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THE DEVELOPMENT OF EULER HYPERBOLE IN HEAVY BARS

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Abstract: Euler [1] discovered the critical buckling force of the bars under an axial load. Euler, however ignored bar's own weight in his calculations. He drew the line diagram of critical strength vs bars slenderness ratio, the so called Euler-Hyperbole. The goal of this paper is to extend Euler-Hyperbole to heavy bars. Furthermore, factors which have effects on critical buckling forces will be analysed. This study may lead to economical gain, since materials are fully utiliesed.

1. INTRODUCTION

A heavy bar hinged at both sides is placed perpendicularly and an axial load is applied as shown in Fiqure 1. The deflection of the bar satisfy the following equation [3]:

$$E\,I\,\frac{d^4w(x)}{dx^4} + (P - \gamma\,A\,x)\,\frac{d^2w(x)}{dx^2} - \gamma\,A\,\frac{dw(x)}{dx} = 0 \quad . \qquad (1)$$

In the above equation, γ represents specific gravity and $p_o = \gamma\ A$ is equal to bar's weight per length. Cross-sectional area of bar is assumed to be constant. If bar's own weight is ignored, $\gamma = 0$ and equation (1) reduces to:

$$E\,I\,\frac{d^4w(x)}{dx^4} + P\,\frac{d^2w(x)}{dx^2} = 0\,, \qquad (2)$$

where E is the Young's modulus, I is the moment of inertia of the cross-section with respect to the neutral axis, A is the cross-sectional area.

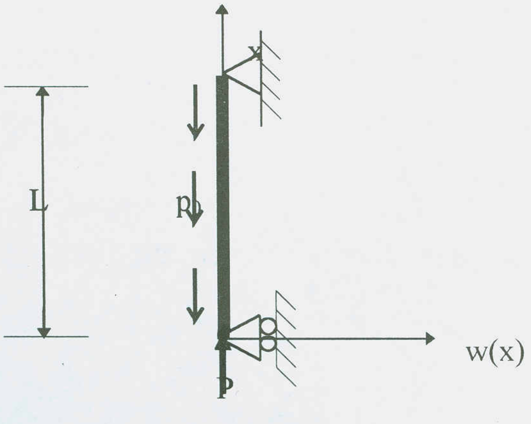

Figure 1: A heavy bar hinged at both sides

The critical buckling force for equation (2) is:

$$P_{cr} = \frac{EI_{\min}\pi^2}{L^2} \tag{3}$$

and the normal stress is

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{\pi^2 E}{\lambda^2} \ . \tag{4}$$

In the above formulation λ represents the bar's slenderness ratio. Defining the inertia diameter $i=\sqrt{\frac{I_{\min}}{A}}$, one can write

$$\lambda = \frac{L}{i} \ . \tag{5}$$

From equation (5), one concludes that as the slenderness ratio increases, the critical normal stress decreases. Furthermore the normal stress is proportional to the Young's modulus and inertia diameter.

2. THE CRITICAL BUCKLING FORCE OF THE HEAVY BARS

The solution of equation (1) valid for heavy bars is as follows:

$$w(x) = C_1 \int \zeta^{\frac{1}{3}}(x)\, s_{0,\frac{1}{3}}(x)\, dx + C_2 \int \zeta^{\frac{1}{3}}(x)\, J_{\frac{1}{3}}(x)\, dx + C_3 \int \zeta^{\frac{1}{3}}(x)\, J_{-\frac{1}{3}}(x)\, dx + C_4 \ , \tag{6}$$

where the following expressions are previously defined [3] :

$$\zeta(x) = \frac{2}{3}\, a^{\frac{1}{2}}\, (b - x)^{\frac{3}{2}},\ a= \frac{\gamma A}{EI} > 0,\ b= \frac{P}{\gamma A} > 0 \ . \tag{7}$$

Here, $s_{0,\frac{1}{3}}(x)$ is the Lommel function and $J_{\pm\frac{1}{3}}(x)$ are the Bessel functions. The deflection w(x) is satisfied by boundary conditions, and critical buckling force P_{cr} is found. As shown below, critical buckling forces P_{cr} and bar lenght L are converted to dimensionless expressions:

$$\overline{P_{cr}} = \frac{P_{cr}}{\sqrt[3]{EI(\gamma A)^2}} \ , \tag{8}$$

$$\overline{L} = L \sqrt[3]{\frac{\gamma A}{EI}} \ . \tag{9}$$

Results can be seen in Table 1. Same results can also be obtained from Airy functions [2].

3. DEVELOPMENT OF EULER HYPERBOLE IN HEAVY BARS

In order to extend Euler Hyperbole to heavy bars, σ_{cr} and λ should be expressed in terms of $\overline{P_{cr}}$ and $\overline{L}$. Therefore, P_{cr} from equation (8) is placed into equation (4):

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{\overline{PL}^2 E}{\left(\dfrac{L}{\sqrt{\dfrac{I}{A}}}\right)^2} \tag{10}$$

or by using equation (5)

$$\sigma_{cr} = \frac{\overline{P}\,\overline{L}^{2} E}{\lambda^{2}} \tag{11}$$

is found. In order to get bar's slenderness ratio in terms of $\overline{P}$ and $\overline{L}$, L from formula (9) is placed into formula (5):

$$\lambda = \frac{L}{\sqrt{\frac{I}{A}}} = \mu\,\overline{L}\,. \tag{12}$$

In the above equation, μ could be defined as "buckling weight parameter" and written explicitly as follows:

$$\mu = \left(\frac{E}{\gamma}\right)^{\frac{1}{3}} \cdot \left(\frac{A}{I}\right)^{\frac{1}{6}}. \tag{13}$$

In order to draw "extended Euler Hyperbole", σ_{cr} and λ are calculated from formula (11) and (12) using $\overline{P}$ and $\overline{L}$ in Table 1 for every μ parameter. If bar's own weight is ignored, then μ is considered to be infinity. New Euler Hyperbole is drawn in Figure 2 for steel and copper bars, hinged at both sides.

4. CONCLUDING REMARKS

In this paper, Euler Hyperbole is extended to heavy bars, specifically hinged both sides. Using this extended hyperbole in design of bars under axial compression would yield economical gains.

Steel and copper bars hinged both sides are analysed in Figure 2. Following results are obtained:

1. For bars made of same material and same length, as the ratio $\frac{I}{A}$ increases, normal stress increases, which is in agreement with classical Euler Hyperbole.

2. For bars which have the same cross-sectional area and length, as the ratio $\frac{E}{\gamma}$ increases, critical normal stress increases.

3. As bar's slenderness ratio increases, economical gain increases. For lower slenderness ratio classical Euler Hyperbole can also be used.

REFERENCES

[1] Euler L., "De Curvis Elasticis", Lausanne-Genf, 1744.

[2] Bernitsas M. M., Kokkinis T., "Buckling of Columns with Nonmovable Boundaries", Journal of Structural Mechanics 11, s.351-370, 1983.

[3] Özdamar A., "Das Knicken schwerer Gestaenge", ISBN 3-930324-71-7, Berlin, 1996.

Table 1: Dimensionless critical buckling forces vs dimensionless bar length

$\overline{P_{cr}}$	$\overline{L}$
4,2503	1,7001
2,923	2,3384
2,648	2,648
2,2268	4,0082
2,0834	5,4168
1,9608	7,451
1,853	10,1915
1,7809	14,2472

Es: Classical Euler Hyperbole for steel
s50: Steel, μ=50
s60: Steel, μ=60
s70: Steel, μ=70
s80: Steel, μ=80
s90: Steel, μ=90

Ec: Classical Euler Hyperbole for copper
c50: Copper, μ=50
c60: Copper, μ=60
c70: Copper, μ=70
c80: Copper, μ=80
c90: Copper, μ=90

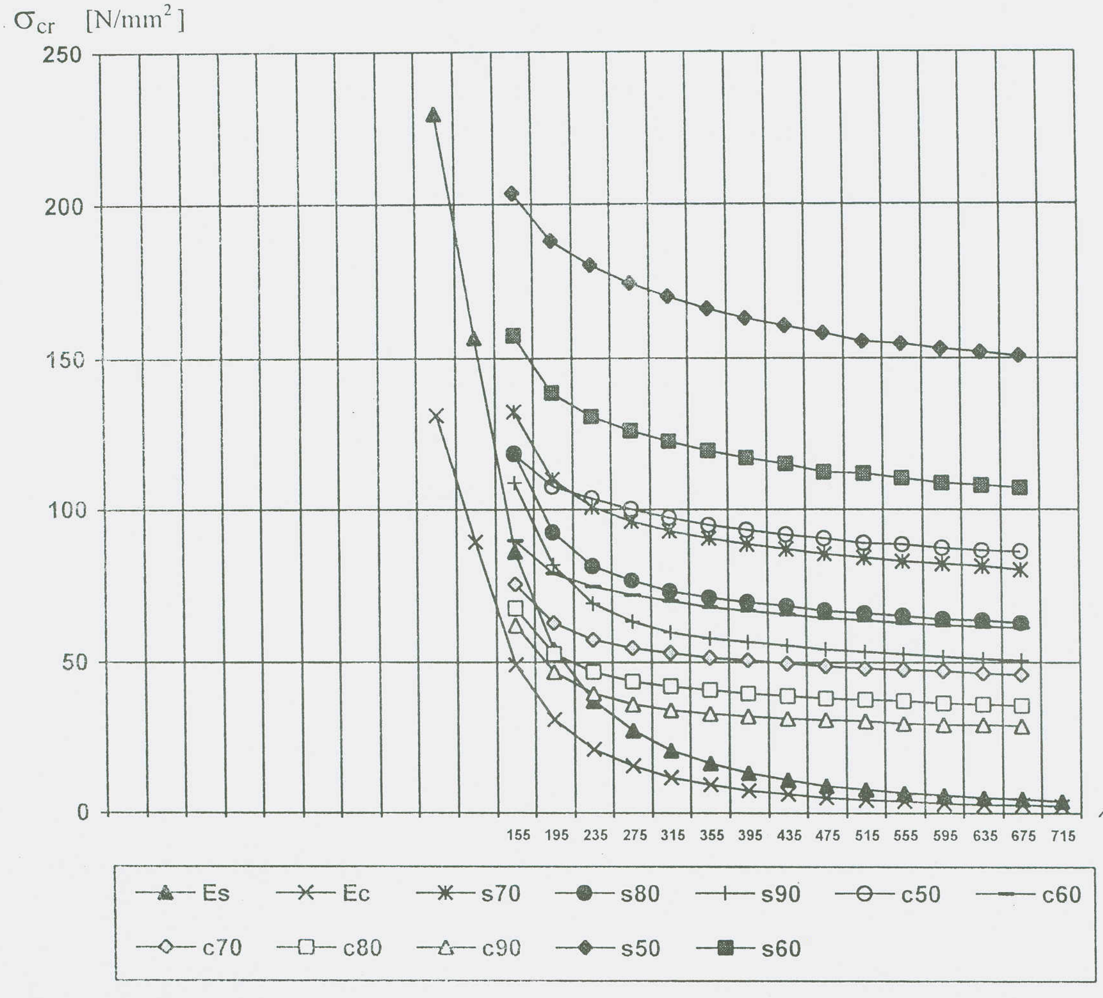

Figure 2: Extended Euler Hyperbole for heavy bars hinged at both sides