b_2(t)=\frac{1}{10}+\frac{1}{40}\sin\sqrt{5}t, \beta_2(t)=4+\cos\sqrt{2}t, H(t)=\frac{1}{10}+\frac{1}{20}\sin\sqrt{3}t$, $\tau=1$,

$c_k=-\frac{1}{2}+\frac{1}{10}\sin\sqrt{2}k, h_k=\frac{1}{5}+\frac{1}{20}\sin\sqrt{3}k$, $P=2$.

It is easy to calculate that

$$\underline{a}=89, \overline{b_1}=3, \underline{b_1}=1, \overline{\beta_1}=0.26, \underline{\beta_1}=0.24, \overline{b_2}=0.125, \underline{b_2}=0.075, \overline{\beta_2}=5, \underline{\beta_2}=3, \overline{H}=0.15,$$

$$-0.6\le c_k\le-0.4, \qquad 0.15\le h_k\le\frac{1}{4}=m, \qquad \overline{H}+\sum_{i=1}^{2}\overline{b_i}<\underline{a},$$

$$M=\frac{2mP}{1-e^{-\underline{a}}}+\frac{1}{\underline{a}e}\sum_{i=1}^{2}\frac{\overline{b_i}}{\underline{\beta_i}}\approx1.05, \qquad \overline{H}<\sum_{i=1}^{2}\underline{b_i}e^{-\overline{\beta_i}M}, \qquad c_kM+h_k\le-0.4\times1.05+0.25<0.$$

By Theorem 1 and Theorem 2, we know equation (5.1) exists a unique almost periodic positive solution $x^*(t)$ satisfying $0<x^*(t)\le M$. Moreover, every solution $x(t)$ of equation (5.1) with initial function $\phi\in U^0$ converges exponentially to $x^*(t)$ as $t\to+\infty$, here $U^0=\left\{\phi\mid\phi\in BPC\left([-\tau,0],R^+\right),0<\phi(t)<M,t\in[-\tau,0]\right\}$.

6. CONCLUSION

Impulsive phenomena exist extensively in natural biological systems, almost periodicity is closer to real world. This paper has studied the almost periodic impulsive Nicholson's blowflies model with linear harvesting term. By applying the contraction mapping fixed point theorem, we obtain sufficient conditions for the existence of unique positive almost periodic solution. By constructing Liapunov functional, we study the exponential convergence of positive almost periodic solution. The dynamic behaviors have close relations to the harvesting term and impulsive term. For the impulsive almost periodic Nicholson's blowflies model with linear harvesting term, we answer the open problem proposed in [9]. Our results complement the previous results of some past literatures.

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8. REFERENCES

1. W. S. C. Gurney, S. P. Blythe, R. M. Nisbet, Nicholson’s blowflies revisited, *Nature*, **287**, 17-21, 1980.
2. A. J. Nicholson, An outline of the dynamics of animal populations, *The Australian Journal of Zoology* **2**, 9-65, 1954.
3. M.R.S. Kulenovic, G. Ladas, Y. G. Sficas, Global attractivity in Nicholson’s blowflies, *Journal of Applied Analysis* **43**(1-2), 109-124, 1992.
4. J.W. Li, C.X. Du, Existence of positive periodic solutions for a generalized Nicholson’s blowflies model, *Journal of Computational and Applied Mathematics* **221**(1), 226-233, 2008.
5. L. Berezansky, L. Idels, L. Troib , Global dynamics of Nicholson-type delay systems with applications, *Nonlinear Analysis: Real World Applications* **12**(1), 436-445, 2011.
6. W. T. Wang, Positive periodic solutions of delayed Nicholson’s blowflies models with a nonlinear density-dependent mortality term, *Applied Mathematical Modelling* **36**(10), 4708-4713, 2012.
7. B. W. Liu, The existence and uniqueness of positive periodic solutions of Nicholson-type delay systems, *Nonlinear Analysis: Real World Applications* **12**(6), 3145-3151, 2011.
8. H. Zhou, W.T. Wang, H. Zhang, Convergence for a class of non-autonomous Nicholson’s blowflies model with time-varying coefficients and delays, *Nonlinear Analysis: Real World Applications* **11**(5), 3431-3436, 2010.
9. L. Berezansky, E. Braverman, L. Idels, Nicholson’s blowflies differential equations revisited: main results and open problems, *Applied Mathematical Modelling* **34**(6), 1405-1417, 2010.
10. W. R. Zhao, C. M. Zhu, H. P. Zhu, On positive periodic solution for the delay Nicholson’s blowflies model with a harvesting term, *Applied Mathematical Modelling* **36**(7), 3335-3340, 2012.
11. V. Lakshmikantham, D. D. Bainov, P. S. Simeonov, *Theory of Impulsive Differential Equations* , World Scientific Press, Singapore, 1989.
12. D. D. Bainov, P. S. Simeonov, *System with Impulsive Effect: Stability, Theory and Applications,* John Wiley and Sons, New York,1986.
13. A. M. Samoilenko, N. A. Perestyuk, *Impulsive Differential Equations,* World Scientific, River Edge, NJ, USA, 1995.
14. A. Fink, *Almost Periodic Differential Equations,* Lecture Notes in Mathematics, vol. 377. Springer, Berlin,1974.
15. J. Hale, *Theory of Functional Differential Equations*, Springer, New York, NY, USA , 1977.

16. F. Long, M. Yang, Positive periodic solutions of delayed Nicholson’s blowflies model with a linear harvesting term, *Electronic Journal of Qualitative Theory of Differential Equations* **41**,1-11, 2011.
17. P. Amster, D. Alberto , Existence of positive T-periodic solutions of a generalized Nicholson’s blowflies model with a nonlinear harvesting term, *Applied Mathematics Letters* **25**(9), 1203-1207, 2012.
18. Q. Zhou, The positive periodic solution for Nicholson-type delay system with linear harvesting terms, *Applied Mathematical Modelling* **37**(8), 5581-5590, 2013.
19. L. Wang, Almost periodic solution for Nicholson’s blowflies model with patch structure and linear harvesting terms, *Applied Mathematical Modelling* **37**(4), 2153-2165, 2013.
20. J.K. Hale, S.M. Verduyn Lunel, *Introduction to Functional Differential Equations*, Springer-Verlag, New York, 1993.
21. B. W. Liu, Global dynamic behaviors for a delayed Nicholson's blowflies model with a linear harvesting term, *Electronic Journal of Qualitative Theory of Differential Equations* **45**, 1-13, 2013.
22. B. W. Liu, Positive periodic solutions for a nonlinear density-dependent mortality Nicholson's blowflies model, *Kodai Mathematical Journal* **37**, 157-173, 2014.
23. B. W. Liu, Global exponential stability of positive periodic solutions for a delayed Nicholson's blowflies model, *Journal of Mathematical Analysis and Applications* **412**(1), 212-221, 2014.